Managing Consumer Attention to Diverse Information Sources in Product Diffusion

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Abstract

Consumers rely on both observational learning and direct search to make purchasing decisions. When directly searching for product information, consumers may have access to different sources of information with different biases and have to choose which type of information to pay attention to minimize decision-making errors. This paper introduces a theoretical framework that examines a firm's dynamic pricing strategy in the context of consumers' allocation of attention amid observational learning. A sequence of short-lived consumers, facing a new product of uncertain quality, must decide whether to search for information, which information source to pay attention to, and whether to buy the product based on the price, purchase decisions by preceding consumers, and the information search outcome. We characterize the optimal pricing strategy for a firm that takes into account the consumers' allocation of attention and search behavior. The analysis reveals that firms can strategically set prices dynamically to influence the type of information consumers direct their attention to, potentially mitigating the inefficiencies typically associated with herding behavior. Surprisingly, firms may want to incentivize consumers to pay attention to information sources that a negative leaning against them, and benefit from the existence of these negative sources. This work contributes to the literature on observational learning and dynamic pricing, offering insights into the value of diverse information sources and the design of pricing policies that can optimize the allocation of consumer attention and improve market efficiency.

Keywords: rational inattention; observational learning; dynamic pricing; consumer search; herding; information bias; product diffusion.

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1 Introduction

When facing new products of uncertain quality, consumers can use two types of information to help their decision-making. First, consumers can engage in observational learning, inferring a product's quality by observing the actions of previous consumers. For example, movie-goers can use a movie's popularity, reflected in its box office performance, as a signal of its quality. Car buyers may choose brands that are popular on the streets. Phone buyers may look at what devices people around them have purchased.

The second source of information is direct information search. Through such search, consumers can gather information about a product's features and quality, though such search is often incomplete and noisy. For instance, a movie-goer might watch trailers to learn about a movie's setting or read critics' reviews before deciding whether to watch it. Car buyers can research a car's specifications and awards or visit a dealership for a test drive. Phone buyers may browse technology websites to watch reviews that evaluate the pros and cons of a phone or try out the product at a local store before making a purchase.

The information consumers receive from observational learning can sometimes make direct searches unnecessary. Prior research has shown that observational learning can lead to informational cascades, where consumers make the same decisions as their predecessors, ignoring their own information about the product (e.g., Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992; Smith and Sørensen 2000). For example, a movie-goer might choose to see a popular movie despite disliking the trailer, deeming the movie's popularity a stronger quality signal than their own assessment. Information cascades can lead to incorrect decisions, resulting in negative ex-post utility for consumers. Hendricks, Sorensen, and Wiseman (2012) demonstrated that herding behavior persists when consumers incur search costs to obtain noisy product information. Moreover, the availability of information through observational learning affects whether consumers decide to engage in further product search or not.

One important aspect of consumer search that the literature on observational learning has understudied is the type of information sources consumers search from. In today's informationrich environment, consumers often encounter multiple sources of information when learning about a product. These sources can vary significantly in perspective—some are heavily positive, highlighting the best features, while others are negative, emphasizing flaws. We refer to these as positive-leaning and negative-leaning sources, respectively.

Positive-leaning sources emphasize the strengths and benefits of a product, which tends to paint a positive picture of the product. Brand websites and advertisements are typical examples as they are designed to present the product in the best possible light. Sponsored influencer reviews also fall under this category, as the overall tone may be supportive due to the brand's involvement. On the other hand, negative-leaning sources highlight problems or shortcomings that tend to paint a negative picture of the product. For example, competitors can act as negative-leaning sources by disseminating information that emphasizes a rival product's weaknesses through comparative advertising. Complaint forums, such as Trustpilot, often present a skewed view that emphasizes negative experiences.

Additionally, different media channels have different positioning, preferences, and motivations, which could also introduce bias/leanings. For example, critical review websites specializing in testing products may have preferred brands and differ in how much they focus on the positive versus negative elements of each brand. A consumer searching for information about a new Apple product may expect to read more positive-leaning information from Apple-friendly outlets such as iMore and MacRumors and more negative-leaning information from outlets such as Android Authority. Influencers also have historical preferences that make them more positive or negative towards specific brands.

The decision of which type of sources to pay attention to is crucial for consumer learning and shapes how information spreads across the market. In observational learning, what one consumer learns can significantly affect the behaviors of other consumers. If consumers pay more attention to positive-leaning sources, they may form more optimistic beliefs about a product, leading future consumers to follow suit without searching, creating informational cascades. Conversely, if negative-leaning sources dominate, they can negatively impact the product's reputation, deterring potential buyers and stifling product diffusion.

Thus, several questions arise when considering consumers' allocation of attention in our context. On the consumer side, what types of information should a consumer focus on in the presence of observational learning? How does it differ for earlier versus later consumers? Does the allocation of attention mitigate or exacerbate inefficiencies from herding behaviors?

On the firm side, how can a firm influence consumers' allocation of attention through pricing? Which information source is preferred by the seller? Particularly, does the seller ever want to incentivize consumers to search from negative-leaning sources in their search? Would the seller be better off if there was no negatively-biased information about the product? And finally, what does the optimal pricing strategy look like?

We assume that information is available from two different types of sources: positive-leaning (PL) sources and negative-learning (NL) sources, analogous to the L-biased and R-biased news sources described in Che and Mierendorff (2019). Positive-leaning sources emit a positive signal whenever the product quality is high and only sometimes send a negative signal when the product quality is low. Similarly, negative-leaning sources always give a negative signal when the product quality is low, but occasionally, in the high-quality state, a positive signal is sent.

When a consumer obtains a signal from an information source consistent with the source's leaning, his belief moves in the direction of the leaning, albeit by a small degree because of source's strong tendency to generate a leaning-consistent signal. Meanwhile, a signal that runs counter to the source's learning, though rare, significantly moves the consumer's belief the opposite direction of the source's leaning. In a continuous time model where a type of information sources is continuously searched, positive-leaning and negative-leaning information sources are equivalent to Poisson processes that generate truth-revealing negative and positive signals, respectively.

We first demonstrate how the consumer's incentives for information search are influenced by the product's price set by the firm. After characterizing consumers' information search strategy, we characterize the firm's optimal dynamic pricing strategy. We demonstrate that the firm can use pricing to influence whether a consumer searches for information and what type of information he pay attention to during the search. This transforms the firm's problem into an optimal control problem, where the firm must decide at each moment whether to incentivize a consumer to search from positive-leaning sources, negative-leaning sources, or stop searching altogether. The firm's pricing strategy may lead subsequent consumers to herd on their predecessors' decisions, or to continue learning about the product's quality.

Common sense may suggest that firms prefer consumers to search from positive-leaning sources rather than negative-leaning sources, and firms are better off without the negative-leaning sources. We found both to be not true. There are indeed cases where firms want to incentivize consumers to search from negative-leaning sources, and the existence of negative-leaning sources is always beneficial to product diffusion.

A firm may want consumers to search from negative-leaning sources for two reasons. First, consumers' search decision depends on the prices they face. Generally, a consumer is more likely to search from positive-leaning sources when the price is lower. When reputation is low, incentivizing search from positive-leaning sources may require the firm to sell at a price below cost, which can be too costly. On the other hand, incentivizing search from negative-leaning sources when the current reputation is low gives the product's reputation a chance to have a drastic improvement that allows the firm to become profitable in the long run.

In equilibrium, when the public belief in product quality is moderately high, the firm sets a relatively low price to incentivize consumers to search from positive-leaning sources. Over time, both the price and the public belief increase unless the truth-revealing negative signal arrives. Conversely, when the public belief is moderately low, the firm sets a relatively high price to encourage consumers to search from negative-leaning sources. In this case, both the price and public belief decrease over time unless the truth-revealing positive signal arrives.

The second reason firms may want to incentivize negative-leaning sources is to prevent consumers from herding. Firms can deter herding by incentivizing a combination of both types of biased signals. After incentivizing searching from either negative-leaning or positive-leaning sources for a period of time, the public belief becomes sufficiently high or low such that consumers no longer have an incentive to continue searching from the same type of source. At this point, the firm can incentivize consumers to search from opposite-leaning sources to nudge consumer belief away from the point of herding. By doing so, the firm prevents herding, and subsequent consumers continue to search until a truth-revealing signal arrives. Under this equilibrium, the aggregate learning is complete.

We also show that such an efficient equilibrium without herding cannot arise if consumers only have access to one type of information sources. Thus, the availability of multiple sources with different leanings or biases and the firm's ability to influence consumer attention through pricing are crucial for limiting herding behaviors in observational learning. These findings offer valuable insights into the importance of information diversity and the mechanisms through which firms can support informed consumer decision-making.

Our results carry significant managerial and economic implications. Firms can use pricing as a strategic tool to influence consumer learning and purchasing behavior. By dynamically adjusting prices to encourage different types of information search, firms can guide the trajectory of their quality reputation and enhance long-term profits. Inadvertently, doing so also helps consumers make more informed decisions, reducing inefficient herding and improving market outcomes.

For a new product, the firm can profit from consumer access to diverse information sources. While it may be tempting to direct consumer attention only to positive-leaning sources, such a "sanitized" environment can backfire. In practice, when launching a new product, besides sponsoring influencers who align with the brand, firms may also want to invite reviews from those more critical of the brand. Comparative advertising from competitors can also help. Firms and policymakers should recognize the important interaction between diverse information availability and dynamic pricing, as their interaction reduces incorrect informational cascades and leads to more efficient learning.

Theoretically, our paper contributes to the literature by studying consumer attention allocation in observational learning contexts. We present a dynamic framework where the firm actively guides consumer information acquisition. This perspective provides new insights into optimal firm behavior and demonstrates how the interplay between pricing, consumer search, and information diffusion can be managed to achieve efficient market outcomes.

The rest of the paper is structured as follows. Section 2 discusses relevant literature. We introduce the model and discuss consumer search behaviors in Section 3. In Section 4, we analyze two benchmarks where only one type of information sources is available to consumers. Section 5 characterizes the equilibrium pricing strategy when both types of information sources are available and how they impact product diffusion and search efficiency. We conclude the paper in Section 6.

2 Literature Review

Previous literature on observational learning has primarily focused on consumers' decision-making processes. A smaller subset of the literature has examined firms' optimal strategies such as pricing when consumers engage in observational learning. Welch (1992), in the context of a sequential IPO, studied the optimal static price. Bose et al. (2006) and Bose et al. (2008) considered observational learning in a monopoly with dynamic pricing. In both papers, if the seller is patient enough and the number of consumers is sufficiently large, a cascade always happens; furthermore, the cascade is wrong with a positive probability. Sayedi (2018) studied the optimal pricing under a duopoly and finds conditions under which cascades may not happen.

Previous papers on dynamic search focus on the decisions of a long-lived agent. An important aspect of this problem is how long the decision-maker should search for information before stopping to take an action. This stopping problem has been studied extensively by many authors using drift-diffusion models where the signal follows a Brownian motion with a drift determined by the state (e.g., Roberts and Weitzman 1981; Branco, Sun, and Villas-Boas 2012). A smaller number of papers have allowed the information structure to be the agent's endogenous choice. Moscarini and Smith (2001) extended the stopping problem by allowing for an endogenous choice of signal precision. Ke and Villas-Boas (2019) studied the optimal dynamic allocation of learning effort on two alternatives. The model closest to ours in terms of information structure is that in

Che and Mierendorff (2019), which studied an agent's dynamic allocation of attention between two Poisson signals. However, all the aforementioned studies have examined the decision of a long-lived agent, while our paper studies the decisions of short-lived agents in an observational learning framework, where the decision of each agent depends on the actions taken by all previous agents.

Our paper also contributes to the literature on rational inattention such as Sims (2003), Matějka and McKay (2015), Steiner, Stewart, and Matějka (2017), and Jerath and Ren (2021). These studies primarily model the allocation of attention in a static fashion, where the cost of information processing is a function of the prior and posterior beliefs. Our study extends the literature by considering consumers' allocation of attention in a dynamic setting where each consumer's prior belief depends on previous consumers' search decisions, and each consumer's search decision affects the beliefs of subsequent consumers.

3 Model

A firm launches a new product of uncertain vertical quality $\omega \in \{0, 1\}$, where 1 denotes the state of high quality and 0 denotes the state of low quality. A continuum of short-lived consumers arrives sequentially on an infinite horizon. Each consumer can be indexed by the time he arrives, $t \in \mathbb{R}_+$. Every consumer values a high-quality product at v_h and a low-quality product at v_l. Normalize the product's marginal cost to 0, i.e., price is negative when it is below the marginal cost. We assume $v_l < 0 < v_h$, which implies that selling a low-quality product at its value to the consumers is loss-incurring to the firm.

Given the novel nature of the product, we assume neither the firm nor the consumers know the true quality of the product. Let x_0 denote the public prior belief about the product's quality. Let $x_t \in [0, 1]$ denote the belief of the consumer arriving at time t upon his arrival. Given a product price of $p_t \in \mathbb{R}$, consumer t has three possible actions: (1) not buying the product; (2) buying the product; (3) making the purchase decision after searching for some product information.

If the consumer does not buy, the consumer gets the reservation utility from the outside option, which is assumed to be 0. If the consumer buys the product without search, the consumer's expected payoff is $\bar{v}(x_t) - p_t$, where $\bar{v}(x_t) := x_t v_h + (1 - x_t) v_l$ is the consumer's expected value for the product given his belief, x_t. Thus, without search, buying the product is optimal if and only if $\bar{v}(x) \geq p_t$.

3.1 Information Sources

If the consumer chooses to search, the consumer searches for a duration of Δt and incurs a cost of $c\,\Delta t$ for some $c > 0$ and $\Delta t > 0$. Consumers who search then receive a signal that is correlated with the product's quality.

We assume that information is available from two different types of sources: positive-leaning sources and negative-leaning sources, similar to the left-biased and right-biased political news sources as motivated in Che and Mierendorff (2019).

The consumer observes either a positive or negative signal from the information source he pays attention to. We assume that a positive-leaning source always sends a positive signal conditional on $\omega = 1$, and sends a negative signal with probability $\lambda\,\Delta t$ conditional on $\omega = 0$. Thus, the positive-leaning source is biased towards sending a positive signal. Table 1(a) characterizes the signal distribution of searching from a positive-leaning source as a statistical experiment. Based on the conditional signal distribution, a negative signal from a positive-leaning source fully reveals $\omega = 0$.

For small $\Delta t > 0$, searching from a positive-leaning source can be equivalently thought of monitoring a Poisson process for the stochastic arrival of a negative signal for a duration of Δt, where the Poisson process has an arrival rate λ. If a negative signal arrives during the duration, then the quality is revealed to be low. A positive-leaning source can be interpreted as one that has been overall favorable to the firm's past products. Thus, when the consumer pays attention to such a source, he expects to see positive information about the product, which will move his belief about the product upwards. However, if this positive-leaning source sends a negative signal

	$\omega = 0$	$\omega = 1$
positive signal	$1 - \lambda \Delta t \in (0, 1)$	1
negative signal	$\lambda \Delta t \in (0, 1)$	0

(a) Positive-leaning source.

	$\omega = 0$	$\omega = 1$
positive signal	0	$\lambda \Delta t \in (0, 1)$
negative signal	1	$1 - \lambda \Delta t \in (0, 1)$

(b) Negative-leaning source.

Table 1: Signal distribution conditional on the product's quality by the two information sources.

about the product, he interprets it as a strong evidence that the product's quality is low.

Similarly, we assume that, given a negative-leaning source, the signal is always negative if $\omega = 0$, and it is positive with probability $\lambda \Delta t$ if $\omega = 1$. Table 1(b) characterizes the signal distribution of searching from a negative-leaning source as a statistical experiment. Based on the conditional signal distribution, a positive signal fully reveals $\omega = 1$.

Similar to the positive-leaning source, for small $\Delta t > 0$, searching from a negative-leaning source can be equivalently thought of monitoring a Poisson process for the stochastic arrival of a positive signal for a duration of Δt, where the Poisson process has an arrival rate λ. If a positive signal arrives during the duration, then the quality is revealed to be high. For example, a negative-leaning source can be interpreted as one that has been overall critical of the firm's other products. Thus, when the consumer pays attention to such information sources, he expects to see negative information about the product, which will move his belief about the product downwards. However, if a negative-leaning source sends a positive signal about the product, he interprets it as a strong evidence that the product's quality is low.

3.2 Consumer Search Incentives

Suppose a consumer's belief is x upon his arrival and the price is p.[1] A consumer benefits from such a search if and only if some realized signal would lead to an optimal action that differs from the optimal action without search.

[1]We drop the subscript t when focusing on an individual consumer.

3.2.1 Incentives to Search from Negative-Leaning Sources

We first consider a consumer's incentive to search from a negative-leaning source. When the consumer chooses to search from a negative-leaning source for a duration of $\Delta t > 0$ and no truth-revealing signal (i.e., positive signal) arrives, the consumer's posterior belief about the product's quality being high is

$$\frac{x(1-\lambda\,\Delta t)}{x(1-\lambda\,\Delta t)+(1-x)}, \tag{1}$$

which approaches x as $\Delta t \to 0$. If $\bar{v}(x) > p$, then the consumer's optimal action without search is to buy the product. When a negative signal arrives, his belief decreases but the updated valuation of the product is still more than p for small enough $\Delta t > 0$. In this case, his optimal action remains the same as his optimal default action. When the realized signal is positive, the updated valuation of the product jumps up to v_h, in which case the consumer's optimal action remains the same as his optimal default action. Hence, for small enough $\Delta t > 0$, the consumer has no incentive to search from a negative-leaning source if $\bar{v}(x) > p$.

Assume $\bar{v}(x) \le p \le v_h$. If a positive signal arrives, the consumer's optimal action is to buy the product, which improves the consumer's payoff from the default action of not buying the product by $v_h - p$. Since the probability of a positive signal arriving is $x\lambda\,\Delta t$, the expected payoff improvement from searching from a negative-leaning source is worth the search cost $c\,\Delta t$ if and only if

$$x\lambda\,\Delta t(v_h - p) \ge c\,\Delta t \implies p \le v_h - \frac{c}{x\lambda}.$$

Thus, the highest price at which a consumer with belief x has an incentive to search from a negative-leaning source is $v_h - \frac{c}{x\lambda}$, and the highest price at which the consumer has an incentive to buy without searching is $\bar{v}(x)$. As $\Delta t \to 0$, searching from a negative-leaning source is more better than not searching if and only if $\bar{v}(x) \le p \le v_h - \frac{c}{x\lambda}$. The price range is nonempty if and only if the following condition holds:

$$\lambda x(1-x)(v_h - v_l) \ge c \tag{2}$$

3.2.2 Incentives to Search from Positive-Leaning Sources

Next, consider a consumer's incentive to search from a positive-leaning source. When a consumer searches from a positive-leaning source for a duration of $\Delta t > 0$ and no truth-revealing signal (i.e., negative signal) arrives, the consumer's posterior belief about the product's quality being high is

$$\frac{x}{x + (1-x)(1-\lambda\,\Delta t)},$$

which goes to x as $\Delta t \to 0$. If $\overline{v}(x) < p$, then the consumer's optimal action without search is not to buy the product. If a negative signal arrives, the updated valuation of the product drops to v_l, in which case the consumer's optimal action is the same as the optimal default action. If a positive signal arrives, the consumer's belief increases but hte updated valuation is still less than p for small enough $\Delta t > 0$, and hence the consumer's optimal action remains the same. Thus, for small enough $\Delta t > 0$, the consumer has no incentive to search from a positive-leaning source if $\overline{v}(x) < p$.

Assume $v_l \leq p \leq \overline{v}(x)$. The optimal action without search is to buy the product. If a negative signal arrives, the consumer's optimal action is not to buy the product, which improves the consumer's payoff from the optimal action without search by $p - v_l$. Since the probability of a signal arriving is $(1-x)\lambda\,\Delta t$, the expected payoff improvement from searching from the positive-leaning source is worth the search cost $c\,\Delta t$ if and only if

$$(1-x)\lambda\,\Delta t(p - v_l) \geq c\,\Delta t \implies p \geq v_l + \frac{c}{(1-x)\lambda}.$$

Thus, the highest price at which a consumer with belief x has an incentive to search is $\overline{v}(x)$ and the highest price at which the consumer has an incentive to buy without search is $\underline{p}(x) := v_l + \frac{c}{(1-x)\lambda}$. As $\Delta t \to 0$, search is better than not searching if and only if $v_l + \frac{c}{(1-x)\lambda} \leq p \leq \overline{v}(x)$. This price range is non-empty if and only if (2) holds.

3.2.3 Parameter Assumptions

Note that (2) does not hold when c is too large. As the left-hand side of (2) is maximized at $x = \frac{1}{2}$, for the rest of the paper, we assume that $\lambda(v_h - v_l) > 4c$. That is, at $x = 1/2$, for any type of information sources, there exists a price such that searching is better than not searching.

Given λ and c, let $\underline{x}$ and $\overline{x}$ be the smallest and the largest x such that (2) holds. We have $\underline{x} + \overline{x} = 1$. Our assumption made in the previous paragraph implies that $\underline{x} < 1/2 < \overline{x}$. Thus $[\underline{x}, \overline{x}]$ is the region where consumer search is possible.

3.3 Information Diffusion

Consumers take actions in the order of their indices. At arbitrary time, we use the term *active consumer* to refer to a generic consumer whose turn it is to take action. We model information diffusion in continuous time by letting Δt as previously defined approach 0. Let $(X_t)_{t \geq 0}$ be the stochastic process where X_t is consumer t's belief about the product's quality being high at the end of time t. At the beginning of time t, the consumer learns the belief of his immediate predecessor.[2] At any arbitrary time t, we use the term *public belief* to refer to the active consumer's initial belief learned from his immediate predecessor at the beginning of time t. Specifically, the public belief at time t is $X_{t-} := \lim_{t' \nearrow t} X_{t'}$.

Let S_t^p and S_t^n denote the two Poisson processes for the arrival of negative and positive signals from positive-leaning and negative-leaning sources, respectively. Let $b_t \in \{-1, 0, 1\}$ denote consumer t's search decision, where $b_t = -1$ if the consumer decides to search from negative-leaning sources, $b_t = 0$ if the consumer does not search, and $b_t = 1$ if the consumer searches from positive-leaning sources. With these notations, we have

$$X_t - X_{t-} = \mathbf{1}(b_t = -1)\omega(S_t^n - S_{t-}^n)(1 - X_{t-}) - \mathbf{1}(b_t = 1)(1 - \omega)(S_t^p - S_{t-}^p)X_{t-}.$$

That is, X_t jumps to 1 if consumer searches from negative-leaning sources and a positive signal

[2] It turns out that it is equivalent to alternatively assume that the consumer can observe all predecessors' purchase decisions and prices.

arrives; X_t drops to 0 if consumer searches from positive-leaning sources and a negative signal arrives.

3.4 Firm's Pricing Problem

At the beginning of time t, the firm observes the public belief X_{t-} and chooses a price $p(X_{t-})$. The firm's profit is $\int_0^\infty e^{-rt} a_t p(X_{t-})\, dt$, where $r > 0$ is the discount rate and $a_t \in \{0, 1\}$ is a function of X_t where $a_t = 1$ if the consumer buys the product and $a_t = 0$ if the consumer does not buy the product. At the beginning of time t, the firm chooses the pricing function $p(\cdot)$ to maximize its expected payoff. We focus on the firm's dynamic pricing strategies that are Markovian in X_{t-}.

We also make some definitions regarding the aggregate learning about the product's quality in equilibrium. Given a Markov equilibrium, we say that the equilibrium aggregate learning is *efficient* at x if on every equilibrium path, the active consumer with belief x is incentivized to search whenever a search-incentivizing price exists (i.e., $x \in [\underline{x}, \overline{x}]$). We say that the equilibrium aggregate learning is *asymptotically complete* at x if the asymptotic belief on every equilibrium path with the initial active consumer's belief being x is degenerate. In other words, an equilibrium is asymptotically complete if and only if consumers never herd on any equilibrium path: the search for information will continue until the truth arrives. Lastly, we say that the equilibrium aggregate learning *herds* at x, if on every equilibrium path, the active consumer with belief x does not search for information.

For reference, we summarize the notations used in this paper in Table 2.

4 Benchmark Analyses

We first analyze two simpler cases where consumers can access only one type of information sources. In the first subsection, consumers only have access to negative-leaning sources. In the second subsection, consumers can only access positive-leaning sources. We then study the case where consumers can access both types of information sources.

Symbol	Description
ω	Product quality
v_h	Utility of a high-quality product
v_l	Utility of a low-quality product
t	Time and consumer index
x_t	Belief that the product quality is high when consumer t makes purchase decision
$\bar{v}(x)$	The consumer's expected value for the product given his belief x
p_t	Price at the beginning of time t
c	Search flow cost
λ	Poisson signal arrival rate
$\underline{p}(x)$	Maximum price that incentivizes buying without search at belief x
$\bar{x}$	Maximum belief at which search can be incentivized
$\underline{x}$	Minimum belief at which search can be incentivized
x_0^*	Break-even belief such that $\bar{v}(x_0^*) = 0$

Table 2: List of Notations

For convenience, let $x_0^* \in (0, 1)$ be the *break-even belief*, defined as the unique belief such that a consumer is indifferent between buying and not buying, $\bar{v}(x_0^*) = 0$, where the uniqueness is implied by the assumption that $v_l < 0 < v_h$. Specifically, x_0^* is the unique solution to the equation $(1 - x_0^*)/x_0^* = -v_h/v_l$. Without search, a consumer with a belief of x buys if and only if $x \geq x_0^*$.

4.1 Negative-Leaning Sources Only

When an active consumer with belief x searches from a negative-leaning source and no positive signal arrives, the belief about the product declines at the rate of $\lambda x(1 - x)$. Given the product price p, the consumer has an incentive to search if $x \in \left[\frac{c}{\lambda}\frac{1}{v_h - p}, \frac{p - v_l}{v_h - v_l}\right]$. If $x \in [\underline{x}, \bar{x}]$, there exists a price that incentivizes searching from negative-leaning sources. Outside this range, no search is possible. In the case where $x \notin [\underline{x}, \bar{x}]$, if $x \geq x_0^*$, the firm's optimal price is $\bar{v}(x)$ and the profit is $\bar{v}(x)/r$; if instead $x < x_0^*$, the firm's optimal price is any price above $\bar{v}(x)$ so that the ensuing consumers do not buy the product, leading to a profit of 0.

For $x \in [\underline{x}, \bar{x}]$, the firm's pricing problem can be transformed into an optimal stopping problem. If the firm wants subsequent consumers to continue searching, then the firm charges any price $p(x) \in [\underline{p}(x), v_h - c/(\lambda x)]$. The firm is indifferent between charing any price that incentivizes search as either the consumer does not buy when the positive signal does not arrive, in which case the price is irrelevant; or the price immediately jumps to v_h when the positive signal arrives, in which case the firm's profit is not affected by the current instantaneous price. If the firm wants subsequent consumers to stop searching, then it charges $p(x) = \overline{v}(x)$ if and only if selling at $p(x) = \overline{v}(x)$ is profitable, i.e., $\overline{v}(x) \geq 0$.

As we focus on Markovian strategies, whenever the firm disincentivizes search, either every following consumer buys the product without search at price $\overline{v}(x)$ and the firm's profit is $\overline{v}(x)/r$, or every following consumer never buys and the firm's profit is 0. When the firm incentivizes consumers to search, consumers before the first consumer that receives the positive signal do not buy the product, and consumers after the positive signal buy the product at the maximum price v_h. When incentivizing search, the firm forgoes extracting surplus at the current public belief for the possibility of charging higher prices to future consumers. The following result characterizes the firm's optimal pricing strategy.

Proposition 1 (Optimal pricing with only negative-leaning source)**.** *Assume consumers only have access to negative-leaning sources. Given any public belief $x \in (\underline{x}, \overline{x}]$ and the active consumer, we have the following characterizations of the firm's optimal pricing strategy at x:*

(a) If $x_0^* \geq \overline{x}$, the firm incentivizes the consumer to search.

(b) If $x_0^* \leq \underline{x}$, the firm incentivizes the consumer to buy without search.

(c) If $x_0^* \in (\underline{x}, \overline{x})$, there exists $x_{nl}^* \in (x_0^*, \overline{x}]$ such that if $x \leq x_{nl}^*$, the firm incentivizes the consumer to search and if $x > x_{nl}^*$, the firm incentivizes the consumer to buy without search.

Figure 1 illustrates the equilibrium pricing and consumer search strategy as a function of the active consumer's belief x upon his arrival. In the case $x_0^* \in (\underline{x}, \overline{x})$, there exists some threshold belief x_{nl}^* such that if the first active consumer's initial belief is not greater than x_{nl}^*, then the firm optimally incentivizes the consumer and the ensuing active consumers to search from negative-leaning sources until the positive signal arrives, or the public belief decreases to $\underline{x}$. In this case, the

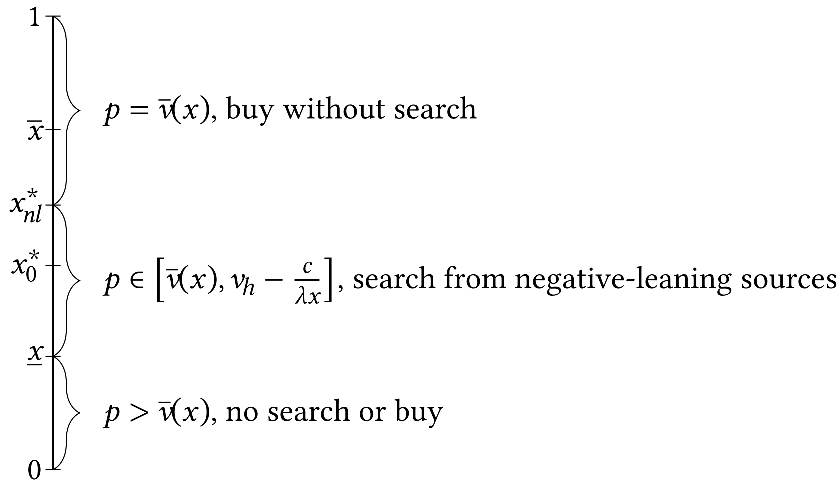


Figure 1: Equilibrium characterization with only negative-leaning sources for $x^*_0 \in (\underline{x}, \overline{x})$

price path will be downward either until the positive signal arrives, at which point the product becomes an instant success and the firm charges the highest price v_h ever since; or no positive signal arrives and the public belief decreases to $\underline{x}$, at which point the firm pulls the product from the market.

For the case $x^*_0 \in (\underline{x}, \overline{x})$, when the public belief is $x \in [\underline{x}, \overline{x}]$, we show that the firm's optimal pricing strategy can be characterized as the choice between two strategies: disincentivizing search immediately, where the active consumer buys if $x \geq x^*_0$ and otherwise does not buy; incentivizing searching from negative-leaning sources until the positive signal arrives or the public belief decreases to $\underline{x}$, at which point the firm removes the product off the market. If the initial belief is lower than x^*_{nl}, then the firm prefers to incentivize searching from negative sources. We illustrate a numerical example of how x^*_{nl} is determined in Figure 2.

Note that in Figure 2, $x^*_{nl} > x^*_0$, where x^*_0 is the point from which the payoff of disincentivizing search starts to become positive. This part of the result implies that when the public belief is not too much above x^*_0, even though selling to the active consumer at price $\overline{v}(x)$ is profitable, the firm optimally forgoes this short-term profit and incentivizes the consumers to search from negative-leaning sources instead, in hope that a positive-signal arrives so that the public belief jumps up, which allows the firm to charge v_h. Another interesting implication is that, when the

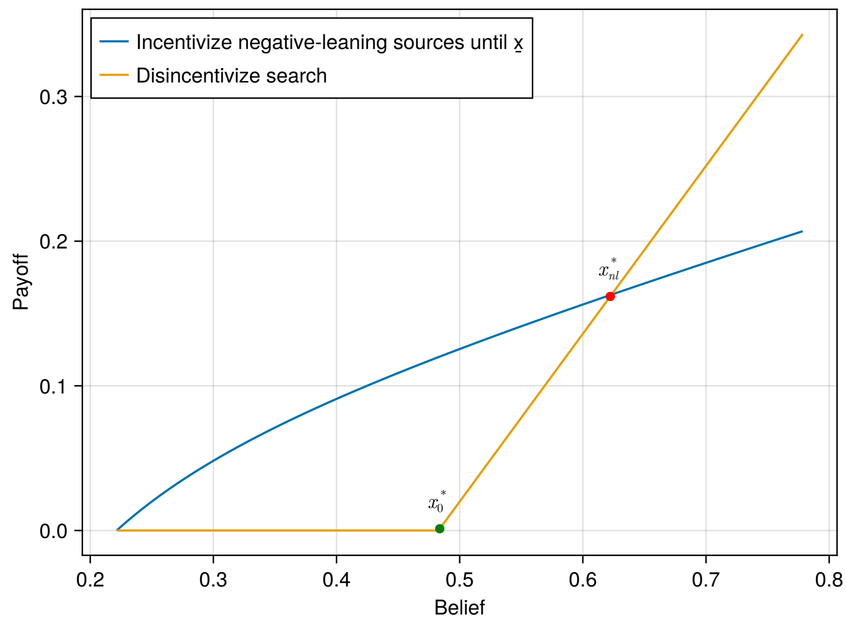


Figure 2: Equilibrium characterization of x_{nl}^* with negative-leaning sources only

active consumer's belief is low ($x < x_0^*$), the firm nonetheless has an incentive to induce the consumer to search from negative-leaning sources, which tends to worsen the product's reputation further. Again, the firm does so in the hope that the product becomes successful when the positive signal arrives.

We provide some intuitions for why $x_{nl}^* > x_0^*$. Fix $x \in (\underline{x}, x_0^*]$. By the choice of x, $\overline{v}(x) \leq 0$. If the firm disincentivizes search at x, the firm optimally incentivizes the consumer not to search or buy, leading to a profit of 0. If the firm continues to incentivize to search from negative-leaning sources until the positive signal arrives or the active consumer's initial belief decreases to $\underline{x}$, then at each point in time t, the active consumer's posterior belief is either 1 or $x_t < x$, with the latter being the posterior belief when no positive signal has arrived by time t. In the event that the belief is 1, the firm's instantaneous revenue flow is $v_h > 0$; if the belief at t is x_t, then the instantaneous revenue flow is 0 since the consumer does not buy when no positive signal arrives. Overall, the firm's expected instantaneous revenue flow at time t is positive. Thus, disincentivizing searching from negative-leaning sources at $x \leq x_0^*$ is strictly dominated and hence $x_{nl}^* > x_0^*$. Intuitively, when $x < x_0^*$, the firm does not wish to sell the product to the active consumer as the maximum price the consumer would buy the product is negative. Thus, by incentivizing searching from negative-leaning sources at $x < x_0^*$, the firm is able to ensure a positive expected revenue flow.

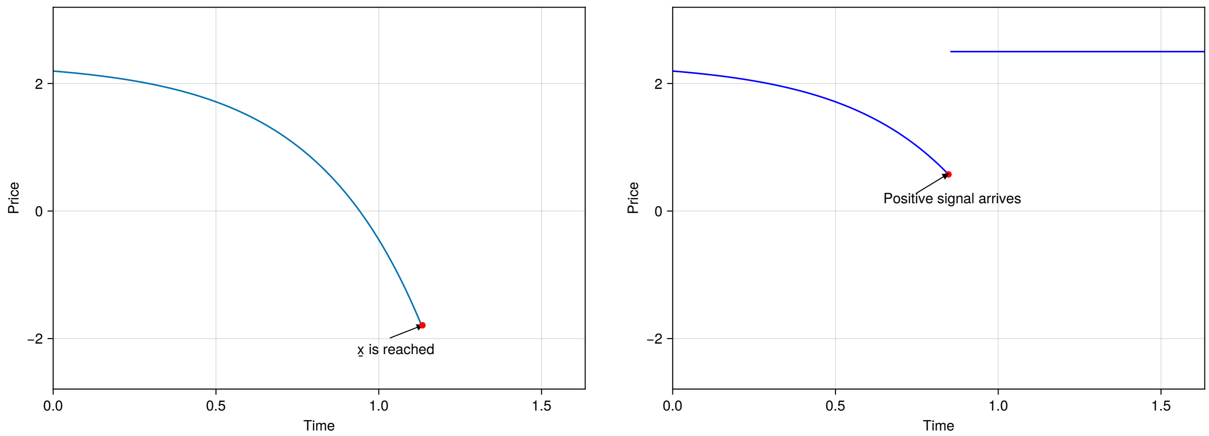
The following result characterizes the degree of aggregate learning about the product's quality, which immediately follows from Proposition 1.

Corollary 1 (Learning with negative-leaning sources only)**.** *Assume consumers only have access to negative-leaning sources. There exists a parameter set such that learning is not efficient for some* $x \in (\underline{x}, \overline{x})$*. Moreover, for every* $x \in [\underline{x}, \overline{x}]$*, the aggregate learning is not asymptotically complete at* x *and herding occurs with a positive probability.*

Specifically, whenever $x_0^* < \underline{x}$ or x_{nl}^* defined in Proposition 1 is above $\underline{x}$, there exists some $x \in (\underline{x}, \overline{x})$ such that the firm optimally disincentivizes search at x. Corollary 1 shows that, even though dynamic pricing leads to more learning than static pricing, the equilibrium aggregate learning is still not asymptotically complete. To see this, observe that based on the characterization in Proposition 1, whenever the firm incentivizes an active consumer to search, there is a positive probability that the active consumer's initial belief decreases to $\underline{x}$, at which point the firm stops incentivizing ensuing consumers to search. In other words, there is always a positive probability that the game ends in herding when consumers only have access to negative-leaning sources.

With regard to the price path in the case with only negative-leaning sources, Proposition 1 implies that once the firm starts to incentivize searching from negative-leaning sources, prices will go down over time until the positive signal arrives, at which point the product becomes an instant success and the firm charges the highest price v_h ever since, or no positive signal arrives and the belief about the product decreases to $\underline{x}$, at which point the firm pulls the product from the market.

For intermediate public beliefs, we illustrate the sample paths of public belief and optimal price in Figure 3. In Figure 3(a), the positive signal never arrives. Thus, belief about the product erodes over time as active consumers keep revising their beliefs further downward. Meanwhile, the firm continues to incentivize consumers to search by lowering its price over time. The firm does so even when it has to charge below the cost. When the public belief reaches $\underline{x}$, the firm can



(a) No positive signal arrives (b) Positive signal arrives before herding

Figure 3: Sample price paths in equilibrium with negative-leaning sources only

no longer incentivize consumers to search, and subsequent consumers herd by not searching or buying. In this case, the firm exits the market, resulting in a failed product launch. In Figure 3(b), the positive signal arrives before herding resulting in a successful product launch that allows the firm to charge v_h ever since.

4.2 Positive-Leaning Sources Only

When an active consumer with belief x searches from a positive-leaning source and no negative signal arrives, the public belief increases at a rate of $\lambda x(1-x)$. Given the product price p, the consumer has an incentive to search if $x \in \left[\frac{p-v_l}{v_h-v_l}, 1-\frac{c}{\lambda(p-v_l)}\right]$. For every $x \in [\underline{x}, \overline{x}]$, there exists a price that incentivizes searching from positive-leaning sources. Outside this range, no search is possible. In the case where $x \notin [\underline{x}, \overline{x}]$, if $x \geq x_0^*$, the firm's optimal price is $p(x) = \overline{v}(x)$ and the profit is $\overline{v}(x)/r$; otherwise, the firm's optimal price is any price above $\overline{v}(x)$ so that the ensuing consumers do not buy the product, and the firm's profit is 0.

For $x \in [\underline{x}, \overline{x}]$, the firm's pricing problem is again transformed into an optimal stopping problem. If the firm wants to incentivize searching from positive-leaning sources, it charges $\overline{v}(x)$, the highest price at which the consumer searches. If the firm wants to disincentivize search, when $\underline{p}(x) \geq 0$, it charges $\underline{p}(x)$, the highest price at which the consumer buys without search; when $\underline{p}(x) < 0$, it charges any $p(x) > \bar{v}(x)$ so that the consumer does not buy the product. If the firm incentivizes searching from positive-leaning sources, consumers before the first consumer that receives the negative signal buy the product at the expected based on the public belief, and consumers after the consumer do not buy the product as the quality is revealed to be low.

When the firm incentivizes the active consumer to buy without search, it ensures the maximum market penetration of the product as the following consumers herd by buying the product without search. In contrast, when the firm incentivizes search and the active consumer receives no negative signal, then the firm not only obtain the instantaneous cash flow from selling to the consumer but also enhances the initial belief of the immediately succeeding active consumer, which allows the firm to charge a higher future price for the product. The following result characterizes the firm's optimal trade-off between maximum market penetration at a lower price and higher future prices at the risk of zero continuation revenue when the negative signal arrives.

Proposition 2 (Optimal pricing with only positive-leaning sources)**.** *Assume consumers only have access to positive-leaning sources. Given any public belief $x \in [\underline{x}, \bar{x})$ and the active consumer, we have the following characterizations of the firm's optimal pricing strategy.*

(a) If $x_0^* \geq \bar{x}$, the firm incentivizes the consumer not to search or buy.

(b) If $x_0^* \leq \underline{x}$, the firm incentivizes the consumer to buy without search.

(c) If $x_0^* \in (\underline{x}, \bar{x})$, there exists $x_{pl}^* \in [\underline{x}, x_0^*)$ such that, if $x < x_{pl}^*$, the firm incentivizes the consumer not to search or buy, and if $x \geq x_{pl}^*$, the firm incentivizes the consumer to search positive-leaning sources.

Figure 4 illustrates the equilibrium pricing and consumer's search strategy as a function of the belief x. In the case $x_0^* \in (\underline{x}, \bar{x})$, there exists some threshold belief x_{pl}^* such that if the first active consumer's initial belief is above x_{pl}^*, then the firm optimally incentivizes the active consumer and the ensuing consumers to search from positive-leaning sources until the negative signal arrives or the belief about the product increases to $\bar{x}$. In this case, the price path will be upward until the

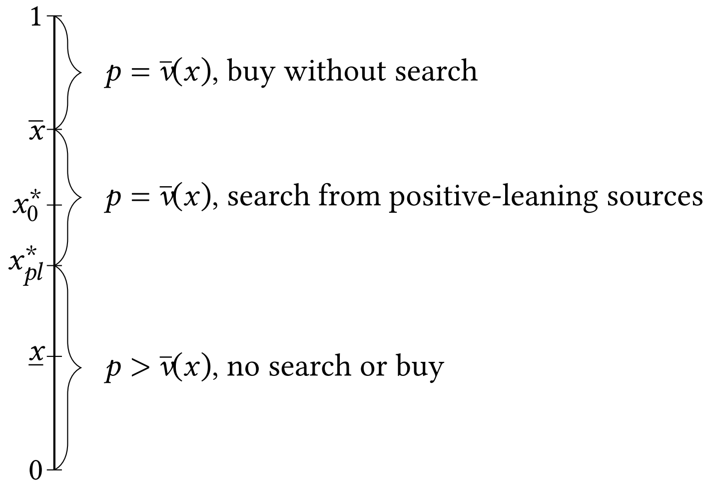


Figure 4: Equilibrium characterizations with only positive-leaning sources for $x_0^* \in (\underline{x}, \overline{x})$

negative signal arrives, at which point the product becomes an instant flop and the firm pulls the product from the market; or no negative signal arrives, and the belief about the product increases to $\overline{x}$, at which point the firm charges $\overline{v}(\overline{x})$ ever since.

In the proof, we show that the firm's optimal pricing strategy involves choosing between incentivizing searching from positive-leaning sources until the negative signal arrives or the active consumer's belief increases to $\overline{x}$, and immediately disincentivizing the search with payoff $\{0, \overline{v}(x)/r\}$. The point at which the firm is indifferent between the two strategies is x_{pl}^* in the proposition. We illustrate a numerical example of how x_{pl}^* is determined in Figure 5.

For exposition, in Figure 5 we also draw the curve for the payoff if the firm is able to obtain a constant cash flow of $\overline{v}(x)$. The intersection of the curve with $x = 0$ characterizes x_0^*. Figure 5 thus also illustrates the result that $x_{pl}^* < x_0^*$. This part of the result implies that when the initial active consumer's belief is not too much below x_0^*, the firm has an incentive to incentivize these consumers to search from positive-leaning sources, even though doing so means that the firm sells to these consumers at prices lower than the marginal cost. The hope of the firm is that there will be no negative signal arriving and the active consumer's belief will eventually increase to $\overline{x}$, at which point the firm can charge a high price $\overline{v}(\overline{x})$ ever since.

We provide some intuitions for why $x_{pl}^* < x_0^*$. Assume there exists $x \in [x_0^*, \overline{x})$. By the choice

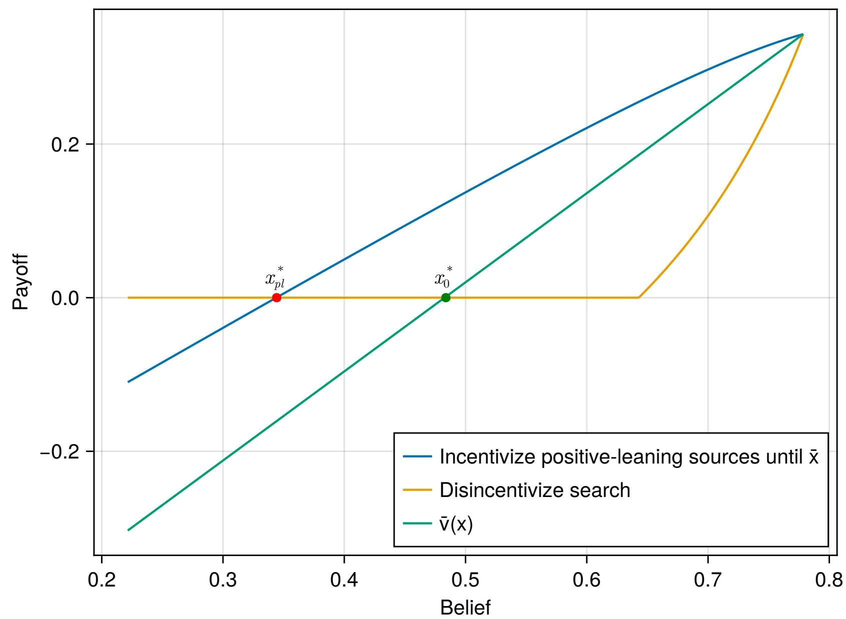


Figure 5: Equilibrium characterization of x^*_{pl} with positive-leaning sources only

of x, $\bar{v}(x) \geq 0$. If the firm disincentivizes the search at x, the firm's payoff is $\underline{p}(x)/r < \bar{v}(x)/r$. Assume that whenever the firm optimally incentivizes searching from positive-leaning sources, it continues doing so until a negative signal arrives or the active consumer's initial belief to $\bar{x}$. In this case, at each point in time t, the belief about the product's quality is either 0 or x_t, the posterior belief if no negative signal has arrived by time t. In the event of the belief being 0, the firm's instantaneous cash flow becomes 0 as there is no profitable price at which the consumer would buy; if the belief is x_t, the instantaneous cash flow is $\bar{v}(x_t) > 0$ since the consumer buys the product. Overall, the firm's expected instantaneous cash flow at time t is larger than $\bar{v}(x_t)$ since $v_l < 0$. Thus, disincentivizing search at x is strictly dominated, and hence $x^*_{pl} < x^*_0$. Intuitively, when $x > x^*_0$ and no negative signal arrives, the firm can extract the maximum surplus from the active consumer, which is more than the marginal cost. When the negative signal arrives, the maximum price the consumer would buy the product is $v_l < 0$, where the firm does not wish to sell the product to the consumer anyway. Thus, by incentivizing the active consumer to search from positive-leaning sources at $x > x^*_0$, the firm is able to obtain a higher profit than the maximum surplus the firm can extract if the consumers' belief stay constant at x.

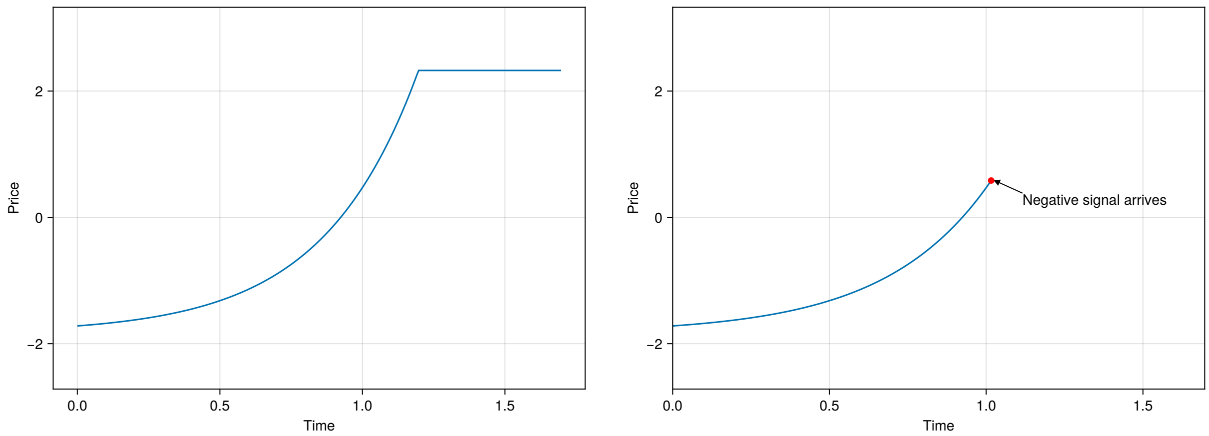
The following result characterizes the degree of aggregate learning about the product's quality, which immediately follows from Proposition 2.

Corollary 2 (Learning with Positive-Leaning Sources Only). *Assume consumers only have access to positive-leaning sources. There exists a parameter set such that learning is not efficient for some $x \in (\underline{x}, \overline{x})$. Moreover, for every $x \in [\underline{x}, \overline{x}]$, the equilibrium aggregate learning is not asymptotically complete at x and herding occurs with a positive probability.*

Specifically, whenever $x_0^* > \overline{x}_{pl}$ or x_{pl}^* as in Proposition 1 is less than $\overline{x}$, there exists $x \in (\underline{x}, \overline{x})$ such that the firm optimally incentivizes the active consumer with belief x not to search, even though there exists a search-incentivizing price at x. Hence, similar to the case with only negative-leaning sources, even though dynamic pricing leads to more learning than static pricing, the equilibrium aggregate learning is never asymptotically complete. To see this, observe that based on the characterization in Proposition 2, whenever the firm incentivizes an active consumer to search, there is a positive probability that the belief about the product increases to $\overline{x}$, at which point the firm stops incentivizing the search and the aggregate learning herds at $\overline{x}$. In other words, there is always a positive probability that the game ends in herding when consumers only have access to positive-leaning sources.

With regard to the price path in the case with only positive-leaning sources, Proposition 2 implies that once the firm starts to incentivize searching from positive-leaning sources, prices will go up over time until the negative signal arrives, at which point the product becomes an instant flop and the firm pulls the product from the market; or no negative signal arrives and the active consumer's initial belief increases to $\overline{x}$, at which point the firm charges the product's expected value $\overline{v}(\overline{x})$ and every ensuing consumer buys the product without search.

For intermediate initial beliefs, we illustrate two equilibrium sample price paths in Figure 6. In Figure 6(a), the negative signal never arrives. Thus, the public belief rises over time as early consumers buy the product. The firm continues to increase the price just enough so that consumers are still incentivized to search. When the belief reaches $\overline{x}$, the firm can no longer incentivize searching from positive-leaning sources, and subsequent consumers herd by buying the product



(a) No negative signal arrives (b) Negative signal arrives before herding

Figure 6: Sample price paths in equilibrium with positive-leaning sources only

Consumer-Incentive Choice	Price	Instantaneous Revenue w/o truth-revealing signal
Negative-leaning sources	$v_h - \frac{c}{x\lambda}$	0
Positive-leaning sources	$\overline{v}(x)$	$\overline{v}(x)$
Buy without search	$\underline{p}(x)$	$\underline{p}(x)$
Not search or buy	v_h	0

Table 3: Firm's Price Choices for each $x \in [\underline{x}, \overline{x}]$

without search. In Figure 6(b), the negative signal arrives before consumers herd, revealing its low quality, causing the firm to exit the market.

5 Analysis with Both Types of Sources

We now assume that each consumer has access to both positive-leaning and negative-leaning sources.

For each $x \in (\underline{x}, \overline{x})$, the firm has the following consumer-incentive choices: incentivize the active consumer to search from negative-leaning sources; incentivize the active consumer to search from positive-leaning sources; incentivize the active consumer to buy without search; incentivize the active consumer not to search or buy. We illustrate the firm's choice of price for each consumer-incentive choice in Table 3.

5.1 Anchoring (Alternate Between Both Types of Sources)

When there are two types of information sources available, there is a new type of search pattern that can emerge. Fix time t and consider two small durations, each with length $\Delta t > 0$. Assume the public belief at t is $x \in [\underline{x}, \overline{x}]$. Suppose the firm incentivizes the consumer active at time t to search from negative-leaning sources for a duration of Δt. If no truth-revealing signal arrives in this duration, the firm then incentivizes the next consumer active at time $t + \Delta t$ to search from positive-leaning sources for a duration of Δt. If no truth-revealing signal arrives in the duration of Δt, then the public belief at $t + \Delta t$ is still x as $\Delta t \to 0$. That is, the firm alternates between incentivizing the two types of information sources to keep the public belief constant.[3] For such a search pattern, we say that the firm *anchors at x*.

Anchoring at x in the context of our model is stochastically equivalent to the case in Che and Mierendorff (2019) where the decision-maker divides his attention between the two types of information sources to keep his belief constant. As the firm can incentivize the consumer to search from any type of information sources for $x \in [\underline{x}, \overline{x}]$, when both types of information sources are available, the firm has the additional action of anchoring at x in its action space.[4]

The firm's payoff from anchoring at x is the firm's limiting payoff as $\Delta t \to 0$. Let $U_a(x)$ be the firm's profit of anchoring at $x \in [\underline{x}, \overline{x}]$, which can be characterized recursively as

$$U_a(x) = \frac{v_h}{r} x\lambda\, dt + \overline{v}(x)dt + [1 - x\lambda\, dt - (1-x)\lambda\, dt - 2r\, dt]\, U_a(x),$$

where the first term is the unconditional expected payoff when the positive signal arrives in the duration of dt, the second and third terms are the unconditional profit in the duration of dt and the continuation profit after the duration when no positive signal arrives in the duration. We can

[3] Based on our benchmark analyses, when the current product price is p, searching from a negative-leaning source is beneficial to the active consumer with initial belief x only if $\overline{v}(x) \leq p$; searching from a positive-leaning source is beneficial to the same consumer only if $\overline{v}(x) \geq p$. Hence, the consumer's choice of information source is not immediately clear when $p = \overline{v}(x)$. As the firm can eliminate the consumer's incentive to search from a negative-leaning source by charging a price infinitesimally below $\overline{v}(x)$, and the incentive to search from a positive-leaning source by charging a price infinitesimally above $\overline{v}(x)$, we assume that the firm decides which type of information sources the active consumer searches when it sets the price at $\overline{v}(x)$.

[4] By adding the action of anchoring at x, we can continue focusing on the firm's Markovian strategies.

solve the above recursive equation to obtain

$$U_a(x) = \frac{(\lambda + r)xv_h + r(1-x)v_l}{r(\lambda + 2r)}. \tag{3}$$

The following result shows that the firm never anchors at beliefs other than $\underline{x}$ and $\overline{x}$.

Lemma 1 (Suboptimality of anchoring at interior beliefs)**.** *For any $x \in (\underline{x}, \overline{x})$, it is suboptimal for the firm to anchor at x.*

Intuitively, the benefit of anchoring at x is that the firm can incentivize search indefinitely. This benefit is not needed when the firm can keep exclusively incentivizing one type of source, as in the case with $x \in (\underline{x}, \overline{x})$. Thus, anchoring at such interior beliefs is suboptimal. More specifically, if the firm anchors at x, then the firm needs to incentivize searching from both types of sources. By our earlier discussions about why it is beneficial for the firm to incentivize search, when $x > x_0^*$, the firm instantly benefits from incentivizing searching from positive-leaning sources but not from incentivizing searching from negative-leaning sources, as the latter means that the firm does not sell to the active consumer even if $\overline{v}(x) > 0$. Thus, in the proof, we show that anchoring at $x \in (x_0^*, \overline{x})$ is always strictly dominated by incentivizing searching from positive-leaning sources only; similarly, anchoring at $x \in (\underline{x}, x_0^*)$ is always strictly dominated by incentivizing searching from negative-leaning sources only.

Note from earlier discussions that anchoring happens when the firm switches the type of sources to incentivize. Lemma 1 shows that doing so must be suboptimal for beliefs in $(\underline{x}, \overline{x})$. If the firm incentivizes searching from positive-leaning sources for some $x' \in (\underline{x}, \overline{x})$, then it must incentivize searching from positive-leaning sources for all $x'' \in (x', \overline{x})$. Similarly, if the firm incentivizes searching from negative-leaning sources for some $x' \in (\underline{x}, \overline{x})$, then it must incentivize searching from negative-leaning sources for all $x'' \in (\underline{x}, x')$. The remaining question is whether the firm should switch the type of sources it incentivizes at $\underline{x}$ and $\overline{x}$, causing the belief to anchor at the two boundaries.

When $x = \underline{x}$ or $x = \overline{x}$, anchoring could be superior to disincentivizing search. For example, in the case with only positive-leaning sources, the firm has to disincentivize search when $x = \overline{x}$ as there is no search-incentivizing price for $x > \overline{x}$. When two types of sources are available, anchoring allows the firm to continue reaping the benefits of incentivizing searching from positive-leaning sources as long as the loss of incentivizing searching from negative-leaning sources is not too large. The following result characterizes the optimality conditions for anchoring when the firm does not incentivize any search.

Lemma 2 (Properties of anchoring)**.** *For $x \in [\underline{x}, \overline{x}]$, we have $U_a(x) \geq 0$ if and only if $-\frac{v_h}{v_l} \geq \frac{r(1-x)}{(\lambda+r)x}$; $U_a(x) \geq \overline{v}(x)/r$ if and only if $-\frac{v_h}{v_l} \leq \frac{(\lambda+r)(1-x)}{rx}$.*

As $\overline{x} + \underline{x} = 1$, Lemma 2 implies that, when the following condition holds, the firm is better of anchoring at $x = \underline{x}$ or $x = \overline{x}$ than disincentivizing search:

$$\frac{r\overline{x}}{(\lambda + r)\underline{x}} \leq -\frac{v_h}{v_l} \leq \frac{(\lambda + r)\underline{x}}{r\overline{x}}. \tag{4}$$

When the magnitude of v_h is large relative to that of v_l (i.e., x_0^* is small), the benefit of keeping the incentives to search from negative-leaning sources (in anchoring) is large since the benefit of incentivizing searching from negative-leaning sources to the firm is increasing in v_h. When the magnitude of v_h is small relative to that of v_l (i.e., x_0^* is large), the benefit of keeping the incentives to search from positive-leaning sources (in anchoring) is large since the benefit of incentivizing searching from positive-leaning sources to the firm is larger for v_l further below 0.

5.2 Optimal Pricing

Let $U_{na}(x)$ be the firm's expected payoff when the current public belief is x and the firm keeps incentivizing searching from negative-leaning sources until the positive signal arrives or the public belief decreases to $\underline{x}$, at which point the firm incentivizes the active consumer not to buy if $U_a(\underline{x}) < 0$ and anchors at $\underline{x}$ if $U_a(\underline{x}) \geq 0$. Let $U_{pa}(x)$ be the firm's expected payoff if the firm incentivizes the active consumer with belief x to search from positive-leaning sources until the negative signal arrives or the public belief increases to $\overline{x}$, at which point the firm incentivizes the

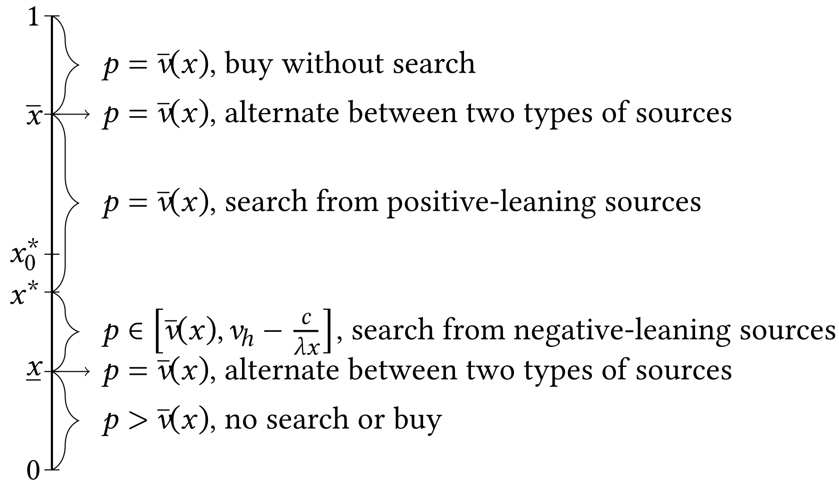


Figure 7: Equilibrium pricing and search strategy with both types of sources when $x^* \in (\underline{x}, \overline{x})$ and (4) both hold.

active consumer with belief $\overline{x}$ to buy without search if $U_a(\overline{x}) < \overline{v}(\overline{x})$ and anchors at $\overline{x}$ if $V_a(\overline{x}) \geq \overline{v}(\overline{x})$. The following result characterizes the firm's optimal pricing strategy for $x \in (\underline{x}, \overline{x})$.

Theorem 1 (Optimal pricing)**.** *Let $U(x)$ for $x \in [\underline{x}, \overline{x}]$ be the firm's expected payoff when the public belief is x according to the firm's optimal pricing strategy. We have $U(x) = \max\{U_{na}(x), U_{pa}(x)\}$. Moreover, if $U_{na}(x^*) = U_{pa}(x^*)$ for some $x^* \in [\underline{x}, \overline{x}]$, then $U_{na}(x) > U_{pa}(x)$ for $x \in [\underline{x}, x^*)$ and $U_{na}(x) < U_{pa}(x)$ for $x \in (x^*, \overline{x}]$.*

Figure 7 illustrates the equilibrium pricing and consumer search strategy as a function of the belief x. The theorem states that the firm's optimal pricing strategy can be characterized by a threshold x^* such that for public beliefs $x \in (\underline{x}, x^*)$, the firm incentivizes searching from negative-leaning sources only; for public beliefs $x \in (x^*, \overline{x})$, the firm incentivizes searching from positive-leaning sources only. Moreover, the last sentence of the theorem implies that the firm is indifferent between incentivizing the two types of sources for at most one belief.

Figure 8 illustrates a numerical example that shows how x^* is determined. Whenever the curve for the payoff of incentivizing searching from negative-leaning sources is above that of incentivizing searching from positive-leaning sources, the firm only incentivizes searching from negative-leaning sources ever since, until the public belief increases to $\underline{x}$, at which point the firm either anchors or disincentivizes any search at $\underline{x}$. Similarly, whenever the curve for the payoff of

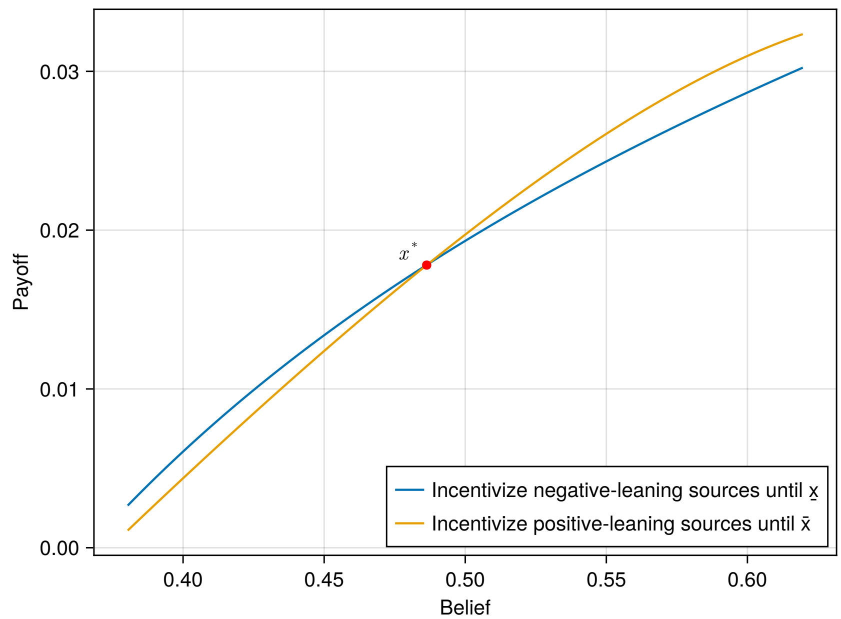


Figure 8: Equilibrium characterization of x^* with both types of sources

incentivizing searching from positive-leaning sources is above the that of incentivizing searching from negative-leaning sources, the firm only incentivizes searching from positive-leaning sources ever since until the belief reaches $\overline{x}$, at which point the firm either anchors or disincentivizes any search at $\overline{x}$. The intersection of the two curves characterizes x^* in the theorem.

When $x^* \in (\underline{x}, \overline{x})$, Theorem 1 implies two possible price paths depending on the initial active consumer's belief. If the initial active consumer's belief is high enough, then the firm incentivizes searching from positive-leaning sources only, until the negative signal arrives or the public belief increases to $\overline{x}$. In this case, the price path will be upward until a truth-revealing signal arrives. Whether the firm incentivizes the active consumer with initial belief $\overline{x}$ to buy without search or anchors at $\overline{x}$ depends on the comparison between $U_a(\overline{x})$ and $\overline{v}(\overline{x})/r$. If the initial public belief is low enough, then the firm incentivizes searching from negative-leaning sources only, until a positive signal arrives or the public belief decreases to $\underline{x}$. In this case, the price path will be downward until a truth-revealing signal arrives. Whether the firm incentivizes the active consumer with belief $\underline{x}$ nor to buy or anchors at $\underline{x}$ depends on the comparison between $U_a(\underline{x})$ and 0. These two comparisons can both be verified through Lemma 2.

The following result characterizes the degree of aggregate learning when consumers have access to both types of information sources.

Corollary 3 (Aggregate learning with both types of sources)**.** *For every $x \in (\underline{x}, \overline{x})$, the equilibrium aggregate learning is efficient at x. Moreover, if* (4) *holds, then aggregate learning is asymptotically complete at every $x \in [\underline{x}, \overline{x}]$, and hence herding never occurs.*

As the active consumer with belief $x \in (\underline{x}, \overline{x})$ is incentivized to search from one type of information sources by Theorem 1, the aggregate learning is efficient at x. To see that the equilibrium aggregate learning can be asymptotically complete at $x \in [\underline{x}, \overline{x}]$, we observe that by Lemma 2, the condition implies that the firm will anchor at $\underline{x}$ and $\overline{x}$, in which case learning will not stop until the product's quality is revealed. Hence, when consumers have access to both types of sources, herding may not happen at all. When the public belief x becomes high enough that consumers would herd by buying without searching, the firm switches to incentivizing searching from negative-leaning sources, which can move the public belief downward so that the immediately next active consumer can be incentivized to search from positive-leaning sources. When the public belief x becomes low enough that consumers would herd by not searching or buying, the firm switches to incentivizing positive-leaning sources, which can move the public belief upward so that the immediately next active consumer can be incentivized to search from negative-leaning sources. In this case, the optimal pricing strategy keeps diverting consumer attention to avoid herding, and consumers continue to learn until the truth is revealed. This result is in stark contrast to the benchmarks with only one type of sources, where herding always occurs with a positive probability.

Since the firm is afforded a larger set of consumer-incentive choices when consumers have access to both types of information sources, the firm's expected payoff is weakly higher than its expected payoff when consumers have access to only one type of sources. In fact, Theorem 1 implies that the firm can be strictly better off when consumers have access to both types of information sources, which we summarize in the following corollary.

Corollary 4 (Profit superiority of both types of sources). *If* (4) *holds, then the firm's expected payoff is strictly larger when consumers have access to both types of information sources than when consumers have access to only one type of information sources.*

Thus, counterintuitively, the firm can strictly benefit from directing consumer attention to negative-leaning sources even when positive-leaning sources are available. A managerial recommendation from this result is that a firm should not only liaise with positive-leaning sources such as sponsored influencers, it could also benefit from reaching out to negative-leaning sources, such as reviewers who tend to be critical of the firm's products.

In the case with only one type of information sources, Proposition 1 and 2 both show that for $x_0^* \in (\underline{x}, \overline{x})$, the firm optimally incentivizes search at x_0^*. The following proposition shows a similar result for the case with both types of information sources. It turns out that the answer depends on whether x_0^* is greater or less than $1/2$ (i.e., whether $v_h + v_l$ is negative or positive).

Proposition 3 (Choice of information source type at x_0^*). *Assume either: (1) $x_0^* \leq \underline{x}$; or (2) $x_0^* \geq \overline{x}$; or (3) $x_0^* \in (\underline{x}, \overline{x})$, $U_{na}(\underline{x}) = U_a(\underline{x})$ and $U_{pa}(\overline{x}) = U_a(\overline{x})$; or (4) $x_0^* \in (\underline{x}, \overline{x})$, $U_{na}(\underline{x}) = 0$ and $U_{pa}(\overline{x}) = \overline{v}(\overline{x})/r$. Fix $x \in [\underline{x}, \overline{x}]$. If $x_0^* < 1/2$, then $U_{na}(x) < U_{pa}(x)$ for every $x \in [\underline{x}, \overline{x}]$ such that $x \geq x_0^*$; if $x_0^* > 1/2$, then $U_{na}(x) > U_{pa}(x)$ for every $x \in [\underline{x}, \overline{x}]$ such that $x \leq x_0^*$.*

The main interpretation of the proposition is that the belief at which the firm is indifferent between incentivizing two types of sources is smaller than x_0^* if $x_0^* < 1/2$ and larger than x_0^* if $x_0^* > 1/2$. We illustrate this pattern in Figure 9, which numerically compares x_0^* and x^* as in Theorem 1 for different ratios of $-v_h/v_l$, where we have $x^* < x_0^*$ if and only if $x_0^* < 1/2$ (which is equivalent to $-v_h/v_l > 1$).

In the case with $x_0^* < 1/2$, for public belief not too much below x_0^*, even though incentivizing searching from negative-leaning sources is instantly beneficial for beliefs below x_0^*, the firm may still have an incentive to incentivize searching from positive-leaning sources instead, leading to short-term losses in the hope of gradually improving the belief about the product. In the case with $x_0^* > 1/2$, for initial beliefs not too much above x_0^*, even given the possibility of incentivizing

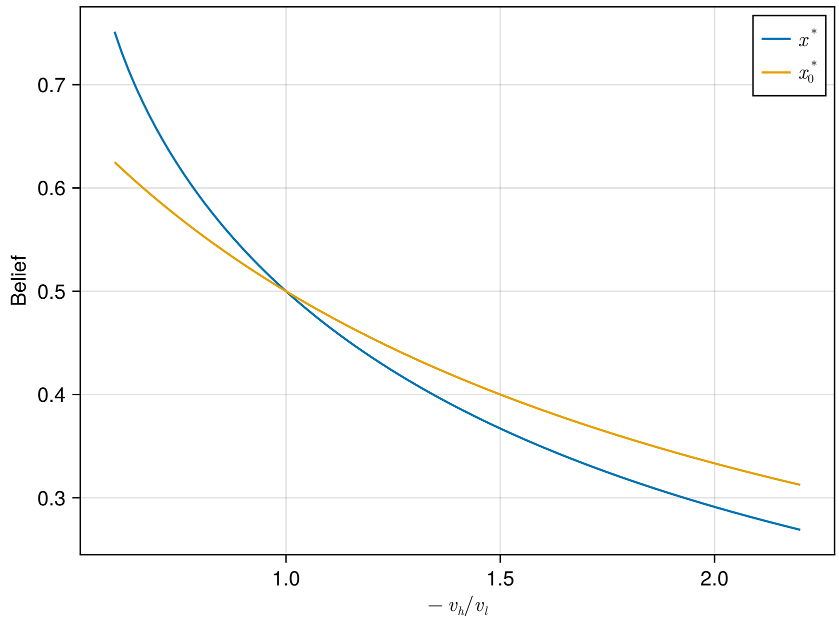


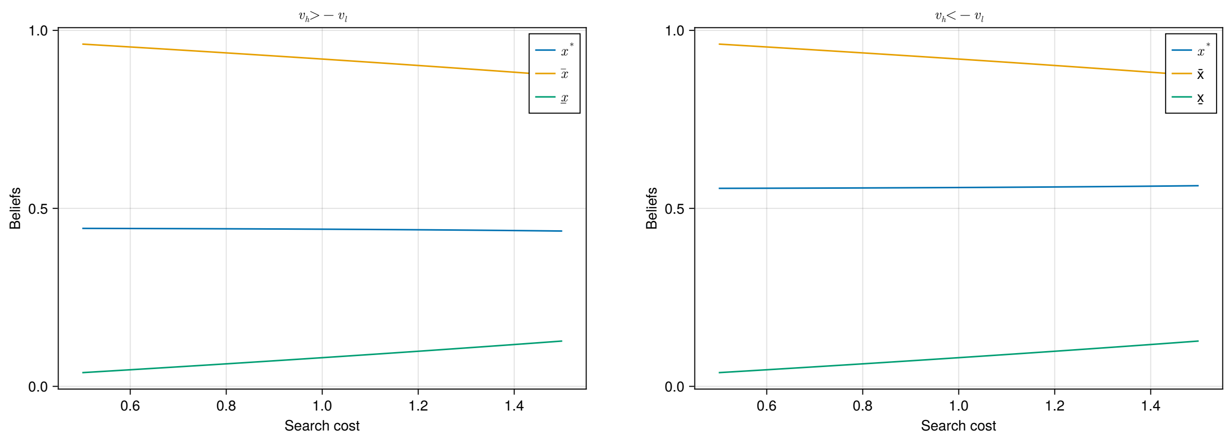
Figure 9: Comparison between x_0^* and x^*

searching from positive-leaning sources, which is instantly beneficial to the firm for beliefs above x_0^*, the firm may still have an incentive to incentivize searching from negative-leaning sources instead, leading to short-term lost revenues from selling to consumers with beliefs above x_0^*, in the hope of the product becoming an instant success when the positive signal arrives.

Proposition 3 reinforces the intuition that searching is beneficial for the firm. When $x_0^* < 1/2$, at x_0^*, the firm is able to exclusively incentivize searching from positive-leaning sources for longer than negative-leaning sources (as indicated by the comparison between the distance from $\underline{x}$ to x_0^* and that from x_0^* to $\overline{x}$), which contributes to the result that the firm incentivizes searching from positive-leaning sources at x_0^*. Similarly, when $x_0^* > 1/2$, at x_0^*, the firm is instead able to exclusively incentivize searching from negative-leaning sources for longer, which contributes to the result that the firm optimally incentivizes searching from negative-leaning sources at x_0^*.

The above intuition is further supported by the following immediate corollary to Proposition 3.

Corollary 5 (Equilibrium thresholds for symmetric utilities)**.** *Assume $x_0^* = 1/2$ (i.e., $v_h = -v_l$). We have $x^* = 1/2$, where x^* is defined in Theorem 1. In other words, if $x \in (\underline{x}, 1/2)$, the firm optimally incentivizes searching from negative-leaning sources; if $x \in (1/2, \overline{x})$, the firm optimally incentivizes*



(a) Case $x_0^* < 1/2$.

(b) Case $x_0^* > 1/2$.

Figure 10: Equilibrium thresholds as functions of c.

searching from positive-leaning sources.

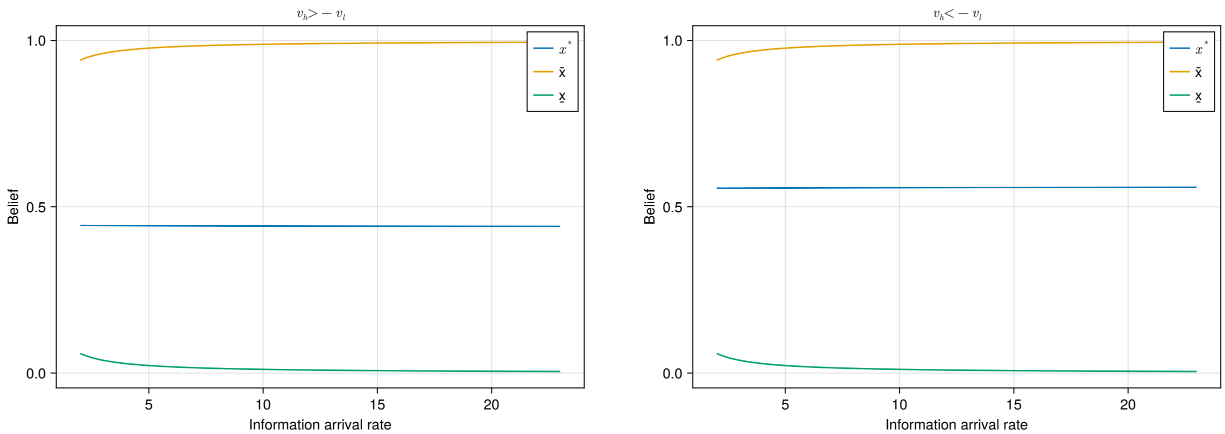
In words, when at $x_0^* = 1/2$, the duration of exclusively incentivizing searching from any type of sources is the same for both types of sources, the firm is indifferent between incentivizing searching from the two types of sources at $x_0^* = 1/2$.

5.3 Comparative Statics

We now examine how the equilibrium thresholds shown in Figure 7, $\bar{x}$, $\underline{x}$, and x^*, depend on the model parameters.

Figure 10 shows how search cost c affects the equilibrium. Intuitively, as the search cost increases, consumers are less willing to search. The range of belief x where the firm can incentivize consumers to search, i.e., $[\bar{x}, \underline{x}]$, shrinks. Consumers will also herd sooner, both when the firm incentivizes searching from negative-leaning sources ($x < x^*$) and when the firm incentivizes searching from positive-leaning sources ($x > x^*$).

Figure 11 shows how the informativeness of the information sources, i.e., λ, affects equilibrium. Intuitively, an increase in λ plays a reverse role from an increase in c. As the informativeness of search increases, consumers are more willing to search for product information. The range of beliefs where the firm can incentivize consumers to search for information, i.e., $[\bar{x}, \underline{x}]$, enlarges.

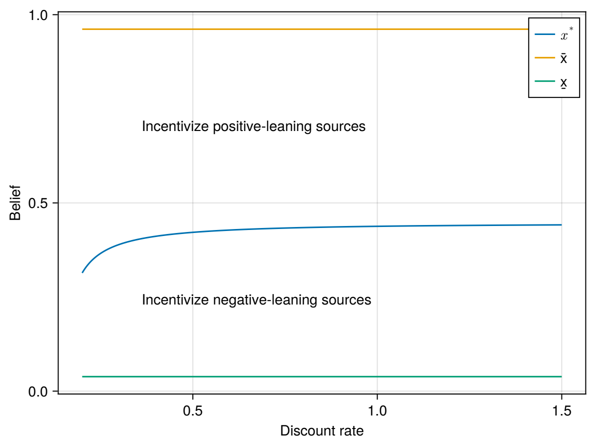


(a) Case $x_0^* < 1/2$. (b) Case $x_0^* > 1/2$.

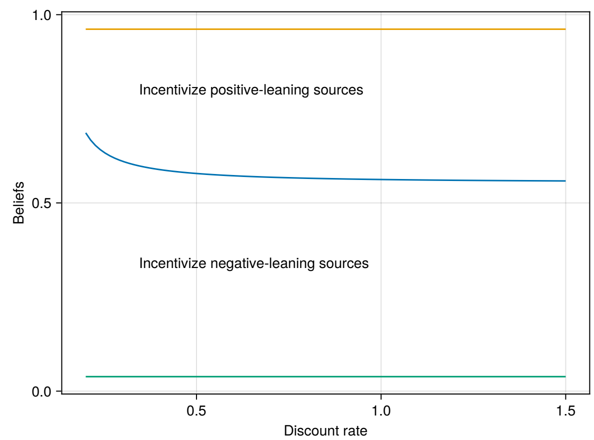
Figure 11: Equilibrium thresholds as functions of λ.

Consumers are less likely to herd, both when the firm incentivizes searching from negative-leaning sources ($x < x^*$) and when the firm incentivizes searching from positive-leaning sources ($x > x^*$).

Figure 12 shows how the firm's discount rate r affects equilibrium. The discount rate does not affect $\bar{x}$ and $\underline{x}$, as consumers are short-lived and their search decisions do not depend on r. Instead, a change in r changes x^*, which determines the range of beliefs for the firm to incentivize searching from each type of sources. When $x_0^* < 1/2$, we have $x^* < x_0^*$ by Proposition 3. In this case, at x^*, the instant benefit is positive for incentivizing searching from negative-leaning sources and negative for positive-leaning sources. Hence, as r increases, the firm puts more weight on the instant benefit and less on the long-term benefit of incentivizing searching from positive-leaning sources for longer, which leads to x^* to become higher towards x_0^*. Similarly, when $x_0^* > 1/2$, we have $x^* > x_0^*$. In this case, at x^*, the instant benefit is negative for incentivizing searching from negative-leaning sources and positive for positive-leaning sources. Hence, as r increases the firm puts more weight on the instant benefit of incentivizing searching from positive-leaning sources and less on the long-term benefit of incentivizing searching from negative-leaning sources for longer, which leads to x^* to become lower towards x_0^*.



(a) Case $x_0^* < 1/2$.



(b) Case $x_0^* > 1/2$.

Figure 12: Equilibrium thresholds as functions of r.

6 Conclusion

We develop a model of consumer attention and pricing in the presence of observational learning, where consumers can directly search for product information from two types of sources, one that is positive-leaning, and the other that is negative-leaning. We have shown how the firm can use its dynamic pricing strategy to control the sequential consumers' search strategy and the evolution of the public belief about the product's quality.

We find that when the public belief about the product is relatively low, incentivizing searching from negative-leaning sources is beneficial to the firm. Through Proposition 1, we have shown that the anticipation of the benefits of incentivizing searching from negative-leaning sources can lead the firm to forgo short-term profits by setting a high price and reduce short-term demand, in the hope of the product becoming an instant success when the positive signal arrives.

We also find that when the public belief is relatively high, incentivizing searching from positive-leaning sources is beneficial to the firm. We have also shown through Proposition 2 that the anticipation of the benefits of incentivizing searching from positive-leaning sources can lead the firm to endure some short-term losses by setting the price below cost and selling to consumers with low beliefs, in the hope of the belief about the product gradually increases above x_0^* and to $\bar{x}$.

When both types of information sources are available, the benefits of searching from any type of sources ensures that the firm always incentivizes search whenever a search-incentivizing price exists, making the aggregate learning efficient. Moreover, the access to both types of information sources gives rise to the possibility that the firm can alternate between the two types of sources to anchor the public belief at some level, which leads to asymptotically complete learning, i.e., no herding arises. With this new structure, we find through Theorem 1 that the firm's optimal pricing strategy can still be concisely characterized by a threshold belief such that the firm incentivizes searching from negative-leaning sources for beliefs below the threshold, and incentivizes positive-leaning sources for beliefs above the threshold. This result stands in contrast to the cases where only one type of information sources is available, in which consumer herding always exists in equilibrium.

A key finding of this research is that firms can benefit from the presence of negative-leaning information sources. These brand-critical information sources provide an economical way to encourage consumer search when the product's reputation is low. Firms can leverage them to encourage consumer search and prevent inefficient herding. In practice, this suggests that for new product launches, firms should also promote diverse reviews from those critical of the brands, instead of only firm-created information or sponsored reviews. A sanitized informational environment with only positive information about the product is detrimental to the diffusion of new products.

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A Proofs

A.1 Proof of Proposition 1

Let $U_{nl}(x)$ be the firm's optimal expected payoff when the public belief is $x \in [\underline{x}, \overline{x}]$. By definition, we have $U_{nl}(\underline{x}) = \max\{0, \overline{v}(\underline{x})/r\}$. Let $U_{nl1}(x)$ be the firm's expected payoff when the firm keeps incentivizing searching from negative-leaning sources from x until the positive signal arrives or the public belief decreases to $\underline{x}$, at which point the firm chooses the optimal action according to $U_{nl}(\underline{x})$. By definition, for $x \in (\underline{x}, \overline{x}]$, $U_{nl1}(x)$ can be written recursively as

$$U_{nl1}(x) = \frac{v_h}{r} x\lambda dt + (1 - x\lambda dt - rdt)[U_{nl1}(x) - \lambda x(1-x)U'_{nl1}(x)dt], \tag{5}$$

where the first term is the unconditional expected payoff when a positive-signal arrives in the duration of dt; the second term is the expected payoff when no positive signal arrives in the duration. The boundary condition of $U_{nl1}(\cdot)$ is $U_{nl1}(\underline{x}) = U_{nl}(\underline{x})$. This recursive equation can be transformed into the following first-order linear differential equation:

$$U_{nl1}(x) = \frac{v_h \lambda x}{r(\lambda x + r)} - \frac{\lambda x(1-x)}{\lambda x + r} U'_{nl1}(x),$$

whose solution is of the form

$$U_{nl1}(x) = L_{nl}(1-x)^{1+r/\lambda} x^{-r/\lambda} + \frac{v_h \lambda x}{r(r+\lambda)}, \tag{6}$$

for some constant L_{nl} that can be determined by the boundary condition.

Assume $x_0^* \geq \overline{x}$, which implies the boundary condition $U_{nl}(\underline{x}) = 0$. We show that in this case, $U_{nl}(x) = U_{nl1}(x)$ for $x \in [\underline{x}, \overline{x}]$. Towards a contradiction, assume that $U_{nl}(x) > U_{nl1}(x)$ for some $x \in [\underline{x}, \overline{x}]$. In this case, it must be that at some public belief $x \in (\underline{x}, \overline{x}]$, the firm is strictly better off disincentivizing search than incentivizing search. Assume without loss of generality that x is such a belief, i.e., $U_{nl}(x) = 0$.

Note that $U_{nl1}(x)$ has the same sign as

$$f_{nl}(x) := L_{nl} + \left(\frac{x}{1-x}\right)^{1+r/\lambda} \frac{\lambda v_h}{r(r+\lambda)}, \tag{7}$$

which is increasing in $x \in (0,1)$. As $f_{nl}(\underline{x}) = 0$ by the boundary condition, we have that $f_{nl}(x) > 0$ and hence $U_{nl1}(x) > 0 = U_{nl}(x)$, a contradiction. Hence, the firm optimally incentivizes searching from negative-leaning sources for every $x \in (\underline{x}, \overline{x}]$.

Assume $x_0^* \leq \underline{x}$. We show that $U_{nl}(x) = \overline{v}(x)/r > U_{nl1}(x)$ for $x \in (\underline{x}, \overline{x}]$. Towards a contradiction, assume that $U_{nl}(x) = U_{nl1}(x)$ for some $x \in (\underline{x}, \overline{x}]$. That is, at x, it is optimal to incentivize searching from negative-leaning sources. Note that we have

$$U_{nl1}(x) - \frac{\overline{v}(x)}{r} = L_{nl} x^{-r/\lambda}(1-x)^{1+r/\lambda} - \frac{r\overline{v}(x) + \lambda(1-x)v_l}{r(r+\lambda)},$$

which has the same sign as

$$g_{nl}(x) := L_{nl} - \left(\frac{x}{1-x}\right)^{1+r/\lambda} \frac{v_h}{r+\lambda} - \left(\frac{x}{1-x}\right)^{r/\lambda} \frac{v_l}{r}.$$

Note that we have

$$g_{nl}'(x) = -\left(\frac{\partial}{\partial x}\frac{x}{1-x}\right) \frac{x^{(r-\lambda)/\lambda}(1-x)^{-r/\lambda}}{\lambda} \overline{v}(x),$$

which has the opposite sign of $\overline{v}(x)$. As $x_0^* \leq \underline{x}$, we have $g_{nl}'(x) < 0$ for $x \in (\underline{x}, \overline{x}]$. Moreover, as $g_{nl}(\underline{x}) = 0$ by the boundary condition, we have $g_{nl}(x) < 0$ and hence $U_{nl1}(x) < \overline{v}(x)/r$, a contradiction. Hence, the firm optimally disincentivizes search for every $x \in (\underline{x}, \overline{x}]$.

Lastly, assume $x_0^* \in (\underline{x}, \overline{x})$, which implies $U_{nl}(\underline{x}) = 0$. Let x_{nl}^* be the supremum of the set of solutions to $U_{nl1}(x) \geq \overline{v}(x)/r$ on $(\underline{x}, \overline{x}]$. We claim that $U_{nl}(x) = U_{nl1}(x)$ for $x \in (\underline{x}, x_{nl}^*]$ and $U_{nl}(x) = \overline{v}(x)/r > U_{nl1}(x)$ for $x \in (x_{nl}^*, \overline{x}]$.

As $f_{nl}(\underline{x}) = 0$ and $f_{nl}'(x) > 0$, we have $U_{nl1}(x) > 0$ for $x \in (\underline{x}, \overline{x}]$. The observation implies that $U_{nl1}(x_0^*) > 0 = \overline{v}(x_0^*)$ Hence, $x_{nl}^* > x_0^*$, and $U_{nl}(x) = U_{nl1}(x)$ for $x \in [\underline{x}, x_0^*]$. As $g_{nl}(x_{nl}^*) \geq 0$ and $g_{nl}'(x) < 0$ for $x > x_0^*$, we have $U_{nl}(x) = U_{nl1}(x) \geq 0$ for $x \in [x_0^*, x_{nl}^*]$. By the definition of x_{nl}^*, we have $U_{nl1}(x) < \bar{v}(x)/r$ for $x \in (x^*_{nl}, \bar{x}]$. As $x^*_{nl} > x^*_0$, we have $U_{nl}(x) = \bar{v}(x)/r > U_{nl1}(x)$. Hence, the firm optimally incentivizes searching from negative-leaning sources if $x \in (\underline{x}, x^*_{nl}]$, and incentivizes buying without search if $x > x^*_{nl}$.

The proof is complete.

A.2 Proof of Proposition 2

Let $U_{pl}(x)$ for $x \in [\underline{x}, \bar{x}]$ be the firm's optimal expected payoff when the current public belief is x. By definition, we have $U_{pl}(\bar{x}) = \max\{0, \bar{v}(\bar{x})/r\}$. Let $U_{pl1}(x)$ be the firm's expected payoff when the firm keeps incentivizing searching from positive-leaning sources from x until the negative signal arrives or the public belief increases to $\bar{x}$, at which point the firm chooses the optimal action according to $U_{pl}(\bar{x})$. By definition, for $x \in [\underline{x}, \bar{x})$, $U_{pl1}(x)$ can be written recursively as

$$U_{pl1}(x) = \bar{v}(x)dt + [1 - (1-x)\lambda dt - rdt][U_{pl1}(x) + \lambda x(1-x)U'_{pl1}(x)dt],$$

where the first term is the unconditional expected payoff in the duration of dt when no negative signal arrives; the second term is the expected payoff after the duration when no negative signal arrives. The recursive equation can be transformed into the following first-order linear differential equation:

$$U_{pl1}(x) = \frac{\bar{v}(x)}{\lambda(1-x)+r} + \frac{\lambda x(1-x)}{\lambda(1-x)+r}U'_{pl1}(x),$$

whose solution is of the form

$$U_{pl1}(x) = R_{pl}x^{1+r/\lambda}(1-x)^{-r/\lambda} + \frac{r\bar{v}(x) + \lambda x v_h}{r(r+\lambda)}, \tag{8}$$

for some constant R_{pl}.

First assume $x^*_0 \geq \bar{x}$, which implies $U_{pl}(\bar{x}) = 0$. We claim that $U_{pl}(x) = 0 > U_{pl1}(x)$ for every $x \in [\underline{x}, \bar{x})$. Towards a contradiction, assume that $U_{pl}(x) = U_{pl1}(x)$ at some $x \in [\underline{x}, \bar{x})$. Note that

$U_{pl1}(x)$ has the same sign as

$$f_{pl}(x) := R_{pl} + \left(\frac{1-x}{x}\right)^{r/\lambda} \frac{v_h}{r} + \left(\frac{1-x}{x}\right)^{1+r/\lambda} \frac{v_l}{r+\lambda}.$$

Moreover, we have

$$f'_{pl}(x) = \left(\frac{\partial}{\partial x}\frac{1-x}{x}\right) \frac{x^{-r/\lambda}(1-x)^{(r-\lambda)/\lambda}}{\lambda} \overline{v}(x),$$

which has the opposite sign as $\overline{v}(x)$. As $x < x_0^*$ we have $f'_{pl}(x) > 0$. Moreover, as $U_{pl1}(\overline{x}) = 0$, we have $U_{pl1}(x) < 0$, a contradiction. Hence, the firm optimally disincentivizes search for every $x \in [\underline{x}, \overline{x})$.

We now assume that $x_0^* < \overline{x}$, which implies $U_{pl}(\overline{x}) = \overline{v}(\overline{x})/r$. We claim that $U_{pl}(x) = U_{pl1}(x)$ for $x \in [\underline{x}, \overline{x}]$. Towards a contradiction, assume that $U_{pl}(x) > U_{pl1}(x)$ for some $x \in [\underline{x}, \overline{x})$. In this case, it must be that at some public belief on $[\underline{x}, \overline{x})$, the firm is strictly better off disincentivizing search than incentivizing search. Assume without loss of generality that x is such a belief, Note that we have

$$U_{pl1}(x) - \frac{\overline{v}(x)}{r} = R_{pl} x^{1+r/\lambda}(1-x)^{-r/\lambda} - \frac{\lambda(1-x)v_l}{r(r+\lambda)},$$

which has the same sign as

$$g_{pl}(x) := R_{pl} - \lambda \left(\frac{1-x}{x}\right)^{1+r/\lambda} \frac{v_l}{r(r+\lambda)}, \tag{9}$$

which is decreasing in $x \in (0,1)$ since $v_l < 0$. Moreover, as $g_{pl}(\overline{x}) = 0$, we have $U_{pl1}(x) > \overline{v}(x)/r$ for $x < [\underline{x}, \overline{x})$, a contradiction. Hence, the firm optimally incentivizes searching from positive-leaning sources for every $x \in [\underline{x}, \overline{x})$.

Lastly, assume that $x_0^* \in (\underline{x}, \overline{x})$. Let x_{pl}^* be the infimum of the set of solutions to $U_{pl1}(x) \geq 0$ on $[\underline{x}, \overline{x}]$. We conjecture that in this case, $U_{pl}(x) = U_{pl1}(x)$ for $x \in [x_{pl}^*, \overline{x}]$; $U_{pl}(x) = 0 > U_{pl1}(x)$ for $x \in [\underline{x}, x_{pl}^*)$.

As $g_{pl}(\overline{x}) = 0$ and $g'_{pl}(x) < 0$, we have $U_{pl1}(x) > \overline{v}(x)/r$ for $x < \overline{x}$. The observation also implies that $U_{pl1}(x_0^*) > \overline{v}(x_0^*) = 0$. Hence, $x_{pl}^* < x_0^*$ and $U_{pl}(x) = U_{pl1}(x)$ for $x \in [x_0^*, \overline{x}]$. As $g_{pl}(x_{pl}^*) \geq 0$

by definition and $g'_{pl}(x) > 0$ for $x \in [x^*_{pl}, x^*_0)$, we have $U_{pl1}(x) > 0$ for $x \in (x^*_{pl}, x^*_0]$. Hence, $U_b(x) = U_{pl1}(x)$ for $x \in [x^*_{pl}, x^*_0]$. By the definition of x^*_{pl}, we have $U_{pl1}(x) < 0$ for $x < x^*_{pl}$. As $x^*_{pl} < x^*_0$, we have $U_{pl}(x) = 0 > U_{pl1}(x)$ for $x \in [\underline{x}, x^*_{pl})$. Hence, the firm optimally disincentivizes search for every $x \in [\underline{x}, x^*_{pl})$, and incentivizes searching from positive-leaning sources for every $x \in [x^*_{pl}, \overline{x})$.

The proof is complete.

A.3 Proof of Lemma 1

There are two cases for x to discuss: $x \in [x^*_0, \overline{x})$ and $x \in (\underline{x}, x^*_0)$.

First assume $x \in [x^*_0, \overline{x})$. When the public belief is $x \in [\underline{x}, \overline{x}]$, let $U_{pa}(x, t)$ be the firm's expected payoff if the firm incentivizes searching from positive-leaning sources for a duration of $t \geq 0$ unless a negative signal arrives; if no negative signal arrives by time t, the firm anchors at the public belief at time t. By definition, we have $U_{pa}(x, 0) = U_a(x)$, the firm's expected payoff of anchoring at x. For small $\Delta > 0$, we have

$$U_{pa}(x, \Delta t) = \overline{v}(x)\Delta t + [1 - (1-x)\lambda\Delta t - r\Delta t]\,[U_a(x) + \lambda x(1-x)U'_a(x)\Delta t] + o(\Delta t).$$

Based on this calculation, by (3), we have

$$\begin{aligned}\frac{U_{pa}(x, \Delta t) - U_{pa}(x, 0)}{\Delta t} &= \overline{v}(x) - [(1-x)\lambda + r]U_a(x) + \lambda x(1-x)U'_a(x) + o(1)\\ &= \frac{r^2\overline{v}(x)}{\lambda + 2r} + o(1),\end{aligned}$$

which has the same sign as $\overline{v}(x)$. Hence, $U'_{pa}(x, 0) > 0$ for $x > x^*_0$. Thus, for $x \in [x^*_0, \overline{x})$, there exists some t such that $U_{pa}(x, t) > U_{pa}(x, 0) = U_a(x)$. That is, anchoring is suboptimal for $x \in [x^*_0, \overline{x})$.

Now assume $x \in (\underline{x}, x^*_0)$. Let $U_{na}(x, t)$ be the firm's expected payoff if the firm incentivizes searching from negative-leaning sources for a duration of $t \geq 0$ unless a positive signal arrives; if no positive signal arrives by time t, the firm anchors at the public belief at time t. By definition, we have $U_{na}(x,0) = U_a(x)$. For small $\Delta t > 0$, we have

$$U_{na}(x,\Delta t) = \frac{v_h}{r} x\lambda\Delta t + [1 - x\lambda\Delta t - r\Delta t]\,[U_a(x) - \lambda x(1-x)U_a'(x)\Delta t] + o(\Delta t).$$

Based on this calculation, by (3), we have

$$\begin{aligned}\frac{U_{na}(x,\Delta t) - U_{na}(x,0)}{\Delta t} &= \frac{v_h}{r}\lambda x - (x\lambda + r)U_a(x) - \lambda x(1-x)U_a'(x) + o(1) \\ &= -\frac{r^2\bar{v}(x)}{\lambda + 2r} + o(1),\end{aligned}$$

which has the opposite sign of $\bar{v}(x)$. Hence, $U_{na}'(x,0) > 0$ for $x < x_0^*$. Thus, for $x \in (\underline{x}, x_0^*)$, there exists some t such that $U_{na}(x,t) > U_{na}(x,0) = U_a(x)$. That is, anchoring is suboptimal for $x \in (\underline{x}, x_0^*)$.

The proof is complete.

A.4 Proof of Lemma 2

Define $\rho = -v_h/v_l$. Fix $x \in [\underline{x}, \bar{x}]$. By (3), $U_a(x) \geq 0$ holds if and only if

$$\frac{(\lambda + r)xv_h + r(1-x)v_l}{r(\lambda + 2r)} \geq 0,$$

which holds if and only if $\rho \geq \frac{r(1-x)}{(\lambda+r)x}$.

Similarly, we have $U_a(x) \geq \bar{v}(x)/r$ if and only if

$$\frac{(\lambda + r)xv_h + r(1-x)v_l}{r(\lambda + 2r)} - \frac{xv_h + (1-x)v_l}{r} \geq 0,$$

which holds if and only if $\rho \leq \frac{(\lambda+r)(1-x)}{rx}$.

The proof is complete.

A.5 Proof of Theorem 1

We first show the single-crossing property of $U_{na}(x)$ and $U_{pa}(x)$ through the following lemma.

Lemma 3. *For every $x' \in [\underline{x}, \overline{x}]$, if $U(x') = U_{na}(x')$, then $U(x) = U_{na}(x)$ for $x \in [\underline{x}, x']$; if $U(x') = U_{pa}(x')$, then $U(x) = U_{pa}(x)$ for $x \in [x', \overline{x}]$.*

Proof of lemma. Assume $U(x') = U_{na}(x')$ for some $x' \in [\underline{x}, \overline{x}]$ and $U(x) > U_{na}(x)$ for some $x \in [\underline{x}, x')$. This assumption implies that we can find some $x \in (\underline{x}, x']$ and $\epsilon > 0$ such that the firm optimally incentivizing searching from negative-leaning sources at x but optimally incentivizes searching from positive-leaning sources for $x'' \in (x - \epsilon, x)$. Therefore, at x, for small $dt > 0$, we have

$$U(x) = \frac{v_h}{r} x\lambda dt + (1 - x\lambda dt - rdt)U(x - \lambda x(1 - x)dt) \tag{10}$$

Since it is optimal to incentivize searching from positive-leaning sources at $x - \lambda x(1 - x)dt$, we have

$$U(x - \lambda x(1 - x)dt) = \overline{v}(x)(1 - x)dt + [1 - (1 - x)\lambda dt - rdt]\, U(x) \tag{11}$$

Plugging (11) into (10), we obtain that

$$(\lambda + 2r)U(x) < \frac{v_h}{r} x\lambda + \overline{v}(x)(1 - x),$$

which implies $U(x) = U_a(x)$, which is a contradiction by Lemma 1. Therefore, if $U(x') = U_{na}(x')$ for some $x' \in [\underline{x}, \overline{x}]$, then $U(x) = U_{na}(x)$ for $x \in [\underline{x}, x']$.

The proof for showing the case that if $U(x) = U_{pa}(x)$, then $U(x) = U_{pa}(x)$ for $x \in [x', \overline{x}]$ is similar and hence omitted.

The proof of the lemma is complete. □

By Lemma 3, we have that $U(x) = \max\{U_{na}(x), U_{pa}(x)\}$ for every $x \in [\underline{x}, \overline{x}]$.

Now assume that $U_{na}(x^*) = U_{pa}(x^*)$ for some $x^* \in [\underline{x}, \overline{x}]$. If $U_{na}(x) \leq U_{pa}(x)$ for some $x \in [\underline{x}, x^*)$, then there must exist some $x \in (\underline{x}, x^*]$ and $\epsilon > 0$ such that $U_{na}(x) = U_{pa}(x)$ and $U_{na}(x') \leq U_{pa}(x')$ for every $x \in (x - \epsilon, x)$. With a similar reasoning as in the proof of Lemma 3, we have

$$U_{na}(x) \leq \frac{v_h}{r} x\lambda + (1 - x\lambda dt - rdt) U_{pa}(x - \lambda x(1-x)dt),$$

and

$$U_{pa}(x - \lambda x(1-x)dt) \leq \overline{v}(x)(1-x)dt + [1 - (1-x)\lambda dt - rdt]\, U_{na}(x),$$

which together imply that

$$(\lambda + 2r) U_{na}(x) \leq \frac{v_h}{r} x\lambda + \overline{v}(x)(1-x),$$

which implies $U_{na}(x) \leq U_a(x)$, a contradiction by Lemma 1. Therefore, we have shown that $U_{pa}(x) < U_{na}(x)$ for $x \in [\underline{x}, x^*)$.

The proof for the case that $U_{na}(x) < U_{pa}(x)$ for $x \in (x^*, \overline{x}]$ is similar and hence omitted.

The proof is complete.

A.6 Proof of Proposition 3

Define $\rho = -v_h / v_l$.

Case $x_0^* \in (\underline{x}, \overline{x})$.

Assume $x_0^* \in (\underline{x}, \overline{x})$. Thus, $\max\{x_0^*, \underline{x}\} = x_0^*$ and $\min\{x_0^*, \overline{x}\} = x_0^*$. By Theorem 1, in this case, it is sufficient to show that $U_{na}(x_0^*) < U_{pa}(x_0^*)$ if $x_0^* < 1/2$ and $U_{na}(x_0^*) > U_{pa}(x_0^*)$ if $x_0^* > 1/2$. By the proof of Proposition 1, $U_{na}(x)$ is of the form in (6); by the proof of Proposition 2, $U_{pa}(x)$ is of the form in (8). Using the same notations in (6) and (8), by the definition of x_0^*, we have

$$U_{pa}(x_0^*) - U_{na}(x_0^*) = R_{pl} x_0^{1+r/\lambda} (1 - x_0^*)^{-r/\lambda} - L_{nl} (1 - x_0^*)^{1+r/\lambda} x_0^{-r/\lambda},$$

which is non-negative if and only if

$$R_{pl} \geq L_{nl} \left(\frac{1 - x_0^*}{x_0^*} \right)^{1+2r/\lambda}. \tag{12}$$

As $x_0^* \in (\underline{x}, \overline{x})$, we have $U_{na}(\underline{x}) = \max\{0, U_a(\underline{x})\}$ and $U_{pa}(\overline{x}) = \max\{U_a(\overline{x}), \overline{v}(\overline{x})/r\}$. We observe that $\overline{x} = 1 - \underline{x}$. Moreover, we have $\rho := \frac{1-x_0^*}{x_0^*} = -\frac{v_h}{v_l}$ since $x_0^* v_h = -(1 - x_0^*)v_l$. By Lemma 2, we have $U_a(\underline{x}) \geq 0$ if and only if $\rho \geq \frac{r}{\lambda+r} \frac{\overline{x}}{1-\overline{x}} =: \rho^*$. By the same lemma, we also have $U_a(\overline{x}) \geq \overline{v}(\overline{x})/r$ if and only if $\rho \leq \frac{\lambda+r}{r} \frac{1-\overline{x}}{\overline{x}} = 1/\rho^*$.

Subcase $\rho^* < \rho < 1/\rho^*$. If $\rho^* < \rho < 1/\rho^*$, we have $U_{na}(\underline{x}) = U_a(\underline{x})$ and $U_{pa}(\overline{x}) = U_a(\overline{x})$. We plug the boundary condition for $U_{na}(\cdot)$ in (6) to get

$$L_{nl}\overline{x}^{1+r/\lambda}(1-\overline{x})^{-r/\lambda} + \frac{v_h \lambda (1-\overline{x})}{r(\lambda+r)} = \frac{(\lambda+r)(1-\overline{x})v_h + r\overline{x}v_l}{r(\lambda+2r)},$$

which we solve for L_{nl} to get

$$L_{nl} = \overline{x}^{-(1+r/\lambda)}(1-\overline{x})^{r/\lambda} \frac{r(1-\overline{x})v_h + (\lambda+r)\overline{x}v_l}{(\lambda+r)(\lambda+2r)}, \tag{13}$$

which is negative as $1 - \overline{x} < x_0^*$. We plug the boundary condition for $U_{pa}(\cdot)$ in (8) to get

$$R_{pl}\overline{x}^{1+r/\lambda}(1-\overline{x})^{-r/\lambda} + \frac{(r+\lambda)\overline{x}v_h + r(1-\overline{x})v_l}{r(\lambda+r)} = \frac{(\lambda+r)\overline{x}v_h + r(1-\overline{x})v_l}{r(\lambda+2r)},$$

which we solve for R_{pl} to get

$$R_{pl} = -\overline{x}^{-(1+r/\lambda)}(1-\overline{x})^{r/\lambda} \frac{(\lambda+r)\overline{x}v_h + r(1-\overline{x})v_l}{(\lambda+r)(\lambda+2r)}, \tag{14}$$

which is negative since $\bar{x} > x_0^*$. By (13) and (14) and the definition of ρ, in this case, (12) is equivalent to

$$f_a(\rho) := (\lambda + r)\bar{x}\rho - r(1 - \bar{x}) - \rho^{1+2r/\lambda}\left[(r + \lambda)\bar{x} - r(1 - \bar{x})\rho\right] < 0$$

We observe that $f_a(\rho) = 0$ and $f_a'(1) = -\frac{2r(\lambda+r)(2\bar{x}-1)}{r} < 0$, which holds since $\bar{x} > 1/2$. Additionally, we have

$$f_a''(\rho) = \frac{2r(\lambda + r)(\lambda + 2r)\rho^{1+2r/\lambda}\left(\rho - \frac{\bar{x}}{1-\bar{x}}\right)}{\lambda^2(1 - \bar{x})},$$

which is negative as $\rho < \bar{x}/(1 - \bar{x})$ by the assumption $x_0^* \in (\underline{x}, \bar{x})$. Moreover, through algebraic manipulation, we observe that

$$f_a(\rho)f_a(1/\rho) = -\rho^{-\frac{2\lambda+2r}{\lambda}}\left[(\lambda + r)\bar{x}\rho - r(1 - \bar{x}) - \rho^{1+2r/\lambda}\left[(\lambda + r)\bar{x} - r(1 - \bar{x})\rho\right]\right]^2,$$

which is negative when $\rho \neq 1$. Thus, $f_a(\rho)$ and $f_a(1/\rho)$ have opposite signs. As $f_a''(\cdot) < 0$, $f_a'(1) < 0$, and $f_a(\rho)f_a(1/\rho) < 0$ for every $\rho \neq 1$, we have $f_a(\rho) < 0$ for $\rho > 1$ and $f_a(\rho) > 0$ for $\rho < 1$. That is, $U_{na}(x_0^*) < U_{pa}(x_0^*)$ if $x_0^* < 1/2$ and $U_{na}(x_0^*) > U_{pa}(x_0^*)$ if $x_0^* > 1/2$.

Subcase $\rho \geq 1/\rho^*$ and $\rho \leq \rho^*$. If $\rho \geq 1/\rho^*$ and $\rho \leq \rho^*$, then we have $U_{na}(\underline{x}) = 0$ and $U_{pa}(\bar{x}) = \bar{v}(\bar{x})/r$. In this case, we can plug the boundary condition for $U_{na}(\cdot)$ in (6) to get

$$L_{nl} = -\bar{x}^{-(1+r/\lambda)}(1 - \bar{x})^{r/\lambda}\frac{\lambda(1 - \bar{x})v_h}{r(\lambda + r)}, \tag{15}$$

which is negative. Similarly, we plug the boundary condition for $U_{pa}(\cdot)$ in (8) to get

$$R_{pl} = \bar{x}^{-(1+r/\lambda)}(1 - \bar{x})^{r/\lambda}\frac{\lambda(1 - \bar{x})v_l}{r(\lambda + r)}, \tag{16}$$

which is negative since $v_l < 0$. By (15), (16), and the definition of ρ, in this case, we see that (12) is equivalent to $\rho^{\frac{2\lambda+2r}{\lambda}} \geq 1$, which holds if and only if $\rho \geq 1$. Thus, $U_{na}(x_0^*) < U_{pa}(x_0^*)$ if $x_0^* < 1/2$ and

$U_{na}(x_0^*) > U_{pa}(x_0^*)$ if $x_0^* > 1/2$.

Case $x_0^* \leq \underline{x}$

Assume $x_0^* \leq \underline{x}$. Thus, $x_0^* < 1/2$ and $\max\{x_0^*, \underline{x}\} = \underline{x}$. By Theorem 1, in this case, it is sufficient to show that $U_{na}(\underline{x}) < U_{pa}(\underline{x})$.

If $U_{na}(x) \geq U_{pa}(x)$ for some $x \in [\underline{x}, \overline{x}]$, then by Theorem 1, we have $U_{na}(\underline{x}) \geq U_{pa}(\underline{x})$. However, we also have $U_{na}(\underline{x}) = \max\{U_a(\underline{x}), \overline{v}(\underline{x})/r\}$, which is a contradiction since $U_{pa}(\underline{x}) > \max\{U_a(\underline{x}), \overline{v}(\underline{x})/r\}$ by Proposition 2 and the proof of Lemma 1. Thus, we have $U_{na}(\underline{x}) < U_{pa}(\underline{x})$.

Case $x_0^* \geq \overline{x}$

Assume $x_0^* \geq \overline{x}$. Thus, $x_0^* > 1/2$ and $\min\{x_0^*, \overline{x}\} = \overline{x}$. By Theorem 1, in this case, it is sufficient to show that $U_{na}(\overline{x}) > U_{pa}(\overline{x})$.

If $U_{na}(x) \leq U_{pa}(x)$ for some $x \in [\underline{x}, \overline{x}]$, then by Theorem 1, we have $U_{na}(\overline{x}) \leq U_{pa}(\overline{x})$. However, we also have $U_{pa}(\overline{x}) = \max\{U_a(\overline{x}), 0\}$, which is a contradiction since $U_{na}(\overline{x}) > \max\{U_a(\overline{x}), 0\}$ by Proposition 1 and the proof of Lemma 1. Thus, we have $U_{na}(\overline{x}) > U_{pa}(\overline{x})$.

The proof is complete.