To Each Their Own: Personalized Product Positioning and Competition

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Abstract

This paper studies a model of competition between two firms that offer personalized product positioning to consumers with horizontally distributed tastes. Firms have private, imperfect signals of each consumer’s ideal location and offer each consumer a personalized positioning and price depending on that signal, without observing the competing firm’s personalized offering. We characterize the equilibrium personalization strategy and examine how the accuracies of firms’ signals affect equilibrium strategy, profits, and consumer welfare. We find that a competing firm charges a higher price and earns a higher profit for a more niche positioning unless the firm’s prediction accuracy is sufficiently low. When both firms have access to the same industry-level prediction accuracy, we find that the average price does not depend on accuracy, that firms charge a higher price when offering a more niche positioning, and that the average level of differentiation first increases and then decreases in the prediction accuracy. Interestingly, equilibrium profits also have an inverse-U shape in the prediction accuracy. A higher accuracy can decrease welfare for consumers with very mainstream tastes. When firms can endogenously invest in prediction accuracy, firms over-invest in equilibrium which results in a prisoner’s dilemma. In such a case, firms can benefit from industry-level self-regulations that restrict their ability to predict individual consumer preferences. The paper also discusses what happens if firms charge subscription pricing or if consumers’ ideal locations are distributed on the Salop Circle, highlighting price discrimination between mainstream and niche consumers as the key driver of our model.

Keywords: personalization, product positioning, pricing, self-regulation.

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1 Introduction

With better predictive technologies and more granular data on individual consumers, firms are increasingly able to conduct marketing mixes at a personal level. Besides personalizing price and communication, firms can also personalize product positioning. Instead of trying to sell the same product with the same positioning to all consumers, firms can present products of different positioning to different consumers based on firms’ knowledge of individual consumers’ preferences.

Examples abound online. In a marketing email, brands often present a small set of products to consumers, and the set can be personalized. Consider a consumer who receives a marketing email from Sephora. Sephora carries a wide range of skincare and cosmetics products that are positioned for consumers of different skin conditions, lifestyles, and aesthetics. Due to an email’s limited space and consumers’ limited attention span, Sephora cannot present its entire catalog in an email. Instead, using its information on each consumer, Sephora can present products of different positioning to consumers with different preferences. Such personalization also takes place when consumers shop online. Consider a consumer who is shopping for sports apparel on Adidas.com. Adidas carries various products with different functionalities, colors, and styles. Using its information on each consumer, Adidas can personalize its website, so that consumers with different preferences are presented with products of different positioning. Similar personalization happens in online advertising when firms showcase specific products in their ad banners. Websites can also personalize search results so consumers searching the same query can see products of different brands/styles/colors (Yoganarasimhan 2020). See Figure 1 for examples of aforementioned personalization in Sephora’s email marketing, online advertising, and e-commerce website.

There are also firms that specialize in such product personalization. Firms such as Stitch Fix and Wantable personalize shopping by picking apparel for their consumers based on each consumer’s preferences in styles, brands, sizes, etc. A consumer who shops through Stitch Fix or Wantable receives a personalized blind box with several items that are picked by their algorithms, and decides whether to buy each of the items.

The ability of firms to present products of different positioning to different consumers is a significant deviation from the traditional concept of product differentiation and warrants additional research. What should a firm’s personalization strategy be when competing with another firm that can also personalize product offerings? What prices do firms charge for...
products of different positioning? More importantly, if personalization is enabled by firms’ abilities to predict individual consumers’ preferences, how do firms’ strategies depend on the accuracies at which they can predict individual consumers’ preferences? Do firms and consumers benefit when firms have more accurate knowledge of individual consumers’ preferences? The latter questions have implications for firms that consider investing in acquiring more data and developing better algorithms, as well as for regulators who are concerned with privacy-related issues on consumer welfare.

To answer these questions, we study the competition between two firms that personalize product positioning to consumers. We assume that each firm has a private, imperfect signal of each consumer’s ideal product positioning, and chooses a personalized positioning and price offer for each consumer based on this private signal. Consumers’ ideal locations are horizontally distributed in the Hotelling line. Consumers located closer to the middle point of the Hotelling line can be seen as having more mainstream tastes, whereas consumers located farther from the middle point can be seen as having more niche tastes.

The private signal of a consumer’s ideal location is a parsimonious representation of a firm’s inference of individual consumer preferences using behavioral data and predictive technology such as machine learning. Given the signals, firms simultaneously choose positioning and price for each consumer. A key difference from the traditional Hotelling-type models (e.g., d’Aspremont et al. 1979) is that we assume firms cannot observe the positioning and
price their rival offers to a consumer, reflecting firms’ inability to observe how competitors personalize to a given consumer on their websites, emails, or ads in practice.

We characterize firms’ equilibrium positioning and pricing strategies. We examine how changes in the prediction accuracies affect strategies, profits, and consumer welfare. We consider both the general, asymmetric case, where the two firms have different prediction accuracies, and the symmetric case, where the two firms have the same accuracy. The symmetric case can also be seen as a situation where both firms can access the same predictive technology. An increase in the symmetric accuracy represents an industry-level improvement in prediction due to technological advancement.

We show that in equilibrium, a firm offers a personalized positioning that hedges between the firm’s signal of the consumers’ ideal positioning and the most mainstream positioning. The range of positioning that a firm offers to consumers increases in its prediction accuracy. In the general case, the firm charges a higher price for a more niche positioning, unless the firm’s prediction accuracy is sufficiently lower than the competitor’s. We find that a firm’s prediction accuracy performs similarly to product quality. A higher accuracy allows a firm to charge higher prices and earn higher a profit.

In the symmetric case, firms always charge a higher price for a more niche positioning, and profit more from consumers with more niche tastes. The degree of product differentiation has an inverse U-shape in the industry-level prediction accuracy. We find that a higher industry-level accuracy first relaxes (intensifies) then intensifies (relaxes) price competition for niche (mainstream) consumers. Equilibrium profits also exhibit an inverse-U shape in the prediction accuracy. A higher accuracy can also decrease welfare for consumers with very mainstream tastes.

We also consider the scenario where firms can endogenously invest their prediction accuracies. We find that firms face a prisoner’s dilemma: a marginal increase in accuracy always increases a firm’s profit but decreases its competitor’s profit, and the net effect on industry profit can be negative. As a result, in the symmetric equilibrium, firms always over-invest in their prediction accuracies compared to the industry-level accuracy that would maximize industry profit. In such a case, firms can mutually benefit from industry-level self-regulations that restrict their ability to predict individual consumer preferences. Such “accuracy-fixing” does not have the negative connotation of price collusion, and more importantly, it can be in line with public interests at a time when regulators are increasingly concerned about the
violation of consumer privacy.

The paper also presents two variations of the main model. In the first variation, a consumer pays a subscription price to a firm before receiving a personalized product, which is a common business model in the content market. In the second variation, consumers’ ideal locations are distributed on the Salop Circle instead of the Hotelling line. In both extensions, firms charge the same price to all consumers in equilibrium, and in the symmetric case, profits do not depend on the industry-level prediction accuracy. These two extensions highlight firms’ ability to price discriminate between mainstream and niche consumers as the main driver behind our main results.

The rest of the paper is organized as follows. After reviewing the literature in Section 2, we present the model in Section 3. The equilibrium is analyzed in Section 4. Section 5 presents three extensions to the main model. Section 6 concludes the paper.

2 Literature Review

Compared to the extensive literature on personalized pricing (e.g., Shaffer and Zhang 1995, Villas-Boas 1999, Fudenberg and Tirole 2000, Chen et al. 2001, Chen and Iyer 2002, Villas-Boas 2004, Choe et al. 2018, Li et al. 2023) and personalized communication (e.g., Ansari and Mela 2003, Iyer et al. 2005, Lambrecht and Tucker 2013, Gardete and Bart 2018, Sahni et al. 2018), there are considerably fewer studies on personalized product positioning. This paper attempts to narrow the gap. An important paper on the topic is Zhang (2011), which considers a two-period customer recognition model where firms offer different positioning for their own customers versus their rivals’ customers in the second period. Pazgal and Soberman (2008) and Li (2021) consider the case where firms can offer different vertical qualities, instead of horizontal positioning, to different consumer segments. Our paper is different from Zhang (2011), Pazgal and Soberman (2008), and Li (2021) in several aspects. First, in the three aforementioned papers, a firm segments consumers into two groups and offers a product positioning for each segment. The segments are determined by consumers’ choices of non-personalized products in the first period. In comparison, we allow firms to offer a different personalized positioning to each individual consumer based on noisy information about each individual’s preferences. Another main difference is in our research questions. Our paper focuses on the effects of firms’ prediction accuracies on personalization strategies, profits, and consumer welfare, which is not studied by Zhang (2011), Pazgal and Soberman
Lastly, in the three aforementioned papers, firms observe their rivals’ positioning strategies before setting prices. We study the case where firms cannot observe rivals’ personalized offerings to each individual, which is typically the case for e-commerce.

Our paper also relates to the literature on product customization (e.g., Dewan et al. 2003, Syam et al. 2005, Syam and Kumar 2006). The key difference between product personalization studied in the current paper and product customization is whether consumers or firms decide the product positioning (Arora et al. 2008). In product customization models, a firm presents a range of product positioning, and each consumer proactively chooses a positioning from that range. The same range of products is presented to all customers. Different consumers purchase products of different positioning from the same firm as a result of their own choices. In the current paper, a firm presents a different positioning to each individual consumer. Hence, consumers purchase products of different positioning from the same firm as a result of the firm’s choices. In reality, e-commerce firms usually present a personalized menu to each consumer, so a consumer’s product choice is both a result of the firm’s personalization and the consumer’s own decision within the menu. In this paper, we focus on the simpler problem where the menu only contains one product. An interesting research question beyond the scope of our paper is what happens when a firm can offer a different range of products to each consumer, effectively combining personalization and customization.

In practice, firms implement personalized product positioning through recommendation algorithms. Thus, our paper is also related to the literature on the strategic effects of personalized recommendation or search ranking (e.g., Hagiu and Jullien 2011, Inderst and Ottaviani 2012, Yang 2013, Hagiu and Jullien 2014, Li et al. 2018, Ke et al. 2022, Teh and Wright 2022, Zhong 2023, Zhou and Zou 2023, Zou and Zhou 2023). The motivations and the contexts are different. The aforementioned papers consider the strategic decisions of an intermediary that hosts sellers and buyers, such as Amazon and Taobao. The intermediary decides which seller to recommend to each consumer, among an existing set of sellers, or decides how to rank the sellers in a search query. In such a context, the intermediary does not have direct control over the sellers’ positioning and pricing decisions. In the aforementioned papers, product positioning and prices are either assumed to be exogenous or decided by sellers on the platform. In contrast, our paper is motivated by individual brands that utilize personalization in their online presence. Firms directly choose positioning and pricing for each individual consumer. The role of competition is also different. In the aforementioned papers, the intermediary
faces no direct competition but has to factor in the competition among sellers when making recommendations. In contrast, our paper studies direct competition between two firms that can personalize positioning and price.

Finally, our paper relates to the literature on targetability, or the accuracy with which a firm can predict individual consumers’ preferences. Previous papers have studied the effects of targetability on personalized pricing (e.g., Chen et al. 2001), personalized search ranking (e.g., Yang 2013 and Zhong 2023), and targeted advertising (e.g., Ning et al. 2023). Our paper contributes to the literature by studying the effects of targetability on personalized positioning, and subsequently, firms’ incentives to invest in or regulate targetability. Our paper echoes the finding of Chen et al. (2001) that a higher industry-wide targetability can decrease industry profit, which suggests that self-regulation can be profitable. In addition, in our model, such a prisoner’s dilemma always materializes when firms endogenously invest in targetability, further highlighting the importance of industry-wide self-regulation.

3 Model

We consider a market with a unit mass of consumers and two firms. Consumer preferences on product positioning are horizontal. Consumers’ ideal locations on product positioning are uniformly distributed, \( X \sim U[0, 1] \). Two firms compete on product positioning and prices. Firms can personalize product positioning, i.e., offering a different positioning in \([0,1]\) for each individual consumer.

Because firms can personalize their product and price offerings to each consumer, we only need to describe the competition over a focal consumer. Firms have incomplete information about the consumer’s ideal location. Before offering a product to the consumer, each firm \( i \) receives a private signal, \( S_i \), about the consumer’s location with accuracy \( \alpha_i \). If the signal is inaccurate, we assume the signal is randomly drawn from \([0,1]\) for simplicity. Thus, if the consumer’s ideal location is \( x \), then

\[
\Pr(\{S_i = X\}) = \alpha_i \quad (1)
\]

\[
\Pr(\{S_i \perp X\}) = 1 - \alpha_i \quad (2)
\]

Each firm’s strategy has two components: personalized product positioning and pricing. Each firm decides what positioning to offer to each consumer, as well as what price to charge for each positioning. Let \( y_i \in [0,1] \) denote firm \( i \)’s personalized positioning to the focal
consumer. We denote \(y_i(s_i)\) as firm \(i\)'s personalized positioning to the focal consumer if firm \(i\)'s signal is \(s_i\). We denote \(p_i(s_i)\) as firm \(i\)'s price to the focal consumer after receiving a signal of \(s_i\).

The consumer's (dis)utility from choosing firm 1 and firm 2, given positioning, \(y_1\) and \(y_2\), and prices, \(p_1\) and \(p_2\), are defined as

\[
U_1 = v - p_1 - t \cdot (y_1 - x)^2
\]

\[
U_2 = v - p_2 - t \cdot (y_2 - x)^2 + \Delta \epsilon
\]

where \((y_i - x)^2\) is the quadratic distance between the consumer's ideal location and firm \(i\)'s personalized positioning, and \(t\) measures the consumer's sensitivity to the mismatch between her idea location and the firm's personalized positioning. We assume that the product utility \(v\) is large enough such that the market is fully covered. The consumer always buys exactly one product. We normalize \(v\) to 0 for the rest of the paper.

The term \(\Delta \epsilon\) represents the consumer’s idiosyncratic preferences that firms cannot observe (de Palma et al. 1985).\(^1\) Such a term is common in empirical discrete-choice models. For simplicity as well as symmetry, we assume \(\Delta \epsilon \sim U[-\theta, \theta]\). Not that, if \(p_1 = p_2\) and \(y_1 = y_2\), then the consumer strictly prefers firm 1 if and only if \(\Delta \epsilon < 0\), and strictly prefers firm 2 if and only \(\Delta \epsilon > 0\). The model remains the same if the term \(\Delta \epsilon\) is in the consumer’s utility from choosing firm 1 instead of firm 2.

The timing of the game is as follows. First, nature draws the focal consumer’s ideal location, \(x\), and the consumer’s idiosyncratic preferences between the two firms, \(\Delta \epsilon\). Then firms receive private signals, \(s_1\) and \(s_2\), of the focal consumer’s ideal location. The two firms then simultaneously choose personalized positioning strategy, \(y_1(s_1)\) and \(y_2(s_2)\), and pricing strategy, \(p_1(s_1)\) and \(p_2(s_2)\). The consumer observes her own ideal locations, \(x\), her idiosyncratic preferences, \(\Delta \epsilon\), and both firms’ positioning and prices. The consumer then chooses the product that returns the highest utility. For tie-breaking, the consumer buys from firm 1 if the consumer is indifferent between the two firms. \(^2\) Each firm’s objective is to maximize its expected profit.

Horizontal positioning models typically assume that positioning and pricing decisions

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\(^1\)In a Hotelling game where firms simultaneously choose both positioning and price, no pure-strategy equilibrium exists (Gabszewicz and Thisse 1992). However, pure-strategy equilibrium can be restored with the addition of an idiosyncratic error term in utility functions which smooths firms’ demand functions (de Palma et al. 1985). Our paper adds the idiosyncratic error term for the same purpose.

\(^2\)All results remain the same if a different tie-breaking rule is used.
are made sequentially (e.g., d’Aspremont et al. 1979, de Palma et al. 1985, Hauser 1988): firms first choose product positioning, then choose prices after observing the opponent’s positioning. The justification is that positioning is hard to adjust once determined, but pricing is more flexible. In our model, the simultaneous move reflects the fact that firms usually cannot observe competitors’ product personalization choices to a given consumer, and firms can adjust such personalization strategy quickly. For example, Walmart’s online store cannot observe Amazon’s personalized offerings to each individual consumer.

For tractability, we assume that the size of the idiosyncratic preferences, \( \theta \), is sufficiently large. This guarantees the existence of pure-strategy equilibrium.

**Assumption 1** \( \theta \geq \underline{\theta} \), where the lower bound \( \underline{\theta} \) is given in the appendix.

We look for pure-strategy Nash equilibrium, which is characterized by a quadruple of functions, \( \{ y_1^*(s_1), p_1^*(s_1), y_2^*(s_2), p_2^*(s_2) \} \).

### 4 Equilibrium Analysis

A consumer chooses firm 1 if

\[
\Delta \epsilon \leq p_2 - p_1 + t \left[ (y_2 - x)^2 - (y_1 - x)^2 \right]
\]

and chooses firm 2 otherwise. Therefore, given \( \{ y_1, y_2, p_1, p_2, x \} \), the consumer’s expected demand for firm 1 can be written as:

\[
Q_1(y_1, y_2, x) + \frac{p_2 - p_1}{2\theta}
\]

and the consumer’s expected demand for firm 2 can be written as:

\[
Q_2(y_2, y_1, x) + \frac{p_1 - p_2}{2\theta}
\]

where

\[
Q_1(y_1, y_2, x) = \frac{1}{2} + \frac{t}{2\theta} \left[ (y_2 - x)^2 - (y_1 - x)^2 \right]
\]

\[
Q_2(y_2, y_1, x) = \frac{1}{2} + \frac{t}{2\theta} \left[ (y_1 - x)^2 - (y_2 - x)^2 \right]
\]
Note that equations (8) and (9) capture the consumer’s expected demand for firm 1 and firm 2 when \( p_1 = p_2 \).

Consider the consumer’s expected demand for each firm \( i \). Suppose firm \( j \) follows a strategy of \( y_j(s_j) \) and \( p_j(s_j) \). Given (6) and (7), firm \( i \)'s expected demand conditional on its signal \( s_i \) can be described by 4 events as follows.

- **Event 1:** Both firms receive the correct signal, i.e. \( s_i = s_j = x \).
  
  This event happens with probability \( \Pr(\text{Event 1}) = \alpha_i \alpha_j \). In this case, firm \( i \)'s expected demand is
  
  \[
  Q_i(y_i, y_j(x), x) + \frac{p_j(x) - p_i}{2\theta}
  \]
  
  where \( Q_i(y_i, y_j(x), x) \) is given in equations (8) and (9).

- **Event 2:** Firm \( i \) receives the correct signal and firm \( j \) receives a wrong one i.e. \( s_i = x, s_j \perp x \).
  
  This event happens with probability \( \Pr(\text{Event 2}) = \alpha_i(1 - \alpha_j) \). In this case, firm \( i \)'s expected demand is
  
  \[
  \int_0^1 Q_i(y_i, y_j(s_j), x) + \frac{p_j(s_j) - p_i}{2\theta} \, ds_j
  = \int_0^1 Q_i(y_i, y_j(s_j), x) \, ds_j + \frac{1}{2\theta} \cdot (E p_j - p_i)
  \]
  
  where \( E p_j = \int_0^1 p_j(s_j) \, ds_j \) is the expected price firm \( j \) would charge under its strategy.

- **Event 3:** Firm \( i \) receives a wrong signal and firm \( j \) receives the correct one i.e. \( s_i \perp x, s_j = x \).
  
  This event happens with probability \( \Pr(\text{Event 3}) = (1 - \alpha_i)\alpha_j \). In this case, firm \( i \)'s expected demand is
  
  \[
  \int_0^1 Q_i(y_i, y_j(x), x) + \frac{p_j(x) - p_i}{2\theta} \, dx
  = \int_0^1 Q_i(y_i, y_j(x), x) \, dx + \frac{1}{2\theta} \cdot (E p_j - p_i)
  \]
  
- **Event 4:** Both firms receive a wrong signal, i.e. \( s_i \perp x, s_j \perp x \).
This event happens with probability $\Pr(\text{Event 4}) = (1 - \alpha_i)(1 - \alpha_j)$. In this case, firm $i$’s expected demand is

$$
\int_0^1 \int_0^1 Q_i(y_i, y_j(s_j), x) + \frac{p_j(s_j) - p_i}{2\theta} \ ds_j \ dx
$$

$$
= \int_0^1 \int_0^1 Q_i(y_i, y_j(s_j), x) ds_j dx + \frac{1}{2\theta} \cdot (E_{p_j} - p_i)
$$

(13)

Define $D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j))$ as firm $i$’s expected demand from a personalized positioning of $y_i$ and price $p_i$, given its signal of the focal consumer’s ideal location is $s_i$ and firm $j$ uses a strategy of $y_j(s_j)$ and $p_j(s_j)$. We can calculate $D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j))$ by summing up (10)-(13), weighted by the probability of each event:

$$
D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_{i}^{np}(y_i|s_i, y_j(s_j))
$$

$$
+ \frac{1}{2\theta} \cdot \left[ \alpha_i \alpha_j \cdot [p_j(s_i) - p_i] + (1 - \alpha_i \alpha_j) \cdot [E_{p_j} - p_i] \right]
$$

(14)

where $D_i^{np}(y_i|s_i, y_j(s_j), p_j(s_j))$ is the sum of the non-pricing terms in equations (10)-(13), weighted by the probability of each event:

$$
D_{i}^{np}(y_i|s_i, y_j(s_j)) = \alpha_i \alpha_j Q_i(y_i, y_j(x), x) + \alpha_i (1 - \alpha_j) \int_0^1 Q_i(y_i, y_j(s_j), x) dx
$$

$$
+ (1 - \alpha_i) \alpha_j \int_0^1 Q_i(y_i, y_j(x), x) dx
$$

$$
+ (1 - \alpha_i) (1 - \alpha_j) \int_0^1 \int_0^1 Q_i(y_i, y_j(s_j), x) ds_j dx
$$

(15)

Note that if there is no price competition, i.e., $p_i = p_j \ \forall s_i, s_j$, then $D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_i^{np}(y_i|s_i, y_j(s_j))$. Thus $D_i^{np}(y_i|s_i, y_j(s_j))$ represents firm $i$’s expected demand without price competition.

Firms $i$’s expected profit can be written as

$$
\Pi_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) = D_i(y_i, p_i|s_i, y_j(s_j), p_j(s_j)) \cdot p_i
$$

$$
= p_i \cdot D_{i}^{np}(y_i|s_i, y_j(s_j)) + p_i \cdot \frac{1}{2\theta} \cdot \left[ \alpha_i \alpha_j \cdot [p_j(s_i) - p_i] + (1 - \alpha_i \alpha_j) \cdot [E_{p_j} - p_i] \right]
$$

(16)

### Personalized Positioning Strategy

Note that $y_i$ enters equation (16) only through the term $D_i^{np}(y_i|s_i, y_j(s_j))$. Thus the value of $y_i$ that maximizes firm $i$’s profit must maximize $D_{i}^{np}(y_i|s_i, y_j(s_j))$. In equilibrium, firms’
personalized positioning strategies must satisfy:

\[ y^*_i(s_i) = \arg\max_{y_i} D^{np}_{i}(y_i|s_i, y_j(s_j)) \quad \forall i, j \in \{1, 2\} \quad i \neq j \]  

(17)

Because the term \( D^{np}_{i}(y_i|s_i, y_j(s_j)) \) does not include prices, one can then solve for \( y^*_i(s_i) \) and \( y^*_j(s_j) \) independently from firms’ pricing decisions. The algebraic derivation is in the Appendix.

**Lemma 1** In equilibrium, firm \( i \) offers a personalized positioning of \( y^*_i = (1 - \alpha_i) \cdot \frac{1}{2} + \alpha_is_i \).

When offering personalized positioning, firms hedge between their signals of the consumer’s ideal location, \( s_i \), and the most mainstream positioning, \( \frac{1}{2} \). How much firms hedge depends on the accuracy of their signals. When firm \( i \) receives a signal \( s_i \neq \frac{1}{2} \), firm \( i \) offers the consumer a more niche positioning that is closer to \( s_i \) when \( \alpha_i \) is higher, and offers a more mainstream positioning that is closer to \( \frac{1}{2} \) when \( \alpha_i \) is lower. This strategy implies that, on average, each firm’s personalized positioning is more mainstream than the focal consumer’s ideal location.

Lemma 1 also implies that firm \( i \)’s product line does not expand the entire range of consumer locations. A firm offers a wider product line as signal accuracy improves, as the product line width, \((1 - \alpha_i) \cdot \frac{1}{2} + \alpha_i \cdot 1) - (1 - \alpha_i) \cdot \frac{1}{2} - \alpha_i \cdot 0 = \alpha_i\), increases in \( \alpha_i \). Firms only offer products with very niche positioning when their predictions of consumer preferences are sufficiently accurate. In the limit as \( \alpha_i \to 0 \), firm \( i \) only offers the most mainstream positioning.

Firms’ equilibrium positioning strategies are intuitive. Note that equation (17) stems from the fact that firms cannot observe each other’s personalized positioning before setting prices, i.e., firms cannot observe how a rival firm personalizes its website, email, ads, etc, to a specific consumer. As a result, prices cannot react to changes in the competitors’ positioning. This eliminates the strategic effect of positioning on subsequent price competition that exists in spatial competition models where firms set prices after observing each other’s positioning. Thus, firms try to position to maximize demand without considering price competition. In other words, firms try to position as close to the consumer as possible given their signals. To do so, firms want to minimize the expected quadratic distance from the focal consumer. Hypothetically, if a firm knows that its signal is accurate, then it should position exactly at its signal, \( s_i \); if a firm knows that its signal is incorrect, then it wants to position at the most
mainstream taste, i.e., $\frac{1}{2}$. Thus the firm balances these two possibilities by positioning at a point between $s_i$ and $\frac{1}{2}$, weighted by the accuracy of its signal.

In spatial competition models, an object of interest is the degree of differentiation, which affects the intensity of price competition. With personalization, the degree of differentiation is probabilistic. It depends both on the focal consumer’s ideal location, $x$, as well as firms’ signals, $s_i$ and $s_j$. Given each firm’s personalized positioning strategy, we can calculate $E[(y_i - y_j)^2]$, the expected degree of product differentiation measured in the quadratic distance:

$$E[(y_i - y_j)^2] = \frac{\alpha_i^2}{12} + \frac{\alpha_j^2}{12} - \frac{\alpha_i^2\alpha_j^2}{6}$$

(18)

Taking derivative with respect to $\alpha_i$, we have

$$\frac{\partial E[(y_i - y_j)^2]}{\partial \alpha_i} = \frac{\alpha_i(1 - 2\alpha_j^2)}{6}$$

(19)

Therefore, if $\alpha_j \in [0, \frac{1}{\sqrt{2}}]$, products differentiation increases with $\alpha_i$; If $\alpha_j \in (\frac{1}{\sqrt{2}}, 1]$, products differentiation decreases as $\alpha_i$ increases.

We further study the symmetric case with $\alpha_i = \alpha_j = \alpha$. The symmetric case can be understood as a situation where both firms can access the same predictive technology, and therefore $\alpha$ represents the accuracy of the common technology. A higher $\alpha$ represents an industry-level increase in prediction accuracy due to technological advancement. In the symmetric case, we obtain the expected differentiation in the symmetric case as

$$E[(y_i - y_j)^2] = \frac{\alpha^2(1 - \alpha^2)}{6}$$

(20)

We present the comparative statics as follows.

**Proposition 1** The expected degree of product differentiation increases in $\alpha_i$ when $\alpha_j$ is low ($\leq 1/\sqrt{2}$) and decreases in $\alpha_i$ when $\alpha_j$ is high ($> 1/\sqrt{2}$). In the symmetric case with $\alpha_i = \alpha_j = \alpha$, the expected degree of product differentiation increases in $\alpha$ for $\alpha \leq \frac{1}{\sqrt{2}}$ and decreases for $\alpha > \frac{1}{\sqrt{2}}$.

Interestingly, how the level of differentiation changes when a firm’s signal accuracy increases depends on the other firm’s accuracy. When the competitor does not have accurate information on consumer preferences, an increase in a firm’s accuracy leads to a higher degree of differentiation. However, when the competitor has accurate enough information on
individual consumers, an increase in the firm’s accuracy always leads to less differentiation in expectation. Intuitively, when the competitor’s prediction accuracy is low, the competitor’s positioning is likely to be far from the consumer’s ideal location. Thus, a higher accuracy, which allows the firm to position closer to the consumer’s ideal location, leads to more differentiation. On the other hand, when the competitor’s accuracy is high, the competitor’s positioning is likely to be close to the consumer’s ideal location. A higher accuracy for the firm then leads to less differentiation.

In the symmetric case, however, the expected degree of differentiation is non-monotonic in signal accuracy, and achieves the highest level when $\alpha = \frac{1}{\sqrt{2}}$. The degree of product differentiation is lowest when $\alpha = 0$ or when $\alpha = 1$. When $\alpha = 0$, both firms always offer the most mainstream positioning at $\frac{1}{2}$. When $\alpha = 1$, both firms know the focal consumer’s ideal location perfectly, thus always providing the same positioning to the consumer. We can then expect the intensity of price competition to also be non-monotonic in $\alpha$. Price competition should be the most intense at the lowest and highest accuracy. Next, we formally solve for firms’ pricing strategies.

**Pricing Strategy**

Given firms’ personalized positioning strategies from Lemma 1, $y^*_i(s_i) = (1 - \alpha_i) \cdot \frac{1}{2} + \alpha_i s_i$, we solve for firms’ equilibrium pricing strategies.

Let $A_i(s)$ denote equation (15) evaluated at equilibrium personalized positioning strategies and at signal $s$, i.e., $A_i(s) = D^np_i(y^*_i(s) | s_i = s, y^*_j(s_j))$. We can then write firm $i$’s profit under the equilibrium positioning strategy as:

$$
\Pi_i(p_i | s_i = s, p_j(s_j)) = p_i A_i(s) + p_i \frac{1}{2\theta} \left[ \alpha_i \alpha_j \cdot [p_j(s) - p_i] + (1 - \alpha_i \alpha_j) \cdot [E p_j - p_i] \right]
$$

Maximizing equation (21) with respect to $p_i$, we get the following conditions regarding equilibrium pricing strategies, $p^*_i(s_i)$ and $p^*_j(s_j)$.

$$
2\theta A_i(s) + \alpha_i \alpha_j p^*_j(s) + (1 - \alpha_i \alpha_j) E p^*_j - 2p^*_i(s) = 0 \quad \forall i, j \in \{1, 2\} \quad i \neq j
$$

which allows us to write $p^*_i(s)$ as a function as $A_i(s)$, $A_j(s)$, $E p_i$, and $E p_j$:

$$
p^*_i(s) = \frac{4\theta}{4 - \alpha_i^2 \alpha_j^2} A_i(s) + \frac{2\theta \alpha_i \alpha_j}{4 - \alpha_i^2 \alpha_j^2} A_j(s) + \frac{\alpha_i \alpha_j (1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} E p^*_i + \frac{2(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} E p^*_j
$$
Using the fact that $E_p = \int_0^1 p_i(s)ds$, we get

$$E_p^* = \int_0^1 p_i^*(s)ds = \frac{4\theta}{4 - \alpha_i^2 \alpha_j^2} B_i + \frac{2\theta \alpha_i \alpha_j}{4 - \alpha_i^2 \alpha_j^2} B_j + \frac{\alpha_i \alpha_j (1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} E_p^* + \frac{2(1 - \alpha_i \alpha_j)}{4 - \alpha_i^2 \alpha_j^2} E_j^*$$  (24)

where $B_i = \int_0^1 A_i(s)ds$. We show in the Appendix that

$$B_i = \int_0^1 A_i(s)ds = \frac{1}{2} + \frac{t}{24\theta} (\alpha_i^2 - \alpha_j^2)$$  (25)

We can then use equations (24) and (25) to derive closed-form solutions for equilibrium average prices:

$$E_p^*_1 = \theta + \frac{t}{36} (\alpha_1^2 - \alpha_2^2)$$  (26)

$$E_p^*_2 = \theta + \frac{t}{36} (\alpha_2^2 - \alpha_1^2)$$  (27)

**Proposition 2** Firm i’s average price in equilibrium increases in the size of idiosyncratic preferences, $\theta$, and its own prediction accuracy, $\alpha_i$, and decreases in the competitor’s prediction accuracy, $\alpha_j$. Firm i’s average price in equilibrium increases in the traveling cost, $t$, if $\alpha_i > \alpha_j$, and decreases in the traveling cost, $t$, if $\alpha_i < \alpha_j$

The comparative statics in Proposition 2 are intuitive. When $\theta$ is higher, the competition between the two firms is softened, causing equilibrium prices to rise. Under personalized positioning, firms cannot observe the competitor’s personalized positioning, and thus cannot price in response to the competitor’s positioning. Instead, a firm’s average price depends on the advantage over the competitor, which is captured by the term $\alpha_i^2 - \alpha_j^2$ in equations (27). When $\alpha_i$ increases, firm i’s personalized positioning becomes closer to the consumer’s ideal location relative to the competitor’s personalized positioning, allowing firm i to charge higher prices. The reverse happens when $\alpha_j$ increases. The traveling cost $t$ magnifies a firm’s advantage/disadvantage. Note that, interestingly, a firm’s prediction accuracy acts similarly to its product quality. At a higher accuracy, a firm’s product delivers a higher expected utility to the consumer, allowing the firm to charge a higher price in equilibrium.

We also examine the symmetric case with $\alpha_i = \alpha_j = \alpha$. By examining equation (27). We see that the equilibrium average prices do not depend on $\alpha$ or $t$, because neither firm has an ex-ante advantage to position closer to the consumer’s ideal location.
Corollary 2.1 In the symmetric case with $\alpha_i = \alpha_j = \alpha$, each firm’s average price in equilibrium does not depend on $t$ or $\alpha$.

This result is somewhat surprising. Proposition 1 shows that the expected degree of differentiation in the symmetric case depends on $\alpha$. We would then expect that the intensity of price competition also depends on $\alpha$. This is, however, not reflected in the average equilibrium price. To better understand how $\alpha$ affects price competition, one has to examine the equilibrium prices for different positioning.

We can then plug in our solutions for $Ep_i^*$ and $Ep_j^*$ into equation (23) to get $p_i^*(s)$. Note that Lemma 1 shows that firm $i$ must offer a different positioning for each different signal, i.e., $y_i^*(s) = y_i^*(s')$ if and only if $s = s'$. Thus, instead of personalized pricing, we may refer to firms’ pricing strategies in this model as product line pricing, because in equilibrium, firm $i$ charges different prices for different positioning, but never charge different prices for the same positioning.

With a slight abuse of notation, we denote $p_i^*(y) = p_i^*(s_i = \frac{y - (1 - \alpha_i) - 1/2}{\alpha_i})$ as firm $i$’s equilibrium price for positioning $y$. The remaining details of the derivation and the closed-form solutions for $p_1^*$ and $p_2^*$ are in the Appendix. We describe firms’ equilibrium pricing strategies by examining the shape of $p_i^*(y)$ (and $p_i^*(s)$).

Proposition 3 Firm $i$’s equilibrium pricing strategy, $p_i^*(y)$ ($p_i^*(s)$), is symmetric around and smooth at $\frac{1}{2}$. Furthermore, there is a threshold $\overline{\alpha_i}(\alpha_j) \in [0,1]$ such that:

- If $\alpha_i < \overline{\alpha_i}(\alpha_j)$, then $p_i^*(y)$ ($p_i^*(s)$) increases as $y \to \frac{1}{2}$ ($s \to \frac{1}{2}$), i.e., firm $i$ charges a higher price when offering a more mainstream positioning (when receiving a more mainstream signal);

- If $\alpha_i = \overline{\alpha_i}(\alpha_j)$, then $p_i^*(y)$ ($p_i^*(s)$) is constant in $y$ ($s$), i.e., firm $i$ charges the same price for all positioning (for all signals).

- If $\alpha_i > \overline{\alpha_i}(\alpha_j)$, then $p_i^*(y)$ ($p_i^*(s)$) decreases as $y \to \frac{1}{2}$ ($s \to \frac{1}{2}$), i.e., firm $i$ charges a higher price when offering a more niche positioning (when receiving a more niche signal).

The threshold $\overline{\alpha_i}(\alpha_j)$ is an increasing function of $\alpha_j$, with $\overline{\alpha_i}(0) = 0$, $\overline{\alpha_i}(1) = 1$, and $\overline{\alpha_i}(\alpha_j) < \alpha_j$ for $\alpha_j \in (0,1)$. 

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Proposition 3 shows that, in equilibrium, firms charge different prices when offering different personalized positioning. Depending on their prediction accuracies, firms may charge higher prices for more mainstream positioning or higher prices for more niche positioning.

Figure 2 shows the range of firm $i$’s personalized positioning and the price it charges for each positioning under different $\alpha_i$ for $\alpha_j = 0.7$. Note that when $\alpha_i$ is low, the pricing schedule is concave, with higher prices for more mainstream positioning, whereas when $\alpha_i$ is high, the pricing schedule is convex, with higher prices for more niche positioning. We can observe that the average price charged by firm $i$ is also increasing in $\alpha_i$, as stated in Proposition 2.

Proposition 3 implies that the pricing pressure a firm faces depends on whether the consumer’s taste is predicted to be niche or mainstream. To gain a more intuitive understanding, we consider two forces that shape firm $i$’s pricing decision upon receiving a signal. When setting a price, firm $i$ cannot observe the competitor’s personalized positioning, so firm $i$’s pricing depends on how close to the consumer’s ideal location it expects to be relative to the competitor’s positioning. Firm $i$ can charge a higher price if it expects its own positioning to be closer to the consumer’s ideal location, or if it expects the competitor to be farther from the consumer’s ideal location.

First, consider the distance between firm $i$’s own positioning and the consumer’s ideal location, given firm $i$’s signal, $s_i$. Suppose firm $i$’s signal is correct, then by Lemma 1, the distance between its positioning and the consumer’s ideal location is closer when $s_i$ (or $y_i^*(s_i)$) is closer to $\frac{1}{2}$. Suppose firm $i$’s signal is wrong, then positioning closer to $\frac{1}{2}$ is also on average closer to the consumer’s ideal location. Thus, overall firm $i$ expects its own positioning to be closer to the consumer’s ideal location when it receives a signal closer to $\frac{1}{2}$. This first
force pushes the firm to charge a higher price when offering a more mainstream positioning (closer to $\frac{1}{2}$).

Second, consider the distance between the competitor’s positioning and the consumer’s ideal location, given firm $i$’s signal, $s_i$. Suppose firm $i$’s signal is correct, then again by Lemma 1, the expected distance between the competitor’s positioning and the consumer’s ideal location is farther when $s_i$ is farther from $\frac{1}{2}$. Suppose firm $i$’s signal is wrong, then $s_i$ is uncorrelated with the distance between the competitor’s positioning and the consumer’s ideal location. Thus, overall firm $i$ expects the competitor’s positioning to be farther when $s_i$ is farther from $\frac{1}{2}$. This second force pushes the firm to charge a higher price when offering a more niche positioning (farther away from $\frac{1}{2}$).

When firm $i$’s prediction accuracy is low, the second force, which is only activated when firm $i$’s signal is correct, is weak. The first force dominates and firm $i$ charges a higher price when offering a more mainstream positioning. When firm $i$’s prediction accuracy is higher, not only the second force becomes stronger, but the first force also becomes weaker: with a higher accuracy, firm $i$’s positioning becomes closer to the consumer’s ideal location across all $s_i$, reducing the effect of $s_i$ on the distance between its own positioning and the consumer’s ideal location. As a result, the firm charges a higher price when offering a more niche positioning when $\alpha_i$ is high. Thus, there arises the threshold on $\alpha_i$ at which firm $i$’s pricing schedule switches from a concave shape to a convex shape. Similarly, when the competitor’s accuracy increases, the second force becomes weaker, because the competitor’s positioning is closer to the consumer’s ideal location across all $s_i$.

We examine the special case where firms are symmetric in their prediction accuracy. One can show that the second force always dominates the first force in the symmetric case, leading to the following result.

**Corollary 3.1** In the symmetric case with $\alpha_i = \alpha_j = \alpha$, firms charge higher prices when offering more niche positioning (receiving more niche signals) for all $\alpha \in (0, 1)$. In the limit as $\alpha \to 1$, firms offer a positioning of $y^*(s_i) = s_i$ and charge a price of $\theta$ for all positioning. In the limit as $\alpha \to 0$, firms always offer a positioning of $\frac{1}{2}$ and charge a price of $\theta$.

Figure 3 shows firm $i$’s range of personalized positioning and pricing schedule for different $\alpha$. As shown in Corollary 2.1, the average price is unaffected by $\alpha$. In the symmetric case, the second force in pricing dominates, pushing firms to charge higher prices when...
offering more niche positioning. In other words, price competition is more intense for more mainstream positioning. This difference in competitive pressure between (predicted) niche and mainstream consumers depends non-monotonically on $\alpha$. At the limit as $\alpha \to 1$, firms charge the same price for all signals and positioning.

Given the characterization of firms’ equilibrium pricing strategies, we are interested in how firms’ prediction accuracies affect pricing. Next, we examine how the equilibrium price for each signal, $p^*(s)$, changes with prediction accuracy.

One might believe that a firm’s unilateral improvement in its signal accuracy enables the firm to charge a higher price for all its product positioning. Interestingly, we find that this intuition does not always hold. Specifically, there are situations in which a firm lowers its price for some product positioning despite its increase in signal accuracy. That is, a higher accuracy intensifies price competition for some consumers and relaxes price competition for others.

To understand the reason, note that each firm uses two levers to win over a consumer: by offering a product positioning that better matches the consumer’s preference, and by charging a price that is more attractive. Consider a certain signal $s$ and its corresponding product positioning $y^*(s)$. As the focal firm’s prediction accuracy improves, it gains a positioning advantage that directly pushes the firm to raise its price. However, the competing firm will respond to its positioning disadvantage by lowering its average price (of all positioning), which indirectly pushes the local firm also to lower its price. We find that the indirect effect can outweigh the direct effect for a mainstream product (i.e., $y$ closer to $\frac{1}{2}$) when the competing firm’s signal accuracy is low. In this situation, the competing firm’s
product offerings center around mainstream positioning. As a result, firms engage in more intense price competition for mainstream consumers when the focal firm gains a positioning advantage through a higher signal accuracy.

Figure 4 plots firm \( i \)'s price for its most mainstream product \( y = \frac{1}{2} \). It is seen that for a relatively low \( \alpha_j \), it is possible that \( \frac{\partial p^*(y=0.5)}{\partial \alpha_i} < 0 \).

![Figure 4: Firm \( i \)'s equilibrium pricing in response to its prediction accuracy (in a setting where \( \{\theta = 2, t = 1, y = 0.5\} \)](image)

For the symmetric case, we can prove the following results regarding how prices change with \( \alpha \):

**Proposition 4** *In the symmetric case with \( \alpha_i = \alpha_j = \alpha \), the impact of the signal precision \( \alpha \) on firm \( i \)'s equilibrium price \( p^*(s) \) for a given signal \( s \) depends on a threshold \( \alpha_0 \).*

- If \( \alpha \leq \alpha_0 \), then \( p^*(s) \) decreases with \( \alpha \) for \( s \in \left( \frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6} \right) \) and increases with \( \alpha \) for \( s < \frac{1}{2} - \frac{\sqrt{3}}{6} \) or \( s > \frac{1}{2} + \frac{\sqrt{3}}{6} \).
- If \( \alpha > \alpha_0 \), then \( p^*(s) \) increases with \( \alpha \) for \( s \in \left( \frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6} \right) \) and decreases with \( \alpha \) for \( s < \frac{1}{2} - \frac{\sqrt{3}}{6} \) or \( s > \frac{1}{2} + \frac{\sqrt{3}}{6} \).

Proposition 4 shows that \( p^*(s) \) is non-monotonic in \( \alpha \), and the direction depend on both \( \alpha \) and \( s \). Note that when firms receive signals that indicate relatively mainstream tastes, \( s \in \left( \frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6} \right) \), prices first decrease then increase in \( \alpha \). In contrast, when firms receive signals that indicate relatively niche tastes, \( s < \frac{1}{2} - \frac{\sqrt{3}}{6} \) or \( s > \frac{1}{2} + \frac{\sqrt{3}}{6} \), prices first increase then

\[ 3 \text{Specifically, } \alpha_0 \approx 0.539 \text{ is the root of the equation } 2 - 4\alpha + \alpha^3 = 0. \]
Figure 5: The effect of industry-level accuracy on equilibrium prices for different signals (in a setting where \( \{\theta = 2, t = 1\} \))

decrease in \( \alpha \). In other words, for relatively mainstream signals, an increase in the industry-level accuracy first intensifies and then relaxes price competition, whereas for relatively niche signals, an increase in the industry-level accuracy first relaxes then intensifies price competition. Figure 5 shows how equilibrium prices at different signals respond to changes in the industry-level accuracy.

From Proposition 1, we know that the expected degree of differentiation first increases then decreases in the industry-level accuracy. This leads us to hypothesize price competition to first relax then intensify as \( \alpha \) increases. However, we do not observe this pattern in the average price per Proposition 2. Proposition 4 provides the answer. While price competition for consumers predicted to have niche tastes indeed first relaxes then intensifies, the pattern for consumers predicted to have mainstream tastes is the opposite.

To understand why the effect of accuracy on price depends on the signal, consider the case where one firm’s signal is correct and its competitor’s signal is wrong. An increase in \( \alpha \) translates to a competitive advantage because the firm positions closer to the consumer. By Lemma 1, the distance between a firm and the consumer, if the firm’s signal \( s \) is correct, is \( (1 - \alpha)|s - 1/2| \), which decreases in \( \alpha \). Similarly, in the case where one firm’s signal is wrong and its competitor’s signal is correct, a higher accuracy translates to a competitive disadvantage because the competitor moves closer to the consumer. However, note that the marginal effect of \( \alpha \) on \( (1 - \alpha)|s - 1/2| \) depends on the signal, \( s \). The effect is larger for niche
signals and smaller for mainstream signals. If \( s = 1/2 \), then the aforementioned competitive advantage disappears, because the firm positions at 1/2 if \( s = 1/2 \), regardless of \( \alpha \). But the aforementioned competitive disadvantage still exists, because the competitor’s signal is uncorrelated with the firm’s wrong signal. Thus, for low \( \alpha \), while an increase in \( \alpha \) relaxes pricing pressure when the firm receives a niche signal, it intensifies pricing pressure when the firm receives a mainstream signal because the competitive benefit from a higher \( \alpha \) is muted.

If both firms’ signals are correct, then both firms offer the same positioning. In this case, the level of price competition does not depend on the location of the signal. This case dominates when \( \alpha \) becomes sufficiently high. In the limit as \( \alpha \to 1 \), firms must charge the same price for all signals, the same as when \( \alpha \to 0 \). Thus, when \( \alpha \) is large, the previous effects of \( \alpha \) on prices reverse. Price competition intensifies for consumers predicted to have niche tastes and relaxes for consumers predicted to have mainstream tastes.

To corroborate with the above intuition, we show in the Online Appendix that conditional on firm \( i \) receiving a signal \( s_i \), firm \( i \)'s expected positioning advantage over firm \( j \) can be written as

\[
E[(y_i - x)^2 - (y_j - x)^2 | s_i] = \frac{\alpha^2(1 - \alpha^2)}{6} \cdot (1 - 6s_i + 6s_i^2)
\]

Note that the term \( \frac{\alpha^2(1 - \alpha^2)}{6} \) is the expected degree of differentiation from Proposition 1, and the term \( (1 - 6s + 6s^2) \) is positive for \( s_i \in \left(\frac{1}{2} - \frac{\sqrt{3}}{6}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right) \) and negative for \( s_i \in \left[0, \frac{1}{2} - \frac{\sqrt{3}}{6}\right) \cup \left(\frac{1}{2} + \frac{\sqrt{3}}{6}, 1\right] \). Thus, for relatively niche signals, a firm’s expected positioning advantage goes in the same direction as the expected level of differentiation, whereas for relatively mainstream signals, a firm’s expected positioning advantage goes in the opposite direction as the expected level of differentiation. This result connects Proposition 1 and Proposition 4.

**Expected Profit**

We first examine firms’ expected profits after receiving a signal \( s \). This allows us to compare the profitability between consumers of different signals. We then proceed to examine firms’ ex-ante expected profits before receiving a signal.

We can calculate firms’ expected profits after receiving signal \( s \) by plugging in firms’ equilibrium pricing strategies into equation (21). First, we can show that not all signals are equally profitable. Let \( \Pi_i^* (s) \) denote the equilibrium expected profit for firm \( i \) when receiving
a signal $s$. On examining $\frac{\partial \Pi_i^*(s)}{\partial s}$, we have the following result:

**Proposition 5** The equilibrium expected profit for firm $i$ when it receives a signal of $s$, $\Pi_i^*(s)$, depends on $\alpha_i$, $\alpha_j$, and $s$:

- If $\alpha_i < \bar{\alpha}_i(\alpha_j)$, then $\Pi_i^*(s)$ increases as $s \to \frac{1}{2}$, i.e., firm $i$’s expected profit is higher when receiving a more mainstream signal.

- If $\alpha_i = \bar{\alpha}_i(\alpha_j)$, then $\Pi_i^*(s)$ is constant in $s$, i.e., firm $i$’s expected profit is the same for all signals.

- If $\alpha_i > \bar{\alpha}_i(\alpha_j)$, then $\Pi_i^*(s)$ decreases as $s \to \frac{1}{2}$, i.e., firm $i$’s expected profit is higher when receiving a more niche signal.

The threshold, $\bar{\alpha}_i(\alpha_j)$, is the same threshold as the one in Proposition 3. Thus, we can draw intuition from Proposition 3. Proposition 3 shows that, when firm $i$’s accuracy is low relative to the competitor’s, firm $i$ charges a higher price when receiving a more mainstream signal (and therefore offering a more mainstream positioning). In such a case, consumers who appear to have more mainstream tastes are more profitable to firm $i$. Conversely, when firm $i$’s accuracy is high relative to the competitor’s, Proposition 3 shows that firm $i$ charges a higher price when receiving a more niche signal (and therefore offering a more niche positioning). As a result, consumers who appear to have more niche tastes are more profitable to firm $i$.

Corollary 3.1 shows that firms charge higher prices for more mainstream positioning in the symmetric case. Thus, we would expect a firm’s expected profit to be higher when receiving a more niche signal in the symmetric case. In the symmetric case with $\alpha_i = \alpha_j = \alpha$, after receiving signal $s$, a firm’s expected profits can be written as:

$$\Pi_i^{sym}(s) = \frac{t^2[(1 - 6s + 6s^2)(-1 + \alpha)^2\alpha^2 - 6t(-2 + \alpha^2)]^2}{72(-2 + \alpha^2)^2\theta}$$

(29)

Checking the derivative, we confirm the following result.

**Corollary 5.1** In the symmetric case with $\alpha_i = \alpha_j = \alpha$, firm $i$’s expected profit is higher when receiving more a niche signal.

Intuitively, a firm receives more profit when its signal is correct while the competitor’s signal is wrong, but Proposition 5 and Corollary 5.1 show that profitability also depends
on where the consumer’s (predicted) ideal location is. When a consumer is predicted to have a more niche taste, the competitor receiving a wrong signal is likely to position farther away from the consumer. Thus firms charge higher prices to and receive higher profits from consumers who appear to have more niche tastes.

We proceed to solve for firm $i$’s ex-ante profit before receiving a signal as $\Pi_i^* = \int_0^1 \Pi_i^*(s)ds$. Below we describe how prediction accuracy affects profits in both the general case and the symmetric case.

**Proposition 6** Firm $i$’s en-ante profit increases with its signal accuracy, $\alpha_i$, and decreases with its competitor’s signal accuracy, $\alpha_j$. In the symmetric case with $\alpha_i = \alpha_j = \alpha$, firm $i$’s profit increases with $\alpha$ for $\alpha \leq \alpha_0$ and decreases with $\alpha$ for $\alpha > \alpha_0$. Here $\alpha_0$ is the same threshold derived in Proposition 4.

The result for the general case is intuitive. As firm $i$’s own prediction accuracy increases, it expects to position closer to the consumer’s ideal location, giving firm $i$ an advantage in competition. When the competitor’s prediction accuracy increases, the competitor is expected to position closer to the consumer’s ideal location, giving the competitor an advantage in competition.

More interestingly, Proposition 6 shows that in the symmetric case, a firm’s profit is not monotonic in the industry-level accuracy, $\alpha$. By extension, the total industry profit is also non-monotonic in $\alpha$. Note that the threshold, $\alpha_0$, is the same threshold as in Proposition 4. Proposition 4 shows that when $\alpha < \alpha_0$, an increase in $\alpha$ intensifies price competition for mainstream consumers but relaxes price competition for niche consumers. Corollary 5.1 shows that symmetric firms’ profits depend more on niche consumers. Thus, a higher $\alpha$ raises profit as the benefit of charging higher prices to niche consumers outweighs the cost of charging lower prices to mainstream consumers. The reverse is true when $\alpha > \alpha_0$. An increase in $\alpha$ relaxes price competition for mainstream consumers but intensifies price competition for niche consumers. The cost of charging lower prices to niche consumers outweighs the benefit of charging higher prices to mainstream consumers, leading to a decrease in profit as $\alpha$ increases.

To better understand why profit decreases as $\alpha$ moves away from $\alpha_0$, we can also consider the two limiting cases. In the limit as $\alpha \to 0$, firms do not personalize. Both firms always offer identical positioning at $\frac{1}{7}$. The complete lack of differentiation leads to the most intense
price competition, minimizing profits. On the other hand, as \( \alpha \to 1 \), firms fully personalize. Because firms can perfectly observe each consumer’s ideal location, firms again offer identical positioning at the consumer’s ideal location, which again eliminates differentiation and minimizes profits. Therefore, an industry-level increase in the ability to predict consumer preferences is not necessarily good for firms. This calls for further investigation on firms’ incentives to invest in better predictions, and the possibility of industry self-regulation as a way to avoid prisoner’s dilemma. We examine these questions in Section 5.

**Social Surplus and Consumer Welfare**

We examine ex-ante social surplus and consumer welfare, with a focus on the symmetric case. If the consumer chooses firm 1, the social surplus is \(-t(y_1 - x)^2\); If the consumer chooses firm 2, the social surplus is \(-t(y_2 - x)^2 + \Delta \epsilon\). The expected social surplus in the symmetric case can be calculated as

\[
SS = -\frac{t(1 - \alpha^2)}{12} + \frac{t^2(1 - \alpha)\alpha^2(20 + 36\alpha - 35\alpha^2 - 23\alpha^3 + 9\alpha^4 + 5\alpha^5)}{360(2 - \alpha^2)^2\theta} - \frac{3\theta}{4}
\]  

(30)

It is then straightforward to show that \( \frac{\partial SS}{\partial \alpha} > 0 \). This is unsurprising, as firms are more likely to position closer to the consumer’s ideal location when prediction accuracy increases. Thus the ex-ante social surplus always increases in the industry-level prediction accuracy.

To get more nuanced results, we investigate the welfare of consumers with different ideal locations. Due to intractability, we use numerical analyses to describe how consumer welfare depends on a consumer’s ideal location and firms’ prediction accuracy.

Let \( U(x) \) denote the ex-ante consumer welfare for a consumer whose ideal location is \( x \). We first examine \( \frac{\partial U(x)}{\partial x} \) in Figure 6. We observe that, for a variety of parameter values, \( \frac{\partial U(x)}{\partial x} \geq 0 \) if and only if \( x \leq \frac{1}{2} \). Therefore, in our observations, consumer welfare always increases as \( x \) moves towards \( \frac{1}{2} \). Consumers with a more mainstream ideal location enjoy a higher consumer welfare ex-ante.
Intuitively, there are three reasons why consumers with more mainstream tastes are better off in equilibrium. First, given the equilibrium positioning strategy in Lemma 1, consumers with more mainstream tastes receive positioning closer to their ideal locations when firms’ predictions are accurate, as firms hedge against the risk of incorrect predictions by positioning between their signals and the location \( \frac{1}{2} \). Second, by Corollary 3.1, firms offer lower prices when they predict consumers to have more mainstream tastes. Third, when predictions are wrong, consumers with more mainstream tastes also on average receive positioning closer to their ideal locations.

We then examine \( \frac{\partial U(x)}{\partial \alpha} \), the effect of industry-level prediction accuracy on consumer welfare. Interestingly, ex-ante welfare for a consumer does not necessarily increase with \( \alpha \). Figure 7 shows that it is possible to have \( \frac{\partial U(x)}{\partial \alpha} < 0 \). Particularly, when the accuracy is not too high, consumers with very mainstream tastes can be worse off ex-ante when \( \alpha \) increases.
To understand this result, consider a consumer whose ideal location is $\frac{1}{2}$. In the limit as $\alpha \to 0$, firms always position at this consumer’s ideal location, because firms seek to occupy more mainstream locations when their prediction power is low. As $\alpha$ increases, firms begin to personalize their positioning, which means that the consumer is more likely to receive a positioning farther from her ideal location. Even though by Proposition 4, her ideal product becomes cheaper as $\alpha$ increases, she is likely to receive a “wrong” positioning with a higher price if predictions are not accurate enough. Thus it is actually possible for the consumer to face worse products with higher prices when firms’ prediction accuracy increases. This drives down the consumer’s ex-ante welfare.

While some consumers can be hurt by an increase in prediction, we confirm that the overall consumer welfare, $\int_0^1 U(x)dx$, increases in $\alpha$. However, this result could change under a different distribution of consumers’ ideal location, given that not all consumers are better off under a higher $\alpha$.

We summarize our numerical observations on consumer welfare below.

**Remark 1** In the symmetric case, consumers with more mainstream tastes have higher utilities ex-ante. When prediction accuracy is not too high, consumers with sufficiently mainstream tastes are worse off when prediction accuracy increases. Overall consumer welfare increases in prediction accuracy.
Summary

In this section, we analyze firms’ personalized positioning strategies, expected product differentiation, pricing strategies, profits, and social and consumer welfare. We show how they depend on the signals that firms receive and firms’ prediction accuracies. Table 1 lists the relevant results.

Table 1: Summary of Results

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5 Extensions

5.1 Investment in Prediction

In the base model, each firm’s prediction accuracy is exogenously given. In this section, we consider the possibility that firms choose their prediction accuracy endogenously through investment in data and algorithms.

The timing of the game is as follows. First, firm $i$ chooses $\alpha_i \in [0, 1]$ and firm $j$ chooses $\alpha_j \in [0, 1]$ simultaneously. Firm $i$ incurs an investment cost of $K(\alpha_i)$ for choosing a prediction accuracy of $\alpha_i$. Then firms compete according to the base model. Let $\Pi^*_i(\alpha_i, \alpha_j)$ denote the equilibrium expected profit of firm $i$ in the base model if the firms’ prediction accuracies are $\alpha_i$ and $\alpha_j$. Firm $i$’s equilibrium expected profit in the investment game is then $\Pi^*_i(\alpha_i, \alpha_j) - K(\alpha_i)$. We assume that the cost function $K(\cdot)$ satisfies $K'(0) = 0$, $K''(\cdot) \geq 0$, and $\frac{d^2[\Pi^*_i(\alpha_i, \alpha_j) - K(\alpha_i)]}{d\alpha_i^2} < 0$. The first condition guarantees that firms always want to have a positive accuracy. The second condition states that the cost function is increasing and convex. The last condition states that there is a diminishing return to investment in accuracy. For parameters $\theta = 2$ and $t = 1$, for example, any cost function that is increasing and convex would also satisfy the last condition.
For simplicity, we focus on symmetric outcomes. Let $\alpha^*$ denote the prediction accuracy in a symmetric equilibrium of the investment game. As a benchmark, we define $\alpha_{\text{coll}}$ as the industry-level prediction accuracy that maximizes industry profit, i.e., the prediction accuracy chosen by collusive firms aiming at maximizing their total profits. More specifically,

$$
\alpha_{\text{coll}} = \arg\max_{\alpha} \Pi^*_i(\alpha, \alpha) + \Pi^*_j(\alpha, \alpha) - 2 * K(\alpha) \tag{31}
$$

We can prove the following result:

**Proposition 7** In a symmetric equilibrium, the equilibrium prediction accuracy must be strictly greater than the industry-level accuracy that maximizes industry profit, i.e., $\alpha^* > \alpha_{\text{coll}}$. Each firm earns a lower profit under equilibrium accuracy $\alpha^*$ than under collusive accuracy $\alpha_{\text{coll}}$. Consumer welfare is high under equilibrium accuracy $\alpha^*$ than under collusive accuracy $\alpha_{\text{coll}}$.

Proposition 7 states that firms always over-invest in prediction accuracy in a symmetric equilibrium, compared to the collusive level of investment that would maximize industry profit. As shown in Proposition 6, higher prediction accuracy can lower industry profit by intensifying price competition for the more profitable, niche consumer segment. This effect is exacerbated by the extra cost of investing in higher prediction accuracy. To gain intuition into this result, consider the marginal value of investing. The marginal value of a higher $\alpha_i$ to firm $i$ is $\frac{d\Pi^*_i(\alpha_i, \alpha_j)}{d\alpha_i} - K'(\alpha_i)$, which is always greater than the marginal value to the industry, $\frac{d\Pi^*_i(\alpha_i, \alpha_j)}{d\alpha_i} + \frac{d\Pi^*_j(\alpha_j, \alpha_i)}{d\alpha_i} - K'(\alpha_i)$, because a higher accuracy for firm $i$ always hurts firm $j$, i.e., $\frac{d\Pi^*_j(\alpha_j, \alpha_i)}{d\alpha_i} < 0$. Because firms do not consider the negative externality their investments impose on competitors, they suffer from the prisoner’s dilemma.

Another implication is that both firms can benefit if they collectively lower their prediction accuracies. Unlike other forms of collusion, collusion between firms to decrease the accuracy of their predictions of consumer preferences can be in line with public interests. As firms gain increasingly granular data on individual consumers, regulators express more concerns over violations of consumer privacy. This means that firms could achieve collusion by self-regulating through mediums such as industry associations, without fearing the public backlash and legal scrutiny other forms of collusion would receive. Proposition 7 shows that firms have incentives to reach an agreement to limit the industry’s prediction accuracy. In the context of the model, consider self-regulation in the form of an upper bound $\tilde{\alpha}$, so that
firms can only choose accuracy between 0 and \( \tilde{\alpha} \).\(^4\) For \( \tilde{\alpha} \) sufficiently close to \( \alpha_{coll} \), both firms strictly prefer playing the game with the upper bound \( \tilde{\alpha} \) rather than without the upper bound.

Note that Remark 1 implies that consumer welfare would decrease if firms self-regulate to achieve a lower industry accuracy. However, our model only considers consumer utility from purchase. If there is any intrinsic value of privacy, such as protection from data breach or preference for anonymity, then self-regulation on prediction accuracy may also improve consumer welfare.

5.2 Subscription Business Model

In the main model, consumers observe each firm’s personalized positioning, \( y_i \), and price, \( p_i \), before making a purchase decision. Such a model is representative of the interactions between consumers and firms in e-commerce. Product personalization is also a common strategy in the content market. Users of platforms such as Netflix or Hulu receive contents of different genres, topics, political views, etc. The business model, however, is different in the content market. Consumers of Netflix or Hulu have to pay subscription fees before seeing what content the algorithms present to them. Netflix or Hulu also learn about consumers’ preferences primarily after they become subscribers. Firms using such a subscription business model also cannot charge consumers different prices for different pieces of content. In this section, we consider a variation of our model that captures this scenario and compares the results to those of the main model.

Consider a modification to the model presented in Section 3 with the following timing. In the first stage, both firms choose their subscription prices, \( p_1 \) and \( p_2 \), and their content personalization algorithms, \( y_1(s_1) \) and \( y_2(s_2) \). In the second stage, the focal consumer, after observing the prices and personalization algorithms of both firms, subscribes to one of the two firms by paying the subscription price. In the last stage, the subscribed firm \( i \) receives a signal \( s_i \) about the consumer’s ideal location and offers a personalized positioning of \( y_i(s_i) \) to the consumer. In reality, consumers do not directly observe each firm’s personalization algorithm but can decide whether to resubscribe based on the observed average performance of the firm’s personalization algorithm. Because we do not model repeated interactions

\(^4\)For example, industries that use consumers’ geo-location data can require all location data only to be accurate to a certain radius.
in this paper, to make consumers’ subscription decisions reactive to firms’ personalization strategies, we assume that each firm’s personalization algorithm is revealed to consumers before subscribing.

We solve the game in the Online Appendix. One can show that in equilibrium, firm $i$’s personalization strategy is

$$y_i^*(s_i) = \alpha_i \cdot s_i + (1 - \alpha_i) \cdot \frac{1}{2},$$

firm $i$’s equilibrium subscription price is

$$p_i^* = \theta + \frac{t}{36} (\alpha_i^2 - \alpha_j^2),$$

and firm $i$’s equilibrium profit is

$$\Pi_i^* = \frac{(36\theta + t(\alpha_i^2 - \alpha_j^2))^2}{2592\theta}.$$ 

Note that firms’ equilibrium positioning strategy and average price are the same as in the main model, which is intuitive. In the main model, because firms cannot observe their rival’s offerings, they choose a positioning strategy that minimizes the expected distance to each consumer’s ideal location. Such a personalization strategy also maximizes a firm’s demand in the subscription model. Also, the average price in the main model depends on the size of a firm’s advantage in prediction over its competitor, with the prediction accuracy performing a similar role to product quality. The same intuition applies here.

However, a crucial difference is that firms in the subscription model can no longer charge different prices for offering content of different positioning. In the main model where firms charge different prices after receiving mainstream and niche signals, we find that firms profit more from consumers predicted to have more niche tastes, and make higher equilibrium profits when price competition for those (predicted) niche consumers softens. Such price discrimination no longer exists for the subscription model. As a result, in the symmetric case with $\alpha_i = \alpha_j = \alpha$, equilibrium profits in the subscription model do not depend on $\alpha$, because equilibrium subscription prices do not depend on $\alpha$. This is different from the main model where equilibrium profits depend on the prediction accuracies.
5.3 The Salop Circular Model

The main model assumes that consumers’ ideal locations are distributed along the Hotelling line. Consumers are considered to have mainstream tastes or niche tastes depending on where they locate on the Hotelling line. In this section, we consider an alternative model where consumers’ ideal locations are distributed along the Salop Circle instead. Consumer tastes on a Salop Circle can no longer be described as mainstream or niche, as any two points on a circle are symmetric to each other.

Consumers’ ideal location on product positioning is uniformly distributed on a circle with a circumference of 1. We use \( r_0 = \frac{1}{2\pi} \) to denote the circle’s radius, and further use polar coordinates to denote a consumer’s ideal location. Specifically, a consumer’s ideal location on product positioning is denoted as \((r_0, x)\), with \( x \sim U[0, 2\pi) \).

Let \( s_i \in [0, 2\pi) \) denote firm \( i \)'s signal of the focal consumer’s ideal location. Let \( y_i \in [0, 2\pi) \) denote firm \( i \)'s positioning to the focal consumer. The consumer’s (dis)utility from choosing firm 1 and firm 2 are

\[
U_1 = -p_1 - t \cdot d(y_1, x) \tag{32}
\]
\[
U_2 = -p_2 - t \cdot d(y_2, x) + \Delta \epsilon \tag{33}
\]

where \( d(y, x) \equiv \min\{(r_0|y - x|)^2, (1 - r_0|y - x|)^2\} \). Same as in the main model, we let \( \Delta \epsilon \sim U[-\theta, \theta] \) and assume \( \theta \) is sufficiently large.

We solve the game in the Appendix. One can show that in equilibrium, firm \( i \)'s personalization strategy is

\[
y_i^* = s_i \tag{34}
\]

firm \( i \)'s equilibrium subscription price is

\[
p_i^* = \theta + \frac{t}{36}(\alpha_i - \alpha_j) \tag{35}
\]

and firm \( i \)'s equilibrium profit is

\[
\Pi^*_i = \frac{(36\theta + t(\alpha_i - \alpha_j))^2}{2592\theta} \tag{36}
\]

Note that the equilibrium when consumers are located on a Salop Circle is very different from the equilibrium when consumers are located on a Hotelling line. First, in the Salop
Circle model, firms always position precisely at their signals. There is no hedging between the signal and the most mainstream point because there are no mainstream or niche locations on the Salop Circle. Because all consumer locations are symmetric to each other, firms also charge the same price for all positioning. Firms do not price discriminate between mainstream and niche consumers because all consumers are equally “niche”. As a result, equilibrium profits also do not depend on the signals that firms receive. In the symmetric case, firms’ equilibrium profits no longer depend on the industry-level accuracy $\alpha$, as equilibrium prices do not depend on $\alpha$.

The comparison between the Salop Circle model and the main model shows that a key driver of the main model is the existence of mainstream versus niche consumer tastes. Thus, the applicability of our results to an industry depends crucially on whether such notions of mainstream and niche consumer tastes exist in that industry.

6 Conclusion

This paper studies the competition between two firms that can personalize product positioning based on imperfect prediction technology. Consumers’ ideal locations are distributed on the Hotelling line. Each firm receives a noisy signal of each consumer’s ideal location, and simultaneously decides what positioning and price to offer to each consumer without observing the competitor’s offer.

We characterize the equilibrium personalization strategy both for the general case where firms have different prediction accuracies and for the symmetric case where firms have the same accuracy. We show that a firm’s personalized positioning is on average more mainstream than consumers’ ideal locations, and the firm charges a higher price for a more niche positioning unless the firm’s prediction accuracy is sufficiently low compared to the competitor’s. In the asymmetric case, a firm’s prediction accuracy is similar to product quality. A higher accuracy allows the firm to charge higher prices and earn higher profits.

Interestingly, in the symmetric case, we find that firms always charge higher prices and earn higher profits when offering more niche positioning, that the degree of product differentiation first increases then decreases in prediction accuracy, and that profits first increase then decrease in prediction accuracy. We find that an increase in the industry-level prediction accuracy first relaxes (intensifies) then intensifies (relaxes) price competition for consumers
predicted to have niche (mainstream) tastes.

If firms can endogenously invest in prediction accuracies, we find that firms over-invest in equilibrium. Competitors can mutually benefit from industry-level self-regulations that restrict their abilities to predict individual consumer preferences. We also discuss what happens if firms operate on a subscription pricing model or if consumer tastes are located on a Salop Circle.

The paper expands the literature on spatial competition and product positioning by incorporating personalization into the positioning decision. Several limitations of the paper highlight the need for future research. First, the paper assumes that firms personalize by only presenting one positioning. It is important to consider what happens if firms can offer a personalized menu to each consumer, and the consumer chooses from the personalized menu. Second, firms’ information about consumers’ preferences is simply modeled as a black-box signal in the paper. It will be interesting to model how such information can be derived from consumers’ online footprints. Finally, the paper only models horizontal preferences. If consumers have heterogeneous vertical preferences, then firms can potentially personalize quality and price using their signals of each consumer’s willingness to pay for quality.
References


