To Each Their Own: Personalized Product Offerings in Competition

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Abstract

We study competition between two firms that personalize product offerings to consumers. Firms have private, imperfect signals of each consumer’s ideal location and offer each consumer a personalized product positioning and price depending on the signals, without observing the competing firm’s personalized offering. We characterize the equilibrium personalization strategy and examine how the accuracies of firms’ signals affect equilibrium strategy, profits, and consumer welfare. We show that a firm charges a higher price and earns a higher profit for a more niche positioning if the firm’s prediction accuracy is not too low, and the reverse is true if its prediction accuracy is sufficiently low. When both firms have the same industry-level prediction accuracy, we find that the average price does not depend on accuracy, that firms charge a higher price when offering a more niche positioning, and that the average level of differentiation first increases and then decreases in the prediction accuracy. Interestingly, equilibrium profits also have an inverse-U shape in the prediction accuracy. A higher accuracy can decrease welfare for very mainstream consumers. When firms can endogenously invest in prediction accuracy, firms over-invest in equilibrium, which results in a prisoner’s dilemma. In such a case, firms can benefit from industry-level self-regulations that restrict their ability to predict individual consumer preferences. The paper also discusses what happens if firms charge subscription pricing or if consumers’ ideal locations are distributed on the Salop Circle, highlighting price discrimination between mainstream and niche consumers as the key driver of our model.

Keywords: personalization, product positioning, pricing, self-regulation.

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1 Introduction

With recent advancements in predictive technologies and the availability of more granular data on individual consumers, firms are increasingly capable of implementing personalized marketing mixes. Common examples of such personalization include individualized pricing and tailored advertising. However, beyond pricing and communication, firms also frequently personalize the products that they present to consumers. Instead of showcasing the same product with the same positioning to all consumers, firms can leverage their diverse product portfolios to present products with different positioning to different consumers based on their predictions of each consumer’s preferences.

Examples are abundant. In marketing emails, brands often present a tailored set of products to each consumer. For instance, consider a consumer receiving a marketing email from Sephora. Sephora offers a wide range of skincare and cosmetics products designed for various skin conditions, lifestyles, and aesthetics. Given the limited space in an email and the consumer’s limited attention span, Sephora cannot present its entire catalog. Instead, Sephora uses information about the consumer to personalize the set of products featured in the email.

This approach is commonplace in e-commerce. For example, a consumer shopping for sports apparel on Adidas.com will encounter a personalized website experience. Adidas, which offers products with diverse functionalities, colors, and styles, uses consumer data to present products that match individual preferences. Similarly, personalization extends to online advertising, where firms display specific products tailored to the consumer’s interests in their ad banners. Additionally, websites personalize search results so consumers searching the same query may see products varying by brand, style, or color (Yoganarasimhan 2020).

Companies like Stitch Fix and Wantable further exemplify this type of personalization by selecting apparel for their customers based on preferences in style, brand, and size. Consumers using Stitch Fix or Wantable receive a curated blind box with items chosen by algorithms, and they can decide whether to purchase each item so that each consumer is making the purchase decision on a different set of items. Figure 1 illustrates examples of such personalization in Sephora’s email marketing, online advertising, and e-commerce websites. These practices are integral to today’s e-commerce landscape. In these examples, the seller’s goal is to increase the likelihood of consumer interest and engagement by offering products that align more closely with the consumer’s preferences. Therefore, we refer to this practice
as personalized product offerings throughout the remainder of the paper.

In practice, firms implement personalized product offerings through recommendation algorithms. However, the objectives and contexts of personalized product offerings differ significantly from those of the recommender systems of e-commerce platforms studied in the literature. The literature on the strategic effects of personalized recommendation or search ranking (e.g., Hagiu and Jullien 2011, Inderst and Ottaviani 2012, Yang 2013, Hagiu and Jullien 2014, Li et al. 2018, Ke et al. 2022, Teh and Wright 2022, Zhong 2023, Zhou and Zou 2023, Zou and Zhou 2023) primarily focuses on the strategic decisions of intermediaries that host sellers and buyers, such as Amazon and Taobao. These intermediaries decide which sellers to recommend to each consumer or how to rank sellers in response to search queries.

In such contexts, the intermediary does not have direct control over the sellers’ positioning or pricing decisions and is not in direct competition with the sellers. In contrast, our paper is motivated by individual brands that use personalization in their online presence. These firms directly choose the positioning and pricing strategies for each consumer. The role of competition is also different. An intermediary does not face direct competition but must consider the competition among sellers when making recommendations. Conversely, a seller that personalizes positioning and pricing must consider direct competition from other sellers who can also personalize their positioning and pricing strategies.

The ability of firms to offer products with different positioning to different consumers
represents a significant departure from the traditional concept of product differentiation and warrants further research. Key questions include: What should a firm’s personalization strategy be when competing with another firm that can also personalize product offerings? What prices should firms charge for products with different positioning?

Moreover, if personalization hinges on firms’ abilities to accurately predict individual consumers’ preferences, how do firms’ strategies vary with the accuracy of these predictions? Do firms and consumers benefit when firms have more precise knowledge of consumers’ preferences? These questions have important implications for firms considering investments in data acquisition and algorithm development, as well as for regulators concerned with the privacy-related impacts on consumer welfare.

To address these questions, we model the competition between two firms that personalize product offerings to consumers. Each firm possesses a private, imperfect signal of each consumer’s ideal product positioning and uses this signal to determine a personalized positioning and price offer for each consumer. Consumers’ ideal locations are horizontally distributed along the Hotelling line, with those closer to the midpoint having more mainstream tastes and those farther from the midpoint having more niche tastes.

While in many cases of personalized product offerings, sellers provide a personalized menu of products to each consumer, incorporating the personalization of menus along with positioning and pricing for each product in the menu adds complexity. Therefore, our model serves as a foundational step by focusing on the simpler scenario where each firm offers only a single product tailored to each consumer. This simplification allows us to better understand the strategic decisions involved in personalized product positioning and pricing in competitive environments.

The private signal of a consumer’s ideal location serves as a parsimonious representation of a firm’s inference of individual consumer preferences using behavioral data and predictive technologies such as machine learning. Based on these signals, firms simultaneously choose the positioning and price for each consumer. A key difference from traditional Hotelling-type models (e.g., d’Aspremont et al. 1979) is that we assume firms cannot observe the positioning and pricing their rivals offer to consumers. This assumption reflects the practical reality that firms cannot see how competitors personalize their offerings on websites, emails, or ads, to each consumer.

We characterize firms’ equilibrium positioning and pricing strategies and examine how
changes in prediction accuracies affect strategies, profits, and consumer welfare. Our analysis considers both the general, asymmetric case, where the two firms have different prediction accuracies, and the symmetric case, where the two firms have the same accuracy. The symmetric case can also be interpreted as a situation where both firms have access to the same predictive technology. An increase in symmetric accuracy represents an industry-level improvement in prediction due to technological advancement.

In equilibrium, a firm chooses a personalized product positioning that balances its signal of the consumer’s ideal positioning and the most mainstream positioning. The range of positioning that a firm offers to consumers expands with its prediction accuracy. In the general case with asymmetric accuracies, a firm charges a higher price for more niche positioning unless its prediction accuracy is significantly lower than that of its competitor. We find that a firm’s prediction accuracy functions similarly to product quality; higher accuracy enables a firm to charge higher prices and earn greater profits.

In the symmetric case where the two firms have the same accuracy, firms always charge higher prices for more niche positioning and profit more from consumers with niche tastes. The degree of product differentiation follows an inverse U-shape with respect to industry-level prediction accuracy. We observe that higher industry-level accuracy initially relaxes then intensifies price competition for niche consumers, while it initially intensifies then relaxes price competition for mainstream consumers. Equilibrium profits also exhibit an inverse U-shape relative to prediction accuracy. Additionally, higher accuracy can decrease welfare for consumers with very mainstream tastes.

We also explore a scenario where firms can endogenously invest in their prediction accuracies. We find that firms face a prisoner’s dilemma: a marginal increase in accuracy always raises a firm’s profit while decreasing its competitor’s profit, with the net effect on industry profit potentially being negative. Consequently, in symmetric equilibrium, firms tend to over-invest in prediction accuracies compared to the industry-level accuracy that would maximize overall industry profit. In such cases, firms could mutually benefit from industry-level self-regulations that limit their ability to predict individual consumer preferences. This “accuracy-fixing” does not carry the negative connotations of price collusion and, more importantly, could align with public interests as regulators become increasingly concerned about consumer privacy violations.

In another extension, we examine the impact of consumers with low search costs who can
bypass a firm’s personalized product offerings and instead find their ideal positioning among all available products. We find that the presence of these low search cost consumers “flattens” the equilibrium prices, reducing firms’ ability to price discriminate between mainstream and niche consumers. This effect results in a more uniform pricing strategy in equilibrium, as firms adjust to the increased possibility of consumers seeking alternatives.

The paper also presents three variations of the main model to illustrate the key drivers of our main findings. In the first variation, firms receive common signals of consumer preferences, which can occur if firms obtain data from the same vendor and apply identical prediction algorithms. In the second variation, a consumer pays a subscription price to a firm before receiving a personalized product, a common business model in the content market. In the third variation, consumers’ ideal locations are distributed on the Salop Circle instead of the Hotelling line. In all three variations, firms charge the same price to all consumers in equilibrium, and in the symmetric case, profits are unaffected by the industry-level prediction accuracy. These variations underscore the significance of individual noise in firms’ predictions and their ability to price discriminate between mainstream and niche consumers as the primary drivers behind our results.

Compared to the extensive literature on personalized pricing (e.g., Shaffer and Zhang 1995; Villas-Boas 1999; Fudenberg and Tirole 2000; Chen et al. 2001; Chen and Iyer 2002; Villas-Boas 2004; Choe et al. 2018; Li et al. 2023) and personalized communication (e.g., Ansari and Mela 2003; Iyer et al. 2005; Lambrecht and Tucker 2013; Gardete and Bart 2018; Sahni et al. 2018), there are considerably fewer studies on personalized product positioning. This paper aims to narrow that gap.

An important paper on the topic is Zhang (2011), which examines a two-period customer recognition model where firms offer different positioning for their own customers versus their rivals’ customers in the second period. Pazgal and Soberman (2008) and Li (2021) explore scenarios where firms offer different vertical qualities, rather than horizontal positioning, to different consumer segments. Our paper diverges from Zhang (2011), Pazgal and Soberman (2008), and Li (2021) in several key aspects. First, in the three aforementioned papers, firms segment consumers into two groups and offer a product positioning for each segment, with segments determined by consumers’ choices of non-personalized products in the first period. In contrast, we allow firms to offer a different personalized positioning to each individual consumer based on noisy information about each individual’s preferences. Another
main difference lies in our research questions. Our paper focuses on the effects of firms’
prediction accuracies on personalization strategies, profits, and consumer welfare, topics not
addressed by Zhang (2011), Pazgal and Soberman (2008), and Li (2021). Lastly, in the three
aforementioned papers, firms observe their rivals’ positioning strategies before setting prices.
We study a case that is more reflective of our motivating examples in e-commerce, where
firms cannot observe rivals’ personalized offerings to each individual consumer.

Our paper also relates to the literature on product customization (e.g., Dewan et al.
2003; Syam et al. 2005; Syam and Kumar 2006). The key difference between the product
personalization studied in this paper and product customization is whether consumers or
firms decide the product positioning (Arora et al. 2008). In product customization models,
a firm presents a range of product positions, and each consumer proactively chooses a position
from that range. The same range of products is presented to all customers, and different
consumers purchase products of different positions from the same firm based on their own
choices.

In contrast, in our current paper, a firm offers a different product positioning to each
individual consumer. Thus, consumers purchase products of different positionings as a result
of the firm’s choices. In practice, e-commerce firms typically present a personalized menu to
each consumer, so a consumer’s product choice results from both the firm’s personalization
and the consumer’s own decision within the menu. In this paper, we focus on the simpler
scenario where the menu contains only one product. An intriguing research question beyond
the scope of this paper is what happens when a firm can offer a different range of products
to each consumer, effectively combining personalization and customization.

Finally, our paper relates to the literature on targetability, or the accuracy with which
a firm can predict individual consumers’ preferences. Previous studies have examined the
effects of targetability on personalized pricing (e.g., Chen et al. 2001), personalized search
ranking (e.g., Yang 2013; Zhong 2023), and targeted advertising (e.g., Ning et al. 2023).
Our paper contributes to this body of literature by exploring the effects of targetability
on personalized positioning, and subsequently, firms’ incentives to invest in or regulate tar-
getability.

Our findings align with those of Chen et al. (2001), indicating that higher industry-
wide targetability can decrease overall industry profit, suggesting that self-regulation could
be beneficial. Additionally, our model shows that a prisoner’s dilemma consistently arises
when firms invest in targetability endogenously, emphasizing the need for industry-wide self-regulation. This self-regulation could help mitigate the competitive pressures that drive firms to over-invest in targetability, aligning firms’ interests with consumer privacy concerns and overall market efficiency.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 analyzes the equilibrium. Section 4 explores extensions involving endogenous investments and consumer search. In Section 5, we discuss the main drivers of our results through three model variations. Section 6 concludes the paper.

2 Model

We consider a market with a unit mass of consumers and two firms. Consumers’ ideal locations on product positioning are uniformly distributed, $X \sim U[0, 1]$. Two firms compete with personalized product positioning and prices.

Because firms can personalize their product and price offerings to each consumer, we only need to describe the competition over one focal consumer. Firms have incomplete information about the consumer’s ideal location. Before offering a product to the consumer, each firm $i$ receives a private signal, $S_i$, about the consumer’s location with accuracy $\alpha_i$. If the signal is inaccurate, we assume the signal is uniformly drawn from $[0, 1]$ for simplicity. Thus, if the consumer’s ideal location is $x$, then

$$\Pr\{S_i = X\} = \alpha_i$$  \hspace{1cm} (1)

$$\Pr\{S_i \perp X\} = 1 - \alpha_i$$  \hspace{1cm} (2)

Each firm does not know whether its own signal is accurate, and it does not observe the other firm’s signal.

Each firm’s strategy has two components: personalized product positioning and pricing. Each firm decides what positioning to offer to each consumer, as well as what price to charge for each positioning. Let $y_i$ denote firm $i$’s personalized positioning and let $p_i$ denote firm $i$’s price to the focal consumer. The consumer’s utility from choosing firm 1 and firm 2, given positioning, $y_1$ and $y_2$, and prices, $p_1$ and $p_2$, are defined as

$$U_1 = v - p_1 - t \cdot (y_1 - x)^2$$  \hspace{1cm} (3)

$$U_2 = v - p_2 - t \cdot (y_2 - x)^2 + \Delta \epsilon$$  \hspace{1cm} (4)
where \( v \) is the base utility, \( t \) measures the consumer’s sensitivity to the mismatch between her idea location and the firm’s personalized positioning, and \( \Delta \epsilon \sim U[-\theta, \theta] \) represents a consumer’s idiosyncratic preferences for brand-level attributes that firms cannot personalize. Not that, if \( p_1 = p_2 \) and \( y_1 = y_2 \), then the consumer strictly prefers firm 1 if and only if \( \Delta \epsilon < 0 \), and strictly prefers firm 2 if and only \( \Delta \epsilon > 0 \). The model remains the same if the term \( \Delta \epsilon \) is in the consumer’s utility from choosing firm 1 instead of firm 2.

We assume that the base utility \( v \) is large enough such that the market is fully covered. The consumer always buys exactly one product. Thus, for simplicity, we ignore the term \( v \) when calculating demand, profit, and consumer welfare in the rest of the paper.

We denote \( y_i(s_i) : [0, 1] \rightarrow [0, 1] \) as firm \( i \)'s positioning strategy. We denote \( p_i(s_i) : [0, 1] \rightarrow [0, \bar{p}] \) as firm \( i \)'s pricing strategy, where \( \bar{p} \) is sufficiently high that it does not bind in equilibrium.

The timing of the game is as follows. First, nature draws the focal consumer’s ideal location, \( x \), and the consumer’s brand-level idiosyncratic preferences, \( \Delta \epsilon \). Then, firms receive private signals, \( s_1 \) and \( s_2 \), of the focal consumer’s ideal location. The two firms then simultaneously choose positioning strategies, \( y_1(s_1) \) and \( y_2(s_2) \), and pricing strategies, \( p_1(s_1) \) and \( p_2(s_2) \). The consumer observes her own ideal locations, \( x \), her idiosyncratic preferences, \( \Delta \epsilon \), and both firms’ positioning and prices. The consumer then chooses the product that returns the highest utility. For tie-breaking, the consumer buys from firm 1 if the consumer is indifferent between the two firms.\(^1\) Each firm’s objective is to maximize its expected profit.

We look for pure-strategy Nash equilibrium, which is characterized by a quadruple of functions, \( \{ y_1^*(s_1), p_1^*(s_1), y_2^*(s_2), p_2^*(s_2) \} \). We assume that the range of brand-level preferences is sufficiently wide, which ensures that all solutions are interior.

**Assumption 1** \( \theta > \bar{p} + t \).

### 2.1 Discussion of Assumptions

Our model differs from classic spatial competition models in several ways. We discuss these differences below.

First, horizontal differentiation models typically assume that positioning and pricing decisions are made sequentially (e.g., d’Aspremont et al. 1979; de Palma et al. 1985; Hauser

\(^1\) All results remain the same if a different tie-breaking rule is used.
In these models, firms first choose product positioning and then set prices after observing their competitors’ positioning. The rationale is that positioning decisions are difficult to change once made, while pricing decisions are more flexible and can be adjusted more readily.

In our model, however, we assume simultaneous moves to reflect the practical reality that firms generally cannot observe competitors’ personalized product offerings to individual consumers. Additionally, firms can adjust their personalization strategies quickly. This simultaneous move assumption captures the nature of e-commerce, where personalized positioning and pricing are determined without direct knowledge of competitors’ specific actions.

For instance, consider the competition between Adidas and Nike. In traditional differentiation models, Adidas and Nike first decide their brand positioning and design. Each firm then sets its price after observing the other firm’s positioning. However, in the context of personalized product offerings, a firm cannot set its price offer to a consumer based on the personalization from its rival. While Adidas knows that Nike also uses personalization and can form expectations about what Nike might present to a consumer in emails, banner ads, and on its website, Adidas cannot directly observe what Nike actually offers to that consumer. Therefore, while potentially correlated with Nike’s, Adidas’ personalization strategy cannot be a direct function of what Nike offers to each consumer.

Second, our model incorporates a term, $\Delta \epsilon$, which represents a consumer’s idiosyncratic preferences for brand-level attributes. While firms can personalize certain product attributes, brand-level attributes often remain unchanged. For instance, all Adidas products inherently carry the same brand-level positioning, distinct from Nike products. Even though Adidas can offer shoes with varied styles and features tailored to individual consumers, these shoes are still inherently different from Nike’s offerings with similar designs.

Note that our model is equivalent to adding a second, brand-level Hotelling dimension, where consumers are uniformly distributed along this dimension, with firms positioned at either end of the Hotelling segment, and consumers incur a linear traveling cost along this dimension. Thus, the term $\Delta \epsilon$ parsimoniously captures these consumer preferences for brand-level attributes.

The term $\Delta \epsilon$ also serves a technical purpose. In a Hotelling game where firms simultaneously choose both positioning and price, no pure-strategy equilibrium exists until the addition of an idiosyncratic error term in utility functions, which smooths firms’ demand.
functions (de Palma et al. 1985). The inclusion of $\Delta \epsilon$ ensures the existence of a pure-strategy equilibrium in our model. Such an error term is also commonly used in empirical discrete-choice models to smooth choice probabilities. While it would be interesting to explore the mixed equilibrium when competitors are not sufficiently differentiated at the brand level, this analysis is beyond the scope of our paper.

Our model also assumes that each firm’s product line spans the entire Hotelling line, enabling firms to offer any personalized positioning. Our model equates to a benchmark where carrying additional products is costless and firms cannot commit to not carrying a product. On the other hand, if each firm’s product line is restricted to a single position, then no personalization is possible, reverting to a model of product differentiation. An interesting future research question is to investigate the implications if each firm carries only a limited product line, and how firms would endogenously choose their product lines.

Another assumption of our model is that a firm only offers a single product positioning to a consumer. In e-commerce, firms typically present a personalized menu of products. However, allowing firms to choose menus significantly enlarges the action space and complicates the problem. Our model addresses the simpler case where the menu contains only one product. An intriguing research question beyond the scope of this paper is what happens when a firm can offer a different menu of products to each consumer.

In practice, some consumers may choose to ignore a firm’s personalized menu and use search functions to find their ideal products within the firm’s entire catalog. We later explore an extension with two segments of consumers: those with high search costs who can only choose between the firms’ personalized positionings, and those with low search costs who can access any product positioning from either firm. We demonstrate that our model remains robust.

3 Equilibrium Analysis

In this section, we analyze firms’ personalized positioning strategies, expected product differentiation, pricing strategies, profits, and social and consumer welfare. We show how they depend on the signals that firms receive and firms’ prediction accuracies. Table 1 provides an index of the relevant results.
Table 1: Table of Index

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A consumer chooses firm 1 if

$$\Delta \epsilon \leq p_2 - p_1 + t \left[ (y_2 - x)^2 - (y_1 - x)^2 \right]$$

(5)

and chooses firm 2 otherwise. Therefore, given \(\{y_1, y_2, p_1, p_2, x\}\), the consumer’s expected demand for firm 1 can be written as:

$$Q_1(y_1, y_2, x) + \frac{p_2 - p_1}{2\theta}$$

(6)

and the consumer’s expected demand for firm 2 can be written as:

$$Q_2(y_2, y_1, x) + \frac{p_1 - p_2}{2\theta}$$

(7)

where

$$Q_1(y_1, y_2, x) = \frac{1}{2} + \frac{t}{2\theta} \left[ (y_2 - x)^2 - (y_1 - x)^2 \right]$$

(8)

$$Q_2(y_2, y_1, x) = \frac{1}{2} + \frac{t}{2\theta} \left[ (y_1 - x)^2 - (y_2 - x)^2 \right]$$

(9)

Note that equations (8) and (9) capture the consumer’s expected demand for firm 1 and firm 2 when \(p_1 = p_2\).

Consider the consumer’s expected demand for each firm \(i\). Suppose firm \(j\) follows a strategy of \(y_j(s_j)\) and \(p_j(s_j)\). Given (6) and (7), firm \(i\)’s expected demand conditional on its signal \(s_i\) can be described by four events: both firms receive the correct signal, i.e. \(s_i = s_j = x\); firm \(i\) receives the correct signal and firm \(j\) receives a wrong one i.e. \(s_i = x, s_j \perp x\); firm \(i\) receives a wrong signal and firm \(j\) receives the correct one i.e. \(s_i \perp x, s_j = x\); and both firms receiver a wrong signal, i.e. \(s_i \perp x, s_j \perp x\).

Adding firm \(i\)’s expected demand under the four events
Define \( D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) \) as firm \( i \)'s expected demand from a personalized positioning of \( y_i \) and price \( p_i \), given its signal of the focal consumer’s ideal location is \( s_i \) and firm \( j \) uses a strategy of \( y_j(s_j) \) and \( p_j(s_j) \). In the Appendix, we calculate \( D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) \) by summing up firm \( i \)'s expected demand under the four events, weighted by the probability of each event:

\[
D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_i^{np}(y_i|s_i, y_j(s_j)) + \frac{1}{2\theta} \cdot \left[ \alpha_i \alpha_j \cdot [p_j(s_i) - p_i] + (1 - \alpha_i \alpha_j) \cdot [E p_j - p_i] \right] \tag{10}
\]

where \( D_i^{np}(y_i|s_i, y_j(s_j), p_j(s_j)) \) is the sum of the non-pricing terms, weighted by the probability of each event:

\[
D_i^{np}(y_i|s_i, y_j(s_j)) = \alpha_i \alpha_j Q_i(y_i, y_j(x), x) + \alpha_i(1 - \alpha_j) \int_0^1 Q_i(y_i, y_j(s_j), x) ds_j + (1 - \alpha_i) \alpha_j \int_0^1 Q_i(y_i, y_j(x), x) dx + (1 - \alpha_i)(1 - \alpha_j) \int_0^1 \int_0^1 Q_i(y_i, y_j(s_j), x) ds_j dx \tag{11}
\]

Note that if there is no price competition, i.e., \( p_i = p_j \forall s_i, s_j \), then \( D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_i^{np}(y_i|s_i, y_j(s_j)) \). Thus \( D_i^{np}(y_i|s_i, y_j(s_j)) \) represents firm \( i \)'s expected demand without price competition.

Firms \( i \)'s expected profit can be written as

\[
\Pi_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) \cdot p_i = p_i \cdot D_i^{np}(y_i|s_i, y_j(s_j)) + p_i \cdot \frac{1}{2\theta} \cdot \left[ \alpha_i \alpha_j \cdot [p_j(s_i) - p_i] + (1 - \alpha_i \alpha_j) \cdot [E p_j - p_i] \right] \tag{12}
\]

### 3.1 Personalized Positioning Strategy

Note that \( y_i \) enters equation (12) only through the term \( D_i^{np}(y_i|s_i, y_j(s_j)) \). Thus the value of \( y_i \) that maximizes firm \( i \)'s profit must maximize \( D_i^{np}(y_i|s_i, y_j(s_j)) \). In equilibrium, firms’ personalized positioning strategies must satisfy:

\[
y_i^*(s_i) = \arg\max_{y_i} D_i^{np}(y_i|s_i, y_j(s_j)) \quad \forall i, j \in \{1, 2\} \quad i \neq j \tag{13}
\]
Because the term $D_{ij}^{np}(y_i|s_i, y_j(s_j))$ does not include prices, one can then solve for $y_i^*(s_i)$ and $y_j^*(s_j)$ independently from firms’ pricing decisions. The algebraic derivation is in the Appendix.

**Lemma 1** In equilibrium, firm $i$ offers a personalized positioning of

$$y_i^* = E_i[x|s_i] = (1 - \alpha_i) \cdot \frac{1}{2} + \alpha_i s_i$$

More generally, for any joint distribution of signals, $F(s_1, s_2)$, and distance function, $d(x, y)$, in a Nash equilibrium, firm $i$’s positioning strategy $y_i^*(s_i)$ must minimize $E_i[d(x, y_i)|s_i]$.

Interestingly, firms’ equilibrium positioning strategies are not strategic. Firms simply position at their respective posterior means of each consumer’s ideal location. This result is driven by the fact that firms cannot observe each other’s personalized positioning before setting prices, i.e., firms cannot observe how a rival firm personalizes its website, email, ads, etc, to a specific consumer before setting prices. As a result, prices cannot react to changes in the competitors’ positioning. This eliminates the strategic effect of positioning on subsequent price competition that exists in spatial competition models where firms set prices after observing each other’s positioning.

One can see this by observing the utility function. For any consumer, the difference in utility between the two products can be written as $U_1 - U_2 = t(x - y_2)^2 - t(x - y_1)^2 + p_2 - p_1 - \epsilon$. Because $p_1$ and $p_2$ are not functions of $y_1$ and $y_2$, it is always optimal for firm $i$ to minimize the term $t(x - y_i)^2$, regardless of $p_i$ and the competitor’s strategy. Thus, in equilibrium, each firm must offer a personalized positioning at its posterior mean, $E_i[x|s_i]$, which minimizes its expected quadratic distance to the consumer, $E_i[t(x - y_i)^2]$.

When firms try to position to maximize demand without considering price competition, firms try to position as close to the consumer as possible given their signals. To do so, firms want to minimize the expected quadratic distance from the focal consumer. Hypothetically, if a firm knows that its signal is accurate, then it should position exactly at its signal, $s_i$; if a firm knows that its signal is incorrect, then it wants to position at the most mainstream taste, i.e., $\frac{1}{2}$. The firm balances these two possibilities by positioning at a point between $s_i$ and $\frac{1}{2}$, weighted by the accuracy of its signal. When firm $i$ receives a signal $s_i \neq \frac{1}{2}$, firm $i$ offers the consumer a more niche positioning that is closer to $s_i$ when $\alpha_i$ is higher and offers a more mainstream positioning that is closer to $\frac{1}{2}$ when $\alpha_i$ is lower. This strategy implies
that, on average, each firm’s personalized positioning is more mainstream than the focal consumer’s ideal location.

Lemma 1 also implies that firm \( i \)'s product line does not expand the entire range of consumer locations. A firm offers a wider product line as signal accuracy improves, as the product line width, \( (1 - \alpha_i) \cdot \frac{1}{2} + \alpha_i \cdot 1 - (1 - \alpha_i) \cdot \frac{1}{2} - \alpha_i \cdot 0 = \alpha_i \), increases in \( \alpha_i \). Firms only offer products with very niche positioning when their predictions of consumer preferences are sufficiently accurate. In the limit as \( \alpha_i \to 0 \), firm \( i \) only offers the most mainstream positioning.

Note that the core finding of Lemma 1, that firms minimize their expected distances to each consumer, is not driven by the particular signal structure or the quadratic distance function. For any distribution \( F(s_1, s_2) \) and any distance function \( d(x, y) \), this result still holds. Solving the full equilibrium with general signal structures and distance functions is difficult and beyond the scope of this paper. Thus, we continue to use the simple signal structure introduced in Section 2 and the quadratic distance function in the remaining of the paper.

Degree of Differentiation

In spatial competition models, an object of interest is the degree of differentiation, which affects the intensity of price competition. With personalized, the degree of differentiation is probabilistic. It depends both on the focal consumer’s ideal location, \( x \), as well as firms’ signals, \( s_i \) and \( s_j \). Given each firm’s personalized positioning strategy, we can calculate \( E[(y_i - y_j)^2] \), the expected degree of product differentiation measured in the quadratic distance:

\[
E[(y_i - y_j)^2] = \frac{\alpha_i^2}{12} + \frac{\alpha_j^2}{12} - \frac{\alpha_i^2 \alpha_j^2}{6} \tag{14}
\]

Taking derivative with respect to \( \alpha_i \), we have

\[
\frac{\partial E[(y_i - y_j)^2]}{\partial \alpha_i} = \frac{\alpha_i(1 - 2\alpha_j^2)}{6} \tag{15}
\]

Therefore, if \( \alpha_j \in [0, \frac{1}{\sqrt{2}}] \), products differentiation increases with \( \alpha_i \); If \( \alpha_j \in (\frac{1}{\sqrt{2}}, 1] \), products differentiation decreases as \( \alpha_i \) increases.

We further study the symmetric case with \( \alpha_i = \alpha_j = \alpha \). The symmetric case can be understood as a situation where both firms can access the same predictive technology, and
therefore $\alpha$ represents the accuracy of the common technology. A higher $\alpha$ represents an industry-level increase in prediction accuracy due to technological advancement. In the symmetric case, we obtain the expected differentiation in the symmetric case as

$$E[(y_i - y_j)^2] = \frac{\alpha^2(1 - \alpha^2)}{6}$$

(16)

We present the comparative statics as follows.

**Proposition 1** The expected degree of product differentiation increases in $\alpha_i$ when $\alpha_j$ is low ($\leq 1/\sqrt{2}$) and decreases in $\alpha_i$ when $\alpha_j$ is high ($> 1/\sqrt{2}$). In the symmetric case with $\alpha_i = \alpha_j = \alpha$, the expected degree of product differentiation increases in $\alpha$ for $\alpha \leq \frac{1}{\sqrt{2}}$ and decreases for $\alpha > \frac{1}{\sqrt{2}}$.

Interestingly, how the level of differentiation changes when a firm’s signal accuracy increases depends on the other firm’s accuracy. When the competitor does not have accurate information on consumer preferences, an increase in a firm’s accuracy leads to a higher degree of differentiation. However, when the competitor has accurate enough information on individual consumers, an increase in the firm’s accuracy always leads to less differentiation in expectation. Intuitively, when the competitor’s prediction accuracy is low, the competitor’s positioning is likely to be far from the consumer’s ideal location. Thus, a higher accuracy, which allows the firm to position closer to the consumer’s ideal location, leads to more differentiation. On the other hand, when the competitor’s accuracy is high, the competitor’s positioning is likely to be close to the consumer’s ideal location. A higher accuracy for the firm then leads to less differentiation.

In the symmetric case, however, the expected degree of differentiation is non-monotonic in signal accuracy, and achieves the highest level when $\alpha = \frac{1}{\sqrt{2}}$. The degree of product differentiation is lowest when $\alpha = 0$ or when $\alpha = 1$. When $\alpha = 0$, both firms always offer the most mainstream positioning at $\frac{1}{2}$. When $\alpha = 1$, both firms know the focal consumer’s ideal location perfectly, thus always providing the same positioning to the consumer. We can then expect the intensity of price competition to also be non-monotonic in $\alpha$. Price competition should be the most intense at the lowest and highest accuracy. Next, we formally solve for firms’ pricing strategies.
3.2 Pricing Strategy

Given firms’ personalized positioning strategies from Lemma 1, \( y_i^*(s_i) = (1 - \alpha_i) \cdot \frac{1}{2} + \alpha_i s_i \), we solve for firms’ equilibrium pricing strategies.

Let \( A_i(s) \) denote equation (11) evaluated at equilibrium personalized positioning strategies and at signal \( s \), i.e., \( A_i(s) = D_1^{np}(y_i^*(s)|s_i = s, y_j^*(s_j)) \). We can then write firm \( i \)'s profit under the equilibrium positioning strategy as:

\[
\Pi_i(p_i|s_i = s, p_j(s_j)) = p_i A_i(s) + p_i \frac{1}{2\theta} \left[ \alpha_i \alpha_j \cdot [p_j(s) - p_i] + (1 - \alpha_i \alpha_j) \cdot [Ep_j - p_i] \right] \tag{17}
\]

Maximizing equation (17) with respect to \( p_i \), we get the following conditions regarding equilibrium pricing strategies, \( p_i^*(s_i) \) and \( p_j^*(s_j) \).

\[
2\theta A_i(s) + \alpha_i \alpha_j p_j^*(s) + (1 - \alpha_i \alpha_j)Ep_j^* - 2p_i^*(s) = 0 \quad \forall i, j \in \{1, 2\} \quad i \neq j \tag{18}
\]

which allows us to write \( p_i^*(s) \) as a function as \( A_i(s), A_j(s), Ep_i \), and \( Ep_j \):

\[
p_i^*(s) = \frac{4\theta}{4 - \alpha_i^2 \alpha_j^2} A_i(s) + \frac{2\theta \alpha_i \alpha_j}{4 - \alpha_i^2 \alpha_j^2} A_j(s) + \frac{\alpha_i \alpha_j(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} Ep_i^* + \frac{2(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} Ep_j^* \tag{19}
\]

Equation (19) pins down the equilibrium pricing function \( p_i^*(s) \). There is no interpretable closed-form solution for \( p_i^*(s) \). Instead, we first describe a firm’s equilibrium average price, \( Ep_i^* \), then describe how the \( p_i^*(s) \) varies by signal \( s \). The latter determines whether firms charge higher prices to more niche consumers (i.e., \( p_i^*(s) \) is a convex “smiley face”) or charge higher prices to more mainstream consumers (i.e., \( p_i^*(s) \) is a concave “grumpy face”).

Average Prices

Using the fact that \( Ep_i = \int_0^1 p_i(s)ds \), we get

\[
Ep_i^* = \int_0^1 p_i^*(s)ds = \frac{4\theta}{4 - \alpha_i^2 \alpha_j^2} B_i + \frac{2\theta \alpha_i \alpha_j}{4 - \alpha_i^2 \alpha_j^2} B_j + \frac{\alpha_i \alpha_j(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} Ep_i^* + \frac{2(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} Ep_j^* \tag{20}
\]

where \( B_i = \int_0^1 A_i(s)ds \). We show in the Appendix that

\[
B_i = \int_0^1 A_i(s)ds = \frac{1}{2} + \frac{t}{24\theta}(\alpha_i^2 - \alpha_j^2) \tag{21}
\]
We can then use equations (20) and (21) to derive closed-form solutions for equilibrium average prices:

\[ E_{p1}^* = \theta + \frac{t}{36} (\alpha_1^2 - \alpha_2^2) \]  
\[ E_{p2}^* = \theta + \frac{t}{36} (\alpha_2^2 - \alpha_1^2) \]  

(22)  

(23)

**Proposition 2** Firm i’s average price in equilibrium increases in the size of idiosyncratic preferences, \( \theta \), and its own prediction accuracy, \( \alpha_i \), and decreases in the competitor’s prediction accuracy, \( \alpha_j \). Firm i’s average price in equilibrium increases in the traveling cost, \( t \), if \( \alpha_i > \alpha_j \), and decreases in the traveling cost, \( t \), if \( \alpha_i < \alpha_j \).

The comparative statics in Proposition 2 are intuitive. When \( \theta \) is higher, the competition between the two firms is softened, causing equilibrium prices to rise. Under personalized positioning, firms cannot observe the competitor’s personalized positioning, and thus cannot price in response to the competitor’s positioning. Instead, a firm’s average price depends on the advantage over the competitor, which is captured by the term \( \alpha_i^2 - \alpha_j^2 \) in equations (23). When \( \alpha_i \) increases, firm i’s personalized positioning becomes closer to the consumer’s ideal location relative to the competitor’s personalized positioning, allowing firm i to charge higher prices. The reverse happens when \( \alpha_j \) increases. The traveling cost \( t \) magnifies a firm’s advantage/disadvantage. Note that, interestingly, a firm’s prediction accuracy acts similarly to its product quality. At a higher accuracy, a firm’s product delivers a higher expected utility to the consumer, allowing the firm to charge a higher price in equilibrium.

We also examine the symmetric case with \( \alpha_i = \alpha_j = \alpha \). By examining equation (23). We see that the equilibrium average prices do not depend on \( \alpha \) or \( t \), because neither firm has an ex-ante advantage to position closer to the consumer’s ideal location.

**Corollary 2.1** In the symmetric case with \( \alpha_i = \alpha_j = \alpha \), each firm’s average price in equilibrium does not depend on \( t \) or \( \alpha \).

This result is somewhat surprising. Proposition 1 shows that the expected degree of differentiation in the symmetric case depends on \( \alpha \). We would then expect that the intensity of price competition also depends on \( \alpha \). This is, however, not reflected in the average equilibrium price.

To better understand how \( \alpha \) affects price competition, one has to examine the equilibrium prices for different signals (or when offering different positioning).
Niche vs. Mainstream Pricing

We can then plug in our solutions for $E p^*_1$ and $E p^*_2$ into equation (19) to get $p^*_i(s)$. Note that Lemma 1 shows that firm $i$ must offer a different positioning for each different signal, i.e., $y^*_i(s) = y^*_i(s')$ if and only if $s = s'$. Thus, instead of standard personalized pricing, where firms charge different prices for the same product to different consumers, firms’ pricing strategies in this model are more akin to product line pricing: in equilibrium, firm $i$ charges different prices for different positioning, but never charge different prices for the same positioning.

With a slight abuse of notation, we denote $p^*_i(y) = p^*_i(s_i = \frac{y-(1-\alpha_i)^{-1/2}}{\alpha_i})$ as firm $i$’s equilibrium price when offering positioning $y$. The remaining details of the derivation and the closed-form solutions for $p^*_1$ and $p^*_2$ are in the Appendix. We describe firms’ equilibrium pricing strategies by examining the shape of $p^*_i(y)$ and $p^*_i(s)$.

Proposition 3 Firm $i$’s equilibrium pricing strategy, $p^*_i(y)$ ($p^*_i(s)$), is symmetric around and smooth at $\frac{1}{2}$. Furthermore, there is a threshold $\alpha_i(\alpha_j) \in [0, 1]$ such that:

- If $\alpha_i < \alpha_i(\alpha_j)$, then $p^*_i(y)$ ($p^*_i(s)$) increases as $y \to \frac{1}{2}$ ($s \to \frac{1}{2}$), i.e., firm $i$ charges a higher price when offering a more mainstream positioning (when receiving a more mainstream signal);

- If $\alpha_i = \alpha_i(\alpha_j)$, then $p^*_i(y)$ ($p^*_i(s)$) is constant in $y$ ($s$), i.e., firm $i$ charges the same price for all positioning (for all signals).

- If $\alpha_i > \alpha_i(\alpha_j)$, then $p^*_i(y)$ ($p^*_i(s)$) decreases as $y \to \frac{1}{2}$ ($s \to \frac{1}{2}$), i.e., firm $i$ charges a higher price when offering a more niche positioning (when receiving a more niche signal).

The threshold $\alpha_i(\alpha_j)$ is an increasing function of $\alpha_j$, with $\alpha_i(0) = 0$, $\alpha_i(1) = 1$, and $\alpha_i(\alpha_j) < \alpha_j$ for $\alpha_j \in (0, 1)$.

Proposition 3 shows that, in equilibrium, firms charge different prices when offering different personalized positioning. Depending on their prediction accuracies, firms may charge higher prices for more mainstream positioning or higher prices for more niche positioning.

Figure 2 shows the range of firm $i$’s personalized positioning and the price it charges for each positioning under different $\alpha_i$ for $\alpha_j = 0.7$. Note that when $\alpha_i$ is low, the pricing
Figure 2: Firm $i$’s equilibrium pricing (in a setting where $\{\theta = 2, t = 1, \alpha_j = 0.7\}$)

schedule is concave, with higher prices for more mainstream positioning, whereas when $\alpha_i$ is high, the pricing schedule is convex, with higher prices for more niche positioning. We can observe that the average price charged by firm $i$ is also increasing in $\alpha_i$, as stated in Proposition 2.

Proposition 3 implies that the pricing pressure a firm faces depends on whether the consumer’s taste is predicted to be niche or mainstream. To gain a more intuitive understanding, we consider two forces that shape firm $i$’s pricing decision upon receiving a signal. When setting a price, firm $i$ cannot observe the competitor’s personalized positioning, so firm $i$’s pricing depends on how close to the consumer’s ideal location it expects to be relative to the competitor’s positioning. Firm $i$ can charge a higher price if it expects its own positioning to be closer to the consumer’s ideal location, or if it expects the competitor to be farther from the consumer’s ideal location.

First, consider the distance between firm $i$’s own positioning and the consumer’s ideal location, given firm $i$’s signal, $s_i$. Suppose firm $i$’s signal is correct, then by Lemma 1, the distance between its positioning and the consumer’s ideal location is closer when $s_i$ (or $y^*_i(s_i)$) is closer to $\frac{1}{2}$. Suppose firm $i$’s signal is wrong, then positioning closer to $\frac{1}{2}$ is also on average closer to the consumer’s ideal location. Thus, overall firm $i$ expects its own positioning to be closer to the consumer’s ideal location when it receives a signal closer to $\frac{1}{2}$. This first force pushes the firm to charge a higher price when offering a more mainstream positioning (closer to $\frac{1}{2}$).

Second, consider the distance between the competitor’s positioning and the consumer’s ideal location, given firm $i$’s signal, $s_i$. Suppose firm $i$’s signal is correct, then again by Lemma 1, the expected distance between the competitor’s positioning and the consumer’s
ideal location is farther when \( s_i \) is farther from \( \frac{1}{2} \). Suppose firm \( i \)'s signal is wrong, then \( s_i \) is uncorrelated with the distance between the competitor’s positioning and the consumer’s ideal location. Thus, overall firm \( i \) expects the competitor’s positioning to be farther when \( s_i \) is farther from \( \frac{1}{2} \). This second force pushes the firm to charge a higher price when offering a more niche positioning (farther away from \( \frac{1}{2} \)).

When firm \( i \)'s prediction accuracy is low, the second force, which is only activated when firm \( i \)'s signal is correct, is weak. The first force dominates and firm \( i \) charges a higher price when offering a more mainstream positioning. When firm \( i \)'s prediction accuracy is higher, not only the second force becomes stronger, but the first force also becomes weaker: with a higher accuracy, firm \( i \)'s positioning becomes closer to the consumer’s ideal location across all \( s_i \), reducing the effect of \( s_i \) on the distance between its own positioning and the consumer’s ideal location. As a result, the firm charges a higher price when offering a more niche positioning when \( \alpha_i \) is high. Thus, there arises the threshold on \( \alpha_i \) at which firm \( i \)'s pricing schedule switches from a concave shape to a convex shape. Similarly, when the competitor’s accuracy increases, the second force becomes weaker, because the competitor’s positioning is closer to the consumer’s ideal location across all \( s_i \).

We examine the special case where firms are symmetric in their prediction accuracy. One can show that the second force always dominates the first force in the symmetric case, leading to the following result.

**Corollary 3.1** In the symmetric case with \( \alpha_i = \alpha_j = \alpha \), firms charge higher prices when offering more niche positioning (receiving more niche signals) for all \( \alpha \in (0, 1) \). In the limit as \( \alpha \to 1 \), firms offer a positioning of \( y^*(s_i) = s_i \) and charge a price of \( \theta \) for all positioning. In the limit as \( \alpha \to 0 \), firms always offer a positioning of \( \frac{1}{2} \) and charge a price of \( \theta \).

Figure 3 shows firm \( i \)'s range of personalized positioning and pricing schedule for different \( \alpha \). As shown in Corollary 2.1, the average price is unaffected by \( \alpha \). In the symmetric case, the second force in pricing dominates, pushing firms to charge higher prices when offering more niche positioning. In other words, price competition is more intense for more mainstream positioning. This difference in competitive pressure between (predicted) niche and mainstream consumers depends non-monotonically on \( \alpha \). At the limit as \( \alpha \to 1 \), firms charge the same price for all signals and positioning.

Given the characterization of firms’ equilibrium pricing strategies, we are interested in
how firms’ prediction accuracies affect pricing. Next, we examine how the equilibrium price for each signal, $p^*(s)$, changes with prediction accuracy.

One might believe that a firm’s unilateral improvement in its signal accuracy enables the firm to charge a higher price for all its product positioning. Interestingly, we find that this intuition does not always hold. Specifically, there are situations in which a firm lowers its price for some product positioning despite its increase in signal accuracy. That is, a higher accuracy intensifies price competition for some consumers and relaxes price competition for others.

To understand the reason, note that each firm uses two levers to win over a consumer: by offering a product positioning that better matches the consumer’s preference, and by charging a price that is more attractive. Consider a certain signal $s$ and its corresponding product positioning $y^*(s)$. As the focal firm’s prediction accuracy improves, it gains a positioning advantage that directly pushes the firm to raise its price. However, the competing firm will respond to its positioning disadvantage by lowering its average price (of all positioning), which indirectly pushes the local firm also to lower its price. We find that the indirect effect can outweigh the direct effect for a mainstream product (i.e., $y$ closer to $\frac{1}{2}$) when the competing firm’s signal accuracy is low. In this situation, the competing firm’s product offerings center around mainstream positioning. As a result, firms engage in more intense price competition for mainstream consumers when the focal firm gains a positioning advantage through a higher signal accuracy.

Figure 4 plots firm $i$’s price for its most mainstream product $y = \frac{1}{2}$. It is seen that for a relatively low $\alpha_j$, it is possible that $\frac{\partial p^*(y=0.5)}{\partial \alpha_i} < 0$. 

![Figure 3: Each firm’s equilibrium pricing in the symmetric case (in a setting where $\{\theta = 2, t = 1\}$)](image)
Proposition 4 In the symmetric case with $\alpha_i = \alpha_j = \alpha$, the impact of the signal precision $\alpha$ on firm $i$’s equilibrium price $p^*(s)$ for a given signal $s$ depends on a threshold $\alpha_0$.\(^2\)

- If $\alpha \leq \alpha_0$, then $p^*(s)$ decreases with $\alpha$ for $s \in \left(\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right)$ and increases with $\alpha$ for $s < \frac{1}{2} - \frac{\sqrt{3}}{6}$ or $s > \frac{1}{2} + \frac{\sqrt{3}}{6}$.

- If $\alpha > \alpha_0$, then $p^*(s)$ increases with $\alpha$ for $s \in \left(\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right)$ and decreases with $\alpha$ for $s < \frac{1}{2} - \frac{\sqrt{3}}{6}$ or $s > \frac{1}{2} + \frac{\sqrt{3}}{6}$.

Proposition 4 shows that $p^*(s)$ is non-monotonic in $\alpha$, and the direction depend on both $\alpha$ and $s$. Note that when firms receive signals that indicate relatively mainstream tastes, $s \in \left(\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right)$, prices first decrease then increase in $\alpha$. In contrast, when firms receive signals that indicate relatively niche tastes, $s < \frac{1}{2} - \frac{\sqrt{3}}{6}$ or $s > \frac{1}{2} + \frac{\sqrt{3}}{6}$, prices first increase then decrease in $\alpha$. In other words, for relatively mainstream signals, an increase in the industry-level accuracy first intensifies and then relaxes price competition, whereas for relatively niche signals, an increase in the industry-level accuracy first relaxes then intensifies price competition. Figure 5 shows how equilibrium prices at different signals respond to changes in the industry-level accuracy.

From Proposition 1, we know that the expected degree of differentiation first increases then decreases in the industry-level accuracy. This leads us to hypothesize price competition

\(^2\)Specifically, $\alpha_0 \approx 0.539$ is the root of the equation $2 - 4\alpha + \alpha^3 = 0$. 

Figure 4: Firm $i$’s equilibrium pricing in response to its prediction accuracy (in a setting where $\{\theta = 2, t = 1, y = 0.5\}$)
to first relax then intensify as $\alpha$ increases. However, we do not observe this pattern in the average price per Proposition 2. Proposition 4 provides the answer. While price competition for consumers predicted to have niche tastes indeed first relaxes then intensifies, the pattern for consumers predicted to have mainstream tastes is the opposite.

To understand why the effect of accuracy on price depends on the signal, consider the case where one firm’s signal is correct and its competitor’s signal is wrong. An increase in $\alpha$ translates to a competitive advantage because the firm positions closer to the consumer. By Lemma 1, the distance between a firm and the consumer, if the firm’s signal $s$ is correct, is $(1 - \alpha)|s - 1/2|$, which decreases in $\alpha$. Similarly, in the case where one firm’s signal is wrong and its competitor’s signal is correct, a higher accuracy translates to a competitive disadvantage because the competitor moves closer to the consumer. However, note that the marginal effect of $\alpha$ on $(1 - \alpha)|s - 1/2|$ depends on the signal, $s$. The effect is larger for niche signals and smaller for mainstream signals. If $s = 1/2$, then the aforementioned competitive advantage disappears, because the firm positions at 1/2 if $s = 1/2$, regardless of $\alpha$. But the aforementioned competitive disadvantage still exists, because the competitor’s signal is uncorrelated with the firm’s wrong signal. Thus, for low $\alpha$, while an increase in $\alpha$ relaxes pricing pressure when the firm receives a niche signal, it intensifies pricing pressure when the firm receives a mainstream signal because the competitive benefit from a higher $\alpha$ is muted.

If both firms’ signals are correct, then both firms offer the same positioning. In this
case, the level of price competition does not depend on the location of the signal. This case dominates when \( \alpha \) becomes sufficiently high. In the limit as \( \alpha \to 1 \), firms must charge the same price for all signals, the same as when \( \alpha \to 0 \). Thus, when \( \alpha \) is large, the previous effects of \( \alpha \) on prices reverse. Price competition intensifies for consumers predicted to have niche tastes and relaxes for consumers predicted to have mainstream tastes.

To corroborate with the above intuition, we show in the Online Appendix that conditional on firm \( i \) receiving a signal \( s_i \), firm \( i \)'s expected positioning advantage over firm \( j \) can be written as

\[
E[(y_i - x)^2 - (y_j - x)^2\mid s_i] = \frac{\alpha^2(1 - \alpha^2)}{6} \cdot (1 - 6s_i + 6s_i^2)
\]

Note that the term \( \frac{\alpha^2(1 - \alpha^2)}{6} \) is the expected degree of differentiation from Proposition 1, and the term \( (1 - 6s + 6s^2) \) is negative for \( s_i \in (\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}) \) and positive for \( s_i \in [0, \frac{1}{2} - \frac{\sqrt{3}}{6}) \cup (\frac{1}{2} + \frac{\sqrt{3}}{6}, 1] \). Thus, for relatively niche signals, a firm's expected positioning advantage goes in the same direction as the expected level of differentiation, whereas for relatively mainstream signals, a firm's expected positioning advantage goes in the opposite direction as the expected level of differentiation. This result connects Proposition 1 and Proposition 4.

3.3 Profit

We first examine firms' expected profits after receiving a signal \( s \). This allows us to compare the profitability between consumers of different signals. We then proceed to examine firms' ex-ante expected profits before receiving a signal.

We can calculate firms' expected profits after receiving signal \( s \) by plugging in firms' equilibrium pricing strategies into equation (17). First, we can show that not all signals are equally profitable. Let \( \Pi_i^*(s) \) denote the equilibrium expected profit for firm \( i \) when receiving a signal \( s \). On examining \( \frac{\partial \Pi_i(s)}{\partial s} \), we have the following result:

**Proposition 5** The equilibrium expected profit for firm \( i \) when it receives a signal of \( s \), \( \Pi_i^*(s) \), depends on \( \alpha_i \), \( \alpha_j \), and \( s \):

- If \( \alpha_i < \overline{\alpha}_i(\alpha_j) \), then \( \Pi_i^*(s) \) increases as \( s \to \frac{1}{2} \), i.e., firm \( i \)'s expected profit is higher when receiving a more mainstream signal.
• If $\alpha_i = \overline{\alpha_i}(\alpha_j)$, then $\Pi_i^*(s)$ is constant in $s$, i.e., firm $i$’s expected profit is the same for all signals.

• If $\alpha_i > \overline{\alpha_i}(\alpha_j)$, then $\Pi_i^*(s)$ decreases as $s \to \frac{1}{2}$, i.e., firm $i$’s expected profit is higher when receiving a more niche signal.

The threshold, $\overline{\alpha_i}(\alpha_j)$, is the same threshold as the one in Proposition 3. Thus, we can draw intuition from Proposition 3. Proposition 3 shows that, when firm $i$’s accuracy is low relative to the competitor’s, firm $i$ charges a higher price when receiving a more mainstream signal (and therefore offering a more mainstream positioning). In such a case, consumers who appear to have more mainstream tastes are more profitable to firm $i$. Conversely, when firm $i$’s accuracy is high relative to the competitor’s, Proposition 3 shows that firm $i$ charges a higher price when receiving a more niche signal (and therefore offering a more niche positioning). As a result, consumers who appear to have more niche tastes are more profitable to firm $i$.

Corollary 3.1 shows that firms charge higher prices for more mainstream positioning in the symmetric case. Thus, we would expect a firm’s expected profit to be higher when receiving a more niche signal in the symmetric case. In the symmetric case with $\alpha_i = \alpha_j = \alpha$, after receiving signal $s$, a firm’s expected profits can be written as:

$$
\Pi_i^{*\text{sym}}(s) = \frac{t^2[(1 - 6s + 6s^2)(-1 + \alpha)^2\alpha^2 - 6t(-2 + \alpha^2)]^2}{72(-2 + \alpha^2)^2\theta}
$$

(25)

Checking the derivative, we confirm the following result.

**Corollary 5.1** In the symmetric case with $\alpha_i = \alpha_j = \alpha$, firm $i$’s expected profit is higher when receiving more a niche signal.

Intuitively, a firm receives more profit when its signal is correct while the competitor’s signal is wrong, but Proposition 5 and Corollary 5.1 show that profitability also depends on where the consumer’s (predicted) ideal location is. When a consumer is predicted to have a more niche taste, the competitor receiving a wrong signal is likely to position farther away from the consumer. Thus firms charge higher prices to and receive higher profits from consumers who appear to have more niche tastes.

We proceed to solve for firm $i$’s ex-ante profit before receiving a signal as $\Pi_i^* = \int_0^1 \Pi_i^*(s)ds$. Below we describe how prediction accuracy affects profits in both the general case and the symmetric case.
Proposition 6. Firm $i$’s en-ante profit increases with its signal accuracy, $\alpha_i$, and decreases with its competitor’s signal accuracy, $\alpha_j$. In the symmetric case with $\alpha_i = \alpha_j = \alpha$, firm $i$’s profit increases with $\alpha$ for $\alpha \leq \alpha_0$ and decreases with $\alpha$ for $\alpha > \alpha_0$. Here $\alpha_0$ is the same threshold derived in Proposition 4.

The result for the general case is intuitive. As firm $i$’s own prediction accuracy increases, it expects to position closer to the consumer’s ideal location, giving firm $i$ an advantage in competition. When the competitor’s prediction accuracy increases, the competitor is expected to position closer to the consumer’s ideal location, giving the competitor an advantage in competition.

More interestingly, Proposition 6 shows that in the symmetric case, a firm’s profit is not monotonic in the industry-level accuracy, $\alpha$. By extension, the total industry profit is also non-monotonic in $\alpha$. Note that the threshold, $\alpha_0$, is the same threshold as in Proposition 4. Proposition 4 shows that when $\alpha < \alpha_0$, an increase in $\alpha$ intensifies price competition for mainstream consumers but relaxes price competition for niche consumers. Corollary 5.1 shows that symmetric firms’ profits depend more on niche consumers. Thus, a higher $\alpha$ raises profit as the benefit of charging higher prices to niche consumers outweighs the cost of charging lower prices to mainstream consumers. The reverse is true when $\alpha > \alpha_0$. An increase in $\alpha$ relaxes price competition for mainstream consumers but intensifies price competition for niche consumers. The cost of charging lower prices to niche consumers outweighs the benefit of charging higher prices to mainstream consumers, leading to a decrease in profit as $\alpha$ increases.

To better understand why profit decreases as $\alpha$ moves away from $\alpha_0$, we can also consider the two limiting cases. In the limit as $\alpha \to 0$, firms do not personalize. Both firms always offer identical positioning at $\frac{1}{2}$. The complete lack of differentiation leads to the most intense price competition, minimizing profits. On the other hand, as $\alpha \to 1$, firms fully personalize. Because firms can perfectly observe each consumer’s ideal location, firms again offer identical positioning at the consumer’s ideal location, which again eliminates differentiation and minimizes profits. Therefore, an industry-level increase in the ability to predict consumer preferences is not necessarily good for firms. This calls for further investigation on firms’ incentives to invest in better predictions, and the possibility of industry self-regulation as a way to avoid prisoner’s dilemma. We examine these questions in Section 5.
3.4 Welfare

We examine ex-ante social surplus and consumer welfare, with a focus on the symmetric case. If the consumer chooses firm 1, the social surplus is \(-t(y_1 - x)^2\); If the consumer chooses firm 2, the social surplus is \(-t(y_2 - x)^2 + \Delta\epsilon\). The expected social surplus in the symmetric case can be calculated as

\[
SS = -\frac{t(1 - \alpha^2)}{12} + \frac{t^2(1 - \alpha)\alpha^2(20 + 36\alpha - 35\alpha^2 - 23\alpha^3 + 9\alpha^4 + 5\alpha^5)}{360(2 - \alpha^2)^2\theta} - \frac{3\theta}{4}
\]  

(26)

It is then straightforward to show that \(\frac{\partial SS}{\partial \alpha} > 0\). This is unsurprising, as firms are more likely to position closer to the consumer’s ideal location when prediction accuracy increases. Thus the ex-ante social surplus always increases in the industry-level prediction accuracy.

To get more nuanced results, we investigate the welfare of consumers with different ideal locations. Due to intractability, we use numerical analyses to describe how consumer welfare depends on a consumer’s ideal location and firms’ prediction accuracy.

Let \(U(x)\) denote the ex-ante consumer welfare for a consumer whose ideal location is \(x\). We first examine \(\frac{\partial U(x)}{\partial x}\) in Figure 6. We observe that, for a variety of parameter values, 

\[
\frac{\partial U(x)}{\partial x} \geq 0 \text{ if and only if } x \leq \frac{1}{2}
\]

Therefore, in our observations, consumer welfare always increases as \(x\) moves towards \(\frac{1}{2}\). Consumers with a more mainstream ideal location enjoy higher consumer welfare ex-ante.

![Figure 6: Ex-ante consumer welfare with ideal location at \(x\) (in a setting where \(\theta = 2, t = 1\)](image)

Intuitively, there are three reasons why consumers with more mainstream tastes are better off in equilibrium. First, given the equilibrium positioning strategy in Lemma 1,
consumers with more mainstream tastes receive positioning closer to their ideal locations when firms’ predictions are accurate, as firms hedge against the risk of incorrect predictions by positioning between their signals and the location $\frac{1}{2}$. Second, by Corollary 3.1, firms offer lower prices when they predict consumers to have more mainstream tastes. Third, when predictions are wrong, consumers with more main mainstream tastes also on average receive positioning closer to their ideal locations.

We then examine $\frac{\partial U(x)}{\partial \alpha}$, the effect of industry-level prediction accuracy on consumer welfare. Interestingly, ex-ante welfare for a consumer does not necessarily increase with $\alpha$. Figure 7 shows that it is possible to have $\frac{\partial U(x)}{\partial \alpha} < 0$. Particularly, when the accuracy is not too high, consumers with very mainstream tastes can be worse off ex-ante when $\alpha$ increases.

Figure 7: The effect of industry-level prediction accuracy on the welfare of consumer located at $x$ (in a setting where $\{\theta = 2, t = 1\}$)

To understand this result, consider a consumer whose ideal location is $\frac{1}{2}$. In the limit as $\alpha \to 0$, firms always position at this consumer’s ideal location, because firms seek to occupy more mainstream locations when their prediction power is low. As $\alpha$ increases, firms begin to personalize their positioning, which means that the consumer is more likely to receive a positioning farther from her ideal location. Even though by Proposition 4, her ideal product becomes cheaper as $\alpha$ increases, she is likely to receive a “wrong” positioning with a higher price if predictions are not accurate enough. Thus it is actually possible for the consumer to face worse products with higher prices when firms’ prediction accuracy increases. This drives down the consumer’s ex-ante welfare.

While some consumers can be hurt by an increase in prediction, we confirm that the overall consumer welfare, $\int_0^1 U(x)dx$, increases in $\alpha$. However, this result could change under a different distribution of consumers’ ideal location, given that not all consumers are
better off under a higher $\alpha$.

Figure 8: The effect of industry-level prediction accuracy on the overall consumer welfare (in a setting where $\{\theta = 2, t = 1\}$)

We summarize our numerical observations on consumer welfare below.

Remark 1 In the symmetric case, consumers with more mainstream tastes have higher utilities ex-ante. When prediction accuracy is not too high, consumers with sufficiently mainstream tastes are worse off when prediction accuracy increases. Overall consumer welfare increases in prediction accuracy.

4 Extensions

4.1 Investment in Prediction

In the base model, each firm’s prediction accuracy is exogenously given. In this section, we consider the possibility that firms choose their prediction accuracy endogenously through investment in data and algorithms.

The timing of the game is as follows. First, firm $i$ chooses $\alpha_i \in [0, 1]$ and firm $j$ chooses $\alpha_j \in [0, 1]$ simultaneously. Firm $i$ incurs an investment cost of $K(\alpha_i)$ for choosing a prediction accuracy of $\alpha_i$. Then firms compete according to the base model. Let $\Pi_i^*(\alpha_i, \alpha_j)$ denote the equilibrium expected profit of firm $i$ in the base model if the firms’ prediction accuracies are $\alpha_i$ and $\alpha_j$. Firm $i$’s equilibrium expected profit in the investment game is then $\Pi_i^*(\alpha_i, \alpha_j) - K(\alpha_i)$. We assume that the cost function $K(\cdot)$ satisfies $K'(0) = 0$, $K''(\cdot) \geq 0$, $K'''(\cdot) \leq 0$, and $K''''(\cdot) > 0$. The cost of acquiring the data $(\alpha_i)$ and the data $(\alpha_j)$ is $K(\alpha_i) + K(\alpha_j)$. Firm $i$'s objective is to maximize its expected profit by choosing $\alpha_i$. The optimal choice of $\alpha_i$ is 

$$
\alpha_i^* = \max_{\alpha_i} \Pi_i^*(\alpha_i, \alpha_j) - K(\alpha_i).
$$

The optimal choice of $\alpha_i$ is obtained by setting the derivative of the expected profit with respect to $\alpha_i$ to zero:

$$
\frac{d\Pi_i^*(\alpha_i, \alpha_j) - K(\alpha_i)}{d\alpha_i} = 0.
$$

Solving this equation yields the optimal choice of $\alpha_i$.
and \( \frac{d^2[\Pi_i^*(\alpha_i, \alpha_j) - K(\alpha_i)]}{d\alpha_i^2} < 0 \). The first condition guarantees that firms always want to have a positive accuracy. The second condition states that the cost function is increasing and convex. The last condition states that there is a diminishing return to investment in accuracy. For parameters \( \theta = 2 \) and \( t = 1 \), for example, any cost function that is increasing and convex would also satisfy the last condition.

For simplicity, we focus on symmetric outcomes. Let \( \alpha^* \) denote the prediction accuracy in a symmetric equilibrium of the investment game. As a benchmark, we define \( \alpha_{\text{coll}} \) as the industry-level prediction accuracy that maximizes industry profit, i.e., the prediction accuracy chosen by collusive firms aiming at maximizing their total profits. More specifically,

\[
\alpha_{\text{coll}} = \operatorname{argmax}_{\alpha} \Pi_i^*(\alpha, \alpha) + \Pi_j^*(\alpha, \alpha) - 2* K(\alpha) \tag{27}
\]

We can prove the following result:

**Proposition 7** In a symmetric equilibrium, the equilibrium prediction accuracy must be strictly greater than the industry-level accuracy that maximizes industry profit, i.e., \( \alpha^* > \alpha_{\text{coll}} \). Each firm earns a lower profit under equilibrium accuracy \( \alpha^* \) than under collusive accuracy \( \alpha_{\text{coll}} \). Consumer welfare is high under equilibrium accuracy \( \alpha^* \) than under collusive accuracy \( \alpha_{\text{coll}} \).

Proposition 7 states that firms always over-invest in prediction accuracy in a symmetric equilibrium, compared to the collusive level of investment that would maximize industry profit. As shown in Proposition 6, higher prediction accuracy can lower industry profit by intensifying price competition for the more profitable, niche consumer segment. This effect is exacerbated by the extra cost of investing in higher prediction accuracy. To gain intuition into this result, consider the marginal value of investing. The marginal value of a higher \( \alpha_i \) to firm \( i \) is \( \frac{d\Pi_i^*(\alpha_i, \alpha_j)}{d\alpha_i} - K'(\alpha_i) \), which is always greater than the marginal value to the industry, \( \frac{d\Pi_i^*(\alpha_i, \alpha_j)}{d\alpha_i} + \frac{d\Pi_j^*(\alpha_j, \alpha_i)}{d\alpha_i} - K'(\alpha_i) \), because a higher accuracy for firm \( i \) always hurts firm \( j \), i.e., \( \frac{d\Pi_j^*(\alpha_j, \alpha_i)}{d\alpha_i} < 0 \). Because firms do not consider the negative externality their investments impose on competitors, they suffer from the prisoner’s dilemma.

Another implication is that both firms can benefit if they collectively lower their prediction accuracies. Unlike other forms of collusion, collusion between firms to decrease the accuracy of their predictions of consumer preferences can be in line with public interests.
As firms gain increasingly granular data on individual consumers, regulators express more concerns over violations of consumer privacy. This means that firms could achieve collusion by self-regulating through mediums such as industry associations, without fearing the public backlash and legal scrutiny other forms of collusion would receive. Proposition 7 shows that firms have incentives to reach an agreement to limit the industry’s prediction accuracy. In the context of the model, consider self-regulation in the form of an upper bound $\hat{\alpha}$, so that firms can only choose accuracy between 0 and $\hat{\alpha}$.\footnote{For example, industries that use consumers’ geo-location data can require all location data only to be accurate to a certain radius.} For $\hat{\alpha}$ sufficiently close to $\alpha_{coll}$, both firms strictly prefer playing the game with the upper bound $\hat{\alpha}$ rather than without the upper bound.

Note that Remark 1 implies that consumer welfare would decrease if firms self-regulate to achieve a lower industry accuracy. However, our model only considers consumer utility from purchase. If there is any intrinsic value of privacy, such as protection from data breach or preference for anonymity, then self-regulation on prediction accuracy may also improve consumer welfare.

### 4.2 Consumer Search

In the base model, consumers can choose only between the two firms’ personalized product offerings. In practice, however, consumers can access more products by using the search functions on a seller’s website. The extent to which consumers rely on personalization versus search in shopping may depend on their search costs. Consumers with higher search costs may rely more on the seller’s personalization, whereas those with lower search costs may use search functions more to find their ideal products.

In this extension, we categorize consumers into two segments based on their search costs. High search costs consumers depend on the personalized product offerings from each firm because searching for alternatives is too costly. They choose between the personalized positionings provided by the two firms. Low search costs consumers are capable of searching through the entire product catalog offered by each firm to find their ideal products. This simple extension allows us to examine how firms’ personalization strategies and competitive dynamics change when factoring in consumer search behavior.

More specifically, high search costs consumers choose between $y_1(s)$ with a price of $p_1(s)$
and \( y_2(s) \) with a price of \( p_2(s) \), same as consumers in the base model. Low search costs consumers can instead choose any positioning \( y_1 \) from firm 1 and pay \( p_1(s) \), or any positioning \( y_2 \) from firm 2 and pay \( p_2(s) \). We assume that there are equal proportions of high search costs and low search costs consumers.

All proofs are in the Online Appendix. Our results on optimal strategies remain qualitatively unchanged. However, the presence of low search cost consumers flattens the equilibrium pricing strategy \( p_i^*(s) \).

In the symmetric case, the base model shows that price competition is more intense at mainstream signals or positionings and more relaxed at niche signals or positionings. The presence of low search cost consumers relaxes price competition at mainstream signals or positioning and intensifies it at niche signals or positioning. This is intuitive. Consider the limit where all consumers have high search costs: this scenario mirrors the base model. Now, consider the limit where all consumers have low search costs: in this case, every consumer will choose their ideal product, rendering firms’ signals and personalized positionings irrelevant. Consequently, firms are no longer differentiated on the personalizable dimension, leading to uniform pricing regardless of the signals received or the positionings offered to consumers, making \( p_i^*(s) \) constant in \( s \).

When there are more high search cost consumers, the results approach those of the base model. Conversely, when there are more low search cost consumers, \( p_i^*(s) \) becomes flatter. This dynamic highlights how consumer search behavior can influence the competitive landscape and pricing strategies in personalized markets.

One caveat to this simple extension is that low search costs consumers pay the same price, \( p_i(s) \), regardless of which positioning they choose from firm \( i \). A more realistic assumption is to allow price to be functions of positioning as well, \( p_i(s, y) \). This does not matter in the base model, because high search costs consumers only view the personalized positioning from each firm.

However, allowing the low search costs consumers to face different prices when selecting different positioning significantly complicates the problem. In the Online Appendix, we discuss a discrete version of the model. Consumers’ ideal locations are either at 0, 1/2, or 1. Each firm also only has three product positionings. Low search costs consumers see different prices when they select niche versus mainstream positioning. We find our results continue to hold qualitatively. The existence of low search costs consumers flatten the price strategy, so
in the symmetric case, firms charge higher prices at the mainstream positioning and lower prices at the niche positionings when there are more low search costs consumers.

One caveat to this simple extension is that low search cost consumers pay the same price, $p_i(s)$, regardless of which positioning they choose from firm $i$. A more realistic assumption is to allow the price to be a function of positioning as well, $p_i(s, y)$. This distinction is irrelevant in the base model, as high search cost consumers only view the personalized positioning from each firm.

However, allowing low search cost consumers to face different prices when selecting different positionings significantly complicates the problem. In the Online Appendix, we discuss a discrete version of the model. Consumers’ ideal locations are at either 0, $\frac{1}{2}$, or 1. Each firm also has only three product positionings. Low search cost consumers see different prices when they select niche versus mainstream positionings.

We find that our results continue to hold qualitatively. The existence of low search cost consumers flattens the price strategy, leading to firms charging higher prices at the mainstream positioning and lower prices at the niche positionings in the symmetric case when there are more low search cost consumers. This demonstrates that while low search cost consumers introduce additional complexity, the core insights of our model remain robust.

5 Exploring Boundary Conditions with Alternative Models

5.1 Subscription Business Model

In the main model, consumers observe each firm’s personalized positioning, $y_i$, and price, $p_i$, before making a purchase decision. Such a model is representative of the interactions between consumers and firms in e-commerce. Product personalization is also a common strategy in the content market. Users of platforms such as Netflix or Hulu receive contents of different genres, topics, political views, etc. The business model, however, is different in the content market. Consumers of Netflix or Hulu have to pay subscription fees before seeing what content the algorithms present to them. Netflix or Hulu also learn about consumers’ preferences primarily after they become subscribers. Firms using such a subscription business model also cannot charge consumers different prices for different pieces of content. In this section, we consider a variation of our model that captures this scenario and compares the
results to those of the main model.

Consider a modification to the model presented in Section 2 with the following timing. In the first stage, both firms choose their subscription prices, \( p_1 \) and \( p_2 \), and their content personalization algorithms, \( y_1(s_1) \) and \( y_2(s_2) \). In the second stage, the focal consumer, after observing the prices and personalization algorithms of both firms, subscribes to one of the two firms by paying the subscription price. In the last stage, the subscribed firm \( i \) receives a signal \( s_i \) about the consumer’s ideal location and offers a personalized positioning of \( y_i(s_i) \) to the consumer. In reality, consumers do not directly observe each firm’s personalization algorithm but can decide whether to resubscribe based on the observed average performance of the firm’s personalization algorithm. Because we do not model repeated interactions in this paper, to make consumers’ subscription decisions reactive to firms’ personalization strategies, we assume that each firm’s personalization algorithm is revealed to consumers before subscribing.

We solve the game in the Online Appendix. One can show that in equilibrium, firm \( i \)’s personalization strategy is
\[
y_i^*(s_i) = \alpha_i \cdot s_i + (1 - \alpha_i) \cdot \frac{1}{2},
\]
firm \( i \)’s equilibrium subscription price is
\[
p_i^* = \theta + \frac{t}{36}(\alpha_i^2 - \alpha_j^2),
\]
and firm \( i \)’s equilibrium profit is
\[
\Pi_i^* = \frac{(36\theta + t(\alpha_i^2 - \alpha_j^2))^2}{2592\theta}.
\]

Note that firms’ equilibrium positioning strategy and average price are the same as in the main model, which is intuitive. In the main model, because firms cannot observe their rival’s offerings, they choose a positioning strategy that minimizes the expected distance to each consumer’s ideal location. Such a personalization strategy also maximizes a firm’s demand in the subscription model. Also, the average price in the main model depends on the size of a firm’s advantage in prediction over its competitor, with the prediction accuracy performing a similar role to product quality. The same intuition applies here.

However, a crucial difference is that firms in the subscription model can no longer charge different prices for offering content of different positioning. In the main model where firms
charge different prices after receiving mainstream and niche signals, we find that firms profit more from consumers predicted to have more niche tastes, and make higher equilibrium profits when price competition for those (predicted) niche consumers softens. Such price discrimination no longer exists for the subscription model. As a result, in the symmetric case with $\alpha_i = \alpha_j = \alpha$, equilibrium profits in the subscription model do not depend on $\alpha$, because equilibrium subscription prices do not depend on $\alpha$. This is different from the main model where equilibrium profits depend on the prediction accuracies.

5.2 The Salop Circular Model

The main model assumes that consumers’ ideal locations are distributed along the Hotelling line. Consumers are considered to have mainstream tastes or niche tastes depending on where they locate on the Hotelling line. In this section, we consider an alternative model where consumers’ ideal locations are distributed along the Salop Circle instead. Consumer tastes on a Salop Circle can no longer be described as mainstream or niche, as any two points on a circle are symmetric to each other.

Consumers’ ideal location on product positioning is uniformly distributed on a circle with a circumference of 1. We use $r_0 = \frac{1}{2\pi}$ to denote the circle’s radius, and further use polar coordinates to denote a consumer’s ideal location. Specifically, a consumer’s ideal location on product positioning is denoted as $(r_0, x)$, with $x \sim U[0, 2\pi]$.

Let $s_i \in [0, 2\pi)$ denote firm $i$’s signal of the focal consumer’s ideal location. Let $y_i \in [0, 2\pi)$ denote firm $i$’s positioning to the focal consumer. The consumer’s (dis)utility from choosing firm 1 and firm 2 are

$$U_1 = -p_1 - t \cdot d(y_1, x) \quad (28)$$
$$U_2 = -p_2 - t \cdot d(y_2, x) + \Delta \epsilon \quad (29)$$

where $d(y_i, x) \equiv \min\{(r_0 |y_i - x|)^2, (1 - r_0 |y_i - x|)^2\}$. Same as in the main model, we let $\Delta \epsilon \sim U[-\theta, \theta]$ and assume $\theta$ is sufficiently large.

We solve the game in the Online Appendix. One can show that in equilibrium, firm $i$’s personalization strategy is

$$y_i^* = s_i \quad (30)$$

firm $i$’s equilibrium subscription price is

$$p_i^* = \theta + \frac{t}{36}(\alpha_i - \alpha_j) \quad (31)$$
and firm $i$’s equilibrium profit is

$$\Pi_i^* = \frac{(36\theta + t(\alpha_i - \alpha_j))^2}{2592\theta}$$

(32)

Note that the equilibrium when consumers are located on a Salop Circle is very different from the equilibrium when consumers are located on a Hotelling line. First, in the Salop Circle model, firms always position precisely at their signals. There is no hedging between the signal and the most mainstream point because there are no mainstream or niche locations on the Salop Circle. Because all consumer locations are symmetric to each other, firms also charge the same price for all positioning. Firms do not price discriminate between mainstream and niche consumers because all consumers are equally “niche”. As a result, equilibrium profits also do not depend on the signals that firms receive. In the symmetric case, firms’ equilibrium profits no longer depend on the industry-level accuracy $\alpha$, as equilibrium prices do not depend on $\alpha$.

The comparison between the Salop Circle model and the main model shows that a key driver of the main model is the existence of mainstream versus niche consumer tastes. Thus, the applicability of our results to an industry depends crucially on whether such notions of mainstream and niche consumer tastes exist in that industry.

### 5.3 Common Signals

In the main model, we assume each firm receives a private signal. Firms can form expectations about their competitors’ signals but cannot directly observe them. While the firms’ signals are correlated with the true consumer preferences, for simplicity, we assume that any errors in these signals are uncorrelated. However, in reality, firms might also make correlated errors. In extreme cases, if firms acquire data from the same vendor and use the same algorithm for inferences, their signals could become perfectly correlated. We discuss the implications of firms receiving common signals below.

For each consumer, two firms receive a common signal, $s$. As before, the signal is correct (i.e., $s = x$) with probability $\alpha$, and is independent of $x$ with probability $1 - \alpha$. The rest of the model remains the same.

We solve the equilibrium in the Online Appendix. The equilibrium positioning strategy follows the same form as in the main model. Firms hedge between the common signal and
the most mainstream positioning:

\[ y_i^*(s) = (1 - \alpha) \frac{1}{2} + \alpha s \] (33)

This implies that firms always offer the same positioning because they receive the same signal. Firms do not strategically differentiate because they cannot set personalized prices based on competitors’ positioning offerings. As shown in Lemma 1, firms always prefer to minimize the expected distance.

Because firms offer the same personalized positioning, price competition is always at its most intense level. The equilibrium prices are

\[ p_i^*(s) = p_j^*(s) = \theta \] (34)

Thus, the price does not depend on the signal. The main model’s pricing results do not hold when firms receive common signals, as common signals eliminate the differences between niche and mainstream consumers.

The equilibrium profits are

\[ \Pi_i^*(s) = \Pi_j^*(s) = \frac{\theta}{2} \] (35)

which is lower than the firms’ equilibrium profits under separate signals. One implication is that sharing data could potentially hurt both firms, as their personalized positionings become homogenized and price competition intensifies.

6 Conclusion

This paper studies the competition between two firms that can personalize product offerings based on imperfect prediction technology. Consumers’ ideal locations are distributed along the Hotelling line. Each firm receives a noisy signal of each consumer’s ideal location and simultaneously decides the positioning and price to offer to each consumer without observing the competitor’s offer.

We characterize the equilibrium personalization strategy for both the general case, where firms have different prediction accuracies, and the symmetric case, where firms have the same industry-level accuracy. We show that a firm’s personalized positioning tends to be more mainstream than consumers’ ideal locations on average, and the firm charges a higher
price for a more niche positioning unless its prediction accuracy is significantly lower than its competitor’s. In the asymmetric case, a firm’s prediction accuracy functions similarly to product quality, allowing higher accuracy firms to charge higher prices and earn greater profits.

In the symmetric case, we find that firms charge higher prices and earn higher profits when offering more niche positioning. The degree of product differentiation first increases and then decreases with prediction accuracy, and profits follow a similar pattern. We also find that an increase in industry-level prediction accuracy first relaxes and then intensifies price competition for consumers predicted to have niche tastes, while the reverse is true for consumers predicted to have mainstream tastes.

If firms can endogenously invest in prediction accuracies, we find that firms over-invest in equilibrium. Competitors can mutually benefit from industry-level self-regulations that restrict their ability to predict individual consumer preferences. We also explore scenarios where some consumers can search, firms operate on a subscription pricing model, consumer tastes are located on a Salop Circle, or firms receive common signals from the same data source.

This paper expands the literature on spatial competition and product positioning by incorporating personalization into the positioning decision. Several limitations highlight the need for future research. First, the paper assumes that firms personalize by presenting only one positioning. It is important to consider what happens if firms can offer a personalized menu to each consumer, allowing the consumer to choose from the menu. Second, firms’ information about consumer preferences is modeled as a black-box signal. Future research could explore how such information can be derived from consumers’ online footprints. Finally, the paper only models horizontal preferences. If consumers have heterogeneous vertical preferences, firms could potentially personalize quality and price using signals of each consumer’s willingness to pay for quality.
References


