Communicating Attribute Importance under Competition

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Abstract

When consumers encounter unfamiliar products, they often face difficulty in understanding which attributes are crucial, leading to challenges in product comparison and potential diminished interest in the category. This study examines how firms strategically communicate the importance of product attributes in a competitive environment. Despite consumer awareness of attributes and their levels, ambiguity regarding their relative importance remains. We analyze a situation where two firms each receive a noisy signal about the true attribute importance and convey this information to consumers through cheap-talk messages. Following these communications, consumers decide whether to incur a cost to further explore the category by visiting stores. Our findings reveal a truthful equilibrium where firms honestly report their received signals. In this equilibrium, when both firms' messages align, their collective messages can credibly convey information about the more important attribute, thereby encouraging store visits and purchase. Interestingly, firms may still find it advantageous to truthfully highlight an attribute, even if it doesn't align with their competitive advantage. Moreover, we show that without competition (i.e., a single firm communicating), this truthful equilibrium does not exist. Thus, the presence of the competition enables the credible communication of information about attribute importance, benefiting both firms by enhancing consumer engagement with the product category.

Keywords: Attribute importance; Two-sender cheap-talk; Competition; Credible communication; Ingredient branding

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1 Introduction

In today's market, consumers often find it overwhelming to choose from the extensive selection of products available. The challenge lies not just in the sheer number of options but also in the complexity of comparing these products, each with its unique set of distinct attributes. The complexity of comparing these diverse products makes it hard for consumers to easily identify which product best suits their needs. The advent of digital technologies and the online platforms has particularly alleviated this challenge by providing consumers with access to details about product attributes. However, choosing between products where no single option clearly outperforms the others remains a challenge. For instance, while Apple's MacBook may excel in design and user interface, Lenovo's ThinkPad may offer superior processor performance. This complexity in the process of comparing products can cause decision fatigue (Ursu et al., 2023), where the search cost, in terms of time and cognitive load, becomes so high that some consumers choose to disengage from the purchasing process altogether, ultimately opting out of the product category. The situation is compounded in categories featuring new or advanced technologies. Novice consumers, especially those unfamiliar with the category, struggle to understand which attribute is more essential in the product category (Dzyabura and Hauser, 2019).

Consider Alex, a first-time buyer in the electric vehicle (EV) category. Alex is initially thrilled at the prospect of buying an electric vehicle but becomes increasingly frustrated after visiting two dealerships. The first Lucid dealer emphasizes its superior battery range and its luxury comfort while the second dealer for Hyundai IONIQ 5 insists that fast charging capabilities are what truly matter.[1] This conflicting advice leaves Alex so confused and overwhelmed that they start to question the practicality of switching to an EV at all. Concerned about making the wrong choice, Alex decides to postpone the purchase indefinitely, sticking with the manufacturers' gas-powered vehicle while lamenting the complexity of the EV market. A similar situation occurs to Jordan, who is a homeowner interested in upgrading to smart home technology. He decided to visit their local Best Buy store to explore the available options firsthand. There, a Google Nest representative highlighted device compatibility, while an Asus ZenWiFi representative focused on superior security features. The contrasting pitches left Jordan unsure which feature was more crucial —compatibility or security.[2] Despite the initial excitement and the visit to gather information, the conflicting advice led to skepticism about the true bene-

[1] "Which Car Brands Offer Full EVs? 7 EV Manufacturers Compared" (https://www.makeuseof.com/which-car-brands-offer-full-evs/).

[2] "The Best Smart Home Devices of 2024, According to Experts" (https://www.goodhousekeeping.com/home-products/a35880026/best-smart-home-device/).

fits of smart home technology. Overwhelmed by the complexity and perceived trade-offs, this skepticism leads to a decision to avoid investing in smart home devices altogether, as Jordan perceives the category as too complicated and fraught with trade-offs.

Without understanding which attributes are most critical and how much weight to allocate across different attributes, detailed attribute information about the products in the category may not be sufficient to make comparisons and, ultimately, purchasing decisions. This highlights the need for firms to streamline the decision-making process by clearly communicating the importance of key product attributes. To prevent consumers from disengaging from the product category, it is in the common interest of the firms in the category to provide consumers with guidance on the significance of key attributes, helping to simplify decision-making and potentially increasing the total demand for the product category.

In practice, we have seen several examples where a specific product attribute becomes a focal point in a category, especially new and advanced technologies which are often unfamiliar for them, significantly influencing consumer behavior. For example, in the early era of personal desktop computers, over 2,700 PC makers all singled out their microprocessors among other components in their advertising campaigns, notably beginning with the Intel Inside campaign in 1991. This collective emphasis enabled consumers to anchor their decision-making process on the microprocessor, significantly contributing to the substantial growth of the PC market (Moon and Darwall, 2002). Similarly, certain attributes gain prominence when multiple firms collectively emphasize their importance. For instance, outdoor apparel brands like The North Face and Patagonia highlight waterproof and breathable fabrics, such as Gore-Tex, guiding consumers to prioritize fabric technology in their decision-making. Additionally, wearable technology companies, including Fitbit and Apple, accentuate health monitoring capabilities. These strategies simplify consumer choices by spotlighting a singular, crucial attribute, aiding in navigating their purchasing decisions.

However, not every company will highlight the same feature due to their unique competitive advantages. For instance, in the electric vehicle market, Tesla is known for its advanced technology and battery range, Hyundai for rapid charging, and Lucid Motors for its luxury design and comfort, each highlighting their unique strengths. This practice becomes evident in sales tactics that focus on specific features, which, although not deceitful, may not fully meet the essential needs of consumers. Taking real estate as an instance, agents significantly influence buyer decisions. When parents search for a new home, prioritizing aspects like school districts and outdoor spaces, agents might direct their attention to the luxurious aspects of a property not located in a preferred school zone, such as a modern kitchen or lavish bathrooms. This demonstrates how sales strategies can influence on defining what attributes are considered more important, potentially shifting the buyers' focus from their initial priorities. In the auto industry, car dealers might highlight a car's safety features over its battery efficiency, influencing buyer perceptions of what is truly important. In consumer electronics, a retailer may promote a phone's camera quality while minimizing its short battery life, potentially obscuring significant drawbacks for the consumer.

The question of credibility in the firm communication arises naturally in these interactions, especially when the importance and prioritization of product attributes are not clearly understood. Mere detailed information on product features may fall short for consumer's effective comparison and decision-making. Companies or salespeople might strategically highlight certain features to present their products favorably. Despite these incentives and potential for mixed messages due to these different focuses, truthful communication can prevail where a collective emphasis on certain key attributes across different firms can provide useful information to consumers. This consensus helps consumers, particularly those new to the category, identify and prioritize the most significant attributes in the category, facilitating more informed shopping decisions.

This study explores how strategic communication by firms in competitive markets can credibly inform consumers about the importance of product attributes and influence their decision-making to engage with a product category. We identify the conditions under which an attribute becomes prominent, even in markets where competing firms possess competitive advantages in different attributes. We employ a two-sender cheap-talk game, where firms send messages highlighting one of two attributes based on a noisy signal about which attribute is more important. A representative consumer, initially uncertain about the attribute's importance, receives these messages, which may either align or conflict on the attribute's importance, and forms beliefs about the relative importance of the attributes. Given each firm's competitive advantage in different attributes, they aim to persuade consumers of the importance of their emphasized attribute. The consumer, with rational expectations about the firms' strategy in equilibrium, decides whether to further engage with the product category by incurring a cost for acquiring more information about both products or to disengage at no cost. If the consumer pays the cost, she observes the idiosyncratic component of her utility from each product not explained by the two main attributes. If the consumer opts to engage, she gains additional information about the product beyond the main attributes communicated, diminishing uncertainty about their true values. Upon partially resolving her uncertainty regarding the attribute importance, she makes an informed decision on whether and which product to buy.

We show that the consumer's uncertainty about the relative importance of attributes can be a critical barrier hindering her further engagement with a product category. Firms can benefit from effectively conveying credible information about attribute importance. Moreover, we identify a truthful equilibrium in which firms, despite their self-interest, truthfully convey the noisy signal about attribute importance, even when it may not align with their competitive advantage (i.e., a firm has a competitive disadvantage in that attribute). In this equilibrium, when both firms' messages align, their collective messages can credibly convey information about the more important attribute, thereby encouraging store visits and purchase. Interestingly, firms may still find it profitable to truthfully emphasize an attribute, even if it doesn't align with their competitive edge.

Should firms diverge in their communications, highlighting only their own superior attributes, consumers are likely to encounter conflicting messages. This inconsistency fails to resolve consumer uncertainty about the relative importance of attributes, leading to consumer confusion. This confusion could prompt consumers to decide against investing time and resources to further explore the products, choosing instead to disengage with the category entirely. Such a scenario is detrimental to both firms, as it prevents them from capitalizing on potential consumer interest and demand, negatively impacting the overall demand within the category.

The impact of competition on communication strategies between firms and consumers is critical, especially regarding attribute importance. Contrary to the lay belief that competition might disadvantage firms by highlighting less favorable attributes, our analysis reveals that the absence of competition (i.e., a monopoly on communication) actually harms the firm more. In situations where only one firm communicates with consumers, the absence of competition undermines the the credibility of communication message, eliminating the possibility of establishing a truthful equilibrium, leading to a breakdown in the credible communication of attribute importance. This scenario restricts consumer engagement, as they remain uninformed about crucial product attributes. Therefore, we demonstrate that competition fosters a disciplined approach to communication, ensuring both firms' messages are more credible. Such credibility is essential for engaging consumers, illustrating that competition, rather than being a hindrance, actually facilitates more effective communication about product attributes, ultimately benefiting all parties involved by enhancing consumer engagement within the product category.

The paper is organized as follows: Section 2 provides a brief overview of the related literature. Section 3 introduces the model setup, including strategies and the equilibrium concept. Section 4 presents the main results and equilibrium analysis. Section 5 demonstrates robustness of the main results in an extended model with endogenous pricing. Section 6 concludes the paper with a summary of findings and discussions. All the technical proofs are provided in Appendix.

2 Related Literature

This paper intersects with several streams of research within the areas of firm strategy, specifically focusing on coopetition, strategic communication, and the dynamics of cheap-talk games. First, it contributes to the literature on the strategic dynamics of competition and cooperation among firms, known as coopetition (Brandenburger and Nalebuff, 1996). In our study, competing firms are shown to engage in rational cooperation to disseminate crucial information to consumers, thereby generating demand for their category. Several papers also explore how firms simultaneously compete and cooperate through various mechanisms such as strategic alliances (Amaldoss et al., 2000; Luo et al., 2007) and licensing agreements (Inkpen, 1996). Particularly relevant to our work is the study by Lu and Shin (2018), which investigates firms' strategic communication about innovations. It posits that sharing information on new innovations enables firms to reduce consumer communication costs significantly, thereby enhancing consumer engagement with new products. However, our focus diverges by focusing on how firms can achieve credibility in their communications through coordinated communication, despite the unverifiable and costless nature of their messages or "cheap-talk" (Farrell and Rabin, 1996).

Also, this paper contributes to the understanding of the credibility of firms' messages in the context of cheap-talk communication games, building on the foundational work by Crawford and Sobel (1982). This includes examining the application of cheap-talk models to the communication between firms and consumers. Shin (2005) demonstrates that a non-binding price claim by a firm, such as "everything priced $19.99 or above," can serve as a credible indicator of low prices, especially when the firm incurs costs in the selling process. Guo (2022) extends this analysis to the credibility of communication about a seller's cost type, considering subsequent buyer learning and bargaining processes. He identifies a separating equilibrium where both high-cost and low-cost sellers' announcements can be credible, influenced by the buyer's optimal information search strategy and its alignment with the seller's intentions.

Recent studies have broadened the scope of cheap-talk communication models in advertising, moving beyond the traditional focus on cost and price to encompass a wider range of product attributes. Chakraborty and Harbaugh (2014) show that a seller's non-verifiable claims, such as "best pizza in town," attain credibility by the mere act of highlighting one attribute (e.g., pizza) at the expense of not mentioning others (e.g., chicken wings). This strategic choice leads consumers to infer positively about the emphasized attribute while drawing negative inferences about the neglected ones. Gardete (2013) studies credibility of a monopolist's advertising content and finds that in markets with different levels of product quality, a seller claiming high quality can appeal to consumers looking for high quality but push away others with a low valuation for quality. Gardete and Bart (2018) analyze the effect of sender's information precision about the receiver's preferences on the outcome of cheap-talk communication game. They find that communication can only be credible when the sender's information is not too precise because if the sender knows too much about the receiver, then the sender cannot commit to saying that the receiver wants to hear, which thus reduces credibility of the sender's claims. Lastly, Gardete and Guo (2021) studies how the possibility of consumers pre-purchase learning affects advertising's credibility. The study reveals that the threat of consumer learning disciplines low-quality firms from falsely representing their products as high-quality.

While these studies focus on the credibility of messages from a single sender about a particular attribute, such as quality, our paper examines how communication between multiple senders and a buyer determines which attribute is most important to the buyer, linking to the broader literature on economics concerning multi-sender cheap-talk games (Ambrus and Takahashi, 2008; Battaglini, 2002; Krishna and Morgan, 2001). These studies investigate the conditions under which fully-revealing equilibria exist in scenarios with multiple senders. They generally conclude that communication from multiple senders can appear more credible than that from a single sender, especially when the senders have conflicting biases (Ambrus and Takahashi, 2008; Krishna and Morgan, 2001) or incentives (Battaglini, 2002). These papers papers analyzes existence of the fully-revealing equilibrium using adverse off-equilibrium beliefs. In contrast, our paper explores the possibility of achieving a separating equilibrium, akin to the fully-revealing equilibrium identified in previous work, without resorting to off-equilibrium beliefs. Instead, we propose that the mechanism for punishing deviations from equilibrium strategies emerges naturally through the buyer's belief updating and subsequent search decisions, offering a novel perspective on credibility in multi-sender communication scenarios.

Additionally, our paper connects with research on the salience of product attributes. Bordalo et al. (2013) propose a framework where a consumer's attention to various attributes and thus, their impact on utility is determined by the relative prominence of these attributes among available products. For example, when price variance is high and quality variance is low, price becomes a more salient attribute for consumers. Similarly, Zhu and Dukes (2017) explore how firms' announcements about prominent attributes can directly influence consumers' utility by altering the weight consumers place on these attributes. Unlike thier work, our approach suggests that attribute importance emerges implicitly and endogenously through consumer inferences drawn from firms' messages, providing a microfoundation for understanding how certain attributes become significant, prominent, or salient to consumers.

Finally, our research underscores the importance of communicating about attribute importance, enabling consumers to make more informed decisions among multiple choices and considerable uncertainty. This aligns with studies on information overload which demonstrate how the abundance of choices or the challenge of processing abundance of information can lead to decision avoidance, procrastination, or diminished satisfaction post-decision (Branco et al., 2016; Iyengar and Lepper, 2000; Jacoby, 1984; Kuksov and Villas-Boas, 2010). Various reasons have been proposed for this phenomenon, including the fear of regretting a bad choice, the anxiety of choice-making, loss aversion (Sarver, 2008; Tversky and Shafir, 1992), or efforts and costs to process information that often comes with a large number of available options (Branco et al., 2016; Ortoleva, 2013). The implications of such consumers' costly information processing or deliberation costs on firms' decision-making have been the focus on several studies in marketing (Branco et al., 2016; Guo, 2016; Guo and Wu, 2016; Guo and Zhang, 2012; Kuksov and Villas-Boas, 2010; Li et al., 2019; Shin and Wang, 2024; Wathieu and Bertini, 2007). In contrast to these studies, our work study how firms can strategically overcome information overload by communicating the most relevant attributes to consumers, thereby aiding in their decision-making process.

3 Model

We consider a market with two firms, 1, 2, and a single representative consumer. Each firm offers a product at an exogenous symmetric price p, which is a common knowledge to both firms and consumer. In Section 5, we analyze a model where the firms set their prices endogenously. The consumer faces a choice of purchasing one of the products or none at all, where the utility of no purchase is normalized to 0. The total consumption value derived from purchasing product i can be expressed as:

$$V_i = U_i + \nu_i - p, \tag{1}$$

where U_i denotes the main utility from the product's primary attributes. This utility component is influenced by the product's essential features, such as battery range and fast charging capabilities in electric vehicles (EVs) or device compatibility and security coverage in smart home technologies. These attributes are crucial in shaping consumer preferences. On the other hand, ν_i represents the idiosyncratic value component of the total consumption value, stemming from secondary attributes like design aesthetics. This element reflects the consumer's unique and personal preferences beyond the primary attributes.

The main utility component U_i for product i is defined as a function of the attribute importance weight ω as follows:

$$U_i(\omega) = \omega \cdot \alpha_i + (1 - \omega) \cdot \beta_i, \tag{2}$$

where $\alpha_i \in \{\alpha_H, \alpha_L\}$ and $\beta_i \in \{\beta_H, \beta_L\}$ represent the levels of two key attributes, with $\alpha_i, \beta_i > 0$ indicating that both attributes contribute positively to utility. These levels are distinguished as high quality (α_H, β_H) or low quality (α_L, β_L). For instance, in the context of electric vehicles (EVs), the α-attribute could relate to battery range, available as long (α_H) or short (α_L), while the β-attribute might refer to charging speed, categorized as fast (β_H) or slow (β_L). For simplicity, we assume $\alpha_H = \beta_H = u_H$ and $\alpha_L = \beta_L = u_L$ (where $u_H > u_L > 0$), implying that each attribute *ex-ante* contributes equally to the total value at their respective quality levels, regardless of being high or low quality. The importance weight ω determines the true relative significance of the first attribute in the product category. Here, we assumes a market-wide consensus on the weight ω.[3]

The model posits two possible states for the attribute importance weight: high (ω_H) or low (ω_L), with $0 \leq \omega_L < \omega_H \leq 1$. Here, ω indicates the importance of the first attribute (α). The state can be high ($\omega = \omega_H$), making the first attribute more important, or low ($\omega = \omega_L$), indicating the second attribute is more important. The true state of ω is drawn from a prior distribution such that $\Pr(\omega = \omega_H) = \mu_0 \in (0, 1)$. Moreover, without loss of generality, we designate firm 1 as having its strength in the first attribute ($\alpha_1 = \alpha_H$) and a weakness in the second attribute ($\beta_1 = \beta_L$). This means that for a given price p, firm 1 offers higher utility to consumers when $\omega = \omega_H$ compared to when ω_L: $U_1(\omega_H) > U_1(\omega_L)$. On the other hand, firm 2 possesses its competitive advantage in the attribute where firm 1 is weaker such that $\alpha_2 = \alpha_L$ and $\beta_2 = \beta_H$. Thus, for a given p_2, firm 2 offers higher utility in the state ω_L than in the state ω_H: $U_2(\omega_L) > U_2(\omega_H)$.

Additionally, we assume that the sum of the attribute importance weights equates to unity, $\omega_H + \omega_L = 1$. This ensures parity in the utility values each firm delivers in their favorable states, making $U_1(\omega_H) = U_2(\omega_L)$ and $U_1(\omega_L) = U_2(\omega_H)$.[4] Such an arrangement posits that while firms 1 and 2 exhibit distinct competitive edges — primarily in the first and second attributes, respectively — their offerings are perceived equally by consumers when evaluating the products

[3]An alternative model could feature consumers with heterogeneous ω weights, the current analysis and outcomes remain robust provided the heterogeneity in ω is minimal.

[4]Here, $U_1(\omega_H) = \omega_H \cdot \alpha_H + (1 - \omega_H) \cdot \beta_L = (1 - \omega_L) \cdot \alpha_H + \omega_L \cdot \beta_L = (1 - \omega_L) \cdot \beta_H + \omega_L \cdot \alpha_L = U_2(\omega_L)$, and $U_1(\omega_L) = \omega_H \cdot \alpha_L + (1 - \omega_H) \cdot \beta_H = (1 - \omega_L) \cdot \alpha_L + \omega_L \cdot \beta_H = (1 - \omega_L) \cdot \beta_L + \omega_L \cdot \alpha_H = U_2(\omega_H)$ because $\alpha_H = \beta_H = u_H$ and $\alpha_L = \beta_L = u_L$.

in their optimal states. Consequently, this setup allows us to focus on the symmetric competition between two differentiated firms.

Each firm independently receives a noisy signal $s_i \in \{H, L\}$ about the true state of ω, where the level of noisiness is captured by $\gamma \in (0, \frac{1}{2})$ such that $\Pr(s_i = H|\omega = \omega_H) = \Pr(s_i = L|\omega = \omega_L) = 1 - \gamma$. While the two firms might receive different signals about the true state of ω, the condition $\gamma < \frac{1}{2}$ ensures that these signals are positively correlated. However, despite the correlation, conditional on the true state of ω, the signals s_1 and s_2 are independent events.

Upon receiving their respective noisy signals $s_i \in \{H, L\}$, each firm communicates a message m_i to the consumer, emphasizing one of the two attributes. Messaging about the first attribute corresponds to reporting that $\omega = \omega_H$, whereas emphasizing the second attribute corresponds to reporting that $\omega = \omega_L$. Consequently, without loss of generality, the firm's message space can be represented by $m_i \in \{h, l\}$, employing lowercase letters for messages (m_i) to differentiate them from the uppercase letters used for signals (s_i). The consumer only knows the prior distribution from which ω is drawn, and she makes inferences about ω after receiving messages from two firms. The noisy signals that the firms receive about the true state introduces information asymmetry between the firms and the consumer about the true state ω. The initial information asymmetry, stemming from the firms' noisy signals about ω, reflects the potential advantage firms have due to their market research efforts, including product testing, which can reveal deeper insights into product attributes and their market valuation (Dzyabura and Hauser, 2019; Shin and Yu, 2021).

This information asymmetry also highlights the consumer's challenge in determining the true relative importance of product attributes without direct experience. For instance, a buyer of an electric vehicle (EV) may only realize the significance of battery range versus fast charging capabilities after experiencing the practical implications of each during their daily use. Similarly, a consumer investing in smart home technology might only understand the relative importance of device compatibility versus security features after encountering various scenarios in their home usage. These scenarios underscore the gap between the firms' marketed attributes and the consumer's eventual realization of attribute importance through usage. These examples highlight the discrepancy between the attributes emphasized by firm's communication and the consumer's ultimate realization of which attributes are truly important, based on actual experience.

The additional term ν_i represents an idiosyncratic shock to the consumer's utility from product i capturing the utility from factors beyond the main attributes described by U_i, such as vehicle design, color options for an EV, or the size of a smart home device to fit in a specific space in the house. This term follows a uniform distribution $U[-1, 1]$, which is known to all market participants. Initially, the consumer is unaware of the exact values of ν_i's but can discover these through direct engagement, by visiting a retailer that showcases both products and incurring a search or traveling cost ($c > 0$).[5] Thus, this idiosyncratic preference is fully resolved if the consumer visits the store by incurring the search cost. Although such visit can reveal those specific details like the exact fit of a smart home device or the color options available for an EV, they do not clarify the consumer's uncertainty about the attribute importance ω as understanding its importance typically comes from post-purchase experience. Moreover, without incurring the search cost $c > 0$, the consumer cannot further engage with the product category,[6] and therefore, the consumer leaves the market with an outside option value set to zero.

Given the consumer needs to incur a positive search cost to visit a retailer for additional product information before deciding to purchase, reducing her uncertainty about the attribute importance (ω) through the firms' announcements becomes critical. As our subsequent analysis will confirm, if the firms fail to credibly communicate which attribute is more important, the consumer may opt not to incur the search cost (c) because the residual uncertainty about the product's utility (due to incomplete knowledge of $U_i(\omega)$) prevents the consumer from paying the cost for visiting a retailer to observe ν_i's alone.[7]

The timeline of the game is as follows: the relative importance parameter $\omega \in \{\omega_H, \omega_L\}$ is drawn from the distribution. Each firm privately receives an independent noisy signal $s_i \in \{H, L\}$ about ω. The firms simultaneously send a message $m_1, m_2 \in \{h, l\}$ to the consumer. Then, the consumer observes (m_1, m_2) and decides whether to pay the cost c and observe (ν_1, ν_2). Finally, the consumer decides whether to purchase either product 1 or 2, or neither. The sequence of events is depicted in Figure 1.

[5] In our model, the cost of visiting is equivalent to the cost of engaging in the product category and obtaining additional information about both sellers' products. Alternatively, we can interpret c as the marginal travel cost to visiting each seller's store. Even then, the main trade-offs that we explore in the current analysis (i.e., a credible communication about the attribute importance) are robust to the alternative setting, and thus the qualitative results are not affected.

[6] This assumption is common in consumer search models where a consumer must at least search one firm in order to buy a product (Armstrong et al., 2009; Armstrong and Zhou, 2011; Zhou, 2014). The rationale behind this assumption, which involves calculating the consumer's expected payoff from a purchase without recognizing ν_i's and ensuring this payoff remains non-positive, is elaborated upon in the Appendix.

[7] Unlike in the current model where the consumer's visit reveals ν_i's independent of the main attributes, we can consider another setting where the consumer is uncertain about the main attributes α_i and β_i and her visit reveals the exact attribute levels. In both settings, the more she knows about the attribute importance, a more informed decision whether to visit the stores she can make. Upon visiting the store, she realizes components of her utility other than ω, which lead her to buy one product or another, or neither. Thus, the main results from the current analysis hold qualitatively under the alternative model specification.

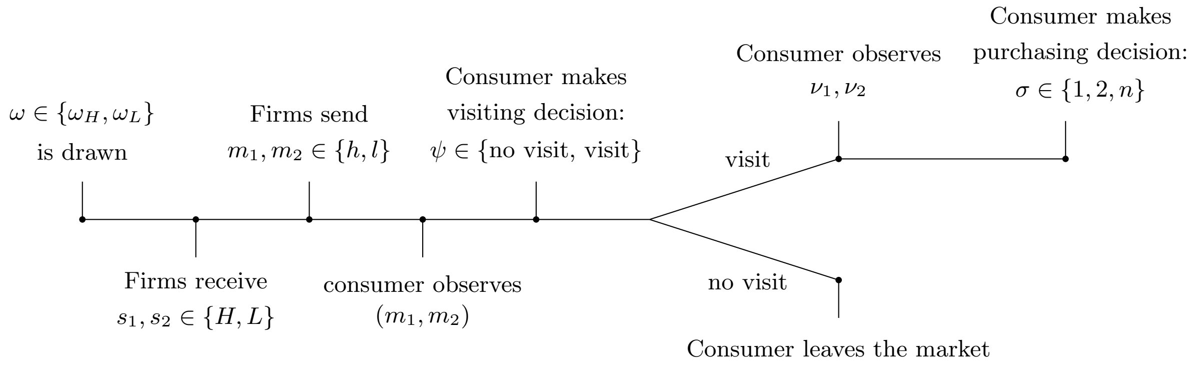


Figure 1: Timeline.

Strategies, Beliefs and Equilibria

The firm i's payoff is p if the consumer purchases the product i, and 0 for otherwise. Each firm's (pure) strategy is a mapping from the firm's private signal about ω to the message of its choice, $m_i : \{H, L\} \to \{h, l\}$, denoted by $m_i(s_i)$. Upon receiving the messages m_1 and m_2, the consumer updates her beliefs about the true state ω. Specifically, she assigns a posterior probability that $\omega = \omega_H$, denoted by $\hat{\mu}(m_1, m_2) = \Pr(\omega = \omega_H | m_1, m_2)$, indicating her revised belief that the first attribute is more important after considering the messages from both firms.[8]

Based on these beliefs, the consumer decides whether to incur the cost $c > 0$ and observe both ν_i's. The consumer's strategy is thus a mapping from the consumer's beliefs to her visit decision $\psi : [0, 1] \to \{\text{visit}, \text{no visit}\}$. Choosing "visit" implies the consumer pays the cost to learn about (ν_1, ν_2), while "no visit" means she opts not to incur the cost, leaving (ν_1, ν_2) unknown. The consumer's purchasing strategy after observing (ν_1, ν_2) is a function, $\sigma : [0, 1] \times [0, 1] \times [0, 1] \to \{1, 2, n\}$, denoted by $\sigma(\hat{\mu}, \nu_1, \nu_2)$,[9] where 1 and 2 indicate purchasing product 1 or 2, respectively, and n means that the consumer does not purchases any product. Without observing (ν_1, ν_2), the consumer can either purchase one of the product or not purchase any product. In equilibrium, we show that consumer leaves the market with a payoff of zero.

Our solution concept is perfect Bayesian equilibrium (PBE) defined more formally below:

Definition 1. ***(Perfect Bayesian Equilibrium)*** *A (pure-strategy) PBE is a triple consisting of senders' strategy (m_1^*, m_2^*), a receiver's strategy (ψ^*, σ^*), and a receiver's belief $\hat{\mu}^*(m_1^*, m_2^*)$ satisfying*

[8] More formally, it is $\hat{\mu} : \{h, l\} \times \{h, l\} \to \Delta\{\omega_H, \omega_L\}$, denoted by $\hat{\mu}(\cdot \mid m_1, m_2)$. With a slight abuse of notation, we use $\hat{\mu}(m_1, m_2)$ to denote the conditional probability that the consumer assigns to ω_H given m_1, m_2.

[9] More formally, the consumer's purchasing strategy should be a function, $\sigma : [0, 1] \times \{[0, 1] \cup \varnothing\} \times \{[0, 1] \cup \varnothing\} \to \{1, 2, n\}$. For example, if the consumer opts not to visit the store, she decides whether to purchase without any information about (ν_1, ν_2). Under such circumstances, $\sigma(\hat{\mu}, \nu_1, \nu_2) = \sigma(\hat{\mu}, \varnothing, \varnothing)$, indicating the strategy accounts for decisions made with and without information about ν_1 and ν_2.

1. *Firms choose* $m_i^*(\omega)$ *for each* $\omega \in \{\omega_H, \omega_L\}$*, to maximize firm* i*'s expected payoff given* m_j^**,* ψ^**,* σ^**, and* $\hat{\mu}^*$*,*

2. *The consumer choose* $\psi^*(\hat{\mu}^*)$ *and* $\sigma^*(\hat{\mu}^*, \nu_1, \nu_2)$ *to maximize her expected utility, given* m_1^**,* m_2^**,* σ^**,* $\hat{\mu}^*$*, and given* m_1^**,* m_2^**,* ψ^**,* $\hat{\mu}^*$*, respectively,*

3. *The belief* $\hat{\mu}^*(m_1, m_2)$ *follows Bayes' rule for each* $m_1, m_2 \in \{h, l\}$*, whenever applicable.*

Moreover, we focus on the truthful equilibrium in which both firms truthfully announce the noisy signals they received. In this equilibrium, the true state can partially be communicated to the consumer through the firms' messages. Also, we strict our focus on equilibrium where the consumer makes a purchase with a positive probability. Given the level of noisiness $\gamma > 0$, firms may receive a signal different from the true state ω, implying that consumers are unable to detect any firm's deviation from the equilibrium strategy. Consequently, there is no off the equilibrium path. The truthful equilibrium is defined as follows:

Definition 2. ***(Truthful Equilibrium)*** *A truthful equilibrium is a PBE, where*

1. *Firms report their received noisy signal truthfully, i.e.,* $m_i(s_i) = s_i$*, and*

2. *The consumer purchases either* 1 *or* 2 *with a strictly positive probability for each* $s_i \in \{H, L\}$*.*

The truthful equilibrium is considered a separating equilibrium because each firm conveys distinct messages based on their signal s_i. As a result, one can expect the most information transmission in the truthful equilibrium. However, other types of equilibria, such as pooling equilibria (in which firms send the same message regardless of its private signal) or a semi-separating equilibrium combining a pooling and a separating equilibrium (in which one firm pooling and the other separating), may exist. At the end of Section 4, we analyze these equilibria and show that (i) as expected, pooling equilibrium always exists, but no information is communicated to the consumer, resulting in zero profit for the firms, and (ii) a semi-separating equilibrium does not exist for all priors $\mu_0 \in (0, 1)$ with the exception that it can only exist in a parameter region of measure zero.

4 Analysis

4.1 Consumer Decisions

Consumer's purchasing decision

We solve the game backwards to analyze consumer behavior. Take a consumer who has visited the stores and observed the idiosyncratic values (ν_1, ν_2) from both firms' products. The consumer's posterior beliefs about the attribute importance based on the firms' messages (m_1, m_2) is $\hat{\mu} = \Pr[\omega = \omega_H | (m_1, m_2)]$. This belief allows the consumer to evaluate the value offered by each product to make a purchasing decision. The consumer's expected total consumption value from product i is

$$\mathbb{E}[V_i] = \bar{U}_i(\hat{\mu}) + \nu_i - p, \tag{3}$$

where $\bar{U}_i(\hat{\mu})$ represents the main expected utility from the primary attributes and ν_i the idiosyncratic values observed during store visits. Therefore, having observed (ν_1, ν_2) at the stores, the consumer's decision to purchase depends on the expected utility from the primary attributes:

$$\bar{U}_i(\hat{\mu}) := \hat{\mu} \cdot U_i(\omega_H) + (1 - \hat{\mu}) \cdot U_i(\omega_L), \tag{4}$$

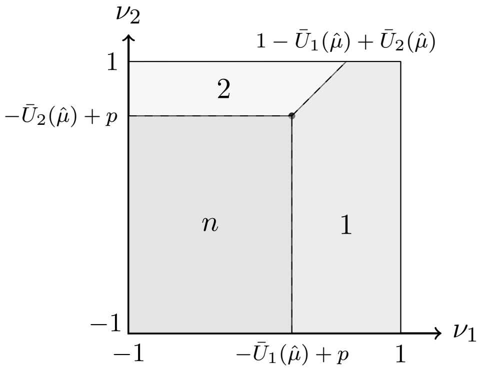
where $U_i(\omega_H) = \omega_H \alpha_i + (1 - \omega_H)\beta_i$, and $U_i(\omega_L) = \omega_L \alpha_i + (1 - \omega_L)\beta_i$, for each $i = 1, 2$.

The consumer's final purchasing decision hinges on their evaluation of expected utility from the main attributes, moderated by their belief $\hat{\mu}$. Upon visiting the retailer and observing the idiosyncratic values (ν_1, ν_2), the consumer weighs these values against the expected utility from the primary attributes, adjusted for the product's price. Specifically, the consumer buys product 1 if $\mathbb{E}[V_1(\nu_1; \hat{\mu})] = \bar{U}_1(\hat{\mu}) + \nu_1 - p$ exceeds both 0 (the value from the outside option) and $\mathbb{E}[V_2(\nu_2; \hat{\mu})] = \bar{U}_2(\hat{\mu}) + \nu_2 - p$; that is,

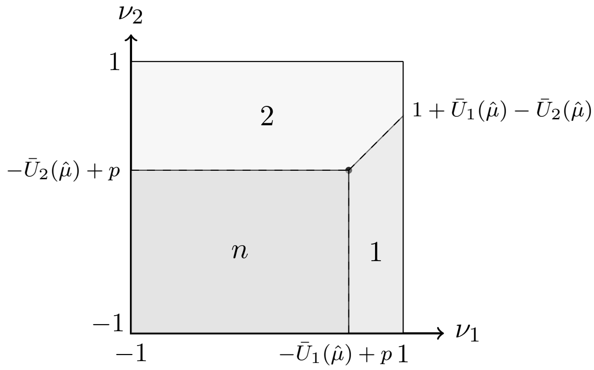
$$\mathbb{E}[V_1(\nu_1; \hat{\mu})] > \max\{0, \mathbb{E}[V_2(\nu_2; \hat{\mu})]\} \tag{5}$$

Similarly, the consumer buys product 2 if $\mathbb{E}[V_2(\nu_2; \hat{\mu})]$ surpasses both 0 and $\mathbb{E}[V_1(\nu_1; \hat{\mu})]$; that is, $\mathbb{E}[V_2(\nu_2; \hat{\mu})] > \max\{0, \mathbb{E}[V_1(\nu_1; \hat{\mu})]\}$. When neither product's adjusted utility convincingly outweighs the other, represented by $0 > \max\{\mathbb{E}[V_1(\nu_1; \hat{\mu})], \mathbb{E}[V_2(\nu_2; \hat{\mu})]\}$, the consumer opts for the outside option, signified by the "n" region in the figure.

Figure 2 illustrates the consumer's purchase zones based on $\hat{\mu}$. For a higher $\hat{\mu}$, implying greater expected utility from product 1's primary attributes, the area favoring product 1 expands



(a) Purchase decision for $\hat{\mu} \geq \frac{1}{2}$



(b) Purchase decision for $\hat{\mu} < \frac{1}{2}$

Figure 2: Consumer Purchasing Decision

(Figure 2-a). In contrast, a lower $\hat{\mu}$, indicating a preference for product 2's primary attributes, enlarges the purchase area for product 2 (Figure 2-b). This mechanism underscores the potential impact of firm messaging in shaping consumer posterior beliefs and subsequent purchasing decision.

The interplay between firm communication, consumer belief updating, and the ultimate market outcome of product adoption or avoidance is crucial. Moreover, the consumer's anticipation of this purchasing rule affects her choice to visit the stores based on her prior beliefs about ν_i's, highlighting the strategic importance of effective communication of the attribute significance.

Search decision

Anticipating the aforementioned purchasing rules, the consumer decides whether or not to visit the stores given the prior beliefs about the ν_i's. Her expected gain for visiting the store (i.e., $\psi(\hat{\mu}) = \text{visit}$) with a cost c is

$$W(\hat{\mu}) := \frac{1}{4} \int_{-1}^{1} \int_{-1}^{1} \max \Big\{ \bar{U}_1(\hat{\mu}) + \nu_1 - p, \bar{U}_2(\hat{\mu}) + \nu_2 - p, 0 \Big\} d\nu_1 d\nu_2, \tag{6}$$

where the factor of 1/4 is due to the pdf, 1/2, of the uniform distribution from which each ν_i is drawn.

The consumer chooses to incur a cost c in order to further engage with the product category by visiting the stores if and only if $W(\hat{\mu}) - c$ is greater than her expected payoff from not visiting the stores, which is zero, i.e., $W(\hat{\mu}) - c > 0$.[10] Thus, the expected gain from visiting,

[10] As a tie-breaking rule, we assume that the consumer does not visit the store and leaves the market when $W(\hat{\mu}) = c$.

$W(\hat{\mu})$, determines the consumer's decision to either engage further with the product category by incurring cost c, or opt out.

Before we establish an important property of $W(\hat{\mu})$ relative to the consumer's posterior beliefs about the attribute importance, we restrict the range of price such that the consumer's visit decision nontrivial. That is, the subsequent analysis assumes that

$$p \in (\omega\alpha_i + (1-\omega)\beta_i - 1,\ \omega\alpha_i + (1-\omega)\beta_i + 1),\ \text{for all } \omega \text{ and for each } i. \tag{7}$$

The implication is straightforward. If the price is so high that $p > \omega\alpha_i + (1-\omega)\beta_i + 1$, the consumer would never visit the stores because even the maximum $\nu_i = 1$ would not be sufficient to persuade her to buy the product. On the other hand, if the price is so low that p falls below $\omega\alpha_i + (1-\omega)\beta_i - 1$, the consumer would purchase a product regardless of a store visit, since even the lowest $\nu_i = -1$ would not prevent her from making a purchase. Thus, we focus on the scenarios where the price is in the intermediate range such that consumer visit decision is nontrivial.

Under this assumption, we characterize the consumer's value function $W(\hat{\mu})$ with the following lemma that reveals its simple yet robust characteristics:

Lemma 1. *The consumer's expected payoff from visiting the stores $W(\cdot)$ is symmetric about $\hat{\mu} = \frac{1}{2}$. Specifically, $W(\cdot)$ strictly decreases within the range $0 \leq \hat{\mu} < \frac{1}{2}$ and strictly increases within $\frac{1}{2} < \hat{\mu} \leq 1$.*

The symmetry of $W(\cdot)$ around $\hat{\mu} = \frac{1}{2}$ stems from the symmetry between the two firms, as illustrated in Figure 3. A more general result in the lemma is that $W(\cdot)$ has a U-shape. This implies that the consumer's expected value from visiting the store diminishes when there is greater uncertainty about the true state (i.e., $\hat{\mu}$ is in an intermediate range). Also, as the consumer's uncertainty decreases (i.e., $\hat{\mu}$ moves to an either end of the interval $[0, 1]$), the expected value from visiting increases. Essentially, the more informed the consumer is regarding the attribute importance ω, the greater the benefit from evaluating the idiosyncratic values ν_i's during a store visit. If the ambiguity about ω persist after observing the ν_i's, the consumer has to make her purchasing decision with considerable uncertainty about ω. Consequently, the consumer's posterior belief about ω significantly influences her willingness to further explore the product category.

Belief updating

Upon observing the firms' messages (m_1, m_2), the consumer updates her beliefs using Bayes' rule. She encounters four possible message combinations, namely $(m_1, m_2) \in \{h, l\} \times \{h, l\}$. In a truthful equilibrium scenario, the consumer's posterior beliefs, $\Pr[\omega = \omega_H | (m_1, m_2)] = \hat{\mu}_{m_1 m_2}$, are specified as:

$$
\begin{aligned}
&\hat{\mu}_{hh} = \frac{(1-\gamma)^2 \mu_0}{(1-\gamma)^2 \mu_0 + \gamma^2 (1-\mu_0)}, \quad \hat{\mu}_{ll} = \frac{\gamma^2 \mu_0}{\gamma^2 \mu_0 + (1-\gamma)^2 (1-\mu_0)}, \\
&\hat{\mu}_{hl} = \hat{\mu}_{lh} = \mu_0.
\end{aligned} \tag{8}
$$

The evaluation of the consumer's value function, given the three feasible sets of messages she might receive, depends on the level of uncertainty, assessed by how significantly the posterior beliefs diverge from the center point of belief, $1/2$, which represents the state of maximal uncertainty for the consumer. This deviation is influenced by the consumer's initial belief μ_0 and the level of noisiness γ present in each firm's signal. We focus on the truthful equilibrium, where the consumer's uncertainty is significantly reduced upon receiving consistent messages rather than conflicting ones, denoted by the conditions $|\, 1/2 - \hat{\mu}_{ll} \,| > |\, 1/2 - \mu_0 \,|$ and $|\, 1/2 - \hat{\mu}_{hh} \,| > |\, 1/2 - \mu_0 \,|$. We ensure this condition by setting

$$
\gamma < \min\{\, \mu_0,\, 1-\mu_0 \,\}. \tag{9}
$$

In contrast, if $\gamma \geq \min\{\, \mu_0,\, 1-\mu_0 \,\}$, it could lead to increased uncertainty for the consumer upon receiving consistent messages compared to when receiving conflicting ones. For example, if $\mu_0 > 1/2$ and $\gamma > \min\{\, \mu_0,\, 1-\mu_0 \,\}$, then $\mu_0 - 1/2 > |1/2 - \hat{\mu}_{ll}|$, implying the consumer's posterior belief after receiving (l, l) shifts closer to $1/2$ compared to her prior belief μ_0. Thus, when γ is high, indicating a significant amount of noise, the messages become less reliable as indicators of the true state diminishes. Consistent messages in such conditions might misrepresent the actual state due to the high noise level in the signal, leading to a paradoxical increase in the consumers' uncertainty. This effect is particularly pronounced when the consumer's initial belief μ_0 is already skewed towards one end of the belief spectrum. Noisy messages in this scenario tend to pull the posterior belief towards the midpoint, $1/2$, where uncertainty is greatest. Given this, we focus on scenarios where $\gamma < \min\{\, \mu_0,\, 1-\mu_0 \,\}$. This allows us to explore natural environments where messages serve as more reliable indicators of the true state, thus offering clearer insights into how consumer beliefs adjust in response to firm communications.

The following lemma maps the consumer's posterior beliefs onto the consumer's value func-

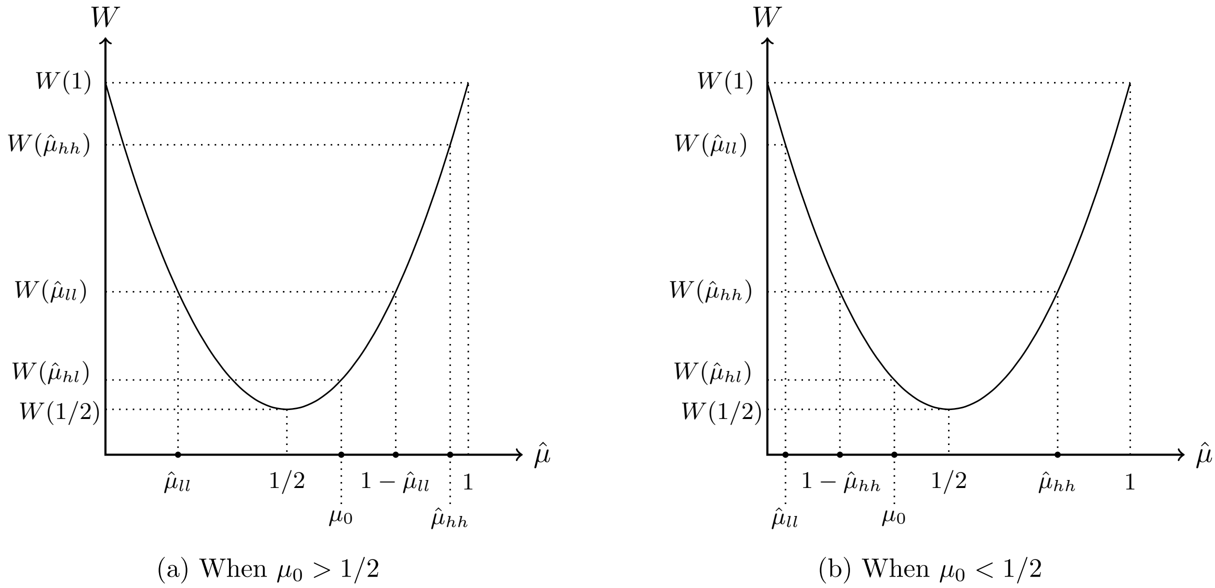


Figure 3: The expected gain for visiting the store: $W(\hat{\mu})$

tions and allows us to make comparisons, which helps us to characterize conditions under which the consumer decides to visit the stores in the truthful equilibrium.

Lemma 2. *The relationship among the posterior beliefs is as follows:* $\hat{\mu}_{ll} < \hat{\mu}_{lh} = \hat{\mu}_{hl} = \mu_0 < \hat{\mu}_{hh}$*. Given that* $\gamma < \min\{\mu_0, 1-\mu_0\}$*, we also find that:*

1. *For* $\mu_0 \geq 1/2$*, we have* $\hat{\mu}_{hh} - 1/2 \geq 1/2 - \hat{\mu}_{ll} > \hat{\mu}_{hl} - 1/2$*, leading to* $W(\hat{\mu}_{hh}) > W(\hat{\mu}_{ll}) > W(\hat{\mu}_{hl})$*.*
2. *For* $\mu_0 < 1/2$*, we have* $1/2 - \hat{\mu}_{ll} > \hat{\mu}_{hh} - 1/2 > 1/2 - \hat{\mu}_{hl}$*, thus* $W(\hat{\mu}_{ll}) > W(\hat{\mu}_{hh}) > W(\hat{\mu}_{hl})$*.*

Figure 3 illustrates $W(\hat{\mu}_{m_1m_2})$ for $\gamma < \min\{\mu_0,\, 1-\mu_0\}$. The left panel (Figure 3-a) is for $\mu_0 > 1/2$, reflecting a situation where the consumer's default belief is that the first attribute (α) is likely to be more significant. Receiving a consistent message (h, h) reinforces the consumer's confidence in the α-attribute's importance. However, a (l, l) message makes the consumer adjust her beliefs more negatively towards the state being ω_L, though with less conviction than after an (h, h) message. In scenarios where consistent messages (h, h) or (l, l) are received, the consumer's level of uncertainty is reduced compared to when conflicting messages (l, h) or (h, l) are encountered. The reduction in uncertainty with consistent messages — whether (h, h) or (l, l) — contrasts against the ambiguity stemming from conflicting messages (l, h) or (h, l), convinces the consumer with enhanced clarity, thus potentially increasing the expected value derived from visiting the stores.

However, the feasibility of such store visits is intricately tied to the cost c. The following lemma establishes a basic relationship between the cost of visiting stores and the consumer's engagement decision with the product category.

Lemma 3. *The consumer's decision to visit stores depends on the comparison between the visiting cost c and the expected value $W(\hat{\mu})$ derived from such visits. Specifically,*

1. *If $c > \max\{W(\hat{\mu}_{hh}), W(\hat{\mu}_{ll})\}$, the cost outweighs the benefits of visiting for all possible messages communicated to the consumer. Thus, the consumer never visits the firms and leaves the product category.*

2. *If $c \leq W(\mu_0)$, the expected benefits of visiting the stores surpass its cost for all possible messages that the consumer receives. Thus, the consumer chooses to visit the firms regardless of the messages received.*

The lemma illustrates the thresholds at which the cost of exploration becomes a barrier or an incentive to seeking additional information by visiting stores. When the cost c is prohibitively high, specifically $c \geq \max\{W(\hat{\mu}_{hh}), W(\hat{\mu}_{ll})\}$, then the consumer never finds it optimal to pay the cost and visit the stores. Consequently, a truthful equilibrium, as outlined in Definition 2, fails to emerge since the likelihood of the consumer making a purchase converges to zero. On the other hand, if $c < W(\mu_0) = W(\hat{\mu}_{hl}) = W(\hat{\mu}_{lh})$, the consumer finds it optimal to visit the stores across all scenarios, irrespective of receiving consistent or conflicting messages from the firms. In such cases, rather than conveying their private signal about the true state honestly, firms are incentivized to announce messages that highlight their competitive strengths — firm 1 favoring $m_1 = h$ and firm 2 leaning towards $m_2 = l$, independent of their actual signals $s_i \in \{H, L\}$. This scenario, nevertheless, prompts the consumer to consistently engage with the product category, driven not by a resolution of uncertainty regarding the true state but by the relatively minimal cost of exploration.

4.2 Equilibrium Results

Building on the results from the previous section, which illustrates how the cost of store visits influences consumer behavior and the firms' strategic messaging, we derive the core equilibrium results of our analysis. The lemma 3 sets a basic condition for the existence of a truthful equilibrium. Specifically, for a truthful equilibrium to exist, c must be in an intermediate range, namely, $W(\mu_0) \leq c \leq \max\{W(\hat{\mu}_{hh}), W(\hat{\mu}_{ll})\}$. This condition is crucial; outside this range, consumers either refrain from searching due to prohibitive costs or indiscriminately purchase based on highest ν, as firms are motivated to emphasize their competitive advantages through strategic messaging.

Next, we analyze the firms' expected profits and incentives. Let $\Pi_i^*(s_i)$ denote firm i's expected profit when it receives a noisy signal $s_i \in \{H, L\}$ in the truthful equilibrium. As the price is exogenous, each firm decides whether to report its signal honestly or dishonestly to maximize its expected demand. Given the symmetry of the two firms, it suffices to analyze firm 1's equilibrium conditions.

Consumer purchasing decisions are based on their beliefs about the true state ω, formed by the messages they receive from both firms. To calculate its expected profit, a firm must compute the conditional probability of the other firm's signal based on its own signal as follows:

$$\begin{aligned} \Pr(s_2 = H | s_1 = H) &= \frac{\mu_0 \cdot (1-\gamma)^2 + (1-\mu_0) \cdot \gamma^2}{\mu_0 \cdot (1-\gamma) + (1-\mu_0) \cdot \gamma} \\ \Pr(s_2 = L | s_1 = L) &= \frac{\mu_0 \cdot \gamma^2 + (1-\mu_0) \cdot (1-\gamma)^2}{\mu_0 \cdot \gamma + (1-\mu_0) \cdot (1-\gamma)}. \end{aligned} \tag{10}$$

Based on these probabilities, the firm predicts consumer response to the messages (m_1, m_2), which determines its demand and profit. If the firms send conflicting messages, the expected profit is zero. Conversely, if the messages align, the firm anticipates positive demand and profit, which is calculated based on consumer evaluations of the competing offers:

$$\begin{aligned} \pi_1^*(h,h) &= p \cdot \Pr\Big(\mathbb{E}[V_1(\nu_1; \hat{\mu}_{hh})] > \max\{\mathbb{E}[V_2(\nu_2; \hat{\mu}_{hh})], 0\}\Big) \\ \pi_1^*(l,l) &= p \cdot \Pr\Big(\mathbb{E}[V_1(\nu_1; \hat{\mu}_{ll})] > \max\{\mathbb{E}[V_2(\nu_2; \hat{\mu}_{ll})], 0\}\Big) \end{aligned} \tag{11}$$

where $\pi_i^*(m_1, m_2)$ is the firm's expected profit conditional when the consumer observes the messages (m_1, m_2). Note that its expected demand here corresponds to the area labeled "1" in Figure 2a (in case the consumer observes messages (h,h)) and in Figure 2b (for messages (l,l)).

When firm 1 receives a private signal s_1, its expected profit under the truthful equilibrium is:

$$\Pi_1^*(H) = \Pr(s_2 = H | s_1 = H) \cdot \pi_1^*(h,h), \text{ and } \Pi_1^*(L) = \Pr(s_2 = L | s_1 = L) \cdot \pi_1^*(l,l). \tag{12}$$

However, if firm 1 misreports its signal, it will only profit if firm 2 coincidentally sends a matching dishonest message:

$$\hat{\Pi}_1(H) = \Pr(s_2 = L | s_1 = H) \cdot \pi_1^*(l,l), \text{ and } \hat{\Pi}_1(L) = \Pr(s_2 = H | s_1 = L) \cdot \pi_1^*(h,h). \tag{13}$$

where $\hat{\Pi}_1(H)$ is the expected profit when firm 1 deviates by misreporting the message $m_i = l$ despite receiving $s_i = H$ and $\hat{\Pi}_1(L)$ is the expected profit when firm 1 deviates by misreporting the message $m_i = h$ after receiving $s_i = L$.

Intuitively, when firm 1 receives the favorable signal $s_i = H$, it has no incentive to mislead the consumer. However, if firm 1 receives the less favorable signal $s_i = L$, it might consider deviating from the truthful strategy to persuade the consumer that the α attribute is more significant, potentially gaining an advantage from this deception. To ensure the truthful equilibrium, we must demonstrate that

$$\Pi_1^*(L) = \Pr(s_2 = L|s_1 = L) \cdot \pi_1^*(l,l) \; > \; \hat{\Pi}_1(L) = \Pr(s_2 = H|s_1 = L) \cdot \pi_1^*(h,h) \tag{14}$$

The following proposition specifies the necessary and sufficient condition for the existence of a truthful equilibrium in this context:

Proposition 1. *There is a threshold $\bar{\gamma} > 0$ such that for $\gamma < \bar{\gamma}$, a truthful equilibrium exists if and only if c falls within the interval $[\underline{c}, \bar{c})$, where $\underline{c} = W(\mu_0)$ and $\bar{c} = \min\{\, W(\hat{\mu}_{hh}),\, W(\hat{\mu}_{ll})\,\}$. Moreover, the expected profits of both firms are positive in this equilibrium: $\mathbb{E}\Pi_1^* > 0$, $\mathbb{E}\Pi_2^* > 0$.*

This proposition demonstrates that the specified range for c and the threshold $\bar{\gamma}$ collectively determine the feasibility of achieving a truthful equilibrium where consumers make informed decisions based on accurate firm disclosures. For c in an intermediate range, the consumer will visit the stores if and only if the firms' messages are consistent, i.e., $(m_1, m_2) = (h, h)$ or (l, l). To have the consumer visit their stores, the firms find it optimal to announce their signals honestly because doing so allows the firms to coordinate on their announced messages. This is because their signals are positively correlated through the true state. This coordination between firms through honest announcement strategies partly resolves the consumer's uncertainty about the true state, thus inducing her visit to the stores and making both firms better off compared to the alternative case where the consumer does not visit.

However, we cannot assume that firms will always follow the equilibrium strategy and communicate truthfully. Specifically, there exists a potential incentive for each firm to deviate from the equilibrium strategy and strategically misrepresent information, highlighting an attribute (or state) where it perceives a competitive edge, regardless of the accuracy of its signal. For example, even when firm 2 communicates its signal truthfully ($m_2 = s_2$), firm 1 might always announce $m_1 = h$. This deviation could be profitable, especially if signal noisiness (γ) is high enough. This is because, in situations where the actual state is $\omega = \omega_L$, there's a non-negligible chance that firm 2 erroneously receives a signal $s_2 = H$. Under such circumstances, firm 1's dishonest deviation could lead to a successful coordination on firm 1's preferred message, leveraging the noise in the signals for strategic advantage. To prevent such deviations, the signals must not be excessively noisy – that is, $\gamma \leq \bar{\gamma}$. This restriction ensures that firms are disciplined to announce its message truthfully, even when it involves acknowledging a competitive shortcoming. With lower noise levels ($\gamma \leq \bar{\gamma}$), even the firm receives an unfavorable signal (for example, $s_1 = L$), it is still better off to report truthfully because both firms are likely to receive the same signals, reducing the incentives to deviate by reporting dishonestly. Even in less favorable scenarios, firms can still attract consumers with high enough idiosyncratic values. Thus, the honest reporting can still lead to purchases under these conditions as shown in $\Pr\Big(\mathbb{E}[V_1(\nu_1; \hat{\mu}_{ll})] > \max\{\mathbb{E}[V_2(\nu_2; \hat{\mu}_{ll})], 0\}\Big)$. Therefore, under this condition, truthful communication becomes the optimal choice for firms, and their collective messages can credibly convey information about the attribute importance, thereby encouraging store visits and purchase.

Finally, we can calculate the overall expected profit for a firm in the truthful equilibrium prior to receiving any signal as follows:

$$\begin{aligned}\mathbb{E}\Pi_1^* &= \Pr(s_1 = H) \cdot \Pi_1^*(H) + \Pr(s_1 = L) \cdot \Pi_1^*(L) \\ &= (\mu_0 \cdot (1-\gamma)^2 + (1-\mu_0) \cdot \gamma^2) \cdot \pi_1(h,h) + (\mu_0 \cdot \gamma^2 + (1-\mu_0) \cdot (1-\gamma)^2) \cdot \pi_1(l,l), \end{aligned} \quad (15)$$

confirming that expected profit is positive.

Although it may seem counterintuitive, firms often promote features where they don't hold a competitive edge. In the electric vehicle market, for example, despite Tesla's superior battery range, competitors like Nissan with its Leaf model and Chevrolet with the Bolt EV still focus on range in their marketing. Even if their vehicles do not match Tesla's performance, these companies emphasize battery range to signal its importance in the decision-making process for an electric vehicle. This strategy not only educates consumers about key product attributes but also helps to reduce buyer uncertainty and increases engagement and consideration (Moon and Darwall, 2002). Ultimately, by reinforcing the significance of these features, all firms contribute to expanding the overall market (Lu and Shin, 2018).

We now explore how the accuracy of the information, γ, influences the equilibrium profits of the firms. This analysis will further clarify the conditions under which the truthful equilibrium is effective and applicable.

Proposition 2. *Suppose that $\gamma < \bar{\gamma}$.*

1. *Conditional on both firms' sending congruent messages, each firm's equilibrium profit changes in γ as follows:*

(a) *When messages are congruent and favorable to the firm, profits decrease as γ increases:* $\frac{\partial \pi_1^*(h,h)}{\partial \gamma}, \frac{\partial \pi_2^*(l,l)}{\partial \gamma} \leq 0$.

(b) *When messages are congruent and unfavorable to the firm, profits increase as γ increases:* $\frac{\partial \pi_2^*(h,h)}{\partial \gamma}, \frac{\partial \pi_1^*(l,l)}{\partial \gamma} \geq 0$.

2. *The firm's overall expected profit prior to observing its signal, $\mathbb{E}\Pi_1$ decreases in γ.*

The first part of the proposition addresses how firm profits are influenced by the accuracy of their messages regarding an important attribute, α. For instance, when both firms send the message (h, h), suggesting that attribute α is significant, Firm 1's profit decreases with noisier messages (i.e., higher γ), indicated by $\frac{\partial,\pi_1^(h,h)}{\partial,\gamma} \leq 0$. Conversely, this same message indicates that the state is likely unfavorable for Firm 2, leading to an increase in its expected profit as message accuracy decreases, shown by $\frac{\partial,\pi_2^(h,h)}{\partial,\gamma} \geq 0$. The same logic also applies similarly to the message pair (l, l).

The second part of the proposition shows that the firm's *ex-ante* expected profit, $\mathbb{E}\Pi_1^*$, can decrease in γ. This decline is mainly due to how γ affects the correlation between the signals from the two firms. Specifically, a lower γ means the firms are more likely to receive the same signal (either $s_1 = s_2 = H$ or $s_1 = s_2 = L$), enhancing potential profits. For example, the probabilities $\Pr(s_1 = H, s_2 = H)$ and $\Pr(s_1 = L, s_2 = L)$ decrease with increasing γ,[11] leading to less frequent congruent signaling. If the signals conflict, the consumer typically does not respond, resulting in zero profit ($\pi_i^*(h,l) = \pi_i^*(l,h) = 0$). Although $\pi_1^*(l,l)$ increases with γ as previously noted, this benefit does not outweigh the negative impacts on the other components of $\mathbb{E}\Pi_1^*$, leading to an overall decrease in expected profit.

Comparative statics

To further refine our understanding, this section explores how changes in key model parameters affect the conditions for a truthful equilibrium. Specifically, the proposition below analyzes how the interval $[\underline{c}, \bar{c}]$, which defines the range of a truthful equilibrium, shifts in response to variations in the parameters γ, p, and u_H $(= \alpha_H = \beta_H)$.

Proposition 3. *The boundaries $\underline{c}$ and $\bar{c}$ exhibit the following characteristics:*

1. *$\bar{c}$ strictly decreases in γ, whereas $\underline{c}$ is independent of γ. Hence, $\bar{c} - \underline{c}$ decreases in γ.*

2. *$\bar{c}$, $\underline{c}$, and $\bar{c} - \underline{c}$ all strictly increase in u_H, where $u_H = \alpha_H = \beta_H$.*

[11]We can directly see this in that $\Pr(s_1 = s_2 = H) = \mu_0 \cdot (1-\gamma)^2 + (1-\mu_0) \cdot \gamma^2$ and $\Pr(s_1 = s_2 = L) = \mu_0 \cdot \gamma^2 + (1-\mu_0) \cdot (1-\gamma)^2$ are increasing in γ for $\gamma < \{\mu_0, 1-\mu_0\}$.

3. $\bar{c}$, $\underline{c}$, and $\bar{c} - \underline{c}$ all strictly decrease in p.

Recall that the lower bound $\underline{c} = W(\mu_0) = W(\mu_{lh})$ represents the consumer's expected utility from searching based on the prior beliefs. The upper bound $\bar{c} = \min\{W(\mu_{hh}), W(\mu_{ll})\}$ represents the consumer's expected utility from visiting the stores given consistent messages. Thus, the difference $\bar{c} - \underline{c}$, or the interval where a truthful equilibrium exists, represents the *marginal value of information* about the attribute importance beyond the prior, communicated through consistent messages from both firms. A higher γ (i.e., noisier signals) reduces $\bar{c}$ due to increased uncertainty while leaving $\underline{c}$ unaffected. Therefore, the difference $\bar{c} - \underline{c}$ decreases in γ, suggesting that less information about the true state ω is communicated, making the consumer less likely to engage in search.

If the level of each firm's stronger attribute, $u_H = \alpha_H = \beta_H$, increases (while holding everything else constant), it is clear that $W(\hat{\mu})$ would also increase for any $\hat{\mu}$.[12] Thus, both $\bar{c}$ and $\underline{c}$ increase in u_H. Furthermore, $\bar{c} - \underline{c}$ also increases in u_H because the value of information about the attribute importance becomes more critical to the consumer. For instance, if $\omega = \omega_H$, the consumer would prefer to buy from firm 1, as the attribute α is more important in this state, and firm 1 offers higher quality in that attribute (i.e., $\alpha_1 = \alpha_H > \alpha_2 = \alpha_L$). This incentives would amplify when the level of the stronger attribute α_H were higher.

The cutoffs $\bar{c}$ and $\underline{c}$ both decrease in p; this reflects that the benefit from search is smaller when consumers face higher prices, irrespective of the consumer's beliefs $\hat{\mu}$. Similarly, the difference $\bar{c} - \underline{c}$ also decreases in price p. This is because the higher prices reduce the net benefit of making a purchase, diminishing the consumer's overall incentives to visit stores. This direct effect of price dominates any indirect effects where higher prices might otherwise motivate consumers to become more informed about the attribute importance and identify a better product.

4.3 Other Equilibria

While our analysis has primarily focused on a truthful equilibrium, we also identify other types of pure strategy equilibria. First, there exists a *pooling equilibrium*, where each firm consistently sends the same message, regardless of the actual state (e.g., firm 1 *always* sends h and firm 2 *always* sends l). Second, there also exists a hybrid type of Perfect Bayesian Equilibrium, *semi-separating equilibrium*, where one firm sends the same message regardless of the state, and the

[12] One can see directly from Equations (4) and (6) that $W(\hat{\mu})$ increase in u_H because $U_i(\omega)$ increases in α_H for all ω.

other firm chooses a different message based on the state (e.g. firm 1 always sends h, and firm 2 sends h when $s_2 = H$ and l when $s_2 = L$).

Proposition 4. *Equilibria other than the truthful separating equilibrium may exist:*

1. *Pooling equilibrium always exists. However, consumers do no purchase in this pooling equilibrium when the condition for a separating equilibrium are met, specifically when* $\gamma < \bar{\gamma}$ *and* $\underline{c} \leq c < \bar{c}$.

2. *Semi-separating equilibrium does not exist except when* $\mu_0 = 1/2$.

In pooling equilibrium, each firm sends the same message regardless of its private signal s_i, thereby failing to convey additional information about the true state ω to the consumer. While the equilibrium exists under all parameter values, consumers never purchase if $c > \underline{c}$, as their uncertainty is too significant to justify paying the cost c for further engaging with the category, resulting in no transaction occurs and zero profit for the firms.

Moreover, there can be a semi-separating equilibrium, which is rare and only exists when μ_0 is precisely 1/2. If one firm's message is completely uninformative, the consumer relies solely on the message from the other firm, which is adopting the separating strategy. In this case, the informative firm has no incentive to coordinate with its competitor and is thus tempted to deviate by highlighting its stronger attribute. This deviation is profitable, thus eliminating the semi-separating equilibrium except when $\mu_0 = 1/2$. Therefore, the separating equilibrium, where information about the attribute is effectively communicated, is the only equilibrium for all $\mu_0 \neq 1/2$, and it is also the only equilibrium in which the transaction occurs with a positive probability.

Impact of competition on messaging and credibility

One may posit that the presence of competing firms with intrinsic desires to highlight opposite attributes excessively restricts each firm's ability to choose its message and eventually make each firm worse off. To analyze this concern, we consider the case in which only one of the firm (say firm 1 without loss of generality) has the opportunity, resources, or right to send a message to the consumer, while firm 2 does not. In this case, we find that a truthful equilibrium, where firm 1 reports its observed signal honestly, does not occur in the current one-shot game.[13] Consequently, critical information about the importance of attributes is not effectively communicated to the consumer. The following proposition establish this this finding.

[13] In a repeated game, Wernerfelt (1994) demonstrates that reputational considerations can maintain honesty in sales communication, even in monopoly scenarios. By contrast, this paper considers a one-shot game where

Proposition 5. *If firm 2 cannot send a message, a truthful equilibrium does not exist.*

The proposition shows that when one firm monopolizes the communication channel by excluding the other firm, its message lose credibility, and the firm becomes worse off than in a scenario where both firms could engage in communication with the consumer. This occurs because the sole communicating firm cannot commit to announcing the message truthfully. It faces a temptation to overemphasize its favored attribute, thus losing the credibility of its announcement. This behavior mirrors the reasons behind the non-existence of a semi-separating equilibrium, where reliance on a single firm for information leads to communication that lacks credibility. When only one firm communicates, it may not have sufficient incentive or ability to convey accurate attribute information, potentially leaving the consumer less informed. Therefore, the presence of competing firms, each able to communicate, is crucial. Their mutual competition disciplines each firm's communication strategies and eventually grants credibility in the market-wide communication between firms and consumers.[14]

5 Extension

So far, we have assumed that the price of product i is exogenously given as p. This section revisits this assumption and examine whether a truthful equilibrium can still exist when pricing becomes endogenous. Our primary purpose here is to establish robustness of our main findings.

Consider the following timeline of a game in which each firm chooses a price endogenously. The true state $\omega \in \{\omega_H, \omega_L\}$ is drawn from the same distribution as before. Each firm privately receives an independent signal $s_i \in \{H, L\}$ about ω. The firms then simultaneously send a message and set their prices, (m_1, p_1), (m_2, p_2). Then, the consumer observes (m_1, p_1), (m_2, p_2) and decides whether to incur the cost c to further investigate the products (ν_1, ν_2). Finally, based on this additional information, the consumer decides whether to purchase either product 1 or 2, or neither. For tractability, we assume that $\gamma = 0$ such that both firms always accurately learn the true state, i.e., $s_i = s = H$ if $\omega = \omega_H$ and $s_i = s = L$ if $\omega = \omega_L$.

In the previous section, we established that each firm's objective was focused on maximizing the likelihood of consumers purchase. However, in this section, each firm maximizes its expected profit through its price and message. The consumer updates her beliefs about ω based on both

the reputation effects seen in repeated games (Jullien and Park, 2014; Kreps and Wilson, 1982) are absent. Without these reputation effects, truthful communication would not naturally occur. Therefore, in our model, the credibility of communication is ensured by the presence of competing firms, which mutually enforce discipline.

[14] Several studies have also identified competition as a key factor in enhancing firm profitability. These studies suggest that competition can be beneficial by either alleviating double-marginalization (Harutyunyan and Jiang, 2019), by reducing the communication costs (Lu and Shin, 2018), or by reducing the intensity of competition (Shin, 2007).

the messages (m_1, m_2) and the prices (p_1, p_2), which then affects the consumer's decision whether to visit the stores *and* make purchases.

While the two firms may choose different prices, we focus on the existence of an extended version of truthful equilibrium in which both firms choose the identical price p^* such that $p_i(s) = p^*$ for $i = 1, 2$ and $s = H, L$. More specifically, we explore a following strategy:

Strategy. *Firm i sends (h, p) when he receives $s_i = H$ and sends (l, p) when he receives $s_i = L$.*

We define the price range that can support a truthful equilibrium, similar to the case with exogenous pricing (Equation (7)), as follows:

$$\begin{aligned} -1 &< \omega_H u_H + \omega_L u_L - p^* < 1 \\ -1 &< \omega_L u_H + \omega_H u_L - p^* < 1 \end{aligned} \tag{16}$$

If $p^* < \omega_H u_H + \omega_L u_L - 1$, the price is so low that the consumer, believing the state to be $\omega = \omega_H$, would opt to buy the less preferred product 2 even if the idiosyncratic match value is minimal, i.e., $\nu_2 = -1$. Thus, firm 2 will find it profitable to deviate from a truthful strategy above and send (l, p) even if $s_i = H$. Similarly, if $p^* < \omega_L u_H + \omega_H u_L - 1$, the price is sufficiently low for a consumer, believing the state to be $\omega = \omega_L$, to buy the less preferred product 1 even if $\nu_1 = -1$. Thus, firm 1 has a profitable deviation by sending a message (h, p) when the true state (and the signal) is $s_i = L$. Conversely, if the price is so high that $p > \omega_H u_H + \omega_L u_L + 1$, the consumer will not buy her preferred product 1 even if $\nu_1 = 1$, leading to zero probability of purchase. This violates the conditions for a truthful equilibrium, which requires a positive probability of purchasing each product under both states $s_i = H$ and L.

The following proposition shows that this strategy can constitute the equilibrium. Specifically, an equilibrium where both firms choose the same price and report their observed signals honestly can be sustained.

Proposition 6. *There exisit constants $\bar{c}$, $\underline{c}$ and a non-empty set S, such that the above strategy constitutes an equilibrium when $\underline{c} \leq c < \bar{c}$ and $(p, u_H, u_L, \omega_H) \in S$.*

This proposition illustrates that when the conditions of market dynamics and firm strategies fall within a defined set, it is feasible for firms to maintain a unified pricing strategy while truthfully communicating their information. The existence of an intermediate interval with endpoints $\bar{c}$ and $\underline{c}$ is similar to the results from the main model with exogenous pricing. If $c > \bar{c}$, then the consumer decides not to explore the category, thus violating the definition of truthful equilibrium. If $c < \underline{c}$, then the consumer will explore the category irrespective of the firms' messages. Thus, each firm finds it profitable to deviate from the strategy defined above and send a message which will induce the consumer's belief more favorable to the firm while ignoring the truly more important attribute. The defined set S represents specific parameter values. While this analysis focuses on the specific scenario where both firms choose identical prices, it serves as a key example demonstrating the robustness of our main findings about truthful equilibrium even when firms choose their prices endogenously.

6 Conclusion

This study has demonstrated how strategic communication between competing firms can significantly influence consumer understanding and engagement with product attributes in a market. We consider the scenario where firms communicate non-verifiable information about product attributes under competition. Each firm inherently seeks to emphasize its distinct competitive advantage in a particular attribute. Despite these competitive pressures, we find that firms are inclined to cooperate by truthfully communicating their information. Truthful messages increase the likelihood of consistent consumer messages, clarifying attribute importance and enhancing consumer beliefs. This encourages consumers to engage more deeply with the product category, which they might otherwise avoid, making both firms better off. Conversely, if a firm misrepresents its competitive attribute, it may likely to lead to inconsistent messages between firms, creating consumer confusion about which attributes are important. This misalignment can deter consumer interaction with the product category, ultimately harming both firms.

Interestingly, our findings suggest that firms benefit from competition because it disciplines their communication strategies. As the sole communicator or monopolist in the product market, the firm can send any message without worrying about what the competitor might say. Witout any mechanism to restrict or verify its communication, the firm's message loses its credibility completely. Therefore, in the absence of a competing firm (or, a competing firm's communication), no information can be conveyed to the consumer, often resulting in no transactions within a wide range of parameters. Therefore, competition serves as a commitment device that enables firms to credibly communicate the importance of product attributes. It is also crucial to recognize that effective competition requires conflicting interests; a competing firm with aligned incentives does not enhance the credibility of communications. In essence, the presence of competition not only enhances the credibility of the information provided but also ensures that firms discipline each other. This finding underscores the beneficial role of competition in markets where firms might otherwise monopolize communication and potentially mislead consumers.

This study has several limitations. First, our analysis extends to scenarios with endogenous pricing, affirming the robustness of our main findings. However, this approach assumes identical pricing between firms to streamline our exploration of endogenous effects. Future research could relax this assumption to fully investigate varying pricing strategies among firms, potentially offering deeper insights into how market competition influences communication and consumer behavior. Second, the accuracy of the firm's signal, denoted as γ, could be made endogenous. Currently, γ can be regarded as the results of the level of effort a firm puts into understanding the most important attributes of their product, such as through market testing or consumer research. Future research could explore how endogenizing γ affects communication strategies and the precision of information shared. This would also provide insights into the optimal level of information precision and the firm's incentives whether to minimize or maximize the uncertainty in its knowledge about attribute importance. Also, we assume that the importance of product attributes is homogeneous for all the consumers. However, different consumers might prioritize different attributes. Investigating scenarios with heterogeneous attribute importance (ω) across consumers could be valuable. Lastly, our analysis assumes consumers are well-informed about the attributes themselves and focuses on the communication of their importance. Future studies could investigate scenarios where consumers also have uncertainties about the attributes, which would add complexity to the credibility of communication. These are important areas for future research, offering significant opportunities to expand our understanding of communication strategies.

APPENDIX

A Proofs

A.1 Preliminary

For simplicity of notation, denote

$$s := U_1(\omega_H) - p = U_2(\omega_L) - p, \text{ and } t := U_1(\omega_L) - p = U_2(\omega_H) - p. \tag{17}$$

since $U_1(\omega_H) > U_1(\omega_L)$ and $U_i(\omega) - p \in (-1, 1)$ from our assumption, we have

$$-1 < t < s < 1.$$

Thus, for any $\hat{\mu} \in [0, 1]$, we have

$$\begin{aligned}
&\bar{U}_1(\hat{\mu}) - p = \hat{\mu} \cdot s + (1 - \hat{\mu}) \cdot t \in (-1, 1), \\
&\bar{U}_2(\hat{\mu}) - p = \hat{\mu} \cdot U_2(\omega_H) + (1 - \hat{\mu}) \cdot U_2(\omega_L) - p = (1 - \hat{\mu}) \cdot s + \hat{\mu} \cdot t \in (-1, 1).
\end{aligned}$$

Next, denote

$$X := \big(-\bar{U}_1(1) + p, -\bar{U}_2(1) + p\big) = (-s, -t) \ , \quad Y := \big(-\bar{U}_1(0) + p, -\bar{U}_2(0) + p\big) = (-t, -s).$$

Then, on a ν_1-ν_2 plane, the point X lies to the left of $\nu_2 = \nu_1$ and Y to the right of $\nu_2 = \nu_1$. Also, X and Y are symmetric with respect to $\nu_2 = \nu_1$.

Now, let $P(\hat{\mu}) := \big(-\bar{U}_1(\hat{\mu}) + p, -\bar{U}_2(\hat{\mu}) + p\big)$. Then, $P(\hat{\mu})$ is the point of internal division of $\overline{XY}$ in the ratio of $1 - \hat{\mu} : \hat{\mu}$, from (4). Thus, as $\hat{\mu}$ increases, $P(\hat{\mu})$ approaches closer to X along $\overline{XY}$. Therefore, in the truthful equilibrium, as $\hat{\mu}$ increases, the consumer's probability of purchasing product 1 increases, while the probability of purchasing product 2 decreases. Figure 4 shows the position of X, Y, and $P(\hat{\mu})$.

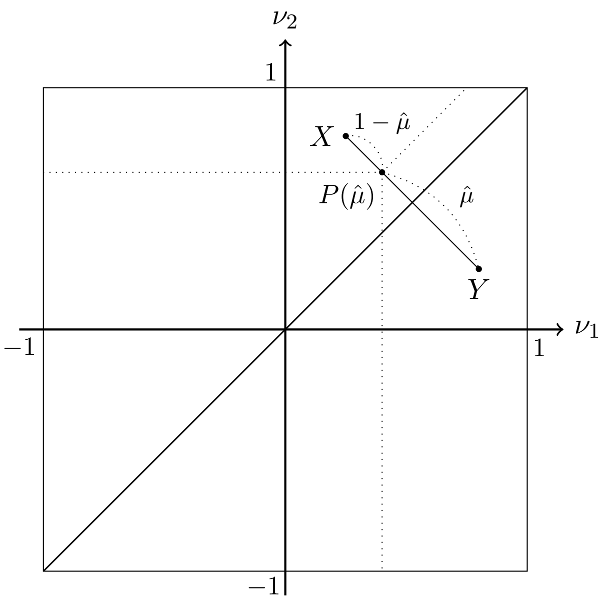


Figure 4: Position of X, Y, and P

A.2 Proof of Lemma 1

STEP 1: Preliminary

Consider $x, y \in (-1, 1)$. Define

$$f(x,y) := \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \max\{x+\nu_1, y+\nu_2, 0\} d\nu_1 d\nu_2. \tag{18}$$

Then, we have

$$W(\hat{\mu}) = f\left(\bar{U}_1(\hat{\mu}) - p, \bar{U}_2(\hat{\mu}) - p\right).$$

If $x \leq y$, we have

$$\begin{aligned}
f(x,y) &= \frac{1}{4}\iint_{x+\nu_1 \geq y+\nu_2,\ 0} (x+\nu_1) d\nu_1 d\nu_2 + \frac{1}{4}\iint_{y+\nu_2 \geq x+\nu_1,\ 0} (y+\nu_2) d\nu_1 d\nu_2 \\
&= \frac{1}{4}\left(\int_{-x}^{1}\int_{-1}^{x-y+\nu_1} (x+\nu_1) d\nu_2 d\nu_1 + \int_{-y}^{1+x-y}\int_{-1}^{-x+y+\nu_2} (y+\nu_2) d\nu_1 d\nu_2 + \int_{1+x-y}^{1}\int_{-1}^{1} (y+\nu_2) d\nu_1 d\nu_2\right) \\
&= \frac{1}{24}(x+1)^2(2x-3y+5) - \frac{1}{24}(x-5)(x+1)^2 + \frac{1}{4}(-x^2-2x+y^2+2y) \\
&= \frac{1}{24}(x^3 - 3x^2y + 6x^2 - 6xy + 9x + 6y^2 + 9y + 10).
\end{aligned}$$

Note that the above expression is strictly positive. To see that, we note that

$$\frac{\partial}{\partial x} f(x,y) = \frac{1}{8}(x+1)(x-2y+3) > 0,$$

where the last inequality holds from $x, y \in (-1, 1)$. Thus, we have

$$0 < \frac{1}{4}(y+1)^2 = f(-1,y) < f(x,y).$$

Similarly, for $x > y$, we have

$$f(x,y) = \frac{1}{24}(y^3 - 3xy^2 + 6y^2 - 6xy + 9y + 6x^2 + 9x + 10) > 0.$$

Therefore, we have $f(x,y) = f(y,x)$.

Step 2: Properties of W

Recall s and t from (17). For $\hat{\mu} > \frac{1}{2}$ (so $\bar{U}_1(\hat{\mu}) > \bar{U}_2(\hat{\mu})$), we have

$$\begin{aligned}
W'(\hat{\mu}) &= \frac{\partial}{\partial \hat{\mu}} f\left(\bar{U}_1(\hat{\mu}) - p, \bar{U}_2(\hat{\mu}) - p\right) \\
&= \frac{1}{4}(s-t)\Big(- (\bar{U}_2(\hat{\mu}) - p)^2 - 3(\bar{U}_2(\hat{\mu}) - p) + 3(\bar{U}_1(\hat{\mu}) - p) + (\bar{U}_1(\hat{\mu}) - p)(\bar{U}_2(\hat{\mu}) - p)\Big) \\
&= \frac{1}{4}(s-t)\Big(- 2(s-t)^2\hat{\mu}^2 + (s-t)(3s-t+6)\hat{\mu} - (s-t)(s+3)\Big) \\
&= -\frac{1}{4}(s-t)^2(2\hat{\mu}-1)\Big((s-t)\hat{\mu} - (s+3)\Big).
\end{aligned}$$

Note that $W'(\hat{\mu})$ is a quadratic function of $\hat{\mu}$ with a negative coefficient for the squared term. And since

$$W'\left(\frac{1}{2}\right) = 0, \; W'(1) = \frac{1}{4}(s-t)^2(t+3) > 0,$$

so W' must be positive on $\left(\frac{1}{2}, 1\right)$. Therefore, $W(\hat{\mu})$ is strictly increasing on $\left(\frac{1}{2}, 1\right)$.

Meanwhile, from (4), we have

$$\bar{U}_1(\hat{\mu}) = \bar{U}_2(1-\hat{\mu}), \; \bar{U}_2(\hat{\mu}) = \bar{U}_1(1-\hat{\mu}).$$

Thus, we have

$$
\begin{aligned}
W(\hat{\mu}) = f\left(\bar{U}_1(\hat{\mu}) - p, \bar{U}_2(\hat{\mu}) - p\right) &= f\left(\bar{U}_2(\hat{\mu}) - p, \bar{U}_1(\hat{\mu}) - p\right) \\
&= f\left(\bar{U}_1(1-\hat{\mu}) - p, \bar{U}_2(1-\hat{\mu}) - p\right) = W(1-\hat{\mu}).
\end{aligned}
$$

Thus, W is symmetric around $\hat{\mu} = \dfrac{1}{2}$.

Finally, since

$$W''(\hat{\mu}) = -\frac{1}{4}(s-t)^2\Big(4(s-t)\hat{\mu} - (3s - t + 6)\Big),$$

and

$$W''\left(\frac{1}{2}\right) = \frac{1}{4}(s-t)^2(s+t+6) > 0,\ W''(1) = \frac{1}{4}(s-t)^2(-s+3t+6) > 0,$$

so W'' must be positive on $\left(\frac{1}{2}, 1\right)$. Therefore, W has a U-shape.

A.3 Proof of Lemma 2

Note that

$$
\begin{aligned}
\mu_0 < \hat{\mu}_{hh} \iff & \mu_0 < \frac{(1-\gamma)^2\mu_0}{(1-\gamma)^2\mu_0 + \gamma^2(1-\mu_0)} \\
\iff & (1-\gamma)^2\mu_0 + \gamma^2(1-\mu_0) < (1-\gamma)^2 \\
\iff & \gamma^2(1-\mu_0) < (1-\gamma)^2(1-\mu_0) \\
\iff & \gamma < (1-\gamma) \iff \gamma < \frac{1}{2}.
\end{aligned}
$$

Similarly, we have

$$\hat{\mu}_{ll} < \mu_0 \iff \gamma < \frac{1}{2}.$$

Since $\gamma < \frac{1}{2}$, so we have $\hat{\mu}_{ll} < \mu_0 < \hat{\mu}_{hh}$.

Step 1: $\mu_0 \geq \frac{1}{2}$ case

First,

$$1-\hat{\mu}_{ll} \leq \hat{\mu}_{hh} \iff \frac{(1-\gamma)^2(1-\mu_0)}{\gamma^2\mu_0+(1-\gamma)^2(1-\mu_0)} \leq \frac{(1-\gamma)^2\mu_0}{(1-\gamma)^2\mu_0+\gamma^2(1-\mu_0)}$$
$$\iff (1-\gamma)^2\mu_0(1-\mu_0)+\gamma^2(1-\mu_0)^2 \leq \gamma^2\mu_0^2+(1-\gamma)^2\mu_0(1-\mu_0)$$
$$\iff 1-\mu_0 \leq \mu_0 \iff \frac{1}{2} \leq \mu_0.$$

Second,

$$\hat{\mu}_{hl} < 1-\hat{\mu}_{ll} \iff \mu_0 < \frac{(1-\gamma)^2(1-\mu_0)}{\gamma^2\mu_0+(1-\gamma)^2(1-\mu_0)}$$
$$\iff \gamma^2\mu_0^2+(1-\gamma)^2\mu_0(1-\mu_0) < (1-\gamma)^2(1-\mu_0)$$
$$\iff \gamma^2\mu_0^2 < (1-\gamma)^2(1-\mu_0)^2$$
$$\iff \gamma\mu_0 < (1-\gamma)(1-\mu_0) \iff \gamma < 1-\mu_0.$$

Step 2: $\mu_0 < \frac{1}{2}$ case
It is similar to Step 1, and we have $1-\hat{\mu}_{ll} \geq \hat{\mu}_{hh}$ always, and $\hat{\mu}_{hh} > \hat{\mu}_{hl}$ if and only if $\gamma < \mu_0$. This completes the proof.

A.4 Proof of Proposition 1

Step 1: $c \geq \max\{W(\hat{\mu}_{ll}), W(\hat{\mu}_{hh})\}$.
In this case, the consumer does not observe (ν_1, ν_2) regardless of the message. Thus, the consumer does not purchase the product 1 or 2. Since the definition of truthful equilibrium includes the condition that the consumer must purchase the product 1 or 2 with strictly positive probability, therefore, the truthful equilibrium does not exist.

Step 2: $\min\{W(\hat{\mu}_{ll}), W(\hat{\mu}_{hh})\} \leq c < \max\{W(\hat{\mu}_{ll}), W(\hat{\mu}_{hh})\}$
Without loss of generality, assume that $W(\hat{\mu}_{ll}) < W(\hat{\mu}_{hh})$. That is, $W(\hat{\mu}_{ll}) \leq c < W(\hat{\mu}_{hh})$.

Suppose that there is a truthful equilibrium and firm 1 observes $s_1 = L$. Due to observational error, firm 2 would have observed either $s_2 = L$ or $s_2 = H$. Therefore, if firm 1 send l, the consumer will receive either (l, l) or (l, h). In both cases, the consumer will leave the market because $W(\hat{\mu}_{lh}) < W(\hat{\mu}_{ll}) \leq c$. Thus, the firm 1's expected payoff when sending l is zero. However, if firm 1 sends h, the consumer receives (h, h) with strictly positive probability. And since $c < W(\hat{\mu}_{hh})$, the consumer will visit the store and purchase the product 1 with strictly positive probability. Therefore, firm 1 will deviate from l to h.

STEP 3: $W(\mu_0) \le c < \min\{W(\hat{\mu}_{ll}), W(\hat{\mu}_{hh})\}$

Consider the truthful equilibrium strategy. We will show that the firms do not deviate from their strategies. Without loss of generality, we assume that $\mu_0 \ge \frac{1}{2}$. Also, note that $\pi_1(h,l) = \pi_1(l,h) = 0$, since $W(\hat{\mu}_{hl}) = W(\mu_0) \le c$.

STEP 3.1: Firm 1

(i) Suppose that firm 1 observes $s_1 = H$. Since $\hat{\mu}_{hh} > \hat{\mu}_{ll}$, according to the last paragraph of Section A.1, the consumer's probability of purchasing product 1 when receiving (h,h) is weakly larger than when receiving (l,l). That is, $\pi_1(h,h) > \pi_1(l,l)$.

Meanwhile, the firm 1's expected payoff for using h and l are

$$\Pi_1^*(H) = \underbrace{\frac{\mu_0(1-\gamma)^2 + (1-\mu_0)\gamma^2}{\mu_0(1-\gamma) + (1-\mu_0)\gamma}}_{\Pr(s_2=H|s_1=H)} \cdot \pi_1(h,h)\ , \quad \hat{\Pi}_1(H) = \underbrace{\frac{\gamma(1-\gamma)}{\mu_0(1-\gamma) + (1-\mu_0)\gamma}}_{\Pr(s_2=L|s_1=H)} \cdot \pi_1(l,l),$$

respectively. Note that $\mu_0 \cdot (1-\gamma)^2 + (1-\mu_0) \cdot \gamma^2 > \gamma \cdot (1-\gamma)$ must hold for $\gamma < \min\{\mu_0, \frac{1}{2}\}$, since

$$\begin{aligned} \mu_0(1-\gamma)^2 + (1-\mu_0)\gamma^2 > \gamma(1-\gamma) &\iff \mu_0 - 2\gamma\mu_0 + \gamma^2 > \gamma - \gamma^2 \\ &\iff (\mu_0 - \gamma)(1-2\gamma) > 0. \end{aligned}$$

Therefore, for $\gamma < \min\{\mu_0, \frac{1}{2}\}$, the first term is larger than the second term, so firm 1 does not deviate to l.

(ii) Suppose that firm 1 observes $s_1 = L$. Similar to (i), the firm 1's expected payoff for using h and l are

$$\hat{\Pi}_1(L) = \underbrace{\frac{\gamma(1-\gamma)}{(1-\mu_0)(1-\gamma) + \mu_0\gamma}}_{\Pr(s_2=H|s_1=L)} \cdot \pi_1(h,h)\ , \quad \Pi_1^*(L) = \underbrace{\frac{(1-\mu_0)(1-\gamma)^2 + \mu_0\gamma^2}{(1-\mu_0)(1-\gamma) + \mu_0\gamma}}_{\Pr(s_2=L|s_1=L)} \cdot \pi_1(l,l),$$

respectively. Since $\pi_1(h,h) > \pi_1(l,l)$, for firm 1 not to deviate, $\Pr(s_2 = H|s_1 = L)$ must be sufficiently smaller than $\Pr(s_2 = L|s_1 = L)$.

Observe that as γ decreases, $\Pr(s_2 = H|s_1 = L)$ converges to 0 and $\Pr(s_2 = L|s_1 = L)$ converges to 1. Moreover, observe that as γ decreases, $\hat{\mu}_{hh}$ converges to 1 and $\hat{\mu}_{ll}$ converges to 0, which implies that $P(\hat{\mu}_{hh})$ converges to X and $P(\hat{\mu}_{ll})$ converges to Y (P, X, and Y are defined in Section A.1). Thus, $\pi_1(h,h)$ and $\pi_1(l,l)$ converge to positive constants. Therefore, for a sufficiently small γ, the second term is larger than the first term, so firm 1 does not deviate to h. That is, there is a $\bar{\gamma}_1 > 0$ such that for $\gamma < \bar{\gamma}_1$, firm 1 does not deviate from the truthful equilibrium strategy.

Step 3.2: Firm 2

(i) Suppose that firm 2 observes $s_2 = L$. Then the firm 2's expected payoff for using h and l are

$$\hat{\Pi}_2(L) = \underbrace{\frac{\gamma(1-\gamma)}{(1-\mu_0)(1-\gamma)+\mu_0\gamma}}_{\Pr(s_1=H|s_2=L)} \cdot \pi_2(h,h)\ ,\quad \Pi_2^*(L) = \underbrace{\frac{(1-\mu_0)(1-\gamma)^2+\mu_0\gamma^2}{(1-\mu_0)(1-\gamma)+\mu_0\gamma}}_{\Pr(s_1=L|s_2=L)} \cdot \pi_2(l,l),$$

respectively. Note that $(1-\mu_0)(1-\gamma)^2 + \mu_0\gamma^2 > \gamma(1-\gamma)$ must hold for $\gamma < \min\{1-\mu_0, \frac{1}{2}\}$, since

$$(1-\mu_0)(1-\gamma)^2 + \mu_0\gamma^2 > \gamma(1-\gamma) \iff (2\gamma-1)(\gamma+\mu_0-1) > 0.$$

Therefore, for $\gamma < \min\{1-\mu_0, \frac{1}{2}\}$, the second term is larger than the first term, so firm 2 does not deviate to h.

(ii) Suppose that firm 2 observes $s_2 = H$. Then, the firm 2's expected payoff for using h and l are

$$\Pi_2^*(H) = \underbrace{\frac{\mu_0(1-\gamma)^2+(1-\mu_0)\gamma^2}{\mu_0(1-\gamma)+(1-\mu_0)\gamma}}_{\Pr(s_1=H|s_2=H)} \cdot \pi_2(h,h)\ ,\quad \hat{\Pi}_2(H) = \underbrace{\frac{\gamma(1-\gamma)}{\mu_0(1-\gamma)+(1-\mu_0)\gamma}}_{\Pr(s_1=L|s_2=H)} \cdot \pi_2(l,l),$$

respectively. Similar to the logic of Step 3.1 (ii), we can conclude that for a sufficiently small γ, firm 2 does not deviate to l. That is, there is a $\bar{\gamma}_2 > 0$ such that for $\gamma < \bar{\gamma}_2$, firm 2 does not deviate from the truthful equilibrium strategy.

Now, let $\bar{\gamma} = \min\{\bar{\gamma}_1, \bar{\gamma}_2\}$. Then, for $\gamma < \bar{\gamma}$, firm 1 and firm 2 do not deviate from the truthful equilibrium strategy.

Step 4: $c < W(\mu_0)$

Suppose that there is the truthful equilibrium. Since $c < W(\mu_0)$, the consumer always observes (ν_1, ν_2) regardless of the message. Thus, $\pi(h,l), \pi(l,h) > 0$.

Suppose that firm 2 observes $s_2 = H$. Then, the firm 2's expected payoff for using h and l

are

$$\Pi_2^*(H) = \frac{\mu_0(1-\gamma)^2 + (1-\mu_0)\gamma^2}{\mu_0(1-\gamma) + (1-\mu_0)\gamma} \cdot \pi_2(h,h) + \frac{\gamma(1-\gamma)}{\mu_0(1-\gamma) + (1-\mu_0)\gamma} \cdot \pi_2(l,h),$$

$$\hat{\Pi}_2(H) = \frac{\mu_0(1-\gamma)^2 + (1-\mu_0)\gamma^2}{\mu_0(1-\gamma) + (1-\mu_0)\gamma} \cdot \pi_2(h,l) + \frac{\gamma(1-\gamma)}{\mu_0(1-\gamma) + (1-\mu_0)\gamma} \cdot \pi_2(l,l),$$

respectively. Since $\hat{\mu}_{ll} < \hat{\mu}_{hl} = \hat{\mu}_{lh} < \hat{\mu}_{hh}$, according to the last paragraph of Section A.1, we have $\pi_2(h,h) < \pi_2(l,h) = \pi_2(h,l) < \pi_2(l,l)$. Thus, we have that $\hat{\Pi}_2(H) > \Pi_2^*(H)$, which implies that firm 2 has an incentive to deviate to l. Therefore, there is no truthful equilibrium.

A.5 Proof of Proposition 3

Step 1: Comparative statics in γ

From Equation (6), it is clear that $\underline{c} = W(\mu_0)$ does not depend on γ. So consider $\bar{c}$. Note that

$$\frac{\partial}{\partial\gamma}\hat{\mu}_{hh} = -\frac{2\gamma(1-\gamma)\mu_0(1-\mu_0)}{\left((1-\gamma)^2\mu_0 + \gamma^2(1-\mu_0)\right)^2} < 0,$$

$$\frac{\partial}{\partial\gamma}\hat{\mu}_{ll} = \frac{2\gamma(1-\gamma)\mu_0(1-\mu_0)}{\left(\gamma^2\mu_0 + (1-\gamma)^2(1-\mu_0)\right)^2} > 0.$$

Thus, $\hat{\mu}_{hh}$ is strictly decreasing in γ, and $\hat{\mu}_{ll}$ is strictly increasing in γ. Thus, we can see from Figure 3 that $W(\hat{\mu}_{hh})$ and $W(\hat{\mu}_{ll})$ are strictly decreasing in γ, regardless of μ_0. Thus,

$$\bar{c} = \min\{W(\hat{\mu}_{hh}), W(\hat{\mu}_{ll})\}$$

is strictly decreasing in γ.

Step 2: $\bar{c}$ and $\underline{c}$

From Equation (6) defining $W(\hat{\mu})$, consider the integrand

$$\max\Big\{\bar{U}_1(\hat{\mu}) + \nu_1 - p, \bar{U}_2(\hat{\mu}) + \nu_2 - p, 0\Big\}. \tag{19}$$

Clearly (19) is decreasing in p. And since $\bar{U}_1(\hat{\mu}) - p \in (-1,1)$ and $\bar{U}_2(\hat{\mu}) - p \in (-1,1)$, (19) is strictly decreasing in p. Also, since

$$\bar{U}_1(\hat{\mu}) = \Big(\omega_H\hat{\mu} + \omega_L(1-\hat{\mu})\Big)u_H + \Big(\omega_H(1-\hat{\mu}) + \omega_L\hat{\mu}\Big)u_L,$$

$$\bar{U}_2(\hat{\mu}) = \Big(\omega_H(1-\hat{\mu}) + \omega_L\hat{\mu}\Big)u_H + \Big(\omega_H\hat{\mu} + \omega_L(1-\hat{\mu})\Big)u_L,$$

we can observe that (19) is strictly increasing in u_H and u_L. Moreover, since (19) is non-negative, for any $\hat{\mu} \in [0,1]$, we can conclude that $W(\hat{\mu})$ is strictly increasing in u_H and u_L, and strictly decreasing in p. Thus, $\underline{c}$ and $\bar{c}$ are strictly increasing in u_H and u_L, and strictly decreasing in p, respectively.

Step 3: $\bar{c} - \underline{c}$

Step 3.1: Comparative statics in u_H

First, we will show that

$$\frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=1/2} > 0.$$

Next, for $\hat{\mu} > 1/2$, we will show that

$$\frac{\partial}{\partial u_H}\frac{\partial}{\partial \hat{\mu}} W > 0,$$

which implies that $\dfrac{\partial}{\partial \hat{\mu}}\dfrac{\partial}{\partial u_H} W > 0$ by linearity. This means that for $\hat{\mu} \in \left[\frac{1}{2}, 1\right)$, W increases more with respect to u_H as $\hat{\mu}$ increases. For $\mu_0 \geq \frac{1}{2}$, this means that

$$\frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\mu_0} < \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=1-\hat{\mu}_{ll}} = \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\hat{\mu}_{ll}} < \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\hat{\mu}_{hh}},$$

where the equality holds because of symmetry of $W(\cdot)$ about $\hat{\mu} = 1/2$. Thus, by definition of $\bar{c}$ and $\underline{c}$, this will prove that $\partial(\bar{c} - \underline{c})/\partial u_H = \partial(W(\hat{\mu}_{ll}) - W(\mu_0))/\partial u_H > 0$.

Likewise, for $\mu_0 < \frac{1}{2}$, we have that

$$\frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=1-\mu_0} = \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\mu_0} < \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\hat{\mu}_{hh}} < \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=1-\hat{\mu}_{ll}} = \frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=\hat{\mu}_{ll}}.$$

Thus, we can conclude that $\partial(\bar{c} - \underline{c})/\partial u_H = \partial(W(\hat{\mu}_{hh}) - W(\mu_0))/\partial u_H > 0$.

It remains to show the following claim.

Claim 1. $\frac{\partial}{\partial u_H} W\Big|_{\hat{\mu}=1/2} > 0$ *and for* $\frac{1}{2} < \hat{\mu}$, $\frac{\partial}{\partial u_H}\frac{\partial}{\partial \hat{\mu}} W > 0$.

Proof. As in Section A.1, denote $s = U_1(\omega_H) - p$ and $t = U_1(\omega_L) - p$ (so $s > t$). Note that

$$\bar{U}_1\left(\frac{1}{2}\right) - p = \bar{U}_2\left(\frac{1}{2}\right) - p = \frac{s+t}{2} \in (-1, 1).$$

Recall f from (18). Then, we have

$$\frac{\partial}{\partial u_H}W\left(\frac{1}{2}\right)=\frac{\partial}{\partial u_H}f\left(\frac{s+t}{2},\frac{s+t}{2}\right)=\frac{\partial}{\partial u_H}\frac{1}{12}\left(-\left(\frac{s+t}{2}\right)^3+3\left(\frac{s+t}{2}\right)^2+9\left(\frac{s+t}{2}\right)+5\right)$$
$$=\frac{1}{2}-\frac{1}{8}\left(\frac{s+t}{2}-1\right)^2>0.$$

Also, note that

$$\frac{\partial}{\partial u_H}\frac{\partial}{\partial\hat{\mu}}W=\frac{1}{4}(\omega_H-\omega_L)\Big(-2(s-t)^2\hat{\mu}^2+(s-t)(3s-t+6)\hat{\mu}-(s-t)(s+3)\Big)$$
$$+\frac{1}{4}(s-t)\Big[-4(s-t)(\omega_H-\omega_L)\hat{\mu}^2+(6s\omega_H-4t\omega_H-4s\omega_L+2t\omega_L+6\omega_H-6\omega_L)\hat{\mu}$$
$$-(\omega_H-\omega_L)(s+3)-(s-t)\omega_H\Big].$$

From Step 2 in the proof of Lemma 1, we know that the first term is positive. Thus, we will show that

$$-4(s-t)(\omega_H-\omega_L)\hat{\mu}^2+(6s\omega_H-4t\omega_H-4s\omega_L+2t\omega_L+6\omega_H-6\omega_L)\hat{\mu}-(\omega_H-\omega_L)(s+3)-(s-t)\omega_H \tag{20}$$

is also positive. Note that (20) is quadratic for $\hat{\mu}$ and concave because the coefficient of the squared term $-4(s-t)(\omega_H-\omega_L)$ is negative. And since (20) has a value 0 at $\hat{\mu}=\frac{1}{2}$ and $(s-t)\omega_L+(t+3)(\omega_H-\omega_L)>0$ at $\hat{\mu}=1$, it must be that (20) is positive on $(\frac{1}{2},1)$. Therefore, $\frac{\partial}{\partial u_H}\frac{\partial}{\partial\hat{\mu}}W$ is positive on $(\frac{1}{2},1)$. □

Step 3.2: Comparative statics in u_L

We repeat the same proof strategy as that for the proof of comparative statics in u_H in Step 3.1. Note that

$$\frac{\partial}{\partial u_L}W\Big|_{\hat{\mu}=1/2}=\frac{\omega_H+\omega_L}{8}\cdot\left(4-\left(\frac{s+t}{2}-1\right)^2\right)>0.$$

And for $\hat{\mu}>1/2$, note that

$$\frac{\partial}{\partial u_L}\frac{\partial}{\partial\hat{\mu}}W(\hat{\mu})=\frac{1}{4}\cdot\frac{\partial}{\partial u_L}(s-t)^2(1-2\hat{\mu})\Big((s-t)\hat{\mu}-(s+3)\Big) \tag{21}$$

$$=\frac{1}{4}(s-t)(1-2\hat{\mu})\Big(3(\omega_L-\omega_H)(s-t)\hat{\mu}-2(\omega_L-\omega_H)(s+3)-\omega_L(s-t)\Big),$$

so we have

$$\frac{\partial}{\partial \hat{\mu}} \frac{\partial}{\partial u_L} W \bigg|_{\hat{\mu}=\frac{1}{2}} = 0.$$

Note that (21) is quadratic for $\hat{\mu}$. Thus, if the derivative of (21) with respect to $\hat{\mu}$ is non-negative at $\hat{\mu} = \frac{1}{2}$, then (21) is positive on $(\frac{1}{2}, 1]$, which implies that

$$\frac{\partial}{\partial u_L} W \text{ in strictly increasing on } \left(\frac{1}{2}, 1\right) \Rightarrow \frac{\partial}{\partial u_L}\left(\bar{c} - \underline{c}\right) > 0.$$

Thus, to decrease $\bar{c} - \underline{c}$ with respect to u_H, it must be that

$$\frac{\partial^2}{\partial \hat{\mu}^2} \frac{\partial}{\partial u_L} W \bigg|_{\hat{\mu}=\frac{1}{2}} = \frac{1}{4}(s-t)(-s\omega_H - 3t\omega_H - 12\omega_H + 3s\omega_L + t\omega_L + 12\omega_L) < 0$$

$$\iff (s + 3t + 12)\omega_H > (3s + t + 12)\omega_L. \tag{22}$$

Moreover, if

$$\frac{\partial}{\partial u_L} \frac{\partial}{\partial \hat{\mu}} W(\hat{\mu}) \bigg|_{\hat{\mu}=1} = \frac{1}{4}(s-t)\Big((s - 3t - 6)\omega_H + (2t + 6)\omega_L\Big) \leq 0$$

$$\iff (-s + 3t + 6)\omega_H \geq (2t + 6)\omega_L, \tag{23}$$

then, $\frac{\partial}{\partial u_L} \frac{\partial}{\partial \hat{\mu}} W < 0$ on $(\frac{1}{2}, 1)$, which implies that

$$\frac{\partial}{\partial u_L} W \text{ is strictly decreasing on } \left(\frac{1}{2}, 1\right) \Rightarrow \bar{c} - \underline{c} \text{ is strictly decreasing in } u_L.$$

We finish this step with the following claim.

Claim 2. *If $\omega_H \geq \frac{2}{3}$, then the condition* (22) *and* (23) *are satisfied for any* (s, t).

Proof. Rewriting the condition (22) using $\omega_H + \omega_L = 1$, it would be as follows:

$$24\omega_H - 12 > (3 - 4\omega_H)s + (1 - 4\omega_H)t. \tag{24}$$

For $\frac{2}{3} < \omega_H \leq \frac{3}{4}$ and $-1 < t < s < 1$, we have

$$(3 - 4\omega_H)s + (1 - 4\omega_H)t < 2 < 24\omega_H - 12.$$

Thus, (24) is satisfied. And for $\frac{3}{4} < \omega_H \leq 1$ and $-1 < t < s < 1$, we have

$$(3 - 4\omega_H)s + (1 - 4\omega_H)t < -4 + 8\omega_H < 24\omega_H - 12.$$

Thus, (24) is also satisfied.

Next, rewriting the condition (23) using $\omega_H + \omega_L = 1$, it would be as follows:

$$12\omega_H - 6 > s\omega_H + (2 - 5\omega_H)t. \tag{25}$$

For $\frac{2}{3} < \omega_H \leq 1$ and $-1 < t < s < 1$, we have

$$s\omega_H + (2 - 5\omega_H)t < 6\omega_H - 2 < 12\omega_H - 6.$$

Thus, (25) is satisfied. $\square$

Note that if $\omega_H < \frac{2}{3}$, then (25) is not satisfied for some (s, t).

Step 3.3: Comparative statics in p

For $\hat{\mu} > \frac{1}{2}$, we have

$$\frac{\partial}{\partial p}\frac{\partial}{\partial \hat{\mu}}W = \frac{1}{4} \cdot \frac{\partial}{\partial p}\Big(- (s-t)^3(2\hat{\mu} - 1)\hat{\mu} + (s-t)^2(s+3)(2\hat{\mu} - 1)\Big) = \frac{1}{4}(s-t)(1-2\hat{\mu}) < 0.$$

For $\mu_0 \geq \frac{1}{2}$, we know that $\frac{1}{2} < \mu_0 < 1 - \hat{\mu}_{ll} < \hat{\mu}_{hh}$, so we have

$$\frac{\partial}{\partial p}W\Big|_{\hat{\mu}=\mu_0} > \frac{\partial}{\partial p}W\Big|_{\hat{\mu}=1-\hat{\mu}_{ll}} > \frac{\partial}{\partial p}W\Big|_{\hat{\mu}=\hat{\mu}_{hh}}.$$

And for $\mu_0 < \frac{1}{2}$, we know that $\frac{1}{2} < 1 - \mu_0 < \hat{\mu}_{hh} < 1 - \hat{\mu}_{ll}$, so we have

$$\frac{\partial}{\partial p}W\Big|_{\hat{\mu}=1-\mu_0} > \frac{\partial}{\partial p}W\Big|_{\hat{\mu}=\hat{\mu}_{hh}} > \frac{\partial}{\partial p}W\Big|_{\hat{\mu}=1-\hat{\mu}_{ll}}.$$

Since W is symmetric around $\hat{\mu} = \frac{1}{2}$, we can conclude that $\bar{c} - \underline{c}$ is strictly decreasing in p.

A.6 Proof of Proposition 2

In the proof of Proposition 3, we show that $\hat{\mu}_{hh}$ is strictly decreasing in γ and $\hat{\mu}_{ll}$ is strictly increasing in γ. Thus, from Section A.1, we can conclude that (i) $\pi_1(h,h)$ and $\pi_2(l,l)$ are strictly decreasing in γ, and (ii) $\pi_2(h,h)$ and $\pi_1(l,l)$ are strictly increasing in γ.

For convenience, denote that

$$\lambda_1 := \Pr[s_2 = H | s_1 = H] = \Pr[s_1 = H | s_2 = H] = \mu_0 \cdot (1-\gamma)^2 + (1-\mu_0) \cdot \gamma^2$$
$$\lambda_2 := \Pr[s_2 = L | s_1 = L] = \Pr[s_1 = L | s_2 = L] = \mu_0 \cdot \gamma^2 + (1-\mu_0) \cdot (1-\gamma)^2.$$

Then, we have

$$\begin{aligned}
\mathbb{E}\Pi_1 &= \frac{\lambda_1}{2} \cdot \pi_1(h,h) + \frac{\lambda_2}{2} \cdot \pi_1(l,l) \\
&= \frac{\lambda_1}{2} \left(2 \left(\bar{U}_1(\hat{\mu}_{hh}) - \bar{U}_2(\hat{\mu}_{hh}) \right) - \frac{1}{2} \left(\bar{U}_2(\hat{\mu}_{hh}) - p - 3 \right) \left(\bar{U}_2(\hat{\mu}_{hh}) - p + 1 \right) \right) \\
&+ \frac{\lambda_2}{2} \left(3 - 2\bar{U}_2(\hat{\mu}_{ll}) + \bar{U}_1(\hat{\mu}_{ll}) + p \right) \left(1 + \bar{U}_1(\hat{\mu}_{ll}) - p \right), \\
\mathbb{E}\Pi_2 &= \frac{\lambda_2}{2} \cdot \pi_2(l,l) + \frac{\lambda_1}{2} \cdot \pi_2(h,h) \\
&= \frac{\lambda_2}{2} \left(2 \left(\bar{U}_2(\hat{\mu}_{ll}) - \bar{U}_1(\hat{\mu}_{ll}) \right) - \frac{1}{2} \left(\bar{U}_1(\hat{\mu}_{ll}) - p - 3 \right) \left(\bar{U}_1(\hat{\mu}_{ll}) - p + 1 \right) \right) \\
&+ \frac{\lambda_1}{2} \left(3 - 2\bar{U}_1(\hat{\mu}_{hh}) + \bar{U}_2(\hat{\mu}_{hh}) + p \right) \left(1 + \bar{U}_2(\hat{\mu}_{hh}) - p \right).
\end{aligned}$$

Meanwhile, let $\lambda_3 = 2 \left(U_1(\omega_H) - U_1(\omega_L) \right) \gamma(1-\gamma)\mu_0(1-\mu_0)$. Then, we have

$$\begin{aligned}
\frac{\partial}{\partial \gamma} \bar{U}_1(\hat{\mu}_{hh}) = -\frac{\lambda_3}{\lambda_1^2}, \quad \frac{\partial}{\partial \gamma} \bar{U}_1(\hat{\mu}_{ll}) = \frac{\lambda_3}{\lambda_2^2} \\
\frac{\partial}{\partial \gamma} \bar{U}_2(\hat{\mu}_{hh}) = \frac{\lambda_3}{\lambda_1^2}, \quad \frac{\partial}{\partial \gamma} \bar{U}_2(\hat{\mu}_{ll}) = -\frac{\lambda_3}{\lambda_2^2}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\frac{\partial}{\partial \gamma} \mathbb{E}\Pi_1 &= \frac{1}{2}(\gamma - \mu_0) \left(2 \left(\bar{U}_1(\hat{\mu}_{hh}) - \bar{U}_2(\hat{\mu}_{hh}) \right) - \frac{1}{2} \left(\bar{U}_2(\hat{\mu}_{hh}) - p - 3 \right) \left(\bar{U}_2(\hat{\mu}_{hh}) - p + 1 \right) \right) \\
&- \frac{\lambda_3}{2\lambda_1} \left(\bar{U}_2(\hat{\mu}_{hh}) - p + 3 \right) \\
&+ \frac{1}{2}(\gamma - (1-\mu_0)) \left(3 - 2\bar{U}_2(\hat{\mu}_{ll}) + \bar{U}_1(\hat{\mu}_{ll}) + p \right) \left(1 + \bar{U}_1(\hat{\mu}_{ll}) - p \right) \\
&+ \frac{\lambda_3}{\lambda_2} \left(3 + 2\bar{U}_1(\hat{\mu}_{ll}) - \bar{U}_2(\hat{\mu}_{ll}) - p \right),
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_2 = {} & \frac{1}{2}(\gamma-(1-\mu_0))\left(2\left(\bar{U}_2(\hat{\mu}_{ll})-\bar{U}_1(\hat{\mu}_{ll})\right)-\frac{1}{2}\left(\bar{U}_1(\hat{\mu}_{ll})-p-3\right)\left(\bar{U}_1(\hat{\mu}_{ll})-p+1\right)\right) \\
& -\frac{\lambda_3}{2\lambda_2}\left(\bar{U}_1(\hat{\mu}_{ll})-p+3\right) \\
& +\frac{1}{2}(\gamma-\mu_0)\left(3-2\bar{U}_1(\hat{\mu}_{hh})+\bar{U}_2(\hat{\mu}_{hh})+p\right)\left(1+\bar{U}_2(\hat{\mu}_{hh})-p\right) \\
& +\frac{\lambda_3}{\lambda_1}\left(3+2\bar{U}_1(\hat{\mu}_{hh})-\bar{U}_2(\hat{\mu}_{hh})-p\right).
\end{aligned}
$$

Note that

$$
\frac{\lambda_3}{\lambda_1},\ \frac{\lambda_3}{\lambda_2} \leq 2\left(U_1(\omega_H)-U_1(\omega_L)\right)\gamma(1-\gamma),
$$

since

$$
\begin{aligned}
\lambda_1 &= (\gamma-\mu_0)^2+\mu_0(1-\mu_0) \geq \mu_0(1-\mu_0), \\
\lambda_2 &= (\gamma-(1-\mu_0))^2+\mu_0(1-\mu_0) \geq \mu_0(1-\mu_0).
\end{aligned}
$$

Thus for $\gamma < \min\{\mu_0, 1-\mu_0\}$, we have

$$
\begin{aligned}
\frac{\lambda_3}{\lambda_1},\ \frac{\lambda_3}{\lambda_2} &< 2\left(U_1(\omega_H)-U_1(\omega_L)\right)\mu_0(1-\mu_0) \\
&< 2\left(U_1(\omega_H)-U_1(\omega_L)\right)\mu_0,\ 2\left(U_1(\omega_H)-U_1(\omega_L)\right)(1-\mu_0).
\end{aligned}
$$

Thus, by reducing μ_0 close to 0 or increasing it close to 1, we can make $\frac{\lambda_3}{\lambda_1}$ and $\frac{\lambda_3}{\lambda_2}$ as close to zero as we want.

Now consider $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_1$. For a sufficiently small μ_0 and $\gamma < \mu_0$, we can observe that the 1st, 2nd, and 4th terms converge to 0, but the 3rd term is bounded away from 0, being a negative number. That is, $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_1 < 0$. And for a sufficiently large μ_0 close to 1 and $\gamma < 1-\mu_0$, we can observe that 2nd, 3rd, and 4th terms converge to 0, but the 1st term is bounded away from 0, being a negative number. That is, $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_1 < 0$. And similarly, we can get same result for $\mathbb{E}\Pi_2$.

Now, suppose that $\mu_0 = \frac{1}{2}$. Then, we have

$$
\hat{\mu}_{hh} = \frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2}, \quad \text{and} \quad \hat{\mu}_{ll} = \frac{\gamma^2}{\gamma^2+(1-\gamma)^2},
$$

so $\hat{\mu}_{hh}+\hat{\mu}_{ll} = 1$. Thus from Figure 4, we have $\pi_1(h,h) = \pi_2(l,l)$ and $\pi_1(l,l) = \pi_2(h,h)$. Note that $\hat{\mu}_{hh} > \hat{\mu}_{ll}$ from $\gamma < \bar{\gamma}$, and $\dfrac{\partial}{\partial \gamma}\hat{\mu}_{hh} < 0$ from Proof of Proposition 3. Also, note that

$$\lambda_1 = \lambda_2 = \frac{\gamma^2 + (1-\gamma)^2}{2}, \quad \text{and} \quad \lambda_1' = \lambda_2' = 2\gamma - 1 < 0.$$

Now consider

$$\mathbb{E}\Pi_1 = \frac{\lambda_1}{2}\pi_1(h,h) + \frac{\lambda_2}{2}\pi_1(l,l) = \frac{\gamma^2 + (1-\gamma)^2}{2} \cdot (\pi_1(h,h) + \pi_2(h,h)).$$

To understand how $\pi_1(h,h)+\pi_2(h,h)$ depends on γ, we need to examine how the area of the rectangle in the bottom left corner of Figure 4, which is obtained by subtracting $\pi_1(h,h)+\pi_2(h,h)$ from the total area, depends on γ. As γ increases, $P(\hat{\mu}_{hh})$ approaches closer to Y, along $\overline{XY}$. And since the slope of $\overline{XY}$ is -1, the top side of the rectangle increases by the same amount that the side decreases. And since $\hat{\mu}_{hh} > 0.5$, thus the area of the rectangle increases. That is, $\pi_1(h,h) + \pi_2(h,h)$ decreases. Therefore, $\mathbb{E}\Pi_1$ decreases. That is, $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_1 < 0$. And similarly, we have $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_2 < 0$. Therefore, by continuity, we can conclude that $\frac{\partial}{\partial\gamma}\mathbb{E}\Pi_1, \frac{\partial}{\partial\gamma}\mathbb{E}\Pi_2 < 0$ for μ_0 close to $\frac{1}{2}$.

A.7 Proof of Proposition 4

Step 1: semi-separating equilibrium

Suppose that there is a semi-separating equilibrium where transactions occur. Without loss of generality, suppose that firm 2 sends h regardless of s_2, firm 1 chooses $m_1 = h$ if $s_1 = H$, and firm 1 chooses $m_1 = l$ if $s_1 = L$. Then the consumer's posterior beliefs are

$$\begin{aligned}\hat{\mu}^h_{hh} &:= \Pr[\omega = \omega_H|(h,h)] = \mu_0(1-\gamma) + (1-\mu_0)\gamma \\ \hat{\mu}^h_{lh} &:= \Pr[\omega = \omega_H|(l,h)] = \mu_0\gamma + (1-\mu_0)(1-\gamma).\end{aligned}$$

Suppose that $\mu_0 > \frac{1}{2}$. Since $\gamma < \frac{1}{2}$, we have

$$1 - 2\hat{\mu}^h_{lh} = (2\mu_0 - 1)(1 - 2\gamma) > 0.$$

And since $s > t$, we have

$$\bar{U}_1(\hat{\mu}^h_{lh}) = \hat{\mu}^h_{lh}s + (1-\hat{\mu}^h_{lh})t < \hat{\mu}^h_{lh}t + (1-\hat{\mu}^h_{lh})s = \bar{U}_2(\hat{\mu}^h_{lh}).$$

Thus, as in Section A.1, the consumer is more likely to buy the product 2 than the product 1 after observing (l, h). Similarly, we can check that the consumer is more likely to buy the product 1 than the product 2 after observing (h, h). Therefore, firm 1 profitably deviates to h from l when firm 1 observes $s_1 = L$.

Similarly, we can check that firm 1 will deviate when $\mu_0 < \frac{1}{2}$.

Step 2: Pooling equilibrium

Without loss of generality, consider the strategy, where firm 1 always sends h and firm 2 always sends l. In this case,

$$\mu_0 = \Pr[\omega = \omega_H | (h, l)].$$

And since the probability of other messages being sent is 0, the receiver's posterior belief regarding these messages remains undetermined. Therefore, to construct equilibrium, we can set this belief as desired. So, let

$$\Pr[\omega = \omega_H | (h, h)] = \Pr[\omega = \omega_H | (l, l)] = \mu_0.$$

If $W(\mu_0) \leq c$, then no firm will deviate because deviating would still result in a payoff of 0. And if $W(\mu_0) > c$, then no firm will still deviate because deviating would not change the payoff.

A.8 Proof of Proposition 5

Suppose that there is the truthful equilibrium. In this case, if the consumer receives h and l, she updates his posterior belief as

$$\hat{\mu}_H = \frac{\mu_0(1-\gamma)}{\mu_0(1-\gamma) + (1-\mu_0)\gamma}, \ \hat{\mu}_L = \frac{\mu_0\gamma}{\mu_0\gamma + (1-\mu_0)(1-\gamma)},$$

respectively.

(i) If the consumer wants to observe (ν_1, ν_2) for only one message among h and l:
Without loss of generality, suppose that the consumer wants to observes (ν_1, ν_2) after he observes H, but does not want after he observes l. Then clearly, firm 1 who observes $s_1 = L$ will deviate from l to h.

(ii) If the consumer wants to observe (ν_1, ν_2) regardless of the message:
Since $\gamma < \frac{1}{2}$, we can check that $\hat{\mu}_L \neq \hat{\mu}_H$. However, since the expected payoffs differ when firm 1 observes $s_1 = H$ compared to when he observes $s_1 = L$, he will deviate from the equilibrium

strategy and send a different message when the expected payoff is lower.

A.9 Proof of Proposition 6

Under the strategy (5), we have

$$\hat{\mu}\Big((h,p),(h,p)\Big) = 1, \ \hat{\mu}\Big((l,p),(l,p)\Big) = 0$$

Recall f from (18). Then, after receiving $((h,p),(h,p))$, the consumer's expected payoff for observing $(\varepsilon_1,\varepsilon_2)$ is $f(U_1(\omega_H)-p, U_1(\omega_L)-p)$. So, let

$$\bar{c} := f\Big(U_1(\omega_H)-p, U_1(\omega_L)-p\Big).$$

If $c \geq \bar{c}$, then the consumer does not observe $(\varepsilon_1,\varepsilon_2)$, no transaction occurs, so there cannot be a truthful equilibrium. Now, we will show that each firm does not deviate from the equilibrium strategy in $\omega=\omega_H$ case. Then by symmetry, each firm does not deviate in $\omega=\omega_L$ case as well.

Step 1: Constructing the firm 1's belief
Since $-1 < U_1(\omega_L)-p < U_1(\omega_H)-p < 1$, the firm 1's expected payoff is

$$p\cdot J, \text{ where } \ J = 2\Big(U_1(\omega_H)-U_1(\omega_L)\Big) - \frac{(U_1(\omega_L)-p-3)(U_1(\omega_L)-p+1)}{2}.$$

Roughly, J is the area of the pentagon corresponding to '1' in Figure 2-a.

Step 1.1: Low price $p' < p$
For each $m_1 \in \{h,l\}$, set

$$\hat{\mu}\Big((m_1,p'),(h,p)\Big) = 1.$$

(i) Let $p' = p-\varepsilon$, where $\varepsilon \in (0, p-U_1(\omega_H)+1]$. It means that $-1 \leq p'-U_1(\omega_H) < p-U_1(\omega_H)$. Then, the firm 1's expected payoff when he deviates to (m_1,p') is

$$(p-\varepsilon)(J+2\varepsilon) = pJ + 2p\varepsilon - J\varepsilon - 2\varepsilon^2. \tag{26}$$

Thus, in order for firm 1 not to deviate from (h,p) to (m_1,p'), it must be that

$$pJ + 2p\varepsilon - J\varepsilon - 2\varepsilon^2 \leq pJ \ \text{ for all } \varepsilon \in (0, p-U_1(\omega_H)+1].$$

In other words, $2p \leq J$ must be satisfied. That is,

$$2p \leq 2(U_1(\omega_H) - U_1(\omega_L)) - \frac{(U_1(\omega_L) - p - 3)(U_1(\omega_L) - p + 1)}{2}$$
$$\iff \quad (p - (U_1(\omega_L) - 3))^2 \leq 4(U_1(\omega_H) - 2U_1(\omega_L) + 3).$$

To exist $p > 0$ that satisfies this, it must be that

$$U_1(\omega_H) - 2U_1(\omega_L) + 3 \geq 0, \tag{27}$$
$$U_1(\omega_L) - 3 + 2\sqrt{U_1(\omega_H) - 2U_1(\omega_L) + 3} > 0 \;\Rightarrow\; 4(U_1(\omega_H) + 1) > (U_1(\omega_L) + 1)^2. \tag{28}$$

With these conditions,

$$p \in \left(0, U_1(\omega_L) - 3 + 2\sqrt{U_1(\omega_H) - 2U_1(\omega_L) + 3}\right) \tag{29}$$

satisfies (26).

(ii) Let $p' = p - \varepsilon$, where $\varepsilon \in (p - U_1(\omega_H) + 1, p)$ (if $U_1(\omega_H) - 1 \leq 0$, we do not need to proceed with this step). It means that $-U_1(\omega_H) < p' - U_1(\omega_H) - p' < -1$. Then, the firm 1's expected payoff when he deviates to (m_1, p') is

$$(p - \varepsilon)\left(4 - \frac{(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon)^2}{2}\right) \tag{30}$$

Since (26) and (30) have the same value at $\varepsilon = p - U_1(\omega_H) + 1$, we only need to check that (30) is decreasing in $\varepsilon \in (p - U_1(\omega_H) + 1, p)$. First, note that

$$\begin{aligned} 1 - U_1(\omega_H) &< 2 - U_1(\omega_H) + U_1(\omega_L) - p < 2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon \\ &< 2 - U_1(\omega_H) + U_1(\omega_L) - p + U_1(\omega_H) - 1 = 1 + U_1(\omega_L) - p < 2. \end{aligned}$$

That is, $1 - U_1(\omega_H) < 2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon < 2$. Next, the derivative of (30) with respect to ε is

$$\frac{(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon)^2}{2} + (p - \varepsilon)(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) - 4. \tag{31}$$

Since $0 < p - \varepsilon < U_1(\omega_H) - 1$, if $(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) \geq 0$, then (31) is lower than

$$\frac{(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon)^2}{2} + (U_1(\omega_H) - 1)(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) - 4. \tag{32}$$

And if $(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) < 0$, then (31) is lower than

$$\frac{(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon)^2}{2} - 4. \tag{33}$$

Observe that both (32) and (33) are quadratic functions of $(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon)$. If (32) is non-positive at $(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) = 2$, then (32) is non-positive on $[0, 2]$, since (32) has a value of -4 at $2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon = 0$ And similarly, if (33) is non-positive at $(2 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) = 1 - U_1(\omega_H)$, then (33) is non-positive on $[1 - U_1(\omega_H), 0]$. These two conditions respectively signify

$$U_1(\omega_H) \leq 2, \; U_1(\omega_H) \leq 1 + 2\sqrt{2}.$$

Thus, if

$$U_1(\omega_H) \leq 2, \tag{34}$$

then both (32) and (33) are negative on a range $(1 - U_1(\omega_H), 1)$. Thus, (31) is also negative. Thus, (30) is decreasing in ε, so firm 1 do not deviate to (m_1, p').

Step 1.2: High price $p' > p$

For $m_1 \in \{h, l\}$, set

$$\hat{\mu}\Big((m_1, p'), (h, p)\Big) = 0.$$

Note that if $U_1(\omega_L) - p' \leq -1$, there is no reason for firm 1 to deviate to p', because the consumer does not purchase the product 1 even after observing $(\varepsilon_1, \varepsilon_2)$. Thus, we only consider the case of $U_1(\omega_L) - p' > -1$. Thus, after receiving $\Big((m_1, p'), (h, p)\Big)$, the consumer's expected payoff for observing $(\varepsilon_1, \varepsilon_2)$ is $f(U_1(\omega_L) - p', U_1(\omega_H) - p)$.

Note that

$$f(U_1(\omega_L) - p', U_1(\omega_H) - p) < f(U_1(\omega_L) - p, U_1(\omega_H) - p) = f(U_1(\omega_H) - p, U_1(\omega_L) - p) = \bar{c},$$

since

$$\frac{\partial}{\partial x} f(x, y) = \frac{1}{2}(x + 1)(x - 2y + 3) > 0$$

for $x < y$. Moreover, since

$$\hat{\mu}\Big((m_1, p'), (h, p)\Big) = 1 \text{ for } p' \leq p \;\text{ and }\; \hat{\mu}\Big((m_1, p'), (h, p)\Big) = 0 \text{ for } p' > p,$$

the firm 1's expected payoff jumps down at p. So, let's take the largest $\bar{p} \leq U_1(\omega_L) + 1$ such that the firm 1's expected payoff is less than or equal to pJ for all $p \leq \bar{p}$ ($\bar{p}$ can be $U_1(\omega_L) + 1$). Now, let

$$\underline{c}^1 = \max\Big\{0, f(U_1(\omega_L) - \bar{p}, U_1(\omega_H) - p)\Big\}.$$

Then if $p' \leq \bar{p}$, firm 1 will not deviate to (m_1, p') since the expected payoff is lower than pJ. And if $p' > \bar{p}$ and $\underline{c}^1 \leq c$, firm 1 will not deviate to (m_1, p') since the consumer does not observe $(\varepsilon_1, \varepsilon_2)$, which implies that the firm 1's expected payoff is zero.

STEP 2: Constructing the firm 2's belief

Since $-1 < U_1(\omega_L) - p < U_1(\omega_H) - p < 1$, the firm 2's expected payoff is

$$p \cdot \frac{(3 - 2U_1(\omega_H) + U_1(\omega_L) + p)(1 + U_1(\omega_L) - p)}{2}. \tag{35}$$

STEP 2.1: Low price $p' < p$

For $m_2 \in \{h, l\}$, set

$$\hat{\mu}\Big((h, p), (m_2, p')\Big) = 1.$$

(i) Let $p' = p - \varepsilon$, where $\varepsilon \in (0, p - U_1(\omega_L) + 1]$. It means that $-1 \leq p' - U_1(\omega_L) < p - U_1(\omega_L)$. Then, the firm 2's expected payoff when he deviates to (m_2, p') is

$$\frac{(p - \varepsilon)(3 - 2U_1(\omega_H) + U_1(\omega_L) + p + \varepsilon)(1 + U_1(\omega_L) - p + \varepsilon)}{2}. \tag{36}$$

Note that for $\varepsilon \in (U_1(\omega_H) - U_1(\omega_L), p - U_1(\omega_L) + 1]$, the firm 2's expected payoff when he deviates to (m_2, p') is lower than (36) (we can check this from Figure 2). Thus, we still only need to check that (36) is lower than (35).

Note that (36) is a cubic equation for ε. Also, the two roots,

$$p - U_1(\omega_L) - 1, \quad -3 + 2U_1(\omega_H) - U_1(\omega_L) - p$$

are negative, since

$$-1 < p - U_1(\omega_L) < 1 \text{ and } 2(U_1(\omega_H) - p) < 2.$$

Thus, in order to prevent deviation, the derivative of (36) must be non-positive at $\varepsilon = 0$. That is,

$$p^2 - 2\Big(2U_1(\omega_H) - U_1(\omega_L) - 3\Big)p + (U_1(\omega_L) + 1)(2U_1(\omega_H) - U_1(\omega_L) - 3) \tag{37}$$

is non-positive (if this condition is satisfied, then the derivative is negative for other values of ε as well). Note that (37) is a quadratic function for p. If $2U_1(\omega_H) - U_1(\omega_L) - 3 \geq 0$, then (37) is positive: the axis of (37) is $p = 2U_1(\omega_H) - U_1(\omega_L) - 3$, and the height of the vertex is

$$-2(2U_1(\omega_H) - U_1(\omega_L) - 3)(U_1(\omega_H) - U_1(\omega_L) - 2) \geq 0,$$

due to $U_1(\omega_H) - U_1(\omega_L) < 2$. Thus, it must be that

$$2U_1(\omega_H) - U_1(\omega_L) - 3 < 0 \tag{38}$$

$$p \in \left(0, 2U_1(\omega_H) - U_1(\omega_L) - 3 + \sqrt{2(2U_1(\omega_H) - U_1(\omega_L) - 3)(U_1(\omega_H) - U_1(\omega_L) - 2)}\right]. \tag{39}$$

Note that $2U_1(\omega_H) - U_1(\omega_L) - 3 + \sqrt{2(2U_1(\omega_H) - U_1(\omega_L) - 3)(U_1(\omega_H) - U_1(\omega_L) - 2)} > 0$.

(ii) Let $p' = p - \varepsilon$, where $\varepsilon \in (p - U_1(\omega_L) + 1, p)$ (if $U_1(\omega_L) - 1 \leq 0$, then we do not need to proceed with this step). It means that $-U_1(\omega_L) < p' - U_1(\omega_L) < -1$. Then, the firm 2's expected payoff when he deviates to (m_2, p') is

$$(p - \varepsilon)\left(4 - \frac{(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon)^2}{2}\right) \tag{40}$$

Since (36) and (40) have the same value at $\varepsilon = p - U_1(\omega_L) + 1$, we only need to check that (40) is decreasing in $\varepsilon \in (p - U_1(\omega_L) + 1, p)$. First, note that

$$\begin{aligned}1 - U_1(\omega_L) &< 2 + U_1(\omega_H) - U_1(\omega_L) - p < 2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon \\ &< 2 + U_1(\omega_H) - U_1(\omega_L) - p + U_1(\omega_L) - 1 = 1 + U_1(\omega_H) - p < 2.\end{aligned}$$

That is, $1 - U_1(\omega_L) < 2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon < 2$. Next, the derivative of (40) with respect to ε is

$$\frac{(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon)^2}{2} + (p - \varepsilon)(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon) - 4. \tag{41}$$

Since $0 < p - \varepsilon < U_1(\omega_L) - 1$, if $(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon) \geq 0$, then (41) is lower than

$$\frac{(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon)^2}{2} + (U_1(\omega_L) - 1)(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon) - 4. \tag{42}$$

And if $(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon) < 0$, then (41) is lower than

$$\frac{(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon)^2}{2} - 4. \tag{43}$$

Observe that both (42) and (43) are quadratic functions of $(2 + U_1(\omega_H) - U_1(\omega_L) - \varepsilon)$. If (42) is non-positive at $(1 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) = 2$, then (42) is non-positive on $[0, 2]$. And if (43) is non-positive at $(1 - U_1(\omega_H) + U_1(\omega_L) - \varepsilon) = 1 - U_1(\omega_L)$, then (43) is non-positive on $[1 - U_1(\omega_L), 0]$. Thus, if

$$U_1(\omega_L) \leq 2,$$

then we can check that both (42) and (43) are negative on a range $(1 - U_1(\omega_H), 1)$. Moreover, Condition (34) implies this condition. Thus, (41) is also negative. In summary, firm 2 do not deviate to (m_2, p').

STEP 2.2: High price $p' > p$

For $m_2 \in \{h, l\}$, set

$$\hat{\mu}\Big((h,p),(m_2,p')\Big) = \frac{1}{2}.$$

And let

$$\underline{c}^2 = f\left(\frac{U_1(\omega_H) + U_1(\omega_L)}{2} - p, \frac{U_1(\omega_H) + U_1(\omega_L)}{2} - p\right).$$

Note that $\underline{c}^2 < \bar{c}$. And note that

$$f\left(\frac{U_1(\omega_H) + U_1(\omega_L)}{2} - p, \frac{U_1(\omega_H) - p}{2} + \frac{U_1(\omega_L) - p'}{2}\right)$$
$$< f\left(\frac{U_1(\omega_H) + U_1(\omega_L)}{2} - p, \frac{U_1(\omega_H) + U_1(\omega_L)}{2} - p\right),$$

since

$$\frac{\partial}{\partial y} f(x,y) = \frac{1}{2}(y+1)(y - 2x + 3) > 0$$

for $x > y$. Thus, for $c \geq \underline{c}^2$, the consumer does not observe $(\varepsilon_1, \varepsilon_2)$ when he receives $\Big((h,p),(m_2,p')\Big)$. Thus, firm 2 will not deviate from (h, p) to (m_2, p').

Finally, let $\underline{c} = \max\{\underline{c}^1, \underline{c}^2\}$. Then if $c \in [\underline{c}, \bar{c})$, neither firm will deviate to a higher price because the expected payoff becomes zero as the consumer no longer observes $(\varepsilon_1.\varepsilon_2)$. Similarly, neither firm will deviate to a lower price because, as previously shown, the expected payoff decreases further.

Now, let define the set S as the set of (p, u_H, u_L, ω_H) that satisfy (27), (28), (29), (34), (38), (39). Note that S is non-empty. To show that, let $U_1(\omega_H) = 1.5$, $U_1(\omega_L) = 1$, and $p = 0.7$. Then we have

- $U_1(\omega_H) - p = 0.8 \in (-1, 1)$, $U_1(\omega_L) - p = 0.3 \in (-1, 1)$,

- $U_1(\omega_H) - 2U_1(\omega_L) + 3 = 2.5 \geq 0$,
- $U_1(\omega_L) - 3 + 2\sqrt{U_1(\omega_H) - 2U_1(\omega_L) + 3} \approx 1.162 > 0$,
- $p < 1.162$,
- $U_1(\omega_H) = 1.5 \leq 2$,
- $2U_1(\omega_H) - U_1(\omega_L) - 3 = -1 < 0$,
- $p < 2U_1(\omega_H) - U_1(\omega_L) - 3 + \sqrt{2(2U_1(\omega_H) - U_1(\omega_L) - 3)(U_1(\omega_H) - U_1(\omega_L) - 2)} \approx 0.732$.

That is, all conditions are satisfied. Now, let $(u_H, u_L, \omega_H) = (1.6, 1, 6/7)$. Then we can check that $U_1(\omega_H) = 1.5$, $U_1(\omega_L) = 1$.

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