Predictive Accuracy, Consumer Search, and Personalized Recommendation

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Abstract

Firms use predictive technology to attract and direct consumer search through personalized product recommendations. This paper examines the firm's recommendation strategy by analyzing a key trade-off: accurate recommendations draw high-search-cost consumers into the search process (the "participation-drawing effect") but may narrow the search breadth of moderate-search-cost consumers (the "search-narrowing effect"). When pricing is inflexible in response to environmental changes, the search-narrowing effect dominates in markets with intermediate predictive accuracy or limited search costs, leading firms to forgo recommendations despite their values in reducing search frictions. However, with pricing flexibility, the no-recommendation strategy is optimal only when both predictive accuracy and search costs are low. Flexible pricing enables firms to capture the surplus from accurate recommendations, strengthening the participation-drawing effect. It also shifts the firm's strategic focus from managing search intensity to managing search participation, increasing the profitability of personalized recommendations. Our findings underscore the dual impacts of personalization recommendations on consumer search behavior and highlight the importance of pricing flexibility in optimizing recommendation strategies. This research provides actionable insights for firms leveraging predictive technologies in customer management.

Keywords: predictive accuracy, personalized recommendation, directed search, pricing flexibility

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1 Introduction

Personalized recommendations have transformed how consumers discover products and services. Many platforms like Netflix, YouTube, and Amazon have demonstrated the immense value of these systems, with recommendations driving 80% of consumption on Netflix (Gomez-Uribe and Hunt, 2016), 70% of YouTube views (Kiros, 2022), and 35% of Amazon sales (MacKenzie et al., 2013). These successes have heightened consumer expectations, with 91% of respondents in a 2018 Accenture survey stating that they are more likely to shop brands that provide relevant offers and recommendations[1]. However, despite widespread consumer demand, many firms remain hesitant to adopt personalized recommendation systems. Only 33% of the companies currently use or plan to use AI-driven product recommendations[2], creating a puzzling gap between demand and supply of personalized recommendations.

The disparity is particularly striking given the availability of affordable, off-the-shelf recommendation solutions. For instance, Google's ML solution, Vertex AI, offers a streamlined process for retailers, including data preparation, algorithm training, and A/B testing, with charges as little as \$0.10 per 1,000 predictions and \$600 free trial credits.[3] Similarly, Shopify, an e-commerce platform serving 4.8 million online businesses, provides ready-to-use website solutions that use purchase histories and product descriptions to generate personalized recommendations automatically on their websites.[4] Despite these accessible and cost-effective solutions, why do many firms hesitate to adopt personalized recommendation systems, a technology that consumers actively seek? This paper seeks to address this question by examining the multifaceted implications of personalized recommendations in markets where consumers engage in active search.

We develop a model in which a firm sells horizontally differentiated products and consumers incur search costs to evaluate the fit of the products. The firm decides whether to offer personalized recommendations based on predicted match values, and consumers search the products sequentially directed by the firm's personalized recommendations, if offered. Importantly, consumers are

[1] https://newsroom.accenture.com/news/2018/widening-gap-between-consumer-expectations-and-reality-in-personalization-signals-warning-for-brands-accenture-interactive-research-finds

[2] https://www.forbes.com/advisor/business/software/ai-in-business/

[3] https://cloud.google.com/retail/docs/pricing

[4] https://help.shopify.com/en/manual/online-store/search-and-discovery/product-recommendations

assumed to have knowledge of the firm's predictive accuracy when making their search decisions. Our analysis reveals a critical tension in the firm's decision to offer personalized recommendations. Although these recommendations can attract consumers to initiate a search (*participation-drawing effect*), they may narrow the breadth (or intensity) of the search of some consumers (*search-narrowing effect*). In addition, the extent of trade-off depends on the accuracy of the predictive technology.

To illustrate this tension, consider a hypothetical example involving AEG Presents, a live entertainment company, and a customer named Alice. After receiving a personalized email from AEG Presents recommending upcoming concerts tailored to her interests, Alice explored the suggested events but found them misaligned with her preferences.[5]. Trusting AEG's knowledge of her tastes, Alice assumed that AEG's other unrecommended shows would also be irrelevant and stopped searching, even though some unrecommended shows might have better matched her preferences. This example highlights how personalized recommendations can simultaneously encourage consumers like Alice to initiate a search while narrowing the breadth (or intensity) of the search, creating a complex trade-off for firms.

We begin our analysis by examining a scenario with inflexible pricing, where the firm's price does not adjust to changes in the firm's predictive technology or recommendation strategy. A possible reason for pricing inflexibility is a company's need to maintain consistent prices across multiple channels. In this case, the firm may opt not to provide recommendations, even when doing so would be socially efficient. The intuition is as follows. Providing recommendations can attract more consumers to participate in search because it is now more likely to find a product well-matched to their preferences. This *participation-drawing effect* increases as predictive accuracy improves. However, providing recommendations discourages consumers from inspecting products not recommended, thereby reducing their search breadth (the *search-narrowing effect*). We show that this loss in demand from the search-narrowing effect reaches its peak at an intermediate level of predictive accuracy and can be large enough to deter the firm from recommending products. Even when the firm prefers recommendation to no recommendation, there exists a range of predictive accuracy where the firm's equilibrium profit decreases as accuracy increases. In that

[5]A discussion of the challenges faced by AEG Presents in implementing personalized recommendations can be found at https://www.lytics.com/assets/aeg-presents.pdf

range, although the participation-drawing effect dominates the search-narrowing effect, more precise recommendations lead to a larger increase in the search-narrowing effect. Moreover, the average search cost increases the dominance of the participation-drawing effect, given most consumers search little at high search costs. When the search cost is above a certain threshold, making recommendations is optimal at all levels of predictive accuracy.

We then extend our analysis to flexible pricing, where the firm can adjust prices in response to predictive technology and recommendation strategies. Pricing flexibility allows the firm to implement personalized recommendations under a broader range of conditions, except in markets with both low search costs and low predictive accuracy. There are two main strategic considerations when a firm sets prices under recommendation: First, the firm may want to charge a lower price to incentivize the consumers to broaden their search breadth. This happens when the firm aims to maximize search intensity by setting relatively low prices. We show that this strategic consideration is only present in markets where there are many consumers tending to search broadly - characterized by low search costs and low predictive accuracy. In such markets, the firm will decrease the price under recommendation to maintain the search intensity. Hence, the quantity demanded is unchanged, making recommendations suboptimal.

Second, the firm may want to charge a higher price under recommendation to capture the improved surplus created by accurate recommendations. This happens when the firm focuses on managing search participation by setting prices to make all consumers participate in the search but not necessarily search for both products. Surprisingly, we find that extracting surplus through price does not always guarantee the optimality of personalized recommendations. In a market with both low average search costs and not-too-large predictive accuracy, making recommendations is suboptimal even if the firm asks for higher prices under recommendations. Low search cost restricts a firm's ability to raise prices following recommendations because recommendations only have limited improvement on marginal consumers' surplus. At the same time, the search-narrowing effect is still at large to overcome under low search costs and not-too-large predictive accuracy. At a high level, even though recommendation creates more surplus by reducing search frictions, it also makes the surplus harder to extract. This is because recommendation creates more heterogeneity in consumer demand between recommended and unrecommended products. As a result, price flexibility only expands the personalized recommendations to markets where either the search cost is intermediate or the predictive accuracy is high enough, in which case the extra surplus created is large enough.

Finally, while our main analysis assumes that consumers have perfect knowledge of the firm's predictive accuracy, we also analyze a model extension where consumers have imperfect information. Our findings show that the key results remain robust, further confirming the interplay between personalized recommendations, consumer search behavior, and predictive accuracy in determining the firm's strategy.

2 Literature Review

This paper is closely related to the literature on marketing communication and consumer search. Early findings in this area focus on settings where the marketing communication does not discriminate across recipients. For example, Butters (1977) studies advertising containing pricing information, showing that increased advertising leads to decreased search because any source can provide the same information. Building on that, much of the literature examines the firms' optimal amount of information to influence consumer search. Mayzlin & Shin (2011) show that the seller may withhold information to signal their high quality and encourage the consumers to search on the product. Shin & Yu (2021) find that firms sometimes send fewer targeted ads as their prediction become more precise to reduce the consumer traffic to search its competitors. Despotakis & Yu (2023) argue that firms can benefit from committing to single-dimensional targeting(rather than multidimensional targeting) to reduce consumers' uncertainty about products' value, thereby increasing click-through rates and purchases. From a platform's perspective, Hagiu & Jullien (2011) identify two motivations for intermediaries to add noise to search: balancing the trade-off between consumer traffic and consumer search depth, and utilizing information to influence strategic choices like price. Zhong (2022) studies the platforms' search design problem that the platform decides both the information precision and price weighting, discussing how this affects the consumer search and its implications on firms' prices and platform revenue. Ke et al. (2022) examine a platform's optimal information design to maximize revenue from sales commission and selling advertising slots. They find that optimal information design may be socially inefficient because imperfect information induces competition for prominence, leading to increased ad revenue. In addition to information design, platforms sometimes deliberately change consumers' search costs. Dukes & Liu (2016) argues that intermediate search costs are optimal for balancing the search breadth across products to avoid competition and search depth for each product to enhance understanding. However, Jiang & Zou (2020) argues that if the platform could optimally adjust its referrals, it would always benefit from lower search costs. Our paper complements the above literature by studying how a multi-product firm can utilize recommendation strategies to influence consumer search at both extensive and intensive margins. We examine the firm's optimal recommendation strategy at different levels of consumer search cost.

This paper contributes to the literature investigating the effect of predictive accuracy on equilibrium outcomes. Empirical studies find that the firm's predictions about consumer preferences are often inaccurate due to imperfect predictive technology(Neumann et al. 2019), the inability to connect information from various sources(Coey & Bailey 2016, Lin & Misra 2022, Trusov et al. 2016), or privacy regulations restricting data collection from consumers (Peukert et al. 2022). Theoretical studies find improving precision can have nuanced effects on equilibrium price. For platforms, Yang (2013) shows that increasing matching accuracy leads to higher prices by increasing the variety of products, which softens the competition. Zhong (2022) finds a non-monotonic effect of accuracy on price due to the interplay between competition and consumers' incentive to search. Zhou & Zou (2023) finds that under the profit-based recommendation system, the equilibrium price first decreases and then increases in the marketplace's consumer profiling accuracy, due to interplay of incentives to compete for higher ranking and to extract consumer surplus. Studies on individual firms implementing predictive technology usually find a higher price with more precise predictions. Shin & Yu (2021) and Ning et al. (2022) study targeting advertising and consumer inference, both showing that firms charge higher prices as targeting precision increases to extracted improved consumer surplus due to more precise targeting. In addition to the surplus-extracting feature, our study identifies another motive for firms to reduce prices to expand consumer search breadth. Unlike sellers on platforms charging a lower price to convert search into sales and prevent further search, a multi-product firm has an incentive to reduce prices to encourage broader search. Whereas individual firms typically raise prices to extract consumer surplus, our paper finds that a multi-product firm sets prices to balance motive to widen consumer search breadth and to extract consumer surplus. As a result, the equilibrium price could decrease with increased predictive accuracy.

Our model builds on the directed search theory (see Armstrong (2017) for an extensive review). Most papers on directed search have focused on the competition among many single-product firms, emphasizing the importance of prominence, or being the first business considered along the consumer search path. In many circumstances, consumers search non-randomly, choosing to first investigate those brands with higher advertising intensity (Haan & Moraga-González 2011), higher ranks by intermediaries (Rhodes 2011), low prices (Armstrong et al. 2009), pre-sale services (Janssen & Ke 2020), or previous purchase interactions (Armstrong & Zhou 2011). Our paper lies within a smaller strand of literature that studies directed searches among products sold by a single firm. Researchers in this area often find that firms have an incentive to induce inefficient consumer search to maximize profits. Petrikaitė (2018) finds the firm often deliberately increases the search cost for some products to enable price discrimination. Nocke & Rey (2024) is closely related to our paper, focusing on the firm's decision to steer consumer search among products with different popularity. In their paper, they find that firms have an incentive to adopt a noisy positioning strategy by placing more popular products alongside less popular ones. Unlike their paper, which studies the steering effect of a common positioning for all consumers, we focus on personalized recommendations that direct consumer search differently. We contribute to this strand of literature by 1. examining product prominence as a result of a firm's equilibrium recommendation strategy and predictive accuracy and 2. finding that better predictive technology does not always lead to more efficient consumer search.

3 Model

3.1 Model Setup

Consider a market that consists of a continuum of consumers with the total market size normalized to one. A monopolistic firm (e.g., a platform) serves the market with two horizontally differentiated products denoted by $j = a, b$. The utility that consumer i receives from product j is given by $m_{ij} - p_j$, in which m_{ij} is product j's match value to consumer i and p_j is its price. We assume the match value m_{ij} depends on consumer preference toward the product's horizontal attributes, taking the value $m_{ij} = 1$ if product j is a *good* match to the consumer and $m_{ij} = 0$ if product j is a *bad* match. For example, consumers may have different preferences for aesthetic features in apparel products, such as color, design, and style. Ex-ante, each product (j) has an equal probability of being a good match or a bad match to a consumer (i), i.e., $P(m_{ij} = 1) = P(m_{ij} = 0) = 0.5$. The match values are identically and independently distributed across products and consumers. For ease of exposition, we omit subscript i throughout the paper.

Prediction Technology and Predictive Accuracy

The firm does not know the actual match values to a consumer; it employs a predictive technology to generate personalized prediction $s = (s_a, s_b) \in \{(better, worse), (worse, better)\}$. A prediction $(s_j, s_{-j}) = (better, worse)$ indicates that product j is predicted to be a better match for the consumer than product $-j$, $j = a, b$. We characterize the prediction generating process as follows:

$$\begin{aligned} &P(s_j = better, s_{-j} = worse | m_j = 1, m_{-j} = 0) = \alpha \\ &P(s_j = better, s_{-j} = worse | m_j = m_{-j}) = 0.5 \end{aligned} \tag{1}$$

where α is the firm's *predictive accuracy*, $j, -j = a, b$. We assume $\alpha \geq 0.5$, thus the prediction is informative about which product is the better match when two products have different match values. When the products have the same match value, either both good or bad, the prediction is purely random.[6]

[6] A typical recommendation system first predicts a score for each alternative (product) indicating its relevance to consumer preference. Then it ranks the alternatives (products) in descending order of the predicted scores. When two products are alike, their predicted scores are likely to be close, making their ranking uninformative. However, when two

Firm's Decisions

The firm decides whether or not to provide consumers with a product recommendation. We denote the firm's recommendation strategy by θ^{σ} where $\sigma \in \{0, 1\}$, $\theta = \theta^1$ indicates the strategy of making recommendations, and $\theta = \theta^0$ indicates no recommendation. When $\theta = \theta^1$, the firm recommends the product that is ranked higher according to the firm's prediction. For example, a website can feature a product on the front page of a website, rank it at the top position of the product placement, or directly communicate its recommendations through a message such as "These are recommended for you based on your shopping history!".

We consider two pricing decision scenarios: *inflexible pricing* and *flexible pricing* scenarios. In the inflexible pricing scenario, the price is fixed at the equilibrium level of a benchmark model (a model without product recommendations). This scenario reflects situations where the firm's pricing decision responds neither to its recommendation decision nor to the change in the firm's predictive accuracy(α). Analyzing the firm's recommendations with a fixed price helps isolate the effects of these two decisions. Such situations are not uncommon in practice. For example, many firms sell products through multiple channels and must keep retail prices consistent across channels. In such a situation, the pricing decision of a firm in the online channel is constrained by the prices in the offline channel. The cost of internal coordination can be another friction to price changes. IT teams may not communicate updates on the algorithm or predictive accuracy with the pricing team, which makes the price invariable to algorithm changes.

In the flexible pricing scenario, the firm optimizes its recommendations and pricing decisions. Since two products are ex ante identical to consumers, the firm charges a uniform price on two products ($p_a = p_j = p$). By comparing the results under the inflexible pricing and the flexible pricing scenarios, we can examine the firm's expected benefit from coordinating its recommendation and pricing decisions as the predictive accuracy evolves.

Consumer Search and Purchases

A consumer may engage in costly searches to learn the match values before purchase. We assume that a consumer can perfectly learn the matching value of a product after searching it once and

products are dissimilar to the consumers, i.e., one is a good match and another is a bad match, preferred products are more likely to get higher scores; ranking is informative in this case.

incurring a search cost c. We also assume that the consumer has perfect recall. At any time during the search process, the consumer can stop the search, either purchasing one of the products she has already searched or exit. The search costs of consumers (c) are heterogeneous and are drawn independently from a uniform distribution $U[0, 2\Delta c]$. A bigger Δc implies a wider range of search costs (from zero). When a market has a large Δc, the average search cost of the market is higher and, given all else being equal, on average consumers are expected to search for fewer products. Consumers observe the recommendation strategy θ^{σ} and are perfectly informed about predictive accuracy α, an assumption that we will extend in the extension.

The firm's recommendation decision can affect a consumer's search behavior: initiating the search ("*search participation*") and deciding how many products to search ("*search intensity*"). Consumers observe the recommendations and (rationally) start the search process with the recommended product. This is consistent with the findings in Ursu et al. (2021) that a large fraction of searches happen in a directed manner and consumer searches are subject to the influence of marketing communication. When a firm does not provide a recommendation, a consumer randomly chooses a product to start the search.

Timeline of the Game

The game proceeds in two stages. Stage 1 is the *marketing stage*. The firm learns its predictive accuracy (α) and decides whether or not to provide a recommendation ($\sigma \in \{0, 1\}$). In the flexible-pricing game, the firm sets a price conditional on its predictive accuracy and recommendation decision. Stage 2 is the *search stage* in which consumers make search and purchase decisions. Consumer decisions include whether to initiate a search and which product(s) to search before making a purchase decision or exit.

Next, we examine consumer search, learning, and purchase behavior.

3.2 Consumer Inference and Search Decision

We start by examining the rational inference of consumers when they receive a personalized recommendation at precision α. When a consumer receives a recommendation, she updates her belief about the recommended product's match value as follows:

$$
\begin{aligned}
& Pr(m_j = 1|(s_j = better, s_{-j} = worse), \alpha) \\
= & \frac{\sum_{m_{-j} \in \{0,1\}} Pr(m_j = 1, m_{-j}) * Pr(s_j = better, s_{-j} = worse | m_j = 1, m_{-j})}{\sum_{m_j \in \{0,1\}} \sum_{m_{-j} \in \{0,1\} \in \{0,1\}} Pr(m_j, m_{-j}) * Pr(s_j = better, s_{-j} = worse | m_j, m_{-j})} \\
= & \frac{0.5 + \alpha}{2}
\end{aligned} \tag{2}
$$

A consumer's belief about the match value of the recommended product is higher with more precise predictions. Note that not providing a recommendation is equivalent to providing a recommendation with pure noise ($\alpha = 0.5$), under which a consumer's belief about the recommended product remains the same as her prior.

Each consumer updates her belief about the recommended product and decides whether or not to participate in the search. The consumer's search participation decision is characterized by the search cost of a marginal consumer who is indifferent between participating and not participating in the search. For this marginal consumer, her search cost equals the expected gain from searching the recommended product. We denote such a threshold value of search cost by $\hat{c}(\alpha, p)$ and compute this threshold value as follows:

$$
\hat{c}(\alpha, p) = (1 - p) \cdot \frac{0.5 + \alpha}{2} \tag{3}
$$

We will refer to $\hat{c}(\alpha, p)$ defined above as the *search participation threshold*. A consumer whose search cost is below the search participation threshold will initiate the search. The search threshold ($\hat{c}(\alpha, p)$) increases with predictive accuracy α and decreases with price p. Intuitively, consumers are more likely to participate in the search when they expect a higher search value - from either higher probability of good match, or higher surplus (lower prices) conditional on finding a good match.

After observing the match value of the recommended product (j), each consumer decides whether or not to continue the search. A consumer will stop the search and purchase the recommended product (j) if it is a good match (as two products have the same price). If the consumer finds the recommended product to be a bad match, she updates her belief about the match value of the remaining product as follows:

$$
\begin{aligned}
&Pr(m_{-j}=1|m_j=0,(s_j=better,s_{-j}=worse),\alpha) \\
&=\frac{Pr(m_j=0,m_{-j}=1)*Pr(s_j=better,s_{-j}=worse|m_j=0,m_{-j}=1)}{\sum_{m_{-j}\in\{0,1\}}Pr(m_j=0,m_{-j})*Pr(s_j=better,s_{-j}=worse|m_j=0,m_{-j})} \\
&=\frac{(1-\alpha)}{(1-\alpha)+0.5}
\end{aligned}
\tag{4}
$$

When the recommended product is a bad match, the consumer draws inferences about two possible causes: the firm's recommendation is wrong, i.e. $(m_j, m_{-j}) = (0, 1)$, or both products are bad matches, i.e. $(m_j, m_{-j}) = (0, 0)$. A distinct feature of inference under recommendation, as shown in Equation (4), is that the belief about the remaining product is more negative when the firm's recommendation is more accurate (higher α). With more accurate recommendations, there is a smaller chance that the firm made a mistake in product recommendation; thus, it is more likely that both products are bad matches. In the extreme case where the recommendation is perfectly accurate ($\alpha = 1$), the consumer will conclude that the other product is also a bad match with certainty.

Equation (4) implies that the consumer's decision about whether to continue the search is characterized by another threshold in search cost:

$$
c(\alpha, p) = (1-p) * \frac{(1-\alpha)}{(1-\alpha)+0.5} \tag{5}
$$

We will refer to $c(\alpha, p)$ defined above as the *broad search threshold*. When the result of the initial search is a bad match, the consumer will continue the search if and only if her search cost is below the broad search threshold. Similarly to $\hat{c}(\alpha, p)$, $(c(\alpha, p))$ decreases with price p as a higher price results in a lower search value. However, unlike $\hat{c}(\alpha, p)$, $c(\alpha, p)$ decreases with the accuracy of prediction α. Thus, with improved predictive accuracy, more consumers will start the search, but fewer consumers will continue the search if the initial search result is a bad match. This tension will be a critical insight that underlies many of our key results.

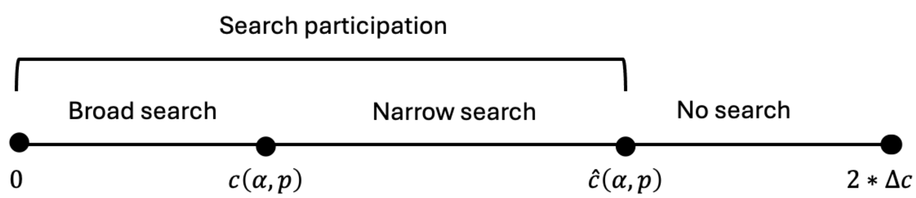
We summarize the above analysis of the search behavior of consumers in Table 1. We also illustrate the cases in Figure 1. There are three levels of search behavior defined by the initial and broad search thresholds: *broad search*, *narrow search*, and *no search*. First, consumers with a low

Table 1: Consumer Search Behavior

Search Cost	Expected Demand	Search Behavior
$c < c(\alpha, p)$	$D^2(\alpha, p) = min\left\{\frac{1-p}{2\Delta c} * \frac{(1-\alpha)}{(1-\alpha)+0.5}, 1\right\} * 0.75$	Broad search
$c(\alpha, p) \leqslant c < \hat{c}(\alpha, p)$	$D^1(\alpha, p) = [min\left\{\frac{1-p}{2\Delta c} * \frac{(0.5+\alpha)}{2}, 1\right\}$ $-min\left\{\frac{1-p}{2\Delta c} * \frac{(1-\alpha)}{(1-\alpha)+0.5}, 1\right\}] * (0.25 + 0.5 * \alpha)$	Narrow search
$c > \hat{c}(\alpha, p)$	0	No search

search cost $(c < c(\alpha, p))$ search broadly. They will continue the search if the recommended product is a bad match. Under a broad search, the probability of finding a good match is 0.75. Second, consumers with an intermediate search cost $(c(\alpha, p) \leqslant c < \hat{c}(\alpha, p))$ search narrowly. They initiate the search of one product, but will not continue and search the other product. Under a narrow search, the probability of finding a good match is $0.25 + 0.5\alpha$. Broad and narrow searches result in different matching probabilities when only one of the two products is a good match. The magnitude of the difference, which is equal to $0.5(1 - \alpha)$, decreases with the accuracy of the recommendation. Finally, consumers with a search cost above the search participation threshold $(c > \hat{c}(\alpha, p))$ will not participate in the search at all.

Figure 1: Consumer's search behavior



3.3 Market Demand

A consumer's search behavior affects her purchase decisions. First, consumers who search broadly will always find the product with a good match, if exists. The expected demand from those consumers is:

$$D^2(\alpha, p) = min\left\{\frac{1-p}{2\Delta c} * \frac{(1-\alpha)}{(1-\alpha)+0.5}, 1\right\} * 0.75$$

Second, those consumers who search narrowly will only purchase from the firm if the recommended product is a good match. The expected demand from such consumers is:

$$D^1(\alpha, p) = \left[min\left\{\frac{1-p}{2\Delta c} * \frac{(0.5+\alpha)}{2}, 1\right\} - min\left\{\frac{1-p}{2\Delta c} * \frac{(1-\alpha)}{(1-\alpha)+0.5}, 1\right\}\right] * (0.25 + 0.5 * \alpha)$$

The no-search consumers will not buy. Overall, given the distribution of consumers in three types of search behavior, the firm's expected demand with product recommendations is

$$D(\alpha, p) = D^1(\alpha, p) + D^2(\alpha, p). \tag{6}$$

The firm's expected demand depends on its predictive accuracy α and price p. A notable insight from the analysis of search behavior is that, while a lower price can increase both search thresholds by enhancing search value, higher predictive accuracy can increase the search participation threshold but decrease the broad search threshold. The firm's total expected profit with recommendation is:

$$\Pi(\alpha, p) = p \cdot D(\alpha, p)$$

We adopt the perfect Bayesian equilibrium as the solution concept, which is defined as follows: (1). the firm's recommendation strategy σ maximizes its expected profit given the consumers' search and purchase decisions; (2). each consumer makes search and purchase decisions to maximize her expected utility, given the firm's recommendation strategy and consumer's belief; and (3). the consumers' belief about the product match value is updated according to Bayes' rule and is consistent with the firm's recommendation strategy. In the main analysis, we assume that consumers have perfect information about the accuracy of the firm's recommendation. Later in the extension, we relax this assumption and incorporate consumer uncertainty about the firm's recommendation precision.

We organize the remainder of this paper as follows. In Section 4, we examine the optimal recommendation strategy of the company under inflexible pricing. Specifically, the price is fixed at the optimal level in the case of no recommendation. In Section 5, we study the firm's recommendation strategy with flexible pricing. Finally, in Section 6 we present a model extension by analyzing a model with consumer uncertainty about the firm's recommendation precision.

4 Optimal Recommendation Strategy with Inflexible Pricing

In this section, we study the firm's recommendation decision when its pricing decision is inflexible. We first examine a benchmark model where the firm does not make any recommendations. We will use the benchmark model's optimal price as the fixed price. We describe the fixed price and corresponding consumer behavior in the following lemma. We use the notation $p^*(0.5)$ because the benchmark case of no recommendation is equivalent to the case of recommendation with α=0.5, under which consumers would ignore the recommendations.

Lemma 1. *(Benchmark Model) Consider the case where the firm does not make a recommendation.*
(i). When $\Delta c > 0.125$, the firm's optimal price is $p^*(0.5) = 0.5$. Those consumers with search cost $c < 0.25$ search broradly, and the rest does not search.
(ii). When $\Delta c \leqslant 0.125$, the optimal price is $p^*(0.5) = 1 - 4 * \Delta c$, and all consumers search broadly.

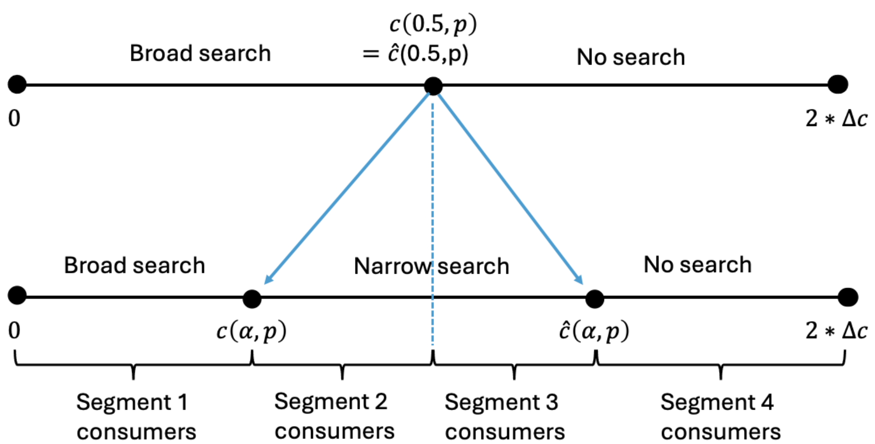
When there is no recommendation, the match values for the two products are i.i.d. and therefore $\hat{c}(\alpha, p) = c(\alpha, p)$. All consumers who participate in the search will search broadly, and the firm's price affects how many consumers will search. When Δc is large ($\Delta c > 0.125$), those consumers who have high search costs do not search. In this case, the firm's expected profit is $p * \frac{\hat{c}(0.5,p)}{2*\Delta c}$ and the optimal price is an interior solution $p^*(0.5) = 0.5$. When Δc is sufficiently small ($\Delta c < 0.125$), the optimal price $p^*(0.5) = 1 - 4 * \Delta c$. Since $\hat{c}(0.5, p) = 2 * \Delta c$, all consumers search broadly and the optimal price is a corner solution.

When the firm makes product recommendations, a recommendation can affect consumer decisions on *search participation* and *search intensity*.[7] We illustrate these two effects in Figure 2, where the top horizontal line is for the benchmark case (no recommendation) and the line at the bottom is for the regular case (with recommendation). Recall that Equations 3 and 5 define two threshold values of search cost, $c(\alpha, p)$ and $\hat{c}(\alpha, p)$, that determine whether consumers engage in broad,

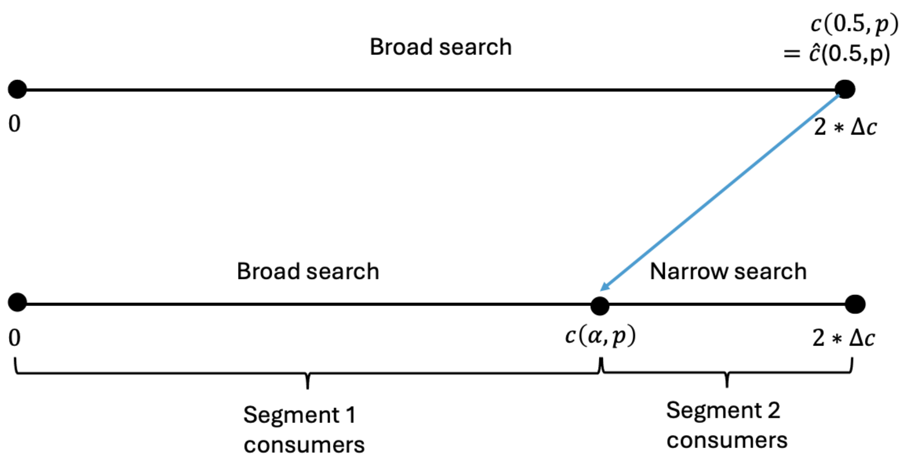
[7] The literature often refers to these two effects as extensive margin and intensive margin effects.

Figure 2: Consumer's search behavior with/without recommendations

(a). $\Delta c > 0.125$



(b). $\Delta c \leqslant 0.125$



narrow, or no search. In the benchmark case, two thresholds are identical ($c(0.5, p) = \hat{c}(0.5, p)$) and consumers engage in a broad search or no search.

We start with the case of a large search cost range ($\Delta c > 0.125$) in Figure 2(a). The figure shows that offering product recommendations would increase the broad-search threshold but decrease the search-participation threshold. Consequently, there are four consumer segments, each segment corresponding to an interval of search cost and one type of change in consumer search behavior due to recommendations. First, consumers in Segment 1 always search broadly regardless of whether the firm makes recommendations; thus, their likelihood of purchasing remains the same.

Consumers in Segment 2 switch from broad searches in the benchmark case to narrow searches in the recommendation case, reducing their likelihood of purchasing from the firm. Consumers in Segment 3 switch from no search to narrow searches and therefore their likelihood of purchase increases. Finally, consumers in Segment 4 never search regardless of the firm's recommendation decision and their purchase likelihood remains zero.

The above discussion points to two opposing effects of personalized recommendations on the firm's demand: increasing the number of consumers who search narrowly but reducing the number of consumers who search broadly. On the one hand, personalized recommendations have a positive *participation-drawing effect* by attracting more consumers to search the firm's products. The magnitude of the firm's gain from this effect depends on the size of Segment 3 and increased demand from no search to narrow searches, specifically,

$$\underbrace{(0.25+\alpha*0.5)}_{\text{increased demand from a consumer in segment 3}} * \underbrace{\left[min\left\{\frac{\hat{c}(\alpha,p)}{2\Delta c},1\right\} - min\left\{\frac{c(0.5,p)}{2\Delta c},1\right\}\right]}_{\text{size of segment 3}}. \tag{7}$$

On the other hand, personalized recommendations have a negative *search-narrowing effect* by switching some consumers from searching broadly to searching narrowly. The magnitude of the firm's loss from this effect depends on the size of Segment 2 and reduced demand when consumers decrease search intensity, specifically,

$$\underbrace{(1-\alpha)*0.5}_{\text{reduced demand for a consumer in segment 2}} * \underbrace{\frac{c(0.5,p)-c(\alpha,p)}{2*\Delta c}}_{\text{size of segment 2}}. \tag{8}$$

The above equation implies an important property for the relation between the search-narrowing effect and predictive accuracy. When predictive accuracy improves, the size of Segment 2 increases but the magnitude of demand reduction for a consumer decreases. More accurate recommendations increase the number of consumers who switch from a broad search to a narrow search. However, when comparing broad and narrow searches, while broad searches always result in higher probabilities of finding good matches, the magnitude of the difference (labeled as "reduced demand for a consumer" in above equation) shrinks as predictive accuracy increases. In the extreme case where $\alpha = 1$, the chance of finding a good match is identical between broad and narrow searches. Overall, the search-narrowing effect should have an inverse-U shaped relation with predictive accuracy, reaching a maximum at an intermediate value of predictive accuracy. This is in stark contrast to the participation-drawing effect which has both components increasing with the predictive accuracy (with the size of segment 3 potentially hitting a ceiling when consumers with the highest search cost participate).

When consumers have low search costs such that $\Delta c \leqslant 0.125$, as shown in Figure 2(b), all consumers search broadly in the benchmark model that provides no recommendation. Only Segment 1 and Segment 2 described above exist when the firm offers recommendations, unlike the 4-segment structure under high search costs ($\Delta c > 0.125$). In this case, the firm will never offer recommendations because a recommendation leads to a loss due to the negative search narrowing effect but no gain from the positive participation drawing effect.

Figure 3 illustrates the trade-off between the participation-drawing effect and the search-narrowing effect at two levels of predictive accuracy ($\alpha = 0.13, 0.16$, both above 0.125). The figures on the left side plot these two contrasting effects, with the solid line representing demand loss due to the (negative) search-narrowing effect and the dotted line representing the demand gain from the (positive) participation-drawing effect. The figures show that the magnitudes of these two effects depend on the firm's predictive accuracy. First, the participation-drawing effect always increases with predictive accuracy (α). More accurate recommendations can improve the value of the search by increasing the probability that consumers find a good match through a narrow search, attracting more consumers to engage in searches (higher $\hat{c}$). The latter effect reaches a ceiling when all consumers participate - leading to a kink poin. Second, the relationship between predictive accuracy α and search-narrowing effect is inverse-U shaped. These results imply that the net benefit of product recommendations can be positive or negative, and the relative magnitudes of two opposing effects depend on search costs and predictive accuracy. We summarize the results on the optimal recommendation strategy of the firm in Proposition 1.

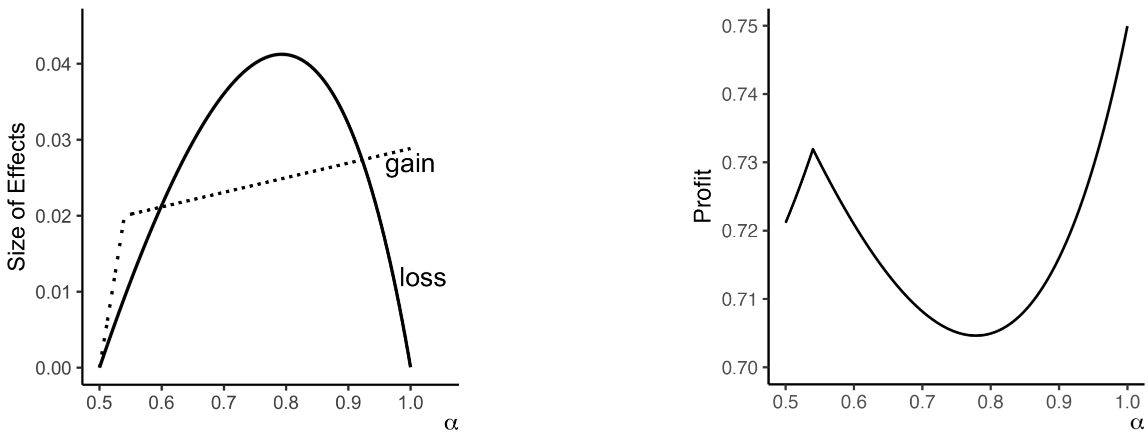
Proposition 1. *(Equilibrium Recommendation Strategy with Fixed Price)*

Under a fixed price given in Lemma 1, the firm's equilibrium recommendation strategy is as follows:

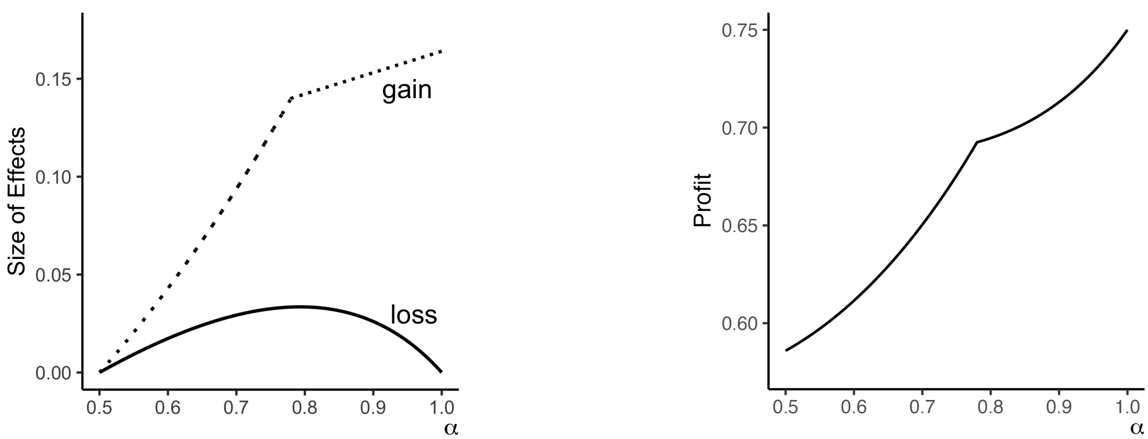
- *When $\Delta c \geqslant 0.133$, it is optimal for the firm to make product recommendations.*

Figure 3: Effects of Recommendation under Fixed Price

(a). $\Delta c = 0.13$



(b). $\Delta c = 0.16$



- *When* $0.125 < \Delta c < 0.133$*, there exist two thresholds* $\alpha_0 < \alpha_1$*, such that the firm will recommend if and only if* $\alpha < \alpha_0$ *or* $\alpha > \alpha_1$.

- *When* $\Delta c \leqslant 0.125$*, it is optimal for the firm not to make recommendations.*

Proof. See the detailed proof in the Appendix □

Proposition 1 indicates that when the search cost range is small ($\Delta c \leqslant 0.125$), the firm will never make recommendations. According to Lemma 1, consumers always search broadly in this case. Thus, the firm will not gain any demand by making recommendations. However, when $\Delta c > 0.125$, only those consumers with $c < 0.25$ would search without a recommendation. In this case, making recommendations, which improves a consumer's search efficiency, can become profitable. However, as implied by the early discussions, this decision is not trivial. Although there is a positive participation-drawing effect on those consumers with high search costs, we expect a negative search-narrowing effect on the consumers with low search costs. The firm's optimal decision on product recommendations depends on the relative magnitude of these two effects as illustrated in Figure 3. Proposition 1 shows that when the range of search costs is large ($\Delta c \geqslant 0.133$), as in Figure 3 (b), the positive effect dominates and it is optimal for the firm to make product recommendations.

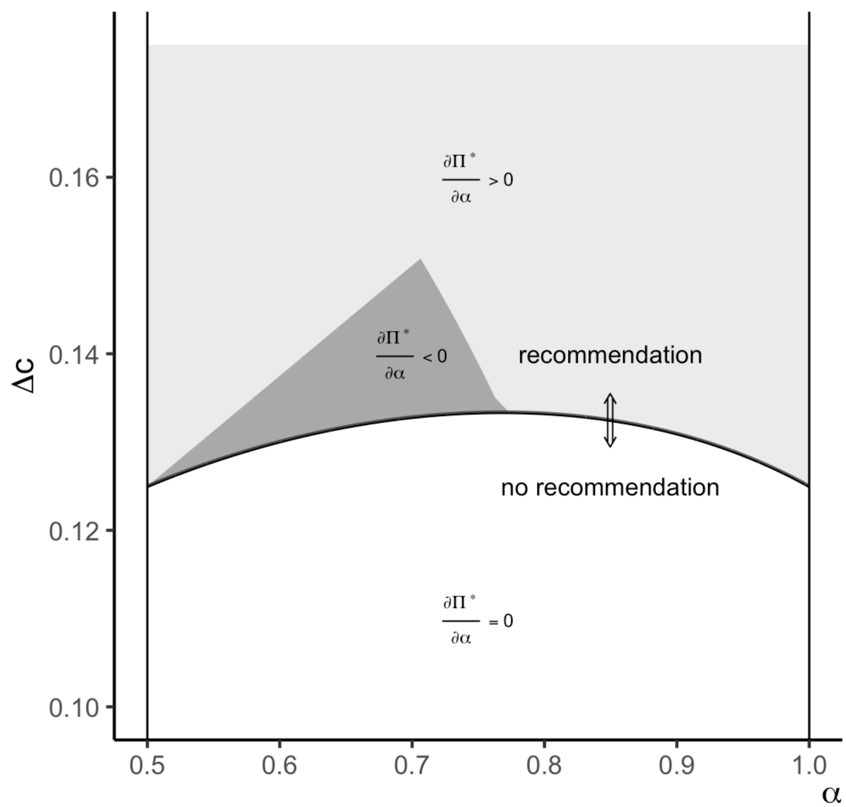
Interestingly, in the markets with intermediate ranges of search costs ($0.125 < \Delta c < 0.133$), a firm may find it profitable to abstain from providing customers with product recommendations when its predictive technology advances in accuracy, from $\alpha < \alpha_0$ to $\alpha_0 < \alpha < \alpha_1$. As depicted in Figure 3 (a), in a market with a relatively small range of search costs ($\Delta c = 0.13$), many consumers would search the firm's products broadly without recommendation. In this case, the firm finds it optimal to provide recommendations when its predictive accuracy is very low (close to 0.5) or very high. However, when the predictive accuracy α increases to an intermediate level, the firm no longer finds it optimal to provide recommendations.[8] Following early discussions, this result arises because the loss due to the negative search-narrowing effect increases to a level that dominates the gain from the positive participation-drawing effect. However, this surprising effect does not hold

[8]In a more realistic setting where the firm has to incur a cost for providing recommendations, the parameter range will shift but the qualitative results should hold.

in markets with a large range of search costs. In these cases, as shown in the right panel of Figure 3 (b), the firm will always find it optimal to provide recommendations.

Next, we examine the effect of recommendation strategy on the firm's equilibrium profits. Given that the firm follows its equilibrium recommendation strategy, Corollary 1 describes how equilibrium profit changes as the firm's predictive accuracy increases.

Figure 4: Firm's recommendation strategy and profit under a fixed price



Corollary 1. *(Effect of Predictive Accuracy on Equilibrium Profit)*

Assume that the firm follows the equilibrium recommendation strategy and the fixed price.

- *If* $\Delta c \geqslant 0.151$*, the equilibrium profit strictly increases with the predictive accuracy* α *(*$\frac{\partial \Pi^*}{\partial \alpha} > 0$*).*
- *If* $0.125 < \Delta c < 0.151$*, the equilibrium profit is nonmonotonic with predictive accuracy* α*. There exist two thresholds* $\bar{\alpha}_0 = 8\Delta c - 0.5 < \bar{\alpha}_1$ *such that the equilibrium profit increases in* α *(i.e.,* $\frac{\partial \Pi^*}{\partial \alpha} > 0$*) if* $\alpha \leqslant \bar{\alpha}_0$ *or* $\alpha \geqslant \bar{\alpha}_1$*, and decreases in* α *(i.e.,* $\frac{\partial \Pi^*}{\partial \alpha} < 0$*) if* $\bar{\alpha}_0 < \alpha < \bar{\alpha}_1$*.*
- *If* $\Delta c \leqslant 0.125$*, the equilibrium profit does not change with* α*.*

Corollary 1 highlights a non-monotonic relation between a firm's equilibrium profit and the accuracy of its predictive technology. The right panel of Figure 3(a) shows that when $\Delta c = 0.13$, the firm's profit with recommendations increases with α at first, then decreases with α, and finally increases with α again. Consistent with early discussions, when the firm's predictive accuracy falls in the middle range, an improvement in the recommendation accuracy could lead to a profit loss for the firm. Basically, the loss from the negative search-narrowing effect accrues faster than the gain from the positive participation-drawing effect. Figure 4 shows that the firm's equilibrium profit can decrease with predictive accuracy only when Δc and α are both at intermediate levels.

When the range of search costs is sufficiently high, improving the accuracy of recommendations always has a positive effect on the firm's profit. This positive relation is clearly illustrated in Figure 3(b). In a market with high Δc, many consumers do not search and hence do not purchase from the firm in the benchmark case of no recommendation. There is limited concern about losing demand due to the negative search-narrowing effect. At the same time, more accurate recommendations could help the firm attract more consumers to participate in search, enhancing the positive participation-drawing effect.

Next, we discuss the effect of predictive accuracy on consumer surplus. Our analysis indicates that the direction of effect, either positive or negative, depends on the firm's recommendation strategy. Corollary 2 summarizes the equilibrium results.

Corollary 2. *(Effect of Predictive Accuracy on Consumer Surplus)*
Assume that the firm follows the equilibrium recommendation strategy with a fixed price.

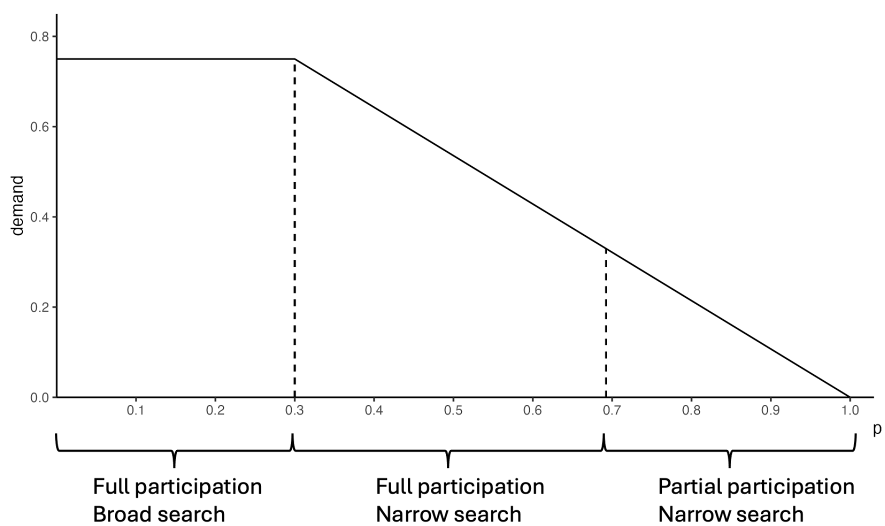
- *If* $\Delta c \geqslant 0.133$, *consumer surplus increases strictly with predictive accuracy.*
- *If* $0.125 < \Delta c < 0.133$, *consumer surplus strictly increases with predictive accuracy when* $\alpha < \alpha_0$ *or* $\alpha > \alpha_1$, *but consumer surplus does not change with predictive accuracy when* $\alpha_0 < \alpha < \alpha_1$.
- *If* $\Delta c \leqslant 0.125$, *consumer surplus does not change with the predictive accuracy.*

The above corollary indicates that when the firm makes product recommendations, a higher predictive accuracy always improves consumer surplus. Recommendation with a higher predictive accuracy not only encourages more consumers to participate in search, but also improves their search efficiency. Naturally, when the company does not make recommendations, consumer surplus is independent of the firm's predictive accuracy. At the margin where a higher predictive accuracy leads the firm to stop making recommendations ($\alpha = \alpha_0$), consumer surplus will drop. At the other end of change ($\alpha = \alpha_1$), consumer surplus jumps when the firm switches to making recommendations.

5 Optimal Recommendation Strategy under Flexible Pricing

In this section, we consider the case in which the company can flexibly adjust the prices in response to advances in its predictive accuracy. We present the results backward, first describing the optimal price and demand before discussing the optimal recommendation strategy.

Figure 5: Piece-wise demand ($\Delta c = 0.1$ and $\alpha = 0.8$)



Optimal Price and Demand

The firm faces a kinked demand function depicted in Figure 5. In general, as the price increases, fewer consumers will participate in product search, and they will search more narrowly. There are two kink points, each corresponding to a change in consumer search behavior. Specifically, when the price exceeds the first kink point, which is represented by $p = 0.3$ in Figure 5, all consumers participate in search activities, but those consumers with the highest search costs (close to $2\Delta c$) change from a broad search to a narrow search. In other words, the demand shifts from the (full participation, broad search) scenario to the (full participation, narrow search) scenario. When the price increases further and exceeds the second kind point ($p = 0.69$ in Figure 5), some consumers change from a narrow search to no search (even for the recommended product). In other words, the demand shifts from the (full participation, narrow search) scenario to the (partial participation, narrow search) scenario. In the remainder of the paper, we will refer to the price at the first kink point by *broad-search binding price* since this price ensures that all consumers to search broadly, and the price at the second kink point by *full-participation binding price* since this price ensures that all consumers search.

The firm's optimal price is either a corner solution equal to one of these two kink points or an interior solution in one of two price intervals defined by the kink points. To explain the logic, we discuss each of the three price intervals defined by the two kink points. First, for any price below the first kink point, all consumers search broadly and find a match if exist, making the demand constant. Thus, the broad-search binding price dominates. Second, for prices between two kink points, all consumers search at least one product. As the price increases, more consumers reduce the search intensity from broad to narrow searches. Thus, the firm balances a larger margin from a higher price with a reduced search intensity. We will thus refer to an interior solution within this interval as *optimal search-intensity price*. Finally, as the price increases beyond the second kink point, some consumers stop participating in the search. Those who do participate also search more narrowly. Within this interval, the firm balances a larger margin from a higher price with fewer consumers participating in narrow searches. We refer to an interior solution within this interval as *optimal search-participation price*. These two interior solutions reflect different tensions that the firm can face in determining its price. Determining the optimal price requires the firm to compare the expected profits corresponding to each candidate solution.

Our analysis indicates that which of the four candidate solutions is optimal depends on the search cost Δc and the predictive accuracy α. Proposition 2 characterizes the firm's optimal pricing strategy when the firm chooses to make recommendations.

Proposition 2. *(Optimal Flexible Pricing under Recommendations)*

Suppose that the firm provides product recommendations. The optimal price is described in the table below:

Search costs	Predictive accuracy	Optimal price	Price type
$\Delta c \leqslant 0.0625$	$\alpha \leqslant \hat{\alpha}_0$	$1 - 2\Delta c \cdot \frac{(1-\alpha)+0.5}{(1-\alpha)}$	broad-search binding price
	$\hat{\alpha}_0 < \alpha < \hat{\alpha}_1$	$\frac{1}{2} + \frac{[\alpha+0.5]\cdot[(1-\alpha)+0.5]}{(1-\alpha)^2} \cdot \Delta c$	optimal search-intensity price
	$\alpha \geqslant \hat{\alpha}_1$	$1 - \frac{4\Delta c}{\alpha+0.5}$	full-participation binding price
$0.0625 < \Delta c < 0.125$	$0.5 \leqslant \alpha \leqslant 1$	$1 - \frac{4\Delta c}{\alpha+0.5}$	full-participation binding price
$\Delta c \geqslant 0.125$	$\alpha \leqslant 8 * \Delta c - 0.5$	0.5	optimal search-participation price
	$\alpha > 8 * \Delta c - 0.5$	$1 - \frac{4\Delta c}{\alpha+0.5}$	full-participation binding price

Proof. See the detailed proof in the appendix. □

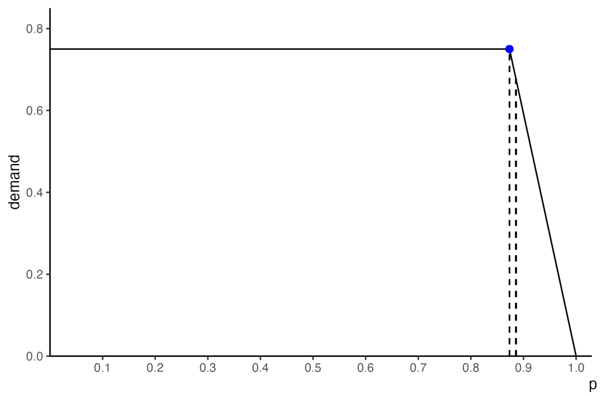
The proposition indicates that in the case of flexible pricing, the firm should adjust its price in response to advances in predictive accuracy. Depending on the values of search cost (Δc) and predictive accuracy (α), the firm can set the price to reach one of the following goals: (i) a *broad-search binding price* ensures that all consumers search broadly, (ii) an *optimal search-intensity price* achieves the proper balance between profit margin and search intensity (broad vs narrow searches), (iii) a *full-participation binding price* ensures that all consumers initiate the search, or (iv) an *optimal search-participation price* achieves the proper balance between profit margin and number of non-searchers. Given all else equal, when the range of search costs (from zero) is large, some consumers search narrowly or even do not search at all. The focus of the firm's pricing goal shifts to encourage search participation; thus, the optimal price is either a full-participation binding price or an optimal search-participation price. On the other hand, when the search cost range is small, all consumers search, and thus the focus of the pricing is to manage the proportion of consumers who search broadly.

We illustrate four types of pricing strategies with Figure 6, where a blue dot represents the optimal price, and two dotted lines indicate binding prices corresponding to two kink points. Panels (a) and (b) show two examples for a small range of search costs ($\Delta c = 0.03 < 0.0625$). In the case of low predictive accuracy ($\alpha = 0.55$ in Figure 6(a)), the firm charges a price that is low enough

Figure 6: Optimal price with product recommendations

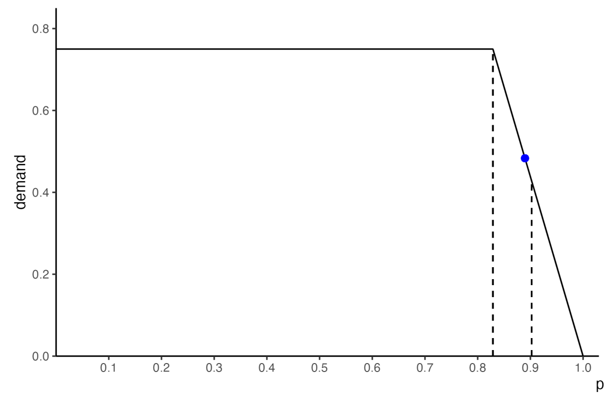
(a). broad-search binding price

($\Delta c = 0.03, \alpha = 0.55$)



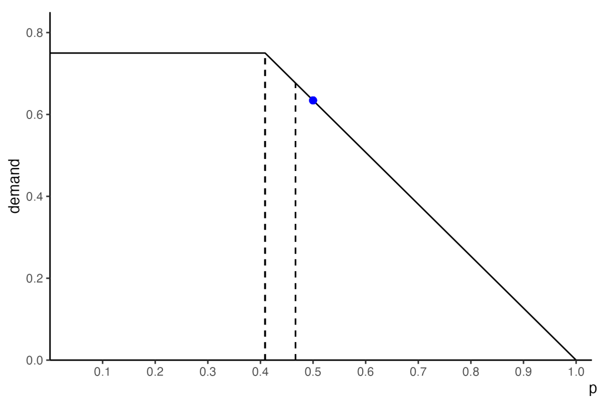
(b). optimal search-intensity price

($\Delta c = 0.03, \alpha = 0.73$)



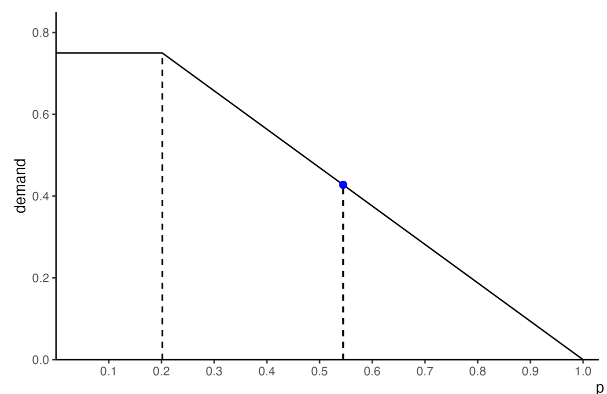
(c). optimal search-participation price

($\Delta c = 0.14, \alpha = 0.55$)



(d). full-participation binding price

($\Delta c = 0.14, \alpha = 0.73$)



Note: The dashed lines divide prices based on their corresponding consumer search behavior: full participation and broad search (left side), full participation and narrow search (middle range), partial participation and narrow search (right side). The blue dot indicates the optimal price under product recommendation.

to ensure that all consumers search broadly. An increase above this broad-search binding price would lead some consumers to search narrowly and hence to a significant drop in demand. In the case of high predictive accuracy ($\alpha = 0.73$ in Figure 6(b)), the firm charges a price that achieves the optimal level of search intensity. When the range of search costs is large ($\Delta c \geqslant 0.125$), the firm focuses on managing the degree of search participation. In the case of low predictive accuracy ($\alpha \leqslant 8 * \Delta c - 0.5$), the firm charges a price to achieve the optimal level of search participation. Figure 6(c) shows such a price at $\alpha = 0.55$. In this case, the equilibrium price remains constant at $p = 0.5$ and is independent of predictive accuracy due to the constant price elasticity. However, in case of high predictive accuracy ($\alpha = 8 * \Delta c - 0.5$), the firm charges a full-participation binding price, as depicted in Figure 6(d).

Some properties of the equilibrium price are worth noting. First, whether equilibrium price and predictive accuracy have a positive or negative relation depends on the type of pricing strategy. To illustrate, consider the case where the range of search costs is small, as shown in Figure 7 for $\Delta c = 0.04$. The firm charges a broad-search binding price for low predictive accuracy. Within this interval of predictive accuracy, the equilibrium price decreases as the firm's predictive accuracy improves because consumers become less willing to search broadly. However, when recommendations are more accurate, it is no longer optimal for the firm to use a low price to ensure a broad search among all consumers. Instead, it is better for the firm to let consumers with high search costs search narrowly. The equilibrium price then increases with predictive accuracy. The positive relation remains when predictive accuracy further increases and the firm switches to full-participation binding price. The flip in the direction of the relationship will be important for the optimal recommendation decision.

Second, when the search cost range is at the intermediate level ($0.0625 < \Delta c < 0.125$), the optimal price is binding for full participation at all values of α. As shown in Figure 8, the equilibrium price increases with predictive accuracy. More precise recommendations improve the probability that consumers find a good match. Consequently, the firm can raise its prices while still ensuring full participation in the search. On the other hand, the equilibrium full-participation binding price decreases with the range of search costs. A larger Δc requires the firm to further reduce its price to ensure that consumers at $2\Delta c$ participate in the search. Finally, the equilibrium price increases at a faster rate in α when Δc is higher, that is, $\frac{\partial^2 p}{\partial \alpha \partial \Delta c} > 0$. As depicted in Figure 8, the equilibrium price increases faster at $\Delta c = 0.12$ than at $\Delta c = 0.07$. Intuitively, with a larger Δc, the firm must provide a higher search value to ensure full participation in the search. Since the marginal consumer searches narrowly, the improved search value following more precise recommendations is proportional to the value of finding a good match $1 - p$. Thus, the marginal effect of increased predictive accuracy on the full-participation binding price is greater with larger Δ.

In summary, the firm's pricing decision in the case of product recommendations is not straightforward. The optimal price can increase or decrease with predictive accuracy α. On the one hand, the firm may raise its price to extract the improved consumer surplus due to more accurate recommendations. This result is particularly evident when the full-participation binding price is optimal. On the other hand, the firm may need to lower its prices to motivate consumers to search broadly. This is because more accurate predictions can reduce consumers' search breadth, and the firm drops the price to counteract such a reduction. The latter effect dominates in a market where many consumers search broadly, and recommendations induce little surplus, characterized by low average search costs and low predictive accuracy. These results underscore strategic roles a firm's prices can play in managing consumer search behavior and how such strategic roles evolve as the firm's predictive technology improves.

Figure 7: Price and demand when $\Delta c = 0.04$

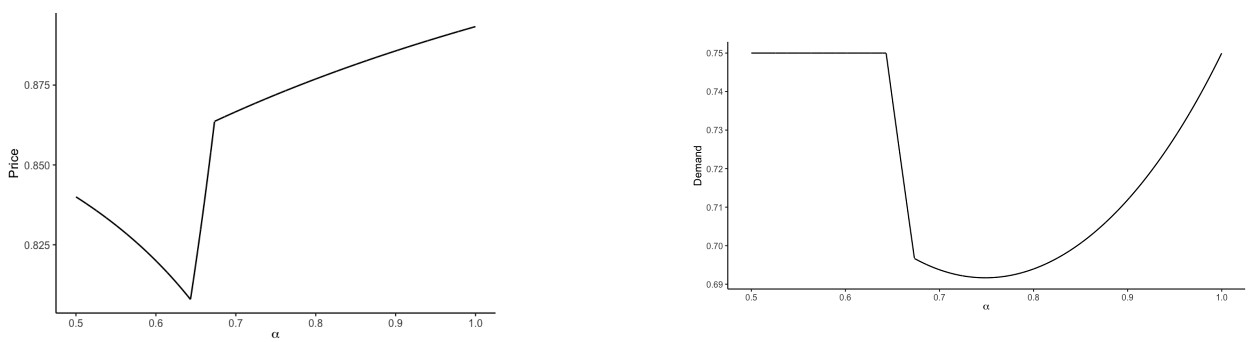
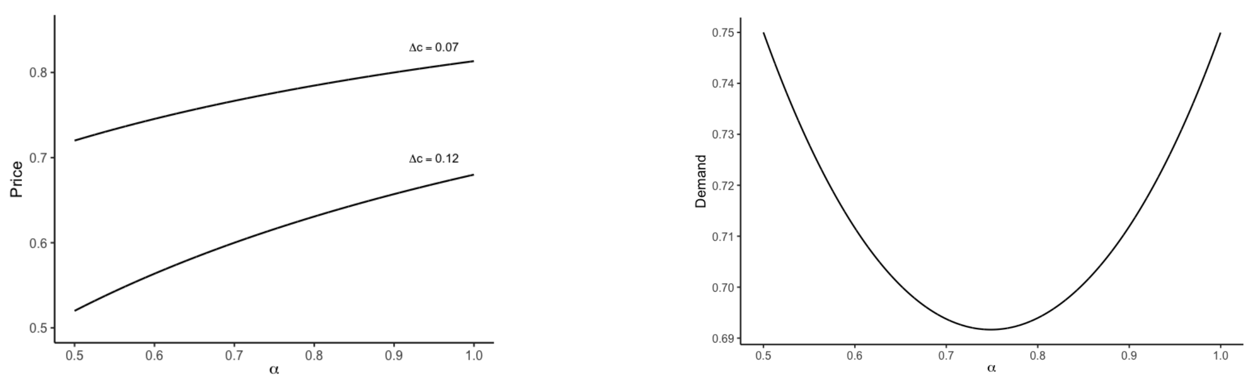


Figure 8: Price and demand when $\Delta c = 0.07$ and $\Delta c = 0.12$



Optimal Recommendation Strategy

We now discuss the firm's recommendation strategy under flexible pricing. We summarize the results on equilibrium recommendation strategies in Proposition 3.

Proposition 3. *(Equilibrium Recommendation Strategy with Flexible Price)*

In the case of flexible pricing, the firm's equilibrium recommendation strategy is as follows:

- *When $\Delta c \geqslant 0.1$, it is always optimal for the firm to recommend.*
- *When $\Delta c < 0.1$, there exists a threshold $\bar{\bar{\alpha}}_1$ that the firm will make recommendations if and only if $\alpha > \bar{\bar{\alpha}}_1$.*

Proof. See the detailed proof in the Appendix □

Proposition 3 indicates that with flexible pricing, the firm is more likely to offer recommendations than the case with fixed price. As depicted in Figure 9, the solid line defines the boundary above which the firm with flexible pricing should make recommendations. Compared to the results with a fixed price (the dotted line), flexible pricing improves the profitability of recommendations in cases where the predictive accuracy α is high and/or the range of search costs Δc is intermediate. The result implies that a firm's pricing rigidity can inhibit product recommendations and constrain a firm's capability to leverage the advances in predictive technology.

To further understand why pricing flexibility is a critical capability to realize the profitability of recommendation strategies, we examine the firm's pricing strategies in the parameter spaces where flexible pricing alters the firm's recommendation decision. We find that the primary driver for the change is the firm's improved ability to appropriate the values created by the recommendations. Consider the case where $0.125 < \Delta c < 0.133$, Proposition 1 does not suggest recommendations if predictive accuracy falls in $\alpha_0 < \alpha < \alpha_1$. With flexible pricing, the equilibrium price is a full-participation binding price equal to $1 - \frac{4\Delta c}{\alpha + 0.5}$. The flexible price is higher than the fixed price 0.5 and increases with α. Thus, while ensuring that all consumers search, the firm increases the profit margin when making recommendations, and more so as its predictive accuracy improves. As illustrated in Figure 10(a), at $\Delta c = 0.13$, the increased margin improves the participation-drawing effect so much that it dominates the negative search-narrowing effect of recommendations. Moreover, the positive impact is greater when α increases. These results significantly alter the patterns under a fixed price shown in Figure 3(a).

When Δc is low ($\Delta c < 0.1$), it is optimal for the firm to make recommendations if and only if α is sufficiently large. If α is small, recall that under inflexible pricing, the price is $1 - 4\Delta c$, the firm does not recommend products because of the resulting negative search-narrowing effect, and all consumers search broadly. Under flexible pricing, the firm focuses on the search intensity (the mix of broad and narrow searches). In this case, the firm may charge the broad-search binding price, which is lower than $1 - 4\Delta c$ and decreases in predictive accuracy because the increased accuracy further reduces search intensity. Thus, it is better for the firm not to make recommendations. However, the firm's flexible pricing shifts the focus to search participation when α is large. As discussed earlier, the shift occurs because the difference in matching probabilities between broad and narrow searches becomes quite small as α is sufficiently large. The optimal price is the full-participation binding price, $1 - \frac{4\Delta c}{\alpha+0.5}$, which is higher than $1 - 4\Delta c$ and increases with α. The higher price allows the firm to extract more search values created for consumers through accurate recommendations. As a result, offering recommendations becomes profitable. We illustrate the above discussion with Figure 10 (b). In essence, it is not just endogenous pricing, but also the corresponding customer management strategies that are critical in enhancing the profitability of product recommendation programs.

Next, we discuss how consumer surplus changes as predictive accuracy improves under flexible pricing.

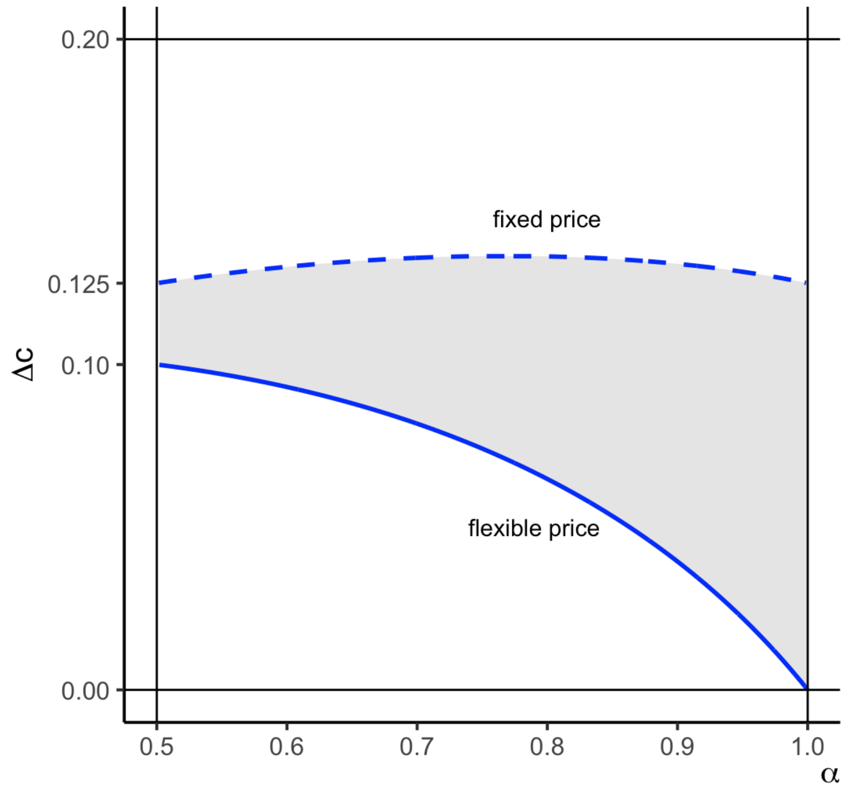
Corollary 3. *(Effects of Flexible Pricing on Consumer Surplus)*
Under flexible pricing,

- *when* $\Delta c \geqslant 0.125$, *consumer surplus is non-monotonic in predictive accuracy. There exists a threshold* $\bar{\alpha}_0 = 8\Delta c - 0.5$ *such that consumer surplus first increases in* α, *that is,* $\frac{\partial CS}{\partial \alpha} > 0$ *when* $\alpha \leqslant \bar{\alpha}_0$, *then strictly decreases in* α, *that is,* $\frac{\partial CS}{\partial \alpha} < 0$ *when* $\alpha > \bar{\alpha}_0$.
- *when* $\Delta c < 0.125$, *consumer surplus is decreasing in* α, *i.e.,* $\frac{\partial CS}{\partial \alpha} \leqslant 0$.

Proof. See the detailed proof in Appendix □

Proposition 3 finds that consumer surplus decreases with predictive accuracy if and only if the firm recommends products and the market size is saturated–meaning that more precise recommendations do not attract additional consumers to search from the firm. Unlike scenarios with fixed pricing, where more precise recommendations always improve consumer surplus, consumer

Figure 9: Firm's recommendation strategy



surplus with flexible pricing declines with increased accuracy as the firm raises its price in response to more precise recommendations. However, when Δc is large and α is low, consumer surplus actually increases with accuracy α. This is because in this region, more precise recommendations expand consumer traffic while the equilibrium price remains unchanged.

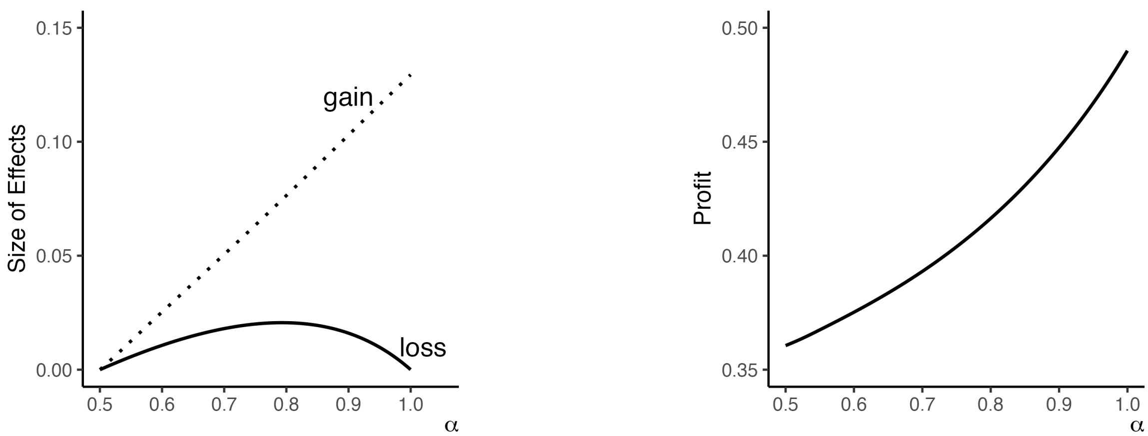
6 Consumer Uncertainty in Predictive Accuracy

So far we have analyzed models in which consumers have perfect information about the accuracy of the firm's predictive technology. In this section, we relax this assumption by allowing consumers to be uncertain about the firm's predictive accuracy. This feature introduces complexity to the model as consumers update their belief about the firm's predictive accuracy within their search process. We study the model under inflexible pricing to focus on the effects of consumers' uncertainty.

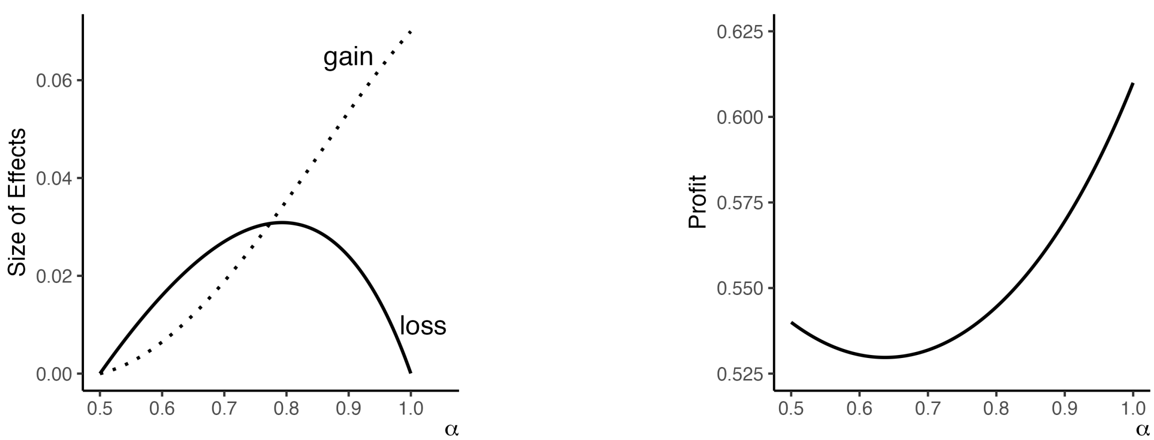
Suppose that the firm draws a predictive accuracy α from the uniform distribution $U[0.5, 1]$ before

Figure 10: Effects of Recommendation under Flexible Price

(a). $\Delta c = 0.13$



(b). $\Delta c = 0.07$



the firm makes recommendation decisions. The market has a continuum of consumers of size 1, of which k proportion of consumers have perfect information about the firm's predictive accuracy ("expert consumers"), and $1-k$ consumers are uncertain about the firm's predictive accuracy ("regular consumers"). Regular consumers, whose prior belief about the firm's predictive accuracy is the same uniform distribution $U[0.5, 1]$, update their beliefs based on the firm's recommendation decision and realized match values. The rest of the model setup is the same as in the main model. We use the perfect Bayesian equilibrium solution concept.

Consumer's Belief Updating

Expert consumers update their beliefs about the match values of the products as in Section 3.2. Regular consumers update their belief about the firm's predictive accuracy as well as the products' match value. After observing the firm's recommendation state $\theta \in \{\theta^1,\ \theta^0\}$, a regular consumer updates her belief about α. The belief updating combines the prior about the firm's predictive accuracy and its recommendation strategy:

$$h(\alpha|\theta) = \frac{h(\alpha)\cdot Pr(\theta|\alpha)}{\int_{0.5}^{1} h(\alpha)\cdot Pr(\theta|\alpha)d\alpha} \tag{9}$$

where $h(\alpha|\theta)$ is the conditional density function of the firm's predictive accuracy (α) given the recommendation state (θ). The term $h(\alpha)$ is the unconditional density of the firm's predictive accuracy (α), i.e., $h(\alpha) = 2$. $Pr(\theta|\alpha)$ is the probability distribution over recommendation state at (α), which is determined by the firm's recommendation strategy $(\sigma(\alpha))$.

Consumers' belief about the products following recommendation now depends on the conditional distribution of firm's predictive accuracy. Her belief about the recommended product is:

$$\begin{aligned}
&Pr(m_j = 1|(s_j = better, s_{-j} = worse), \theta^1) \\
&= \int_{0.5}^{1} Pr(m_{-j} = 1|m_j = 0, (s_j = better, s_{-j} = worse), \alpha)\cdot h(\alpha|\theta^1)d\alpha \\
&= \int_{0.5}^{1} \frac{0.5+\alpha}{2}\cdot h(\alpha|\theta^1)d\alpha \\
&> 0.5 = Pr(m_j = 1|(s_j = better, s_{-j} = worse), \theta^0)
\end{aligned} \tag{10}$$

Her belief about the other product following a bad match is:

$$
\begin{aligned}
& Pr(m_{-j}=1|m_j=0,(s_j=better,s_{-j}=worse),\theta^1) \\
=& \frac{\int_{0.5}^{1} Pr(m_{-j}=1,m_j=0,(s_j=better,s_{-j}=worse),\alpha)\cdot h(\alpha|\theta^1)d\alpha}{\int_{0.5}^{1} Pr(m_j=0,(s_j=better,s_{-j}=worse),\alpha)\cdot h(\alpha|\theta^1)d\alpha} \\
=& \frac{\int_{0.5}^{1} 0.5*(1-\alpha)h(\alpha|\theta^1)d\alpha}{\int_{0.5}^{1} 0.25+0.5*(1-\alpha)h(\alpha|\theta^1)d\alpha} \\
<& \, 0.5 = Pr(m_{-j}=1|m_j=0,(s_j=better,s_{-j}=worse),\theta^0)
\end{aligned}
\tag{11}
$$

Unlike the perfect information case, the consumer here takes into account all firm types who could send out personalized recommendation to update her belief. However, if a firm does not send recommendation, the consumers correctly perceive there is no information.

We first consider an extreme case in which all consumers in the market are uncertain about the precidion of the firm's prediction. In this case, consumers' search behavior is defined by two threshold in search cost. A consumer choose to participate in search at recommendation state θ if and only if the search cost is below threshold $\hat{c}(\theta,p)$:

$$\hat{c}(\theta,p) = Pr(m_j=1|(s_j=better,s_{-j}=worse),\theta)\cdot(1-p)$$

Conditional on an initial bad match, a consumer chooses to search on the other product if and only if the search cost is below threshold $c(\theta,p)$:

$$c(\theta,p) = Pr(m_{-j}=1|m_j=0,(s_j=better,s_{-j}=worse),\theta)\cdot(1-p)$$

Considering consumers' belief updating and search decisions, the firm optimizes its recommendation decision. The firm's tradeoff is similar to the one under fixed price: on the one hand, the recommendation can expand the demand by having more consumers participate and find a match in the initial search. On the other hand, having recommendations reduces the search intensity beyond the recommended product. The following proposition characterizes the firm's optimal recommendation strategy.

Proposition 4. *(Equilibrium Recommendation Strategy with $k=0$)*

When all consumers are uncertain about the firm's predictive accuracy, there exists a unique Perfect Bayesian Equilibrium in which

- *when* $\Delta c \geqslant 0.1458$*, it is always optimal for the firm to recommend.*
- *when* $0.125 < \Delta c < 0.1458$*, there exists a threshold* $\alpha^{uncertainty}$*, the firm will recommend if and only if* $\alpha > \alpha^{uncertainty}$*.*
- *when* $\Delta c \leqslant 0.125$*, it is always optimal for the firm not to recommend.*

Proof. See the detailed proof in the Appendix □

Proposition 4 shows that the firm sometimes still find it optimal to not recommend when consumers are uncertain about the firm's predictive accuracy. At intermediate level of precision, only firms whose precision above a threshold will recommend on equilibrium. Next, Proposition 5 studies the firm's recommendation strategy when most of the consumers know the firm's predictive accuracy, i.e., k is close to 1.

Proposition 5. *(Equilibrium Recommendation Strategy with Large k)*
When k is sufficiently large, there exists a unique Perfect Bayesian Equilibrium in which

- *when* $\Delta c \geqslant 0.1458$*, it is always optimal for the firm to recommend.*
- *when* $0.133 < \Delta c < 0.1458$*, there exists a threshold* $\bar{\alpha}^{uncertainty}$*, the firm will recommend if and only if* $\alpha > \bar{\alpha}^{uncertainty}$*.*
- *when* $0.125 < \Delta c \leqslant 0.133$*, there exists thresholds,* $\bar{\alpha}_1^{uncertainty} < \bar{\alpha}_2^{uncertainty} < \bar{\alpha}_2^{uncertainty}$*, the firm will recommend if and only if* $\bar{\alpha}_1^{uncertainty} < \alpha < \bar{\alpha}_2^{uncertainty}$ *or* $\alpha > \bar{\alpha}_3^{uncertainty}$*.*
- *when* $\Delta c \leqslant 0.125$*, it is always optimal for the firm not to recommend.*

Proof. See the detailed proof in the Appendix □

When $0.133 < \Delta c < 0.1458$, the firm will recommend when all consumers have perfect information about α, but will not recommend when the firm's precision α is low and some consumers are uncertain about the α. This is because consumers who are uncertain about precision will form belief about precision based on recommendation decision and revelation of product match value. Therefore, if the firm with low precision recommend, they will be pooled with other firms with higher precision who also recommend. On one hand, it can benefit the firm with low precision by improving demand at the extensive margin. On the other hand, it hurts the low-precision firm by reducing demand at intensive margin. When Δc is not sufficiently large, $0.133 < \Delta c < 0.1458$, the latter effect dominates, making recommendation not optimal for low-precision firm. On the contrary, the former effect dominates when the market has sufficient consumers who do not search from the firm without recommendation, i.e., $\Delta c > 0.1458$. As a result, firm always find it optimal to recommend.

7 Discussion and Concluding Remarks

This paper investigates the impact of technology-driven personalized recommendations on consumer search behavior. Firms leverage customer data and predictive technologies to predict consumer preferences, enabling them to make personalized recommendations that direct consumers toward products better aligned with their needs. We study how consumers infer the match values of a firm's products based on the accuracy of the firm's predictive technology and how such inferences may influence their within-firm search behavior. Additionally, we examine how advancements in predictive technology shape the firm's recommendation strategies.

Our analysis reveals that personalized recommendations influence consumer search in nuanced and non-trivial ways. While recommendations can reduce search frictions and encourage more consumers to participate in the search (the "participation-drawing effect"), they can also narrow the breadth of consumer search by discouraging exploration of non-recommended products (the "search-narrowing effect"). The interplay between these opposing effects creates complex implications for firms' optimal recommendation strategies. For instance, in firms with inflexible pricing policies, it may be optimal for the firm to forgo personalized recommendations, even when such recommendations are socially efficient. This occurs when the search-narrowing effect is strong - either in markets with intermediate predictive accuracy or in markets where consumers already engage in broad searches without recommendations (characterized by a high concentration on low search cost). Surprisingly, when firms prefer to provide personalized recommendations, their expected profits may decrease as predictive technology becomes more precise, highlighting the intricate trade-offs involved.

When firms have pricing flexibility, they can better capture the values created by personalized recommendations, but only under specific conditions. In markets with intermediate search costs or high predictive accuracy, firms can raise prices sufficiently to offset losses from the search-narrowing effects. However, in markets with low search costs and very low predictive accuracy, firms may need to lower prices to maintain demand, rendering recommendations suboptimal. Even when firms adjust prices in response to recommendations, surplus extraction may remain inefficient, particularly when average search costs or predictive accuracy are not sufficiently low,

Our findings underscore the importance of understanding the dual effects of personalized recommendations on consumer search and how these effects evolve with advancements in predictive technology. Firms must carefully evaluate their consumer base, distinguishing between high-search-cost consumers (who are more likely to increase purchases following recommendations) and low-search-cost consumers (who may reduce their search breadth). When high-search-cost consumers dominate, firms can confidently implement personalized recommendations across all levels of predictive accuracy. However, when a significant segment of consumers has lower search costs, firms must strategically balance the impact of recommendations across different consumer segments. Our paper provides a framework for determining firms' optimal recommendation strategies based on predictive accuracy and consumer composition.

Furthermore, our results stress the critical role of pricing flexibility in leveraging predictive technologies effectively. As firms enhance their recommendation algorithm or predictive capabilities, they must adapt their prices to capture the improved consumer surplus. This flexibility enables firms to shift the strategic role of recommendations from managing search breadth to managing search participation, fostering a more proactive approach to developing predictive technologies. However, the firm must also recognize the limitations of price flexibility, particularly in markets with low search costs and low predictive accuracy. Decisions regarding personalized recommendations should not be driven solely by the potential for higher prices or profit margins. Instead, firms must assess whether the efficiency of surplus extraction can compensate the demand loss caused by the search-narrowing effect. Firms should aim to harness the full potential of predictive technologies while mitigating unintended consequences on consumer search behavior.

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Appendix

Proof for Proposition 1

Proof. At $\alpha = 0.5$, all consumers who participate in search will also continue search when the first product is a bad match. This is because those two products are i.i.d.

Case 1: $\Delta c < 0.125$

When $\Delta c < 0.125$, the exogenous price is making the consumer with the highest search cost indifferent between search and no search. All consumers will participate in search and are willing to continue search. Having recommendation will not improve the extensive margin given all consumers search without recommendation, but recommendation will negatively affect the intensive margin–less consumers are willing to continue search. As a result, the firm always find it optimal not to recommend.

Case 2: $\Delta c > 0.125$

When $\Delta c > 0.125$, the exogenous price is an interior solution 0.5, and not all consumers participate in search when there is no recommendation($\alpha = 0.5$). When $\alpha < \alpha_0 = 8\Delta c - 0.5$, not all consumers participate in search under recommendation with precision α. Given a recommendation with precision α, the gain is

$$
\begin{aligned}
gain(\alpha, 0.5) &= \underbrace{(0.25 + \alpha * 0.5)}_{\text{demand from a consumer in segment 3}} * \underbrace{\left[\frac{\hat{c}(\alpha, p = 0.5)}{2\Delta c} - \frac{c(0.5, p = 0.5)}{2\Delta c}\right]}_{\text{size of segment 3}} \\
&= (0.25 + \alpha * 0.5) * \frac{0.5}{2\Delta c} * \left[\frac{\alpha + 0.5}{2} - \frac{1 - \alpha}{1 - \alpha + 0.5}\right]
\end{aligned}
$$

which is increasing and convex in α

$$
\frac{\partial gain(\alpha, 0.5)}{\partial \alpha} = \frac{0.25}{2\Delta c} * \left[\frac{\alpha + 0.5}{2} - \frac{1 - \alpha}{1 - \alpha + 0.5}\right] + (0.25 + \alpha * 0.5) * \frac{0.5}{2\Delta c} * \left(0.5 + \frac{0.5}{[1 - \alpha + 0.5]^2}\right) > 0
$$

$$
\frac{\partial^2 gain(\alpha, 0.5)}{\partial^2 \alpha} = \frac{0.5}{2\Delta c} * \left(0.5 + \frac{0.5}{[1 - \alpha + 0.5]^2}\right) + (0.25 + \alpha * 0.5) * \frac{0.5}{2\Delta c} * \frac{1}{[1 - \alpha + 0.5]^3} > 0
$$

therefore, we have

$$\frac{\partial gain(\alpha,0.5)}{\partial\alpha} \geqslant \frac{\partial gain(0.5,0.5)}{\partial\alpha} = \frac{3}{16*\Delta c}$$

Given a recommendation with precision α, the loss is

$$loss(\alpha,0.5) = \underbrace{(1-\alpha)*0.5}_{\text{reduced demand for a consumer in segment 2}} * \underbrace{\frac{c(0.5,p=0.5)-c(\alpha,0.5)}{2*\Delta c}}_{\text{size of segment 2}}$$
$$= (1-\alpha)*0.5*\frac{1-p}{2\Delta c}*\left(0.5-\frac{1-\alpha}{1-\alpha+0.5}\right)$$

which is inverse U-shaped and concave in α

$$\frac{\partial loss(\alpha,0.5)}{\partial\alpha} = -0.5*\frac{1-p}{2\Delta c}*\left(0.5-\frac{1-\alpha}{1-\alpha+0.5}\right)+(1-\alpha)*0.5*\frac{1-p}{2\Delta c}*\frac{0.5}{[1-\alpha+0.5]^2}$$
$$\frac{\partial^2 loss(\alpha,0.5)}{\partial^2\alpha} = -\frac{1-p}{2\Delta c}*\frac{0.5}{[1-\alpha+0.5]^2}+(1-\alpha)*0.5*\frac{1-p}{2\Delta c}*\frac{1}{[1-\alpha+0.5]^3}$$
$$= -(1-\frac{1-\alpha}{1-\alpha+0.5})*\frac{1-p}{2\Delta c}*\frac{0.5}{[1-\alpha+0.5]^2} < 0$$

therefore, we have

$$\frac{\partial loss(\alpha,0.5)}{\partial\alpha} < \frac{\partial loss(0.5,0.5)}{\partial\alpha} = \frac{1}{32\Delta c}$$

By combining with the fact that $gain(0.5,0.5) = loss(0.5,0.5) = 0$, we have

$$gain(\alpha,0.5) > loss(\alpha,0.5)\ \forall\alpha < \alpha_0 = 8\Delta c - 0.5$$

which makes the recommendation optimal for the firm, moreover the firm has no incentive to obfuscate the information given $gain(\alpha,0.5) - loss(\alpha,0.5)$ is increasing in α: the size of gain increases at a faster rate than loss, thus the net gain from recommendation is increasing in α.

When $\alpha > \alpha_0$, the loss from recommendation remain the same, except the gain changes because the size of consumer participating in search no longer increase with α. The gain is:

$$gain(\alpha,0.5) = \underbrace{(0.25+\alpha*0.5)}_{\text{demand from a consumer in segment 3}} * \underbrace{\left[1-\frac{c(0.5,p=0.5)}{2\Delta c}\right]}_{\text{size of segment 3}}$$
$$= (0.25+\alpha*0.5)*\left[1-\frac{0.5}{2\Delta c}*\frac{1-0.5}{1-0.5+0.5}\right]$$

Since the gain is linear in α and loss is concave in α, the firm always benefits from higher precision α if

$$\frac{\partial gain(\alpha_0, 0.5)}{\partial \alpha} \geqslant \frac{\partial loss(\alpha_0, 0.5)}{\partial \alpha}$$

For market with higher Δc, the market composition has more consumers from segment 3 and less consumers from segment 2, thus the firm's loss increases at a slower speed as α increases, whereas the firm's gain increases at a faster speed as α increases:

$$\begin{aligned}
&\frac{\partial^2 gain(\alpha_0, 0.5)}{\partial \alpha \partial \Delta c} > 0 \\
&\frac{\partial^2 loss(\alpha_0, 0.5)}{\partial \alpha \partial \Delta c} < 0 \text{ when } \frac{\partial loss(\alpha_0, 0.5)}{\partial \alpha} > 0
\end{aligned}$$

Given a threshold $\Delta c = 0.151$ satisfying

$$\frac{\partial gain(\alpha_0, 0.5)}{\partial \alpha} = \frac{\partial loss(\alpha_0, 0.5)}{\partial \alpha}$$

when combined with the fact that (1). α_0 is increasing in Δc; (2). For market with higher Δc, the firm's loss increases at a slower speed as α increases, whereas the firm's gain increases at a faster speed as α increases;(3). gain is linearly increasing in α, while loss is concave in α we have for any $0.125 < \Delta c < 0.151$;

$$\frac{\partial gain(\alpha_0, 0.5)}{\partial \alpha} < \frac{\partial loss(\alpha_0, 0.5)}{\partial \alpha}$$

therefore the firm's profit from recommendation is non-monotonic in α as α increases from α_0 when $0.125 < \Delta c < 0.151$. As the firm's recommendation makes all consumers to participate in search($\alpha > \alpha_0$), the firm firstly experience a dip in profit from recommendation as α increases from α_0, and eventually increases in α. Moreover, the dip in profit is smaller for larger Δc because the gain is increasing in Δc and loss is decreasing in Δc, i.e., $\frac{\partial gain}{\partial \Delta c} > 0$, $\frac{\partial loss}{\partial \Delta c} < 0$. As depicted in Figure 3, the drop in profit with α after $\alpha > \alpha_0$ is larger when $\Delta c = 0.13$ than $\Delta c = 0.14$. Therefore, we could conclude that there is a threshold in Δc, such that when Δc is below such threshold, the dip is large enough such that there is an intermediate range of precision under which the firm find it optimal not to recommend, i.e., $gain - loss < 0$. Such threshold is $\Delta c = 0.133$. Therefore, the firm's recommendation strategy is:

- when $\Delta c \geqslant 0.133$, it is always optimal for the firm to recommend.
- when $0.125 < \Delta c < 0.133$, there exist two thresholds $\alpha_0 < \alpha_1$, the firm will recommend if and only if $\alpha < \alpha_0$ or $\alpha > \alpha_1$
- when $\Delta c < 0.125$, it is always optimal for the firm not to recommend.

□

Proof for Corollary 1

Proof. From the proof of Proposition 1, we could divide our analysis into three cases.

Case 1: $\Delta c \leqslant 0.125$. The firm will not recommend on equilibrium at all levels of predictive accuracy, therefore its equilibrium profit does not change with α.

Case 2: $\Delta c \geqslant 0.151$. The firm always recommend on equilibrium. Moreover, the gain from recommendation always increases at a faster rate than the loss from recommendation as α improves, i.e., $\frac{\partial gain}{\partial \alpha} \geqslant \frac{\partial loss}{\partial \alpha}$. As a result, the firm's equilibrium profit is always increasing in its precision α.

Case 3: $\Delta c \in (0.125, 0.151)$. The firm will recommend when $\alpha < \alpha_0 = 8\Delta c - 0.5$, moreover the gain from recommendation always increases at a faster rate than loss from recommendation as α improves. Therefore, its equilibrium profit is always increasing in precision α when $\alpha \leqslant \alpha_0 = 8\Delta c - 0.5$. When $\alpha > \alpha_0$, the firm initially experience a drop in equilibrium profit due to $\frac{\partial gain}{\partial \alpha} < \frac{\partial loss}{\partial \alpha}$. In such case, the firm will continue to recommend and have equilibrium profit decreasing in α. As the precision continues improves, the firm sometimes experience significant decrease in profit and even find it optimal to stop recommending. In such case, the firm experience a temporary no change in equilibrium profit with respect to α. As precision becomes sufficiently large, profit from recommendation again surpasses the profit from no recommendation. The firm starts to recommend and equilibrium profit again increasing in precision α. □

Proof for Corollary 2

Proof. When the firm choose not to provide recommendations, the consumer surplus does not change with the firm's precision.

Now we focus on the consumer surplus when firm choose to recommend. Consider the case when the firm's precision increases from α to $\alpha + \Delta\alpha$. If the demand from extensive margin increases, we could divide the demand into three segment and separately analyze the change in consumer surplus for those consumers.

First, for consumers whose search cost $\Delta c \in [\hat{c}(\alpha, p), \hat{c}(\alpha + \Delta\alpha, p)]$, they gain positive consumer surplus gain from participating in search.

Second, for consumers whose search cost $\Delta c \in [c(\alpha, p), \hat{c}(\alpha, p)]$, their search behavior are the same at both levels of precision: they only search for the first product. Their consumer surplus is improved with higher precision because their search cost remain the same, but higher precision helps them match with the correct product with higher probability.

Third, for consumers whose search cost $\Delta c \in [c(\alpha + \Delta\alpha, p), c(\alpha, p)]$, they change their search behavior at two levels of precision: they search both products at precision α and only search recommended product at precision $\alpha + \Delta\alpha$. Their consumer surplus is also improved with higher precision. Because if the consumer surplus to search only recommended product is higher than search both products at precision $\alpha + \Delta\alpha$ by the optimality of consumer search decision. And consumer surplus to search both product is higher at precision $\alpha + \Delta\alpha$ than at precision α due to improved matching efficiency while keeping search cost the same. By transitivity of optimality, we have consumer surplus to search once at $\alpha + \Delta\alpha$ to be higher than consumer surplus to search twice at α.

Lastly, for consumers whose search cost $\Delta c \leqslant c(\alpha + \Delta\alpha, p)$, they always search twice at both levels of precision. Their consumer surplus is improved with higher precision because better matching efficiency while keeping search cost the same.

To summarize, under exogenous price, the consumer surplus is increasing in precision α. When the firm switch from recommend to no recommend, the consumer surplus discountinuously drop due to a discontinuous drop in precision. Vice versa, when the firm switch from no recommendation to recommendation, the consumer surplus discountinuously increase due to a discontinuous improvement in precision. $\square$

Proof for Proposition 2

Proof. At each precision α, the firm could choose price to maximize its profit

$$\max_{p} \; p * \left\{0.75 * \min\left\{\frac{c(\alpha,p)}{2*\Delta c}, 1\right\} + (0.25 + 0.5*\alpha) * \left[\min\left\{\frac{\hat{c}(\alpha,p)}{2*\Delta c}, 1\right\} - \min\left\{\frac{c(\alpha,p)}{2*\Delta c}, 1\right\}\right]\right\}$$

There are four potential solutions for the optimal prices: (1). a corner solution where all consumers search from the firm and the consumer with the highest search cost is indifferent at search from the firm:

$$p^{corner1} = 1 - \frac{4\Delta c}{\alpha + 0.5}$$

(2). a corner solution where all consumers search deeply and the consumer with the highest search cost is indifferent at search deeply and shallow

$$p^{corner2} = 1 - 2\Delta c \cdot \frac{1 - \alpha + 0.5}{1 - \alpha}$$

(3). an interior solution where all consumers search from the firm

$$p^{interior1} = \min\left\{0.5 + \frac{[\alpha + 0.5] \cdot [1 - \alpha + 0.5]}{(1-\alpha)^2} \cdot \Delta c, 1\right\}$$

(4). an interior solution where not all consumers search from the firm

$$p^{interior2} = 0.5$$

Our analysis consists of three cases.

Case 1: $\Delta c > 0.125$. When $\Delta c > 0.125$, optimal price is not $p^{interior1}$ because $p^{interior1} > 1$. If $p^{interior2}$ is not optimal price, the optimal price will be $p^{corner1}$ because the unconstrained optimal price when all consumers participate in search is $p^{interior1} > 1$, leading the optimal price is binding at the upper boundary $p^{corner1}$ when consumers with the highest search cost is indifferent between participate in search or not. Now we discuss when $p^{interior2}$ is optimal price. As

$$0.5 = \arg\max p * \left\{0.75 * \frac{c(\alpha,p)}{2*\Delta c} + (0.25 + 0.5*\alpha) * \left[\frac{\hat{c}(\alpha,p)}{2*\Delta c} - \frac{c(\alpha,p)}{2*\Delta c}\right]\right\}$$

$p^{interior2} = 0.5$ is optimal price if and only if

$$\hat{c}(\alpha, 0.5) = (1-p) \cdot \frac{\alpha + 0.5}{2} \leqslant 2\Delta c \Leftrightarrow \alpha \leqslant 8\Delta c - 0.5$$

Therefore, we have optimal price $p^* = 0.5$ when $\alpha \leqslant 8\Delta c - 0.5$ and $p^* = p^{corner1} = 1 - \frac{4\Delta c}{\alpha + 0.5}$ when $\alpha > 8\Delta c - 0.5$.

Case 2: $\Delta c < 0.125$. When $\Delta c < 0.125$, we have $8\Delta - 0.5 < 0.5$, therefore $p^{interior2} = 0.5$ will not be optimal price. Given $p^{interior1}$ is the unconstrained optimal price when all consumers participate in search. Sometimes, it is bind by $p^{corner1}$ when $p^{interior1} > p^{corner1}$. Sometimes, it is bind by $p^{corner2}$ when $p^{interior1} < p^{corner1}$. To understand when those two conditions are met, we want to point out the following facts:

$$\frac{\partial p^{corner1}}{\partial \alpha} > 0$$
$$\frac{\partial p^{corner2}}{\partial \alpha} < 0$$
$$\frac{\partial p^{interior1}}{\partial \alpha} > 0$$
$$p^{corner1} = p^{corner2} \text{ when } \alpha = 0.5$$
$$p^{interior1} > p^{corner1} > p^{corner2} \text{ when } \alpha = 1$$

When $p^{interior1} < p^{corner1} = p^{corner2}$ when $\alpha = 0.5$, the optimal price are initially $p^{corner2}$, then $p^{interior1}$ and eventually $p^{corner1}$ as precision α increases from 0.5 to 1. When $p^{interior1} > p^{corner1} = p^{corner2}$ when $\alpha = 0.5$, the optimal price is $p^{corner1}$ at all levels of precision α. We find that

$$p^{interior1} < p^{corner1} = p^{corner2} \text{ when } \alpha = 0.5 \Leftrightarrow \Delta c < 0.0625$$

□

Proof for Proposition 3

Proof. **Case 1**: First, we consider the corner solution 1 where the consumer with search cost $2\Delta c$ is indifferent between participate in search. Under such solution, we have

$$p^{interior1} = 1 - \frac{4\Delta c}{\alpha + 0.5}$$
$$D^{interior1} = 0.75 - 0.5 * (1-\alpha) * \left[1 - \frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5}\right]$$

For the ease of exposition, we get rid of super-script for the analysis in this section. How the firm's profit from interior solution 1 changing with precision α is determined by the first order derivative:

$$\begin{aligned}\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha} &= \frac{4\Delta c}{(\alpha+0.5)^2} * D + \left(1 - \frac{4\Delta c}{\alpha+0.5}\right) * \frac{\partial D}{\partial \alpha} \\ &= \frac{\partial D}{\partial \alpha} + \left(\frac{4}{(\alpha+0.5)^2} * D - \frac{4}{\alpha+0.5} * \frac{\partial D}{\partial \alpha}\right) * \Delta c\end{aligned}$$

where

$$\begin{aligned}\frac{\partial D}{\partial \alpha} = 0.5 &\left[1 - \frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5}\right] \\ &- 0.5(1-\alpha) * \frac{2}{(\alpha+0.5)^2} * \frac{1-\alpha}{1-\alpha+0.5} \\ &- 0.5(1-\alpha) * \frac{2}{\alpha+0.5} * \frac{0.5}{(1-\alpha+0.5)^2}\end{aligned}$$

Notice, $\frac{\partial D}{\partial \alpha}$ is increasing in α, and $\frac{\partial D}{\partial \alpha} = -0.5 < 0$ at $\alpha = 0.5$, and $\frac{\partial D}{\partial \alpha} = 0.5 > 0$ at $\alpha = 1$. And if we define $B(\alpha) = \frac{4}{(\alpha+0.5)^2} * D - \frac{4}{\alpha+0.5} * \frac{\partial D}{\partial \alpha}$, we have

$$\begin{aligned}B(\alpha) &= \frac{4}{(\alpha+0.5)^2} * \left[D - (\alpha+0.5) * \frac{\partial D}{\partial \alpha}\right] \\ &= \frac{4}{(\alpha+0.5)^2} * \{0.75 - (0.5(1-\alpha) + 0.5(\alpha+0.5)) * \left[1 - \frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5}\right] \\ &+ 0.5 * (1-\alpha) * \frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5} + \frac{0.5 * (1-\alpha)}{(1-\alpha+0.5)^2}\} \\ &= \frac{4}{(\alpha+0.5)^2} * \left\{\frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5} + 0.5 * (1-\alpha) * \frac{2}{\alpha+0.5} * \frac{1-\alpha}{1-\alpha+0.5} + \frac{0.5 * (1-\alpha)}{(1-\alpha+0.5)^2}\right\}\end{aligned}$$

which is decreasing in α and is positive at all levels of α, given $B(1) = 0$.

By simulation, we have $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha}$ increasing in α at $\Delta c = 0.125$. Therefore, $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha}$ increasing in α for any $\Delta c < 0.125$. Since $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha} > 0$ for $\alpha = 1$, we only need to decide the sign of $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha}$ at $\alpha = 0.5$ to decide whether the profit from interior solution 1 is always increasing in α. $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha} = -0.5 + 5\Delta c$. Therefore, we have $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha}$ is firstly negative and then positive as precision α increases when $\Delta < 0.1$, therefore profit from corner solution 1 is inverse U-shaped in precision when $\Delta c < 0.1$. When $\Delta c \geqslant 0.1$, we have $\frac{\partial p}{\partial \alpha} * D + p * \frac{\partial D}{\partial \alpha} > 0$, because it stands for special case $\Delta c = 0.1$, and the first derivative is increasing in Δc at range when $\frac{\partial D}{\partial \alpha} < 0$, therefore profit from corner solution 1 is increasing in α.

Case 2: we consider the interior solution 1 where

$$p^{interior1} = 0.5 + \frac{(\alpha + 0.5) \cdot (1 - \alpha + 0.5)}{(1 - \alpha)^2} \cdot \Delta c$$

To see how profit from interior solution 1 changes with α, we consider its first order derivative:

$$\frac{\partial \pi^*}{\partial \alpha} = p \cdot \frac{\partial D}{\partial \alpha} = \frac{0.5 \cdot p}{2\Delta c} \cdot \left\{ 2\Delta c - (1 - p) \cdot \left[1 + \frac{0.5}{(1 - \alpha) + 0.5} \right] \cdot \left[1 - \frac{0.5}{(1 - \alpha) + 0.5} \right] \right\}$$

we find the above first order derivative is negative only if α small given $2\Delta c - (1 - p) \cdot \left[1 + \frac{0.5}{(1-\alpha)+0.5} \right] \cdot \left[1 - \frac{0.5}{(1-\alpha)+0.5} \right]$ is increasing in α(given price is increasing in α). By looking at the sign of $\frac{\partial \pi^*}{\partial \alpha}$ at $\alpha = 0.5$, we have: when $\Delta c < 0.075$, profit from interior solution 1 is inverse-U shaped in α. When $\Delta c > 0.075$, profit from interior solution is increasing in α.

Case 3: the profit from corner solution 2 is decreasing in α, given price is dropping and demand remain the same in α.

Case 4: the profit from interior solution 2 is increasing in α given previous analysis for exogenous price.

From proposition 2, when $\Delta c \leqslant 0.0625$, if the firm implement recommendation, the profit firstly decreases in α given the firm firstly implement corner solution 2, then transits to interior solution 1. The profit could either increasing or decreasing in α. Depending on the condition of profit when the firm implements interior solution 1, we divide the analysis into three cases: 1. if the profit continues to decrease before transit to corner solution 1, then profit will move back to the original level under corner solution 1 given demand at $\alpha = 0.1$ is the same as the demand at $\alpha = 0.5$, while the price at $\alpha = 0.5$ is strictly less than the price at $\alpha = 1$, i.e., $1 - 4\Delta c < 1 - \frac{4\Delta c}{1.5}$. Moreover, there exists a threshold in precision $\bar{\alpha}$, such that the firm's profit under recommendation is higher than no recommendation iff $\alpha > \bar{\alpha}$. 2. If the profit when the firm implements interior solution 1 has a range of α where profit increases with α, if the profit does not back to the level of profit at $\alpha = 0.5$ before transit to corner solution 1, the argument is the same as before, the firm's profit will later move back to original level under corner solution 1 given demand at $\alpha = 0.1$ is the same as the demand at $\alpha = 0.5$, while the price at $\alpha = 0.5$ is strictly less than the price at $\alpha = 1$, i.e., $1 - 4\Delta c < 1 - \frac{4\Delta c}{1.5}$. Moreover, there exists a threshold in precision $\bar{\alpha}$, such that the firm's profit under recommendation is higher than no recommendation iff $\alpha > \bar{\alpha}$. 3. If the profit when the firm implements interior solution 1 increases back the level of profit at $\alpha = 0.5$ before transit to corner solution 1, the firm's profit will continue to increases after the firm transit to corner solution 1 given profit from corner solution 1 is inverse U-shaped in α and the profit from corner solution at $\alpha = 0.5$ is the same as the the profit from corner solution 2 at $\alpha = 0.5$. In summary, when $\Delta c \leqslant 0.0625$, there exists a threshold $\bar{\alpha}$, such that the firm find it optimal to recommend if and only if $\alpha > \bar{\alpha}$.

When $0.0625 < \Delta c < 0.1$, the firm only implements corner solution if they recommend. Since we know that the profit from corner solution 1 is inverse U-shaped in precision when $\Delta c < 0.1$, we conclude the firm's recommendation strategy is: there exists a threshold $\bar{\alpha}$, such that the firm will recommend if and only if $\alpha > \bar{\alpha}$.

When $\Delta c \geqslant 0.1$, the firm either implements interior solution 2 or corner solution 1, which both generate positive increasing profit w.r.t. α. Therefore, the firm always find it optimal to recommend.

□

Proof for Proposition 3

Proof. Depending on the equilibrium, we divide the analysis into three cases.

Case 1: $\Delta c > 0.125$ & $\alpha <= \alpha_0 = 8\Delta c - 0.5$. Under such case, the firm always recommend and charge a price at $p^* = 0.5$. This is similar to the exogenous price situation where price does not change to precision α, under which consumer surplus strictly increases in precision α.

Case 2: $\Delta c < 0.1$ & $\alpha < \bar{\bar{\alpha}}_1$. Under such case, the firm does not recommend products on equilibrium. Consumer surplus remain constant.

Case 3: Other parameter range other than Case 1 and Case 2. Under those case, the firm chooses to recommend and charge a price at $p^* = 1 - \frac{4\Delta c}{\alpha+0.5}$. Consumer surplus from consumer who search shallow is:

$$
\begin{aligned}
CS_1(\alpha, p) &= \left(1 - \frac{c(\alpha, p)}{2\Delta c}\right) * [(0.25 + 0.5 * \alpha) * (1 - p) - E(c|c(\alpha, p) \leqslant c \leqslant 2 * \Delta c)] \\
&= \left(1 - \frac{2}{\alpha + 0.5} * \frac{(1-\alpha)}{1 - \alpha + 0.5}\right) * \left[(0.25 + 0.5 * \alpha) * \frac{4 * \Delta c}{\alpha + 0.5} - 0.5 * (2\Delta c + \frac{4\Delta c}{\alpha + 0.5} * \frac{1-\alpha}{1-\alpha+0.5})\right] \\
&= \left(1 - \frac{2}{\alpha + 0.5} * \frac{(1-\alpha)}{1 - \alpha + 0.5}\right) * (\Delta c) * \left[(0.25 + 0.5 * \alpha) * \frac{4}{\alpha + 0.5} - 0.5 * (2 + \frac{4}{\alpha + 0.5} * \frac{1-\alpha}{1-\alpha+0.5})\right] \\
&= \left[1 - \frac{2}{\alpha + 0.5} * \frac{(1-\alpha)}{1 - \alpha + 0.5}\right]^2 * (\Delta c)
\end{aligned}
$$

Consumer surplus from consumers who search deep is:

$$
\begin{aligned}
CS_2(\alpha, p) &= \frac{c(\alpha, p)}{2\Delta c} * [0.75 * (1 - p) - (1 + 0.25 + 0.5 * (1 - \alpha))E(c|c \leqslant c(\alpha, p))] \\
&= \frac{2}{\alpha + 0.5} * \frac{(1-\alpha)}{1-\alpha+0.5} * \left[0.75 * \frac{4 * \Delta c}{\alpha + 0.5} - (1 + 0.25 + 0.5 * (1 - \alpha)) * 0.5 * (\frac{4\Delta c}{\alpha + 0.5} * \frac{1-\alpha}{1-\alpha+0.5})\right] \\
&= \frac{2}{\alpha + 0.5} * \frac{(1-\alpha)}{1-\alpha+0.5} * (\Delta c) \\
&* \left[0.75 * \frac{4}{\alpha + 0.5} - (1 + 0.25 + 0.5 * (1 - \alpha)) * 0.5 * (\frac{4}{\alpha + 0.5} * \frac{1-\alpha}{1-\alpha+0.5})\right]
\end{aligned}
$$

Notice consumers' surplus is proportional to Δc, therefore whether consumer surplus increases or decreases with respect to α under the price $p^* = 1 - \frac{4\Delta c}{\alpha+0.5}$ is independent of Δc. If we simulate the consumer surplus when $\Delta c = 0.11$, we have $\frac{CS}{\partial \alpha} < 0$. Therefore, we could conclude that $\frac{CS}{\partial \alpha} < 0$ for all levels of Δc in Case 3.

Consider a marginal change of α from α_1 to $\alpha_1 + \Delta\alpha$. We could divide the market into three segments. Segment 1 who always search deeply, segment 2 who switch from search deeply to search shallow, and segment 3 who always search shallow. For segment 3 consumer, their consumer surplus remain the same:

$$
\left(1 - \frac{c(\alpha_1, p)}{2\Delta c}\right) * [(0.25 + 0.5 * (\alpha_1 + \Delta\alpha)) * (1 - p(\alpha_1 + \Delta\alpha)) - E(c|c(\alpha_1, p) \leqslant c \leqslant 2 * \Delta c)]
$$

because $(0.25 + 0.5 * (\alpha_1 + \Delta\alpha)) * (1 - p(\alpha_1 + \Delta\alpha)) = (0.25 + 0.5 * (\alpha_1)) * (1 - p(\alpha_1))$

For segment 1 consumers, their consumer surplus at α_1 is:

$$\frac{c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [0.75*(1-p(\alpha_1)) - (1+0.25+0.5*(1-\alpha_1))E(c|c \leqslant c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))]$$

their surplus at $\alpha_1 + \Delta\alpha$ is:

$$\frac{c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [0.75*(1-p(\alpha_1+\Delta\alpha)) - (1+0.25+0.5*(1-\alpha_1-\Delta\alpha))E(c|c \leqslant c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))]$$

where price increased but search efficiency decreased. To take a difference, we have the change in consumer surplus is

$$\begin{aligned}
&\frac{c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [-0.75*(p(\alpha_1+\Delta\alpha)-p(\alpha_1)) + 0.5*\Delta\alpha*E(c|c \leqslant c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))]\\
&= \frac{c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * \left[-0.75*\frac{4\Delta c}{(\alpha_1+0.5)^2}*\Delta\alpha + 0.5*\Delta\alpha*0.5*(\frac{4\Delta c}{\alpha_1+\Delta\alpha+0.5}*\frac{1-\alpha_1-\Delta\alpha}{1-\alpha_1-\Delta\alpha+0.5})\right]\\
&= \frac{c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * \Delta\alpha * \left[\frac{-\alpha_1^2+3.5*\alpha_1-4}{(\alpha_1+0.5)*(1-\alpha_1+0.5)}\right] < 0
\end{aligned}$$

For segment 2 consumers, their change in consumer surplus is

$$\begin{aligned}
&\frac{c(\alpha_1, p(\alpha_1)) - c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [(0.25+0.5*(\alpha_1+\Delta\alpha))*(1-p(\alpha_1+\Delta\alpha)) - 0.75*(1-p(\alpha_1))\\
&+ (0.25+0.5*(1-\alpha_1-\Delta\alpha))*E(c|c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha)) \leqslant c \leqslant c(\alpha_1, p(\alpha_1)))]\\
&\frac{c(\alpha_1, p(\alpha_1)) - c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [-0.5*(1-\alpha_1)*(1-p(\alpha_1)) + (0.25+0.5*(1-\alpha_1))*c(\alpha_1, p(\alpha_1))]\\
&= \frac{c(\alpha_1, p(\alpha_1)) - c(\alpha_1+\Delta\alpha, p(\alpha_1+\Delta\alpha))}{2\Delta c} * [-0.5*(1-\alpha_1)*\frac{4\Delta c}{\alpha+0.5}\\
&+ (0.25+0.5*(1-\alpha_1))*(\frac{4\Delta c}{\alpha_1+0.5}*\frac{1-\alpha_1}{1-\alpha_1+0.5})] = 0
\end{aligned}$$

As a result, the total consumer surplus is decreasing in precision α from segment 1 consumers who always search deeply. □

Proof for Proposition 4

Proof. Consider the market has k consumers who have perfect information and $1-k$ consumers who are complete uncertain about α.

Under complete uncertainty $k = 0$, the firm's demand from recommendation is:

$$
\begin{aligned}
&0.75 * \frac{(1-p) * Pr(m_{-j} = 1|m_j = 0, (s_j = better, s_{-j} = worse), \theta)}{2\Delta c} \\
&+ (0.25 + 0.5 * (1-\alpha)) * [\min\left\{\frac{(1-p) * Pr(m_j = 1)|(s_j = better, s_{-j} = worse), \theta)}{2\Delta c}, 1\right\} \\
&- \frac{(1-p) * Pr(m_{-j} = 1)|m_j = 0, (s_j = better, s_{-j} = worse), \theta)}{2\Delta c}]
\end{aligned}
$$

Notice the demand is increasing in α. Therefore, the firm's optimal recommendation strategy should be defined by a threshold in precision $\bar{\alpha}$, such that the it is optimal. Next, we decide under which Δc, some firms find it optimal to recommend while others do not, i.e., $0.5 < \bar{\alpha} < 1$.

The gain from recommendation at α is:

$$
\begin{aligned}
gain &= \frac{\min\{(1-p) * \int_{\alpha}^{1} \frac{x+0.5}{2} * \frac{1}{1-\alpha} dx, 2\Delta c\} - 0.5 * (1-p)}{2\Delta c} * (0.25 + 0.5 * \alpha) \\
&= \frac{\min\{\frac{(1-p)*(2+\alpha)}{4}, 2\Delta c\} - 0.5 * (1-p)}{2\Delta c} * (0.25 + 0.5 * \alpha)
\end{aligned}
$$

where the price is fixed at $p = 0.5$. Notice, the gain is always increasing in α, because higher α expands the extensive margin and has higher efficiency.

The loss from recommendation is:

$$
loss = \frac{0.5(1-p) - (1-p) * \frac{\int_{\alpha}^{1} 1 - x dx}{\int_{\alpha}^{1} 1 - x + 0.5 dx}}{2\Delta c} * (0.5 * (1-\alpha))
$$

The loss is is inverse U-shaped in α, which is less obvious, because although the consumer base switching from search deeply to search shallow increases with α, but the difference in demand from consumer search deeply and search shallow are smaller with higher α. If we look closer at the loss, we have loss to be:

$$
loss = \frac{0.5(1-p) - (1-p) + \frac{(1-p)}{2-\alpha}}{2\Delta c} * (0.5 * (1-\alpha))
$$

which is inverse U shaped in α

$$\frac{\partial loss}{\partial \alpha} = \frac{0.5(1-p) - \frac{1-p}{2-\alpha} + \frac{(1-p)*(1-\alpha)}{(2-\alpha)^2}}{4\Delta c} = \frac{1-p}{4\Delta c*(2-\alpha)^2}*(0.5*(2-\alpha)^2-(2-\alpha)+1-\alpha)$$
$$= \frac{1-p}{4\Delta c*(2-\alpha)^2}*(0.5*(2-\alpha)^2-1)$$
$$\frac{\partial^2 loss}{\partial^2 \alpha} = -\frac{1-p}{2\Delta c*(2-\alpha)^3} < 0$$

Firstly, we focus on $\alpha = 0.5$. When $\frac{(1-p)*(2+0.5)}{4} = 0.3125 > 2\Delta c \Leftrightarrow \Delta c < 0.15625$, we have

$$gain = \frac{2\Delta c - 0.5*(1-p)}{2\Delta c}*(0.25+0.5*\alpha) = \frac{2\Delta c - 0.25}{2\Delta c}*(0.25+0.5*0.5)$$

and

$$loss = \frac{0.25 - 0.5 + \frac{0.5}{1.5}}{2\Delta c}*(0.5*0.5)$$

it is straightforward to check that $loss > gain$ at $\alpha = 0.5$ when

$$\frac{-0.25+\frac{1}{3}}{2\Delta c}*0.25 > \frac{2\Delta c - 0.25}{2\Delta c}*0.5 \Leftrightarrow \Delta c < \frac{7}{48} = 0.1458$$

Therefore, when $\Delta c < 0.1458$, we have a threshold $\bar{\alpha}$, such that $gain > loss$ iff $\alpha > \bar{\alpha}$. For $\Delta c > 0.1458$, we need to further investigate the change in gain and loss w.r.t α at $\alpha = 0.5$. We have

$$\frac{\partial gain}{\partial \alpha}_{\alpha=0.5} = 0.5*\frac{2\Delta c - 0.5*0.5}{2\Delta c}$$
$$\frac{\partial loss}{\partial \alpha}_{\alpha=0.5} = \frac{0.5*0.5 - \frac{0.5}{1.5} + \frac{0.5*0.5}{1.5^2}}{4\Delta c}$$

after some calculation we have when $\Delta c > \frac{5}{36} = 0.1389$, we have $\frac{\partial gain}{\partial \alpha}_{\alpha=0.5} > \frac{\partial loss}{\partial \alpha}_{\alpha=0.5}$. Therefore, when $\Delta c > 0.1458$, we not only have $gain > loss$ at $\alpha = 0.5$, we also have $\frac{\partial gain}{\partial \alpha}_{\alpha=0.5} > \frac{\partial loss}{\partial \alpha}_{\alpha=0.5}$. Therefore, when $\Delta > 0.1458$, the firm will always find it optimal to recommend. Especially for $\Delta c > 0.15625$, because the numerator is increasing in Δc. Also the numerator is increasing at a faster rate in α because of improvement in matching efficiency and expansion of extensive margin. When $\Delta c \in (0.1389, 0.1458)$, we have $gain < loss$ at $\alpha = 0.5$ and $\frac{\partial gain}{\partial \alpha}_{\alpha=0.5} > \frac{\partial loss}{\partial \alpha}_{\alpha=0.5}$, therefore there exists a threshold $\alpha^{uncertainty}$ such that the firm will recommend if and only if $\alpha > \alpha^{uncertainty}$. When $0.125 < \Delta c < 0.1389$, we have $gain < loss$ at $\alpha = 0.5$ and $\frac{\partial gain}{\partial \alpha}_{\alpha=0.5} < \frac{\partial loss}{\partial \alpha}_{\alpha=0.5}$, therefore there exists a threshold $\alpha^{uncertainty}$ such that the firm will recommend if and only if $\alpha > \alpha^{uncertainty}$.

When $\Delta c < 0.125$, the firm will always have $gain < loss$ at all levels of α, therefore the firm will always not recommend.

$\square$

Proof for Proposition 5

Proof. When $\Delta c \geqslant 0.1458$, it is always optimal for the firm to recommend with/without uncertain customers. It is straightforward to prove that there is a Perfect Bayesian Equilibrium that the firm recommends at all levels of precision on equilibrium. On equilibrium, uncertain customers believe that firms at all levels of precision $\alpha \geqslant 0.5$ recommends, generating a demand strictly larger than no recommendation: $gain > loss$ at $\alpha \geqslant 0.5$. Customers who knows the firms' actual precision will also generate a demand strictly greater than no recommendation.

When $0.133 < \Delta c < 0.1458$, we consider the equilibrium belief where all firms whose precision $\alpha \geqslant \bar{\alpha}$ recommend. Denote demand generated from consumers with complete information $D^{complete\ information}$ and demand from consumers with uncertainty $D^{uncertainty}$. We consider demand for marginal firm $\alpha = \bar{\alpha}$. For k large enough, the shape of $k * D^{complete\ information}(\bar{\alpha}) + (1-k) * D^{uncertainty}(\bar{\alpha})$ is the same as $D^{complete\ information}$ which is increasing and decreasing and finally increasing in $\bar{\alpha}$ according to Corollary 1. However, $k * D^{complete\ information}(\bar{\alpha}) + (1-k) * D^{uncertainty}(\bar{\alpha})$ at $\bar{\alpha} = 0.5$ is strictly less than demand from no recommendation $D^{completeinformation}(0.5)$. It is because for uncertain consumers their belief when there is recommendation is that all firms recommend $\alpha \geqslant \bar{\alpha}^{uncertain} = 0.5$ which changes discontinuous from knowing $\alpha = 0.5$ when there is no recommendation. Combined with our understanding of $k * D^{complete\ information}(\bar{\alpha}) + (1-k) * D^{uncertainty}(\bar{\alpha})$ is the same as $D^{complete\ information}$ which is increasing and decreasing and finally increasing in $\bar{\alpha}$ and $k * D^{complete\ information}(\bar{\alpha}) + (1-k) * D^{uncertainty}(\bar{\alpha}) < D^{complete\ information}(0.5)$, there exists a threshold $\bar{\alpha}^{uncertainty}$ such that $k * D^{complete\ information}(\bar{\alpha}^{uncertainty}) + (1-k) * D^{uncertainty}(\bar{\alpha}^{uncertainty}) = D^{complete\ information}(0.5)$. Moreover, the firm will recommend iff $\alpha > \bar{\alpha}^{uncertainty}$, i.e., $k * D^{complete\ information}(\bar{\alpha}^{uncertainty})$ $(1-k) * D^{uncertainty}(\bar{\alpha}^{uncertainty}) > D^{complete\ information}(0.5)$.

When $\Delta \leqslant 0.125$, the firm always find it optimal to not recommend given it already achieve highest demand 0.75 at precision $\alpha = 0.5$.

$\square$