Content Marketing: Why Do Firms Handicap Themselves with Brand-Neutral in a Competitive Environment?

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Feburary 2023

Abstract

To attract prospective consumers, brands are embracing a content marketing strategy – playing the role of content publishers by producing brand-neutral content without promoting brand information. Despite the fact that brand-neutral content generates web traffic, it is unclear why and to what extent symmetric firms should adopt this type of handicapping content marketing strategy in a competitive environment even if they can enhance brand-level matching by including brand-specific information. To answer the question, we develop a game-theoretical model that captures one strategic role of brand-neutral content in facilitating search prominence while suppressing one dimension of information. Our results suggest that firms opting for content marketing as a dominant strategy when content marketing can serve as "dual commitment" device: The prominent content marketer can induce its competitor's commitment problem of exploiting the searching consumer, preventing the consumer to search further. Meanwhile, the content marketer can alleviate its own commitment problem since it makes consumer search incomplete by handicapping itself and the presence of an inactive competitor credibly prevents itself from exploiting the consumer. Consequently, the content marketer who refrains from touting brand information counterintuitively capture the consumer with certainty. Then both firms opt for content marketing as a dominant strategy. We extend the analysis to the cases of price commitment, asymmetric firms, third-party information source, and general distributions and investigate the conditions for when and which firm should opt for or prioritize brand-neutral content marketing.

Keywords: Content marketing, inbound marketing, search prominence, advertising, information disclosure, Perfect Bayesian Equilibrium, D1 criterion

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1 Introduction

Aiming at *pulling* prospective customers toward a business, *inbound marketing*, as opposed to outbound marketing that pushes products to consumers, has gained increasing popularity among marketers. A primary driver of successful inbound marketing is valuable content (Steenburgh et al., 2011). To this end, brands now are embracing one content marketing strategy – playing the role of content publishers by producing brand-neutral information to educate and attract consumers without promoting their branded products. Patagonia started "The Cleanest Line" in 2005 on its website, which primarily focuses on material-related environmental issues and interest-based content produced by independent and in-house contributors. Content marketing is no longer a big brand's privilege. Acorns - a young fintech start-up of micro-investing aimed at millennials - launched its educational platform "Grow" in 2016, producing over 800 articles, essays, and videos about micro-investing that have been viewed by over 3.6 million visitors since its inception.[1]

Consumer acceptance of products will not occur without primary need recognition in the information funnel (Lu and Shin (2018), Choi et al. (2022)). Take sustainable fashion as an example: Consumers may not proceed to the purchase stage until they are convinced that recycled plastics indeed reduce ocean pollution. Rather leaving consumers to other information sources or completely uninformed, brands shift toward the brand-neutral content marketing strategy by supplying relevant information to address consumers' primary informational needs.

However, it is puzzling to both academics and practitioners as to strategic implications of becoming a content marketer or a brand advertiser. A recent industry survey demonstrated this industry-wide dilemma that nearly 60% of $1,798$ surveyed companies agree to prioritize audiences' informational needs over organizations' promotion and sales messages, whereas the rest agree with the opposite.[2] Though generating web traffic, promotion-free educational content cannot guarantee conversions to sales. Even worse, the content marketer paves the way for a competing brand to capture educated consumers. This undesirable consequence is particularly acute when the competitor is equally strong in the brand name but heavily advertises its branded products. This dilemma is exemplified in Patagonia and Nike's divergent strategies. Free-riding on Patagonia's content that

[1]Acorns is now valued at $860 million following a $105 million funding round, Business Insider, January 2019, https://www.businessinsider.com/acorns-approaching-unicorn-status-2019-1.

[2]B2C Content Marketing 2020: Benchmarks, Budgets, and Trends, Content Marketing Institute, December 2019.

educates consumers to buy in the concept of recycled plastics, Nike's strategy is to focus on the superior performance of its Flyknit material, which is also made with recycled plastics. Such asymmetry in content provision clearly leads to a brand advantage for Nike. Although Patagonia's digital creative director took a stand that "our content stays away from the hard sell," many brands are still facing a dilemma: They can either make great content or try to sell products, but not both.[3]

This research aims to contribute to the understanding of this brand-neutral content marketing phenomenon by addressing questions concerning when and why symmetric firms should adopt brand-neutral content marketing in a competitive environment even if they can enhance brand matching by advertising brand-specific information.

To answer these questions, we develop a game-theoretical framework that captures the strategic role of content in inbound marketing. More specifically, we recognize the role of brand-neutral content in facilitating prominence in consumer information search processes. Armstrong et al. (2009) coin the term "prominence" in the search process by supposing that the prominent firm will be visited first. Our model endogenizes the origin of firms' prominence from the information perspective. We consider that consumers first resolve their need recognition in their information funnel (Lu and Shin (2018), Choi et al. (2022), etc.). Thus, a brand that provide content who provides information that helps consumers to recognize their need and make decisions at the early stage of the purchase funnel becomes the first-stop for consumers.

However, gaining search prominence seems to come at the cost of suppressing one important dimension of information. Brand-level information can improve a brand's profit as it communicates brand advantages and facilitates differentiation. This concern over search prominence and information suppression leads to an under-explored question that invites an investigation of the impact of asymmetric information provision on search outcomes. To the best of our knowledge, an analysis of firms' potential asymmetric content provision departs from classic prominence literature (e.g., Armstrong et al. (2009), Arbatskaya (2007), Haan and Moraga-González (2011), Zhou (2011), etc.) that assume firms are symmetric in search markets except for the exogenous or endogenous prominence. In contrary, we intend to capture the fact that prominence only arises under firms' asymmetric information provision. Moreover, we investigate the above tradeoff in a competitive

[3]Inside Patagonia's Content Machine, Digiday, January 2013, https://digiday.com/marketing/inside-patagonias-content-machine

environment by considering two firms to compete in content provision and product prices.

We characterize a firm's content design as a mixed strategy profile that prescribes a probability distribution over brand-neutral content marketing and brand-specific advertising, where the first type of content resolves consumer need recognition and brand advertising potentially enhance brand matching. One can interpret mixed strategy as coverage or frequency of different types of content.

The primary insight of our research is that content marketing becomes a dominant strategy when it can service as "dual commitment" device. A content marketer who provides information that helps consumers to recognize their need and make decisions at the early stage of the purchase funnel becomes the first-stop for consumers. Once consumers recognize their need, to generate information values and attract the consumer to search further, the competing firm can only provide brand-specific information. Brand-specific information could have enabled the brand advertiser to extract surplus from the consumer who highly values brand-level matching. Anticipating the brand advertiser's incentive, consumers trade off between the value of seeking brand-specific information and surplus exploitation once they incur search costs (*information-exploitation* tradeoff). As the first firm to be visited, it can strategically set its price to induce firm 2's incentive to fully exploit the searching consumer, resulting in the hold-up problem between its competitor and the consumer. Then a consumer forgoes further searching as the negative impact of being exploited dominates the benefit from seeking information value. In this way, the content marketer can retain the consumer once it has been searched. Even the content marketer also faces the commitment problem, it can commit not to exploit consumers. Its commitment power comes from the fact that it handicaps itself without touting brand content. In doing so, it faces a different hold-up problem in the way consumer search is incomplete. The presence of an inactive competitor alleviates the content marketer's deviation incentive to extract surplus after consumers have searched it. In a nutshell, we capture one strategic benefit of content marketing as "dual commitment" device: By facilitating search prominence and making consumer search incomplete, content marketing enable the content marketer to induce a commitment problem for its competitor while resolving its commitment issue.

Next, we demonstrate that the content marketer's advantage can still arise when firms commit on prices, for example, through displayed ads links. We further investigate questions concerning the extent to which firms should prioritize content marketing or brand advertising. The answer hinges on when content marketing or brand advertising leads to price and equivalently profit premium. We show that the firms' price difference can be either positive or negative, depending on the strengths of the two forces in consumers' *information-exploitation* tradeoff. The brand advertiser can exploit a consumer's revealed preference and charge a high price so long as it expects the consumer who values brand-level information shows up in its site. When brand-level uncertainty reduces (increases), the information gain from search diminishes (enlarges) and the post-search exploitation problem exacerbates (alleviates) as continuing in searching reveals higher (lower) preferences. The content marketer is in a favoring situation in the sense that its higher price (than the brand advertiser's price) can strategically "help" the brand advertiser to alleviate the surplus exploitation curse. Whether the brand advertiser can exploit the searching consumer and obtain price premium depends on the right tail of the preference distribution so that the consumer who sufficiently values brand-specific information continues to search even knowing she will be exploited. Moreover, we find that the firm that wins the price premium is also able to capture a higher profit. We then compare and contrast our insights with previous literature and demonstrate that the way we capture pre-search heterogeneity in terms of information preferences leads to fundamentally different equilibrium predictions.

We recapitulate the results into two managerial insights. The first insight deals with the "why" question and can be summarized into one line: "Attract consumers with non-branded content, corner your competitors to tout brand information, just let them turn off consumers, and consumers are yours." As a result, both firms choose content marketing as a dominant strategy. The second managerial insight answers the "to what extent" question. Sometimes the content marketer may lose a high-value consumer to the competitor. This consequence happens when the horizontal differentiation in the market is substantial, and the additional value of brand-specific information is important in consumers' decision-making processes.

To check the robustness of our insights, we extend the analysis to more general forms of distributions. We show the prevalence and the boundary condition of the emergence of dual commitment device. In the case of price commtiment, we identify a general condition for the rise of the content marketer's premium in the notion of mean-preserving spread and confirm that price premium implies profit premium.

The rest of the paper is organized as follows: We briefly review literature in Section 2 and introduce the model in Section 3. In the main analysis, Section 4.1 examines the key insights without price commitment and Section 4.2 investigates firms' optimal content design when prices can be committed. Then we extend the analysis to asymmetric firms in Section 5.1, third-party information source in Section 5.2, and general distributions in Section 5.3. We conclude and acknowledge our limitations in Section 6.

2 Literature Review

First, our research provides a model of inbound marketing in which the content marketer uses brand-neutral content to attract consumers to search it first. Hence, we contribute to the understanding of the source of search prominence and its effect on market outcomes. Armstrong et al. (2009) provides a framework of modeling prominence, in which one prominent firm will be first visited by all consumers. Their work treats prominence to be exogenously given. Further research attributes sources of prominence to price advertising (Armstrong and Zhou (2011)), advertising intensity (Haan and Moraga-González (2011)), firms' incentives to buy prominence through intermediaries (Armstrong and Zhou (2011), Athey and Ellison (2011)) and past purchases (Armstrong and Zhou (2011)). This research proposes a new source of prominence from the perspective of consumers' hierarchical information needs and firms' asymmetry in information provision. However, this informational perspective introduces new complexity. To the best of our knowledge, previous work mainly focuses on characterizing symmetric equilibrium. Unlike their framework, our informational perspective captures *firm-level asymmetry* and thus investigates asymmetric market outcomes, enabling us to generate competitive insights.

Our paper joins recent literature on incorporating ex-ante consumer heterogeneity in sequential search. The presence of pre-search heterogeneity complicates the problem because it generates search order and heterogeneous search paths for consumers. Arbatskaya (2007) considers consumers to be heterogeneous in their search cost, whereas Zhou (2011) and Choi et al. (2018) capture pre-search heterogeneity in terms of consumer knowledge of partial product information. We capture consumer heterogeneity in needs for information. In contrast to the monotonic relationship between price and search order predicted in Zhou (2011) and Arbatskaya (2007), we find that a prominent firm's price premium can be either positive or negative, depending on the market environment. The distinction in the source of consumer heterogeneity leads to qualitatively different equilibrium predictions, which we will elaborate on at the end of Section §4.2.

Our research captures how content – generic versus brand-oriented – influences consumer search. In this regard, this paper is related to a stream of literature on advertising content and consumer search. Ke and Lin (2018) study a setting in which the two products share similar attributes so that part of the information is also treated as generic, and the two firms are symmetric in offering product information to consumers. Their information structure is additive, whereas ours is hierarchical. In addition, we consider that firms are asymmetric in information provision. Hence, we approach the problem from a different angle: A firm that refrains from providing brand-specific information can obtain search prominence and strategically causes a hold-up trap for its competitor, thereby benefiting the content marketer. Anderson and Renault (2006) demonstrates the hold-up problem when a monopoly firm discloses product matching to consumers. If an advertisement fully reveals matching information, consumers' search behavior reveals that their matching values exceed search costs for any price consumers anticipate. Then the monopoly firm can hold up consumers who visit by raising its price. In our context, we investigate the hold-up problem from a different competitive perspective. When consumers are heterogeneous in their preferences toward brand-specific values, a content marketer can induce and take advantage of the hold-up tension between consumers and its competitor by fully resolving primary-level uncertainty. Anderson and Renault (2009) analyze the competitive effect of advertising content in a duopoly market. However, in their paper, consumers incur no search costs. As a result, advertising content does not lead to asymmetric prices, irrespective of which firm advertises brand matching.

On a higher level, generic consumer education can be viewed as goodwill services provided by a content marketer that allows competing firms to free ride. Such free-riding service is prevalent in various industries. Shin (2007) first analyzes the strategic effect of free-riding service in the showrooming context and provides an explanation of why free riding benefits both free-riding retailers and the retailer who provides that service. Lu and Shin (2018) examine the free-riding problem in the marketing communication process that builds consumers' category demand.

3 Model

Consumer Utility Formulation A consumer t derives the following utility:

$$U(t) = \chi \cdot (u^0 + t \cdot v_j - p_j) \tag{1}$$

A consumer has two levels of uncertainty. On the primary level, she is not aware of her need. With probability λ, $\chi = 1$ such that a consumer recognizes the need for the product; with probability $1-\lambda$, $\chi = 0$ so that the consumer has no need and derives zero utility.

Once a consumer recognizes her need, she then derives product-level utility for $u^0 + t \cdot v_j - p_j$ for product $j \in \{1, 2\}$, where u_0 is the base utility and p_j is product j's price. On the secondary level, a consumer then needs to resolve product-level uncertainty on v_j. To highlight the main story, we consider the two firms are ex-ante symmetric in the main model. That is, v_j, is independently and identically distributed according to a well-defined distribution $F(\cdot)$ in the support $[-\underline{v}, \overline{v}]$ with mean v^0. To illustrate our main ideas, we start with a discrete-uniform case (Section §4.1 and §4.2), in which $v_j \in \{\overline{v}, -\underline{v}\}$ with probability μ and $1-\mu$. The key insights set the building blocks for the analysis of the case of general distributions in Section §5.3.

Moreover, we capture a consumer's pre-search heterogeneity $t \sim U[0, T]$ in the utility weight over the brand-specific value.

Content Provision A firm's content strategy is defined as a mixed strategy profile that prescribes a probability distribution $\{\sigma, 1-\sigma\}$ over generic and brand-specific content, where generic content reveals χ and brand-specific information reveals v_j. One can interpret σ as coverage or frequency of different types of content. To differentiate the roles of content, the brand-neutral content that resolves primary need recognition facilitates the creation of a market benefiting both firms whereas the brand-level content potentially enhances match value.

Search Prominence Firms' content provision and the consumer's information needs endogenously generates a search market and search prominence. Prominence only arises under asymmetric content provision - one firm provides generic content ("content marketer") while the other firm provides brand-specific content ("brand advertiser"). The content marketer, who provides generic content that resolve primary-level need recognition, becomes the prominent source to be visited first in the search process. We denote the content marketer as firm 1 and the brand advertiser as firm 2. Once the consumer searches a content marketer by incurring a search cost c, she discovers

χ and p_1. If $\chi = 0$, the game ends. If $\chi = 1$, the consumer decides whether to purchase product 1 immediately or incur c to continue the second-round search. If the consumer further searches the brand advertiser, she realizes v_2 and p_2. Then the consumer decides whether to purchase product 2 or go back to purchase product 1.

Under symmetric content provision, no prominence is induced. If both firms provide generic content, a consumer randomly search one firm. If both firms provide brand content, there is no market for the product, resulting in zero demand for both firms.

The tie-breaking rule we use is that a consumer prefers the current option if she is indifferent. For example, a consumer forgoes further searching if she is indifferent between search and no search. Given a consumer has searched two products, she buys the second product without returning if she is indifferent.

We conclude this section by describe the timeline of the game. In Stage 1, the firms simultaneously determine content strategies, mixing over two types of content. In Stage 2, once content provision is realized, the firms choose prices. In Stage 3, the consumer observes her type t and sequentially searches for products and prices by incurring cost c.

4 Analysis

4.1 Content Marketing as a "Dual Commitment" Device

The goal of this section is to study the strategic benefit of becoming a content marketer in a symmetric setting. We solve from the backwards. The realizations of the firms' mixed content strategies include asymmetric content provision ($\{C, B\}, \{B, C\}$) and symmetric ones ($\{C, C\}, \{B, B\}$). Prominence arises under asymmetric content provision – a consumer will first visit the firm who chooses content marketing to resolve primary need recognition. Under symmetric content provision, no prominence is induced. We use the Perfect Bayesian Equilibrium (PBE) as the solution concept. To refine the off-the-equilibrium belief, we apply the D1 criterion (Banks and Sobel (1987)).[4]

[4]Concerning continuous consumer types, the D1 criterion provides a stronger refinement than the intuitive criterion (Cho and Kreps (1987)).

Prominence under Asymmetric Content Provision

A consumer's search decision is driven by pre-search heterogeneity characterized by her type t. For any price pair $\{p_1, p_2\}$, we can find the marginal type $\tilde{t}$ so that a consumer with $t > \tilde{t}$ engages in the first-round search.[5] In the last stage, given that a searching consumer has figured out primary need recognition ($\chi = 1$) from the content marketer (firm 1), she needs to decide whether to further search for brand-level information provided by the brand advertiser (firm 2). Consumer t continues searching firm 2 if the search benefit, $V(t)$, exceeds the search cost c:

$$V(t) \equiv \mu \max\{t(\overline{v} - v^0) + (p_1 - p_2), 0\} + (1-\mu)\max\{-t(\underline{v} + v^0) + (p_1 - p_2), 0\} > c, \tag{2}$$

where the mean value $v^0 = \mu\overline{v} - (1-\mu)\underline{v}$.

Notice that the price difference, $\Delta p \equiv p_2 - p_1$, results in different search incentives and thus who will search firm 2. If firm 2's price is sufficiently low such that the search benefit $V(t) = \Delta p < -c$, all consumer types continue to search and purchase from firm 2 due to a lower price, irrespective of the product match. Conversely, for $\Delta p \geq -c$, the search benefit in the second round is $V(t) = \max\{0, \mu(t(1-\mu)(\overline{v} + \underline{v}) - \Delta p)\}$, which is driven by the tradeoff between the value of brand-level information, $\mu(1-\mu)(\overline{v} + \underline{v})$, and the price difference, Δp, a consumer needs to pay for firm 2's product. A consumer continues to search firm 2 so long as she sufficiently values product-level information, i.e., $t > \hat{t} \Leftrightarrow V(t) > c$, where the marginal type of consumer, $\hat{t}$, who is indifferent between continuing and forgoing further search is as follows,

$$\hat{t} \equiv \frac{1}{(1-\mu)(\overline{v} + \underline{v})}\left(\frac{c}{\mu} + \Delta p\right). \tag{3}$$

Upon the searching consumer has arrived at firm 2, firm 2 can condition its ex-post optimal pricing strategy, denoted as p_2', on the realization of brand-level matching and the revealed information $t > \max\{\tilde{t}, \hat{t}\}$. In a competitive environment, firm 2's ex-post incentive depends on firm 1's pricing strategy. To keep a mismatched consumer, firm 2 has to charge a price lower than firm 1's. If firm 1's price is not too high, racing down the price to retain a mismatched consumer is less profitable compared to an alternative strategy of exploiting a matched consumer. To exploit a

[5] We will verify the monotonicity of the search rule in the appendix.

matched consumer, for any price p_2 that leads to a consumer with $t > \max\{\tilde{t}, \hat{t}\}$ searches firm 2, firm 2 can at least increase its price by $\frac{c}{\mu}$ without losing any matched consumer. Then any p_2 that induces the consumer with $t > \hat{t}$ to search is not sequential rational, i.e., $p_2 \neq p_2'$. If the consumer with $t > \max\{\tilde{t}, \hat{t}\}$ rationally anticipates firm 2's ex-post exploitation incentive, then at least some consumer with $t \leq \max\{\tilde{t}, \hat{t}\} + \frac{c}{\mu(1-\mu)(\overline{v}+\underline{v})}$ would not search, rendering firm 2's belief to be inconsistent. Consequently, a consumer would forgo searching firm 2 and stay with firm 1 if she expects firm 2's exploitation incentive. Anderson and Renault (2006) study this classic hold-up problem in a monopoly setting. In contrary, our focus is that in a competitive setting, firm 1, as the first firm to be visited, it can strategically set its price to induce firm 2's exploitation incentive, resulting in the hold-up problem between its competitor and the consumer. Then a consumer forgoes further searching as the negative impact of being exploited dominates the benefit from seeking information value. In this way, firm 1 can retain the consumer once it has been searched.

Having said that, the aforementioned benefit is contingent on consumers' first-stage search decisions. If a consumer can rationally anticipate the second-round hold-up, she would also have anticipated that firm 1 may not commit on the price and hold her up once she incurs search cost. To understand the role of content marketing in resolving firm 1's commitment problem, we first establish the following result and then explain how firm 1 can commit not to hold up the consumer in the first round.

Lemma 1. *Under asymmetric content provision, when $u^0 > \overline{u}^0$ so that the search market exists (i.e., at least some consumer type engages in costly searching), firm* 1 *can commit to charge $p_1^* = \frac{2-\mu}{1-\mu}\mu(\overline{v} + \underline{v})T$, and a searching consumer buys from firm* 1 *and receives positive surplus.*

It is worth noting that on- and off-the equilibrium path, firm 1's price is independent of who searches in the first round. Put differently, despite a consumer may have revealed her type through the first-round search, firm 1 will not react to that information. In sharp contrast, as shown earlier, firm 2 who provides brand-specific information cannot commit on not using the revealed information about consumer type t. In the next corollary, we first explain the rationale why the strategic interplay between firm 1 and firm 2 can enable firm 1 to commit on p_1^*. Then we will discuss how the types of information (brand-neutral vs. brand-specific) and firm 1's knowledge of t affects the commitment power.

Corollary 1. *The equilibrium of the subgame described in Lemma 1 survives the D1 refinement and is unique.*

In the equilibrium where no consumer further searches firm 2, firm 1's price strategically influences firm 2's rational interpretation of the consumer's out-of-equilibrium searching behavior and thereby its subsequent pricing strategy - exploiting a matched consumer or retaining a mismatched consumer. If firm 1 charges a price such that $p_1 \leq \frac{2-\mu}{1-\mu}\mu(\overline{v}+\underline{v})T$, the consumer's continuation in searching delivers the following speech to firm 2: *"Given that firm* 1 *offers a good price to me and I know your best response price can be higher than firm* 1 *for some beliefs, the fact that I am willing to incur costs to search you should convince you that I am more likely to be a higher type as I will find a wider range of your prices that justify my search."* Iteratively applying the D1 criterion, the only type that cannot be eliminated upon observing the out-of-equilibrium searching behavior is the highest type. Given firm 1's price and the rational out-of-equilibrium belief, firm 2's best response is to exploit the searching consumer once she finds a match. As a result, a consumer's deviation payoff is equilibrium dominated. Conversely, if firm 1 charges a too high price, i.e., $p_1 > \frac{2-\mu}{1-\mu}\mu(\overline{v}+\underline{v})T$, searching firm 2 may not necessarily suggest a higher type. With a D1-refined belief, firm 2 is better off accommodating a mismatched consumer, which then rationalizes a consumer's deviation behavior, which leaves firm 1 with zero payoff. Therefore, by deliberatively choosing its price, firm 1 can cause firm 2's problem by inducing firm 2 to exploit a matched consumer. A consumer's strategic incentive to avoid being held up once she searches the brand advertiser prevents her from leaving the content marketer, which enables the content marketer to capture the demand. Though firm 2 is an inactive firm[6], potential competition from firm 2 limits firm 1's pricing power.

Even the first-round search may reveal consumer type t to firm 1, firm 1's commitment power comes from the fact that it handicaps itself without touting brand content. In doing so, firm 1 faces a different hold-up problem in the way consumer search is incomplete.[7] The presence of an inactive firm 2 alleviates firm 1's deviation incentive to extract surplus after a consumer searches it because firm 1's pricing strategy is to induce firm 2's exploitation incentive.

Moreover, the search market exists so long as firm 1 has the commitment power of charging

[6]As suggested in Arbatskaya (2007), an inactive firm may be present if fixed costs are conditional on positive sales.

[7]If firm 1 also provides brand-specific information so that search for a matched consumer is complete, it will not always find profitable by committing itself not to exploit the matched consumer.

p_1^* and some consumer expects positive surplus, that is $u^0 > \overline{u} \equiv \frac{c}{\lambda} + \left(\frac{1}{1-\mu}v^0 + \frac{2-\mu}{1-\mu}\underline{v}\right)T$. As v^0 increases, $\overline{u}^0$ increases. That is, when firm 1 knows that the searching consumer is more likely to be of a higher type, is less likely to commit, which backfires firm 1.

Competitive Content Strategies

The preceding section shows that content marketing can serve as a commitment device when there exists a firm who promotes brand, allowing the content marketer to take the advantageous position. However, when firms compete in content strategies, their competitive motive to capture such advantage leads both to choose content marketing with probability one, dissipating such premium. To see this, when both firms choose content marketing, both firms have commitment problems as consumer search is complete by visiting one firm. Firms cannot commit not to exploit the revealed information so long as no consumers leaves it. One interesting implication is that content marketing can serve as a commitment device only with the presence of one firm who advertises its brand.

Proposition 1. *When prices are uncommitted, both firms choose brand-neutral content marketing as a dominant strategy.*

The above discussion suggests that firms should opt for brand-neutral content marketing as a dominant strategy due to its strategic role as *"dual commitment" device.* Content marketing facilitates search prominence. Besides the direct benefit of prominence in bringing traffic, the prominent position enables a content marketer to induce a commitment problem for its competitor and keep the consumer. Second, brand-neutral content marketing serves as a commitment device not to hold up the consumer in the first-round search as consumer search is incomplete at the prominent firm such that the prominent firm uses competitor as a credible commitment device to prevent itself from deviation.

The analysis of no commitment case sharpens our understanding of one rationale why firms should opt for content marketing. In what follows, we will extend our analysis to the cases of price commitment, asymmetric firms and general distributions and examine optimal content strategies.

4.2 Optimal Content Strategies with Price Commitment

A natural question arises: With price commitment, to what extent the content marketer can still enjoy the strategic benefit from prominence. This section addresses this question and the problem of optimal content design.

We first analyze the search subgame following asymmetric content provision. Recall that when deciding whether to search for brand-level information, a consumer trades off between the value of information from further searching and the negative consequence of being exploited by the revealed information preference. Whether the prominent firm 1 can keep the consumer depends on its ability to influence consumer's *information-exploitation* tradeoff. When prices are not committed, firm 1 can induce firm 2's incentive to fully exploit the searching consumer by holding her up. In this case, the negative consequence of surplus extraction predominantly outweighs the benefit of information value, discouraging a consumer to search at all. Thus firm 1's strategic benefit from prominence is maximized. With price commitment, firm 2 still can partially exploit a consumer's revealed type as it anticipates that a consumer with higher valuations toward brand-level information searches it. To enrich our understanding, this section focuses on the parameter space when no force in the *information-exploitation* tradeoff dominates, leading to the emergence of type-dependent search behavior in equilibrium.[8]

In the type-dependent search equilibrium, a consumer with $t \leq \hat{t}(\Delta p)$ forgoes further searching and buys brand 1, where $\hat{t}$ is given in Equation (3); whereas a consumer with $t > \hat{t}(\Delta p)$ continues to search and returns to buy brand 1 if and only if $v_2 = -\underline{v}$. As λ only plays a role in the first-round search, we safely drop λ in the analysis of the second-round search. Given $\chi = 1$, the two firms maximize the following payoffs $\Pi_1 = p_1 \cdot Q_1 = p_1 \cdot \left(1 - \mu\left(1 - \frac{\hat{t}(\Delta p)}{T}\right)\right)$ and $\Pi_2 = p_2 \cdot Q_2 = p_2 \cdot \mu\left(1 - \frac{\hat{t}(\Delta p)}{T}\right)$, respectively, which leads to the equilibrium prices,

$$p_1^* = \frac{(2-\mu)(1-\mu)(\overline{v}+\underline{v})T}{3\mu} + \frac{c}{3\mu}, \quad p_2^* = \frac{(1-\mu)^2(\overline{v}+\underline{v})T}{3\mu} - \frac{c}{3\mu}. \tag{4}$$

The equilibrium price difference should satisfy $\Delta p^* \in \left[-c, (1-\mu)(\overline{v}+\underline{v})T - \frac{c}{\mu}\right)$, and no firm has a unilateral incentive to deviate. In the online appendix, we give the detailed existence proof

[8] When the negative impact of surplus exploitation dominates, no consumer continues searching, which is similar to the no-commitment case. On the other hand, when the benefit of information value dominates, all consumer types continue searching.

of the equilibrium parameter space.

Next, we analyze the subgame of symmetric content provision. When both are content marketers, with price commitment, undercutting the competitor's price by an infinitely small amount leads to a discrete jump in demand. Thus, the two firms engage in the Bertrand competition and end up with zero payoffs. Under symmetric content provision when both are brand advertisers, no search can be induced. Both firms expect zero payoffs. In Section 5.2, we consider a scenario when search could be induced and analyze price competition under symmetric adoption of brand advertising.

In the appendix, we examine the first-round search decision and show that anticipating the equilibrium prices and following the optimal second-round search rule, a consumer with any t prefers to search in the first round under the market-existing condition $u^0 > \overline{u}^0$ given in Lemma 1.

Based on the analysis of equilibrium search and pricing strategies following all possible content strategies, we characterize the equilibrium optimal content provision in the next proposition.

Proposition 2. *There exists two pure strategy equilibrium in which symmetric firms choose asymmetric content strategies and one mixed strategy equilibrium in which firms mix brand-neutral content marketing with probability* $\sigma^* = \frac{\Pi_1^*}{\Pi_1^*+\Pi_2^*}$ *and brand advertising with probability* $1-\sigma^* = \frac{\Pi_2^*}{\Pi_1^*+\Pi_2^*}$*, where* $\Pi_1^* = \frac{c\left(1+(2-3\mu+\mu^2)\psi\right)^2}{9\mu(1-\mu)\psi}$ *and* $\Pi_2^* = \frac{c\left(1-(1-\mu^2)\psi\right)^2}{9\mu(1-\mu)\psi}$*, where* $\psi \equiv \frac{(\overline{v}+\underline{v})T}{c}$.

Proposition 2 restates firms' motive to opt for content marketing as a dominant strategy due to its commitment role. If prices can be committed, firms have no need to handicap themselves to commit, and thus content marketing will not arise as a dominant strategy. Consider pure-strategy equilibrium. Even with symmetric firms, an asymmetric equilibrium in which firms differ in their content strategies can emerge. In a mixed strategy equilibrium, the type of content a firm should prioritize (i.e., $\sigma \geq \frac{1}{2}$, a higher proportion of coverage) depends on whether the content marketer or the brand advertiser can obtain profit premium.

Proposition 3. *Firms should prioritize brand-neutral content marketing* ($\sigma^* \geq \frac{1}{2}$) *if content marketing leads to profit (price) premium, which is equivalent to the following conditions,*

- *when* $\psi > 16$*, and* $\mu \leq \frac{3}{4} - \frac{1}{4}\sqrt{1-\frac{16}{\psi}}$*, or* $\mu \geq \frac{3}{4} + \frac{1}{4}\sqrt{1-\frac{16}{\psi}}$*;*
- *or when* $\psi \leq 16$.

Otherwise, firms should prioritize brand advertising.

Figure 1 depicts the equilibrium prices and the content marketer's premium as a function of the probability μ. Notice that both firms' prices drop as μ increases. This is because the type-dependent search equilibrium arises only when $\mu > \frac{1}{2}$. As the matching probability becomes more certain (μ increases), the information gain from search diminishes, reducing a consumer's incentive to search firm 2. Thus, the two firms' head-to-head competition for the consumer intensifies.

Recall a consumer's tradeoff between information value and surplus extraction. Whether the content marketer or the brand advertiser obtains premium depends on the *information-exploitation* tradeoff. For small μ, the information value is large. A consumer is more likely to search firm 2 (i.e., $\hat{t}$ in Equation (3) decreases with information value). Rather than lowering the price to prevent a consumer from further searching, firm 1 can facilitate searching by charging a high price and then capture most of its demand from the returning consumer. As shown in Armstrong et al. (2009), the returning demand is less elastic, enabling firm 1 to charge a premium price. Meanwhile, as continuation in searching does not reveal high t, firm 2 cannot exploit the searching consumer either. Hence, when $\mu \leq \frac{3}{4} - \frac{1}{4}\sqrt{1 - \frac{16}{\psi}}$, firm 1 charges a higher price and obtains a higher profit than firm 2.

To the opposite, for large μ, the information value from further searching diminishes but the surplus exploitation problem exacerbates as continuation in searching reveals even higher types. Then a consumer is more likely to forgo furthering search, enabling firm 1 to charge a high price. To attract a consumer to search, firm 2 has to use a low price strategy. Thus, when $\mu \geq \frac{3}{4} + \frac{1}{4}\sqrt{1 - \frac{16}{\psi}}$, firm 1 charges a higher price and obtains a higher profit than firm 2.

When μ is moderate, the information value is moderate and a consumer's further searching behavior signals her higher types, favoring firm 2 to charge a high price. Meanwhile, firm 1's returning demand reduces. Hence, when μ is moderate, $\mu \in \left(\frac{3}{4} - \frac{1}{4}\sqrt{1 - \frac{16}{\psi}}, \frac{3}{4} + \frac{1}{4}\sqrt{1 - \frac{16}{\psi}}\right)$, it is firm 2 who obtains price and profit premium. Certainly, this scenario happens when consumer should exhibit sufficient heterogeneity in propensity to search ($\psi > 16$) so that the information-exploitation tradeoff vary across different consumer types and there exists enough high types to pay for firm 2's price premium.

Hence, even with price commitment, our main insight is still kept that the content marketer can sometimes benefit from prominence by strategically taking advantage of the exploitation tension between its competitor and the consumer. Moreover, we identify market conditions when the content

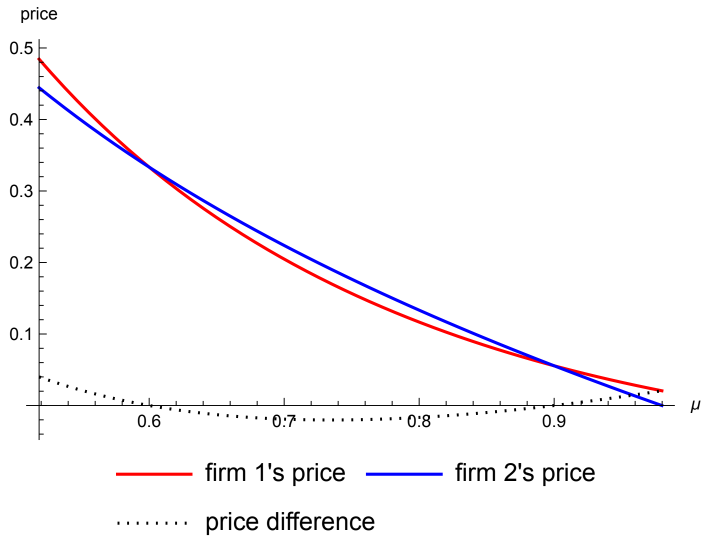

Figure 1: Comparison of Equilibrium Prices ($\psi = 25$)

marketer or the brand advertiser obtains premium, which shed light on managerial implications as to when a firm should prioritize which type of content.

It is worthwhile comparing with a benchmark case where search cost is zero and prominence no longer matters. When $\mu > \frac{1}{2}$, the content marketer ends up charging a lower price and obtaining less profit than the brand advertiser. Nonetheless, when prominence do play a strategic role, the content marketer can gain price and profit premium in the market

Relation to Literature. We compare and contrast our findings on the equilibrium price difference $p_1^* - p_2^*$ with previous literature. In Anderson and Renault (2009), consumers incur no search costs to acquire information. Then search order plays no role. They find that given symmetric distributions, the equilibrium prices are symmetric, irrespective of which firm provides brand-level information. If we set $\mu = \frac{1}{2}$ and $c = 0$, we obtain their same results. The presence of search costs and search order contributes to the asymmetric outcome of price equilibrium.

Armstrong et al. (2009) consider homogenous consumers (except their section 4) and find that the price charged by the prominent firm is lower than other firms. This is because fresh demand is more elastic than returning demand, and the prominent firm's demand is composed of a higher proportion of fresh demand compared with other firms. If we consider that firm 1 also reveals brand-level information, their insight is kept even in the presence of heterogeneous consumers. Hence, the content marketer is unable to keep the price premium if it also provides the brand content. Arbatskaya (2007) and section 4 of Armstrong et al. (2009) consider heterogeneous consumers in terms of search costs and find that the price of the first searched firm is higher because earlier firms can capture consumers with high search costs. Our case of a duopoly market is a special case of the ordered search in Arbatskaya (2007). The assumption on consumer heterogeneity on preferences toward different types of information leads to qualitatively different equilibrium predictions. We show that the price difference between the prominent firm and the non-prominent firm can be either higher or lower, depending on consumers' information-exploitation tradeoff. Section §5.3.1 generalizes the findings in Proposition 8 with a continuous distribution of v_j. So the above discussions on comparisons of findings with previous literature are robust.

5 Extensions

5.1 Asymmetric Firms

The main model treats two firms to be symmetric before the game. The purpose is to highlight (1) the strategic benefit of becoming a prominent content marketer and (2) the possibility of asymmetric equilibrium outcome. In this section, we investigate the case of asymmetric firms and address the problem as to which firm should opt for content marketing. To characterize firm-level asymmetry, we consider the two firms to differ in *strength* in terms of first-order stochastic dominance and in *appeal* in terms of mean-preserving spread.

5.1.1 Strong vs. Weak firms

Firm s is considered to be a stronger one if the distribution of its brand-specific value, v_s, first-order stochastic dominates that of the weaker firm, v_w. With a discrete random variable, we characterize first-order stochastic dominance by setting $\mu_s > \mu_w$, $\overline{v}_j = \overline{v}$ and $\underline{v}_j = \underline{v}$ for all $j \in \{s, w\}$ with no loss of generality (?). Moreover, we denote the difference in strength as $\Delta\mu \equiv \mu_s - \mu_w$.

The equilibrium outcome depends on when content marketing becomes a dominant strategy for which firm. Without price commitment (Section 4.1), due to the motivation to capture dual commitment benefit, content marketing is the dominant strategy for both firms. This finding persists in the case of asymmetric firms. We show in the online appendix that content marketer can commit so long as $u^0 > \frac{c}{\lambda} + T\frac{\mu_2}{1-\mu_2}\left(\overline{v}+\underline{v}\right) + T\underline{v}$, where μ_2 refers to the second firm's strength. This condition is more likely to hold when the weaker firm is the prominent firm. Hence, the following result immediately follows.

Proposition 4. *The weaker firm has more commitment power than the stronger firm.*

Recall that commitment power comes from handicapping itself and compensate for consumer's incomplete search. It costs more for the weaker to commit more than the stronger firm.

With price commitment (Section 4.2), symmetric firms will never find content marketing to be the dominant strategy, leading to an asymmetric outcome in which multiple equilibria arise. In what follows, with asymmetric firms, we would like to study when content marketing will be a dominant strategy for which firm. It is sufficient to study the two firms' incentives to choose content marketing over brand advertising given the competing firm chooses content marketing. To facilitate the analysis, we define $\mathcal{I}_i^C \equiv \Pi_i^{C*} - \Pi_i^{B*}$, for $i = \{s, w\}$, as the difference between firm i's payoffs as a content marketer and as a brand advertiser if the competing firm is a content marketer. A firm chooses content marketing if $\mathcal{I}_i^C > 0$.

If both firms choose content marketing, a consumer only searches once, and the two firms compete for the first-round searcher. The equilibrium payoffs for the stronger and the weaker firms depend on the relative strength $\Delta\mu$, i.e., $\Pi_s^{C*} = \frac{4T\Delta\mu}{9}$ and $\Pi_w^{C*} = \frac{T\Delta\mu}{9}$, respectively. The competition intensifies as they are getting closer in brand strength, resulting in lower payoffs to both firms. Note that the symmetric model in Section 4.2 represents the limiting case of $\Delta\mu \to 0$.

Under asymmetric content provision, the search sequence of the stronger and the weaker firm has significant impact on a consumer's optimal search rule and thereby the equilibrium payoffs. Recall that whether the content marketer or the brand advertiser earns premium depends on the information-exploitation tradeoff the consumer faces. A higher value of information incentivizes the consumer to search further, enabling the brand advertiser to raise the price level and better exploits the matching consumer. Let us denote a content marketer as firm 1 and a brand advertiser as firm 2. Following equation (2), with asymmetric firms, we revise a consumer's search benefit as follows: $V(t) = \mu_2 \max\left\{t(1-\mu_1)\left(\overline{v}+\underline{v}\right) - \Delta p, 0\right\} + (1-\mu_2)\max\left\{-t\mu_1\left(\overline{v}+\underline{v}\right) - \Delta p, 0\right\}$, where $\Delta p = p_2 - p_1$. The value of information from further searching, $\max\left\{\left(\mu_2-\mu_1\right)t\left(\overline{v}+\underline{v}\right), \mu_2(1-\mu_1)t\left(\overline{v}+\underline{v}\right)\right\}$, becomes larger (smaller) if the prominent firm is weaker (stronger). Moreover, from the consumer's perspective, the value of information from further searching decreases with relative strength $\Delta\mu$ if the second firm is the weaker one but increases with $\Delta\mu$ if the second-firm is the stronger one. Consistent with the analysis in Section 4.2, we consider type-dependent second-round search rule and formally present the equilibrium second-round search rule in Lemma A2 in the appendix.

Proposition 5. *For $\mu_s > \underline{\mu}$ and $T \leq \overline{T}$, there exists a parameter space in which one of the following equilibrium outcomes can arise:*

- *when $T < \min\{T_s, T_w\}$ and for any $\Delta\mu$, $\{C, C\}$-equilibrium arises where both firms adopt content marketing as a dominant strategy;*
- *when $\min\{T_s, T_w\} \leq T < \max\{T_s, T_w\}$, asymmetric content provision emerges in equilibrium,*
  - *when the two firms' brand strengths are relatively close ($\Delta\mu < \widehat{\Delta\mu}$), for $T_s \leq T < T_w$, $\{C, B\}$-equilibrium arises where the stronger firm opts for content marketing while the weaker firm opts for brand advertising;*
  - *when the two firms' brand strengths are relatively far ($\Delta\mu \geq \widehat{\Delta\mu}$), for $T_w \leq T < T_s$, $\{B, C\}$-equilibrium arises where the stronger firm opts for brand advertising while the weaker firm opts for content marketing;*
- *when $T \geq \max\{T_s, T_w\}$ and for any $\Delta\mu$, neither firm chooses content marketing as a dominant strategy,*

where $\mathcal{I}_i^C \geq 0$ when $T \leq T_i(\Delta\mu)$ and $T_s = T_w$ at $\Delta\mu = 0$ and $\Delta\mu = \widehat{\Delta\mu} = \frac{9(1-\mu_s)\mu_s}{25-9\mu_s}$.

Let us turn to the firm side. As $\Delta\mu$ increases, the stronger firm's expected payoff as a brand advertiser increases due to increased likelihood of being searched (i.e., a higher value of information) and a higher chance of converting the searching consumer (i.e., a higher matching probability μ_s). And the magnitude of payoff increase as a brand advertiser outweighs that as a content marketer for a sufficiently large relative strength.

For the weaker firm, as $\Delta\mu$ increases, it has a relative less incentive to become a brand advertiser as opposed to a content marketer, i.e., $\mathcal{I}_w^C$ increases in $\Delta\mu$. In contrast with the stronger firm, due to lower value of information it provides and a low chance of converting the consumer, the weaker

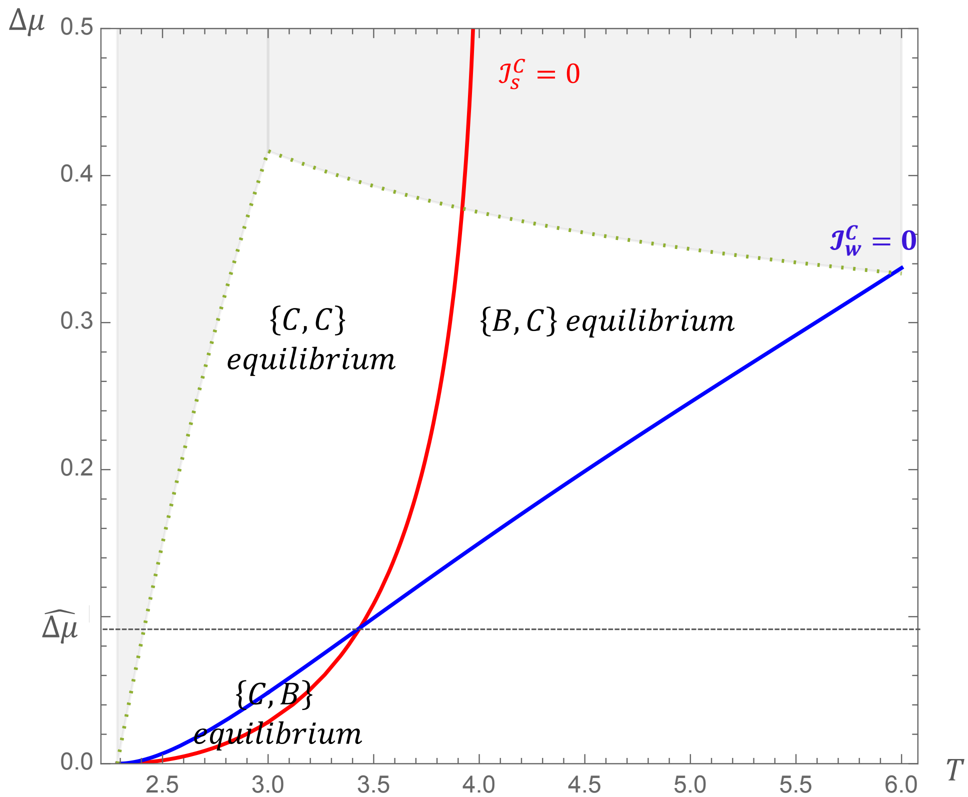

Figure 2: Equilibrium Outcomes When Firms Are Asymmetric

firm has to even lower the price to attract and keep a second-time searcher. The weaker firm would rather become the content marketer as an increase in $\Delta\mu$ softens the competition under symmetric content provision.

Next, we discuss when the parameter space described in Proposition 5 can arise.

Corollary 2. *When consumer heterogeneity T increases, $\frac{\partial \mathcal{I}_s^C}{\partial T} < \frac{\partial \mathcal{I}_w^C}{\partial T}$, a weaker firm has a stronger incentive to do content marketing.*

Consider the situation where the stronger firm is a brand advertiser. It should have a better chance to attract and exploit a searching and matched consumer by charging a high price. Nonetheless, the weaker firm that is in the prominent position would charge a low competing price to retain the consumer because it knows the chance of having a returning consumer is low. The competitive force hinders the stronger firm to fully exploit the consumer, even resulting in a lower price than the weaker firm. This price competition effect is more severe as T, μ_s and $\Delta\mu$ become lower. The competition effect is softened otherwise.

5.1.2 Mass-appealing vs. Niche-appealing firms

Set $v_1 = v_2 = v_0$ and suppose $\mu_1 = \mu_2 = \mu$. Denote the term spread as $S = \overline{v} + \underline{v}$. The brand advertiser's payoff is always increasing in spread. Therefore, a firm with a larger spread is more likely to become a brand advertiser.

Proposition 6. *Compared with a niche-appealing firm, a mass-appealing firm has a stronger incentive to become a content marketer.*

5.2 Symmetric Adoption of Brand Advertising

Following Armstrong et al. (2009), the main model considers prominence as the ability to attract all consumers to search the prominent firm first. In our context, the source of prominence comes from firms' asymmetric information provision and consumers information funnel, which is often prevalent in markets where ensuing consumer demand requires large-scale consumer education. The main model highlights the strategic benefit of prominence. Nonetheless, one can argue that such benefit always dominates due to no induced demand when both firms choose brand advertising. In this extension, we consider one practice where firms often sponsor third-party information sources to induce consumer acceptance, e.g., PSA, NGOs, KOL(key opinion leaders), etc. The extension addresses this possibility and investigates questions that may interest many practitioners as to whether firms should opt for content marketing themselves or sponsor third-party sources and focus on advertising brand.

Symmetric content provision leads to no prominence in the market. We follow the literature (e.g., Wolinsky 1986, Anderson and Renault 1999) and assume passive belief updating in that if a consumer observes a seller's deviation from the equilibrium price, she does not update her belief about the prices of other sellers that she has not visited yet.

Under symmetric brand advertising where there exists a third-party information source that provides free information on v, following classic literature a la Wolinsky (1986), we consider symmetric price equilibrium in which both firms charge p^*. In a symmetric equilibrium, a consumer randomly picks one firm to start searching. To solve the game, we consider that one firm deviates locally from p^* to $\widehat{p}$ while the other one sticks to the equilibrium price. One firm's demand possibly comes from a consumer who searches it in the first place or in the second place. As shown in the proof of Lemma 2, a local upward price deviation, $\widehat{p} - p^* \leq \frac{c}{\mu}$, only affects the demand from the consumer who visits the deviating firm in the first place and considers to search for another firm while has no influence on the demand from the consumer who has sunk the search cost. Thus, a brand advertiser's incentive to hold up a high-type consumer who visits it in the second place is counterbalanced by the cost of losing a high-type consumer who visits in the first-place.

Lemma 2. *Under symmetric adoption of brand advertising,*

- *both firms charge* $p^* = \frac{(2-\mu)\mu\left(T\overline{v}+u^0\right)-c(1-\mu)}{(3-\mu)\mu}$ *and expect a profit of* $\Pi^* = \frac{\left((2-\mu)\mu\left(T\overline{v}+u^0\right)-c(1-\mu)\right)^2}{2(3-\mu)^2\mu T\overline{v}}$*;*
- *a consumer with* $t > \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$ *randomly chooses one firm to start searching and purchases the first matched product; while a consumer with* $t \leq \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$ *does not search nor purchase.*

Under asymmetric content provision with the presence of third-party information source, due to verifiability of the information, information provided by the content marketer and the sponsored third-party information source services the same purpose. As a consumer needs to incur search cost for each place she visits, to resolve primary uncertainty, she still starts from the content marketer.

Proposition 7.

- ***No Price commitment.*** *If u^0 is sufficiently large so that the market is fully covered under asymmetric content provision, content marketing is still a dominant strategy. Otherwise, if u^0 is not large enough so that the market is partially covered, symmetric adoption of brand advertising can emerge in equilibrium for sufficiently large μ and T.*

- ***Price Commitment.*** *For sufficiently large μ and T, brand advertising is a dominant strategy; otherwise, the equilibrium content strategy is given in Proposition 2.*

Under symmetric adoption of BA, the results are the same with and without commitment assumption. What is new in this extension is firms' incentives to conduct brand advertising given the competitor is a brand advertiser. The firm trades off between the benefit from prominence and loss of the opportunity to promote its brand value. It is straightforward that large T and μ increases the benefit of becoming a brand advertiser, and brand advertising arises as a dominant strategy for both firms. These results also provide managerial insights when to resort to third-party information source for content marketing.

5.3 General Distributions

In this section, we conduct robustness checks by extending the discrete distribution of v_j to more general continuous distributions. We start with the case of observable prices and identify general conditions under which the content marketer obtains price and profit premium so that a firm should prioritize content marketing over brand advertising. Then we examine the boundary condition under which firm 2 has an incentive to hold up the consumer. One may wonder whether the reason for the hold-up effect is is due to the specification of discrete distribution of v_j such that upward price deviations do not generate demand response. Nonetheless, a firm's hold-up incentive does not rely on unchanged demand; rather a general result attributed to the fact that the demand from the consumer who values brand-specific information is less responsive to the price increase. No demand loss can be viewed as the extreme situation due to the specification of discrete distribution of brand-specific shocks.

5.3.1 Price Commitment

Again, we start from analyzing a consumer's optimal second-round search behavior with observable prices. In more detail, if $\Delta p \leq 0 \Leftrightarrow p_2 \geq p_1$, a consumer with $t < \frac{-\Delta p}{\overline{v}-\mu}$ would always purchase product 1 regardless search outcomes because $u^0 + t\mu - p_1 \geq u^0 + t\overline{v} - p_2$ always holds. So the consumer will buy firm 1 immediately. A consumer with $t \geq \frac{-\Delta p}{\overline{v}-\mu}$ continues to search if the expected net utility exceeds the current option: $U(t) \geq u^0 + t\mu - p_1$, where $U(t)$ is specified as follows,

$$\begin{aligned} U(t) &= \int_{\underline{v}}^{\frac{-\Delta p}{t}+\mu} (u^0 + t_i\mu - p_1) dF(v_{i2}^b) + \int_{\frac{-\Delta p}{t}+\mu}^{\overline{v}} (u^0 + tv_{i2}^b - p_2) dF(v_{i2}^b) - c \\ &= u^0 + t\mu - p_1 + t\int_{\frac{-\Delta p}{t}+\mu}^{\overline{v}} (1 - F(x))\, dx - c, \end{aligned} \tag{5}$$

The integral part of the above equation captures the value of information by continuing to search firm 2. The marginal search type as the solution $\hat{t}(\Delta p)$ to

$$V(t) \equiv t\int_{\frac{-\Delta p}{t}+\mu}^{\overline{v}} (1 - F(x))\, dx = c, \tag{6}$$

As one can easily see that $V(t)$ increases in t under $\Delta p \leq 0$ and $\frac{-\Delta p}{\overline{v}-\mu} < \hat{t}(\Delta p)$, a consumer with $t \leq \hat{t}(\Delta p)$ purchases product 1 immediately without further search; whereas a consumer with $t > \hat{t}(\Delta p)$ continues to search firm 2. Note that Δp should not be too large such that no consumer searches.

If $\Delta p > 0$, a consumer with $t < \frac{\Delta p}{\mu-\underline{v}}$ would always prefer firm 2, and she searches if firm 1's price is overly priced such that $\Delta p > c$. For a consumer with $t \geq \frac{\Delta p}{\mu-\underline{v}}$, she searches if and only if $V(t) \geq c$. We summarize a consumer's optimal behavior for information search in the next lemma.

Lemma 3. *(Optimal Second Round Search with Price Commitment)*

Let $\Delta p \equiv p_1 - p_2$,

- *If* $\Delta p \leq \min\left\{c, T\left(\mu - M^{-1}\left(\frac{c}{T}\right)\right)\right\}$, *no consumer goes second-round search.*
- *If* $T\left(\mu - M^{-1}\left(\frac{c}{T}\right)\right) < \Delta p \leq c$, *a consumer with* $t \leq \hat{t}(\Delta p)$ *does not search; whereas a consumer with* $t > \hat{t}(\Delta p)$ *searches.*
- *If* $\Delta p > c$, *all consumer types continue with second-round search,*

where M is a decreasing function defined as $M(y) = \int_y^{\overline{v}} (1 - F(x))\, dx$ for $y \in [\underline{v}, \overline{v}]$.

We relegate the proof in the online appendix. Again, a consumer's search behavior indicates that she has strong $t > \hat{t}\,(\Delta p)$, which then can be exploited by firm 2.

Noted in Choi et al. (2018) and Ke and Lin (2018) that it is generally difficult to obtain closed-form solutions due to the existence of the returning demand, we generalize Proposition 2 by focusing on the general condition under which the content marketer obtains or loses brand premium. We show derivations of the equilibrium pricing strategies in the appendix.

Proposition 8. *When $q^* \le \frac{c}{T}$, then $p_1^* \ge p_2^*$ and $Q_1^* \ge Q_2^*$ so that $\sigma^* = \frac{\Pi_1^*}{\Pi_1^* + \Pi_2^*} \ge \frac{1}{2}$; otherwise, $p_1^* < p_2^*$ and $Q_1^* < Q_2^*$ so that $\sigma^* < \frac{1}{2}$, where $q^* = \left(1 - \frac{1}{2}\frac{1}{1-F(\mu)}\right) \int_{\mu}^{\overline{v}} [1 - F(x)]\, dx$.*

First, q^* represents the net value of information accounted for the preference exploitation, which can be interpreted from the notion of mean-preserving spread. Fixing the mean μ, suppose the distribution F rotates counterclockwise such that the new distribution is a mean-preserving spread of the old distribution, suggesting a higher variance and thereby a larger value of information from search. According to the formula of q^*, mean-preserving spread results in a higher q^*, suggesting that all else equal, firm 1 has to reduce price to compensate for a consumer's value of information.

Next, $\frac{c}{T}$ measures the market average search cost. Large T indicates that there is a broader range of consumer types in the market who have high information preferences for brand-level content. The higher t, the lower the average search cost becomes. Hence, all else equal, a consumer is more likely to search, restricting firm 1 from charging a high price.

Whether firm 1 or firm 2 charges a premium price depends on the size of net value of information and the average search cost. The first part of the above proposition examines the scenario when the value of information net of post-search preference exploitation does not outweigh the search cost, firm 1 can benefit from being the first firm to be visited. It not only charges a premium price but also captures a higher proportion of the demand than the competitor. In opposite, so long as the benefit from value of information outweighs the average search cost, firm 2 can charge premium price and still attract a wider range of consumer types to search for the brand-level information even a consumer of a higher type knows she will be exploited after search.

5.3.2 Unobservable Prices

Next, we investigate the case where prices are not observable to a consumer before she searches and firms cannot commit on their prices. A consumer decides her search decision solely based on her beliefs on firms' pricing strategy. Even if firm 2 charges a price other than consumers' conjecture, such price can only be observed by the consumer who continues her search but not known by who forgoes search. Suppose a consumer anticipates that firm 2 would charge p_2 such that $\Delta p = p_1 - p_2$ satisfies $T\left(\mu - M^{-1}\left(\frac{c}{T}\right)\right) < \Delta p \leq c$. According to Lemma 3, those consumers with $t \leq \hat{t}\left(\Delta_p\right)$ forgo search, where $\hat{t}(\Delta p)$ is given in equation (6). They do not observe the actual price $\widehat{p}_2$ charged by firm 2. Those consumers with $t > \hat{t}\left(\Delta_p\right)$ go further search and observe $\widehat{p}_2$. They choose to buy brand 2 if and only if $u^0 + t\mu - p_1 < u^0 + tv_2 - \widehat{p}_2 \Leftrightarrow v_2 > \mu - \frac{1}{t}\left(p_1 - \widehat{p}_2\right) = \mu - \frac{1}{t}\left(\Delta p - \left(\widehat{p}_2 - p_2\right)\right)$. The resulting demand for firm 2 is given by,

$$Q_2\left(\widehat{p}_2, \hat{t}\left(\Delta_p\right)\right) = \frac{1}{T}\int_{\hat{t}(\Delta_p)}^{T}\left[1 - \Pr\left(v_2 \geq \mu - \frac{\Delta p - \left(\widehat{p}_2 - p_2\right)}{t}\right)\right] dt. \tag{7}$$

Notably, the conjectured price p_2 determines a consumer's search decision (i.e., through $\hat{t}\left(\Delta_p\right)$, whereas the actual price $\widehat{p}_2$ determines the purchase decision. To exemplify firm 2's ex-post incentive for holding up the searching consumer, we derive its optimal ex-post price that maximizes its profit $\widehat{p}_2 Q_2\left(\widehat{p}_2\right)$ given p_1 and $\hat{t}\left(\Delta p\right)$ from the following first-order condition for $\widehat{p}_2$,

$$Q_2\left(\widehat{p}_2, \hat{t}\left(\Delta_p\right)\right) + \widehat{p}_2 \frac{\partial Q_2\left(\widehat{p}_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \widehat{p}_2} = 0. \tag{8}$$

To illustrate the main insights, consider the case where v_j is uniformly distributed on the support $[\underline{v}, \overline{v}] = [0, V]$. Then equation (8) leads to

$$\widehat{p}_2 = \frac{V}{4}\frac{1 - \frac{\hat{t}(\Delta_p)}{T}}{\ln \frac{T}{\hat{t}(\Delta_p)}} + \frac{p_1}{2}. \tag{9}$$

It is straightforward to show that $\widehat{p}_2$ increases in $\hat{t}\left(\Delta_p\right)$ and thus increases in p_2, which implies that firm 2 finds more profitable to raise price from any price level that a consumer conjectured before search. Such ex-post hold up incentive arises due to two facts. First, firm 2 can exploit the consumer who reveals high preferences toward brand-specific values ($t > \hat{t}(\Delta p)$) through continued search. Second, from ex-post perspective, the size of these consumer types $(1 - \hat{t}(\Delta p)/T)$ is taken as given because firm 2 does not need to internalize the effect of an increased price on search.

However, in equilibrium, a consumer should have a correct belief and factors in firm 2's ex-post hold-up incentive. Expecting $\widehat{p}_2$ and thereby a smaller price difference $\widehat{\Delta}_p \equiv p_1 - \widehat{p}_2 < \Delta p$, a consumer with an even higher private types $t > \hat{t}\left(\widehat{\Delta}_p\right)$ goes further search, which triggers another round of ex-post price increase for firm 2. Consider v_j follows the discrete distribution, this chain of logics lead to the outcome stated in Section §4.1 that no p_2 can be rationally anticipated with the absence of price commitment and thereby all consumer types would forgo searching firm 2. Considering continuous distributions of v_j, firm 2's price that a consumer can rationally anticipate may exist in an interior solution. To obtain the interior solution, we follow the standard literature (e.g., Armstrong et al. (2009), Zhou (2011)) and set $\widehat{p}_2 = p_2$ in equation (8), which leads to

$$Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right) + p_2 \cdot \left.\frac{\partial Q_2\left(\widehat{p}_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \widehat{p}_2}\right|_{\widehat{p}_2 = p_2} = 0. \tag{10}$$

Next, we turn to firm 1's problem. Given a consumer has visited it, firm 1's demand can be specified as,

$$Q_1\left(p_1, \hat{t}\left(\Delta_p\right)\right) = \frac{1}{T}\left(\hat{t}(\Delta p) + \int_{\hat{t}(\Delta_p)}^{T}\left[1 - \Pr\left(v_2 < \mu - \frac{\Delta p}{t}\right)\right] dt\right). \tag{11}$$

Then firm 1's profit maximization problem is as follows,

$$Q_1\left(p_1, \hat{t}\left(\Delta_p\right)\right) + p_1 \cdot \frac{dQ_1\left(p_1, \hat{t}\left(\Delta_p\right)\right)}{dp_1} = 0. \tag{12}$$

The interior solutions $\{\tilde{p}_1, \tilde{p}_2\}$ to equation (10) and (12) can be an equilibrium when $\tilde{\Delta} p = \tilde{p}_1 - \tilde{p}_2$ should at least induce some consumer types to go further search. Otherwise, such hold-up equilibrium may not exist. Nonetheless, without the ability to commit, firm 2's post-search hold-up incentive may discourage some consumer types to search it in the first place. To see this, we take a total derivative of $Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)$ with respect to p_2 and obtain:

$$\begin{aligned}
\frac{dQ_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)}{dp_2} &= \frac{\partial Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)}{\partial p_2} + \frac{\partial Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \hat{t}\left(\Delta_p\right)} \frac{d\hat{t}\left(\Delta_p\right)}{dp_2} \\
&= -\frac{1}{VT} \ln \frac{T}{\hat{t}\left(\Delta_p\right)} + \left(\frac{1}{2T} + \frac{\Delta p}{VT} \frac{1}{\hat{t}\left(\Delta_p\right)}\right) \times \underbrace{\frac{d\hat{t}\left(\Delta_p\right)}{d\Delta_p}}_{(-)}.
\end{aligned}$$

The first component captures the ex-post price effect on purchases while the second part reflects the ex-ante price effect on search (through $\hat{t}\left(\Delta_p\right)$). Moreover, compared with the commitment case in Section §5.3.1, firm 2's price determined in equation (10) is higher. This can be shown by evaluating the following marginal profit at $\tilde{p}_2$:

$$\begin{aligned}
\left.\frac{d\left[p_2 Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)\right]}{dp_2}\right|_{p_2=\tilde{p}_2} &= Q_2\left(p_2, \hat{t}\left(\Delta p\right)\right) + p_2 \cdot \left.\frac{\partial Q_2\left(\widehat{p}_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \widehat{p}_2}\right|_{\widehat{p}_2=\tilde{p}_2} \\
&\quad + p_2 \times \frac{\partial Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \hat{t}\left(\Delta_p\right)} \frac{d\hat{t}\left(\Delta_p\right)}{dp_2} \\
&= p_2 \times \underbrace{\frac{\partial Q_2\left(p_2, \hat{t}\left(\Delta_p\right)\right)}{\partial \hat{t}\left(\Delta_p\right)} \frac{d\hat{t}\left(\Delta_p\right)}{dp_2}}_{(-)} < 0.
\end{aligned}$$

In the commitment case, the above marginal profit is zero when evaluated at the equilibrium prices. The negativity of the marginal profit evaluated at $\tilde{p}_2$ suggests that the firm 2's price a consumer can rationally anticipate in the no-commitment case is higher than the equilibrium price when firm 2 has the commitment power. Whether such a high price $\tilde{p}_2$ can be sustained as an equilibrium in the no-commitment case crucially depends on the right tail of the distribution that represents consumer preferences toward brand-specific values. For small T, no consumer would sufficiently value brand-level information; rather the concern for being held up by firm 2 may lead to an equilibrium in which no one further searches. Then firm 1 becomes a *de facto* monopolist.

Proposition 9. *The boundary condition under which the hold-up effect arises is $VT < 8c$. Otherwise, the interior solutions $\{\tilde{p}_1, \tilde{p}_2\}$ to equation (10) and (12) constitute an equilibrium.*

Proposition 9 shows the prevalence and boundary condition of the hold-up problem. In the online appendix, we provide a complete proof. In the case when v^b is continuously distributed, any arbitrarily tiny change in firm 2's posted price moves the demand with smaller magnitude, and firm 2's can improve profitability by adjusting price upwardly if there is no price commitment. Anticipating the hold-up effect, a consumer is unwilling to continue further search on brand-level, which in turn enhance the market power of the focal brand.

6 Conclusion

A content marketing strategy that focuses on providing brand-neutral and promotion-free content becomes a popular trend among brands to attract consumers. Nonetheless, neither practitioners nor academics have a full understanding of why and to what extent brand-free content marketing can build brand preference and ultimately benefit the brand. This research makes an endeavor to fill in this gap.

Our research has a few limitations that we invite future research. We do not consider a content marketer's optimal information design problem. Future research can explore this domain by considering flexible formulations of information disclosure on both primary-level and brand-level information. Moreover, our work assumes the content marketer always provides unbiased information. Nonetheless, its incentive can be further investigated.

Appendix

Proof of Lemma 1 and Corollary 1.

Proof. We break down the proof into three parts. In Part I, we show that no prices that induce finer partition of consumer types in the second-round search can be sequential rational. In Part II, we prove the equilibrium characterized in Lemma 1 survives D1 refinement. Lastly, we prove uniqueness by showing Lemma A1.

Part I. Let us first show the monotonicity of the search rule in the first-round. Consumer t's expected utility in the first round is $EU_1(t) = \lambda\left(V(t) - c + u^0 + tv^0 - p_1\right) - c$, where $V(t)$ is defined in Equation (2). Since $V(t)$ increases in t and $v^0 \geq 0$, $EU_1(t)$ increases in t as well. Thus, we can find a marginal type $\tilde{t}$ such that $EU_1(\tilde{t}) = c$, a consumer with $t \geq \tilde{t}$ searches in the first round. As λ only plays a role in the first-round search, we safely drop λ in the analysis of the second-round search.

In the last stage, upon observing p_1 and anticipating some price p_2 such that a consumer $t \in \mathbb{T} \equiv \left[\max\{\tilde{t}, \hat{t}\}, T\right]$ chooses to visit firm 2, where $\hat{t} = \frac{1}{(1-\mu)(\overline{v}+\underline{v})}\left(\frac{c}{\mu} + \Delta p\right)$. We show the tuple $\{p_1, p_2, \mathbb{T}\}$ never emerges in equilibrium. Let us first define the equilibrium: $\{p_1, p_2, \mathbb{T}\}$ can constitute a Perfect Bayesian Equilibrium iff (1) upon visitation of firm 2, $\{p_1, p_2\}$ are sequential rational given the belief that a consumer $t \in \mathbb{T}$; (2) the belief should be consistent such that a consumer with $t \in \mathbb{T}$ searches firm 2 given prices p_1 and p_2.

In the first step, we check sequential rationality. After the consumer incurred the sunk cost c to visit firm 2, firm 2 does not need to take into account c in its pricing decision. It can increase price to exploit a searching consumer with $\overline{v}$ or decrease price to accommodate a searching consumer with $-\underline{v}$. Denote this ex-post optimal price as p_2'. Sequential rationality requires $p_2 = p_2'$ in equilibrium. We analyze these two strategies separately.

Strategy 1 – Exploitation. In the exploitation case, $p_2' > p_1$ such that only the consumer who receives $\overline{v}$ considers purchasing from firm 2. More specifically, the consumer purchases firm 2 iff $u^0 + t\overline{v} - p_2' \geq u^0 + tv^0 - p_1 \Leftrightarrow t > \frac{p_2' - p_1}{\overline{v} - v^0}$. Firm 2 chooses p_2' by maximizing the following expected profit:

$$p_2' \in \arg\max_{p_2' > p_1} E\left[\Pi_2 \mid t \in \mathbb{T}\right] = p_2' \cdot \mu \cdot \min\left\{\max\left\{0, 1 - \frac{p_2' - p_1}{T(\overline{v} - v^0)}\right\}, 1 - \frac{\max\{\tilde{t}, \hat{t}\}}{T}\right\},$$

where $\hat{t} = \frac{1}{\overline{v} - v^0}\left(\frac{c}{\mu} - p_1 + p_2\right)$ and $\tilde{t}$ is a function of p_1 and p_2.

From the above equation, we notice that p_2 determines the consumer types who arrive at the firm 2's place while p_2' determines the consumer types who decide to stay with firm 2. As a result, $p_2' = \max\left\{\frac{1}{2}\left(p_1 + (\overline{v} - v^0)T\right), p_1 + (\overline{v} - v^0)\max\{\tilde{t}, \hat{t}\}\right\}$. Given a positive draw, the cutoff consumer type who is indifferent between firm 1 and 2 is $\hat{t}' \equiv \frac{p_2' - p_1}{\overline{v} - v^0} = \max\left\{\frac{1}{2}\left(T - \frac{p_1}{\overline{v} - v^0}\right), \max\{\tilde{t}, \hat{t}\}\right\}$. And the searching consumer with $t \geq \hat{t}'$ stays with firm 2 while the searching consumer with $t < \hat{t}'$ returns to firm 1. Given a negative draw, the searching consumer returns to firm 1.

Strategy 2 – Accommodation. In this case, $p_2' \leq p_1$ such that the consumer who receives $-\underline{v}$ considers purchasing from firm 2. More specifically, the consumer purchases firm 2 iff $u^0 - t\underline{v} - p_2' > u^0 + tv^0 - p_1 \Leftrightarrow t < \frac{p_1 - p_2'}{\underline{v} + v^0}$. Firm 2 chooses p_2' by maximizing the following expected profit:

$$\begin{aligned} p_2' &\in \arg\max_{0 < p_2' \leq p_1} E\left[\Pi_2 \mid t \in \mathbb{T}\right] \\ &= p_2' \cdot \left[\mu\left(1 - \frac{\max\{\tilde{t}, \hat{t}\}}{T}\right) + (1 - \mu)\min\left\{\max\left\{0, \frac{p_1 - p_2'}{T(\underline{v} + v^0)} - \frac{\max\{\tilde{t}, \hat{t}\}}{T}\right\}, 1 - \frac{\max\{\tilde{t}, \hat{t}\}}{T}\right\}\right]. \end{aligned}$$

If firm 2 is better off with a price that only keeps the consumer with $\overline{v}$, the accommodation strategy is strictly dominated by the exploitation strategy. To keep the consumer with $-\underline{v}$, $p_2' = \max\left\{\frac{1}{2}\left(p_1 + \frac{(\mu T - \max\{\tilde{t}, \hat{t}\})(\underline{v} + v^0)}{1 - \mu}\right), p_1 - T\left(\underline{v} + v^0\right)\right\}$. Given a negative draw, the marginal consumer type who is indifferent between firm 1 and firm 2 is $\hat{t}' \equiv \frac{p_1 - p_2'}{\underline{v} + v^0} = \min\left\{\frac{1}{2}\left(\frac{p_1}{\underline{v} + v^0} - \frac{\mu T - \max\{\tilde{t}, \hat{t}\}}{1 - \mu}\right), T\right\}$. And the consumer with $t > \hat{t}'$ returns to firm 1 while the consumer with $t \leq \hat{t}'$ stays with firm 2. Given a positive draw, the searching consumer stays with firm 2. Moreover, $\hat{t}' > \max\{\tilde{t}, \hat{t}\}$ should hold; otherwise, no consumer with $t > \hat{t}$ searches firm 2 if she anticipates that she will return to firm 1 even firm 2 accommodates a mismatched outcome.

In the second step, we show that there is no consistent belief $t \in \mathbb{T} = \left[\max\{\tilde{t}, \hat{t}\}, T\right]$ given p_1 and $p_2'(t \in \mathbb{T})$. That is, no sequential rational price $p_2'(t \in \mathbb{T})$ can support that $U(t, p_2'(t \in \mathbb{T})) > u^0 + tv^0 - p_1$ holds for $t \in \mathbb{T}$ but does not hold for $t \notin \mathbb{T}$. We show this result case by case.

Suppose the exploitation pricing strategy is sequential rational. Anticipating $p_2'(t \in \mathbb{T})$, the consumer t continues searching iff

$$V(t) = \mu\left(u^0 + t\overline{v} - p_2'\right) + (1-\mu)\left(u^0 + tv^0 - p_1\right) - \left(u^0 + tv^0 - p_1\right) > c$$
$$\Leftrightarrow t > \frac{c}{(\overline{v}-v^0)\mu} + \max\left\{\frac{1}{2}\left(T - \frac{p_1}{(\overline{v}-v^0)}\right), \max\{\tilde{t}, \hat{t}\}\right\}.$$

The above condition suggests that anticipating an exploitation price $p_2'\,(t \in \mathbb{T})$, the consumer with $t \in \left(\max\{\tilde{t}, \hat{t}\}, \frac{c}{(\overline{v}-v^0)\mu} + \max\left\{\frac{1}{2}\left(T - \frac{p_1}{(\overline{v}-v^0)}\right), \max\{\tilde{t}, \hat{t}\}\right\}\right]$ would never search firm 2 . Hence, firm 2's belief is inconsistent.

Suppose the accommodation pricing strategy is sequential rational. First, anticipating $p_2'\,(t \in \mathbb{T})$, a consumer with $t \in \left(\hat{t}', T\right]$ continues searching iff:

$$V(t) = \mu\left(u^0 + t\overline{v} - p_2'\right) + (1-\mu)\left(u^0 + tv^0 - p_1\right) - \left(u^0 + tv^0 - p_1\right) > c$$
$$\Rightarrow t > \frac{1}{\overline{v}-v^0}\left(\frac{c}{\mu} - p_1 + p_2\right) + \frac{1}{\overline{v}-v^0}\left(p_2' - p_2\right) = \hat{t} + \underbrace{\frac{1}{\overline{v}-v^0}\left(p_2' - p_2\right)}_{<0}.$$

Hence, the above condition always holds for $t \in \left[\hat{t}', T\right]$.

Second, a consumer with $t \in \left(\hat{t}, \hat{t}'\right)$ continues to search iff

$$U(t) = u^0 + tv^0 - p_2' - c > u^0 + tv^0 - p_1.$$

The above condition is independent of t: if the condition holds for $t \in \left(\hat{t}, \hat{t}'\right]$, it should also hold for $t \le \hat{t}$. If $\tilde{t} < \hat{t}$, anticipating an accommodating price $p_2'\,(t \in \mathbb{T})$, a consumer with $t \in [\tilde{t}, \hat{t}]$ would have searched firm 2. Conversely, if the condition does not hold, $t \in \left(\hat{t}, \hat{t}'\right)$ should not have searched. Hence, firm 2's belief is inconsistent. If $\tilde{t} \ge \hat{t}$, anticipating the accommodation pricing strategy, the consumer who has searched for the first round should continue searching for the second round. Firm 2's belief is consistent in this subgame.

In conclusion, for any consumer with $t > \tilde{t}$ who has searched in the first round, only when $\tilde{t} \ge \hat{t}$ there can arise an equilibrium where firm 2 is better off accommodating the searching consumer and a consumer with $t > \tilde{t}$ searches for firm 2. In other words, no prices that induce finer partition of consumer types in the second-round search can be sequential rational.

Part II. In an equilibrium where a consumer with $t > \tilde{t}$ forgoes searching firm 2, a consumer obtains an equilibrium payoff, $u^*(t) = u^0 + tv^0 - p_1^*$, given $\chi = 1$. The search market exists if $\lambda u^*(T) > c \Leftrightarrow p_1^* < u^0 + Tv^0 - \frac{c}{\lambda}$. Since, visiting firm 2 is off-the-equilibrium path, we apply the D1 criterion to eliminate unreasonable beliefs for firm 2 when the consumer deviates to search it. Notice that the elimination process is independent of $\tilde{t}$.

The first step is to determine the consumer type who is most likely to deviate, denoted as $\mathbb{T}^{**}(Search)$. To facilitate the analysis, we define a set of best responses of the firm 2 to the consumer's deviation such that consumer type t strictly prefers further searching to the equilibrium strategy:

$$D(t, Search) = \{p_2 \in BR(f(t), Search) \mid u^* < u(Search, p_2, t)\},$$

where $f(\cdot)$ is a probability density function that represents any belief firm 2 holds about the type who makes the deviation.

We also need to define a set of best responses of the firm 2 such that the consumer is indifferent between the deviation and the equilibrium payoff:

$$D^0(t, Search) = \{p_2 \in BR(f(t), Search) \mid u^* = u(Search, p_2, t)\}.$$

The D1 criterion puts zero probability on t if there exists some other t' such that the following holds:

$$D(t, Search) \cup D^0(t, Search) \subset D(t', Search). \tag{13}$$

By iteratively applying the condition (13), we can reach a subset of types $\mathbb{T}^{**}(Search)$ that cannot be eliminated by the above procedure.

To find $D(t, Search)$ and $D^0(t, Search)$, we need to further determine firm 2's possible best responses $BR(f(t), Search)$. For any t, firm 2 can decide whether to hold up the consumer by charging $p_1^* + t(\overline{v} - v^0)$ or accommodating the consumer by charging $p_1^* - t(\underline{v} + v^0)$. Firm 2 is better off with accommodating the consumer t iff $\max\{0, p_1^* - t(\underline{v} + v^0)\} \geq \mu\left(p_1^* + t(\overline{v} - v^0)\right) \Leftrightarrow p_1^* \geq \frac{2-\mu}{1-\mu} t\mu(\overline{v} + \underline{v})$. If $p_1^* \leq \frac{2-\mu}{1-\mu} T\mu(\overline{v} + \underline{v})$, $BR(f(t), Search) = \left[\max\{0, p_1^* - T(\underline{v} + v^0)\}, p_1^* + T(\overline{v} - v^0)\right]$. If $p_1^* > \frac{2-\mu}{1-\mu} T\mu(\overline{v} + \underline{v})$, for any belief of t, charging a price higher than p_1^* will not be a best response. The set of available responses is $BR(f(t), Search) = \left[p_1^* - T(\underline{v} + v^0), p_1^*\right]$. Notice that

$p_1^* - T(\underline{v} + v^0) > 0$ because $\frac{2-\mu}{1-\mu}T\mu(\overline{v} + \underline{v}) > T(\underline{v} + v^0) \Leftrightarrow \frac{2-\mu}{1-\mu} > 1$ for any μ. Hence, depending on p_1^*, firm 2's off-the-equilibrium belief refinement is different.

Case 1: $p_1^* \leq \frac{2-\mu}{1-\mu}T\mu(\overline{v} + \underline{v})$**.** Consumer t deviates to search firm 2 iff

$$u\left(Search, p_2, t\right) = \mu \max\left\{u^0 + t\overline{v} - p_2, u^*(t)\right\} + (1-\mu)\max\left\{u^0 - t\underline{v} - p_2, u^*(t)\right\} - c > u^*(t)$$

For each t, we look for all p_2 such that the above condition holds. For $t > \frac{c}{\mu(\overline{v}+\underline{v})}$, if $p_2 \geq p_1^* - t(\underline{v} + v^0)$, consumer t deviates to search iff $u\left(Search, p_2, t\right) = u^0 + t\overline{v} - p_2 - \frac{c}{\mu} > u^0 + tv^0 - p_1^* \Leftrightarrow p_2 < p_1^* + t(\overline{v} - v^0) - \frac{c}{\mu}$. If $p_2 < p_1^* - t(\underline{v} + v^0)$, consumer t deviates to search iff $u\left(Search, p_2, t\right) = u^0 + tv^0 - p_2 - c > u^0 + tv^0 - p_1^* \Leftrightarrow p_2 < p_1^* - c$, which always holds under $p_2 < p_1^* - t(\underline{v} + v^0)$ and $t > \frac{c}{\mu(\overline{v}+\underline{v})}$. Hence, for $t > \frac{c}{\mu(\overline{v}+\underline{v})}$, a non-empty set $D\left(t, Search\right) = \left[\max\left\{0, p_1^* - T(\underline{v} + v^0)\right\}, p_1^* + t(\overline{v} - v^0) - \frac{c}{\mu}\right)$, which increases in t.

Next, for $t \leq \frac{c}{\mu(\overline{v}+\underline{v})}$, when $p_2 < p_1^* - c$, consumer t deviates because $p_2 < p_1^* - c$ implies $p_2 < p_1^* - t(\underline{v} + v^0)$ for $t \leq \frac{c}{\mu(\overline{v}+\underline{v})}$. When $p_2 \geq p_1^* - c$, $t \leq \frac{c}{\mu(\overline{v}+\underline{v})}$ implies $p_2 \geq p_1^* + t(\overline{v} - v^0) - \frac{c}{\mu}$, which means the consumer does not deviate. Thus, for $t \leq \frac{c}{\mu(\overline{v}+\underline{v})}$, a non-empty set $D\left(t, Search\right) \cup D^0\left(t, Search\right) = \left[\max\left\{0, p_1^* - T(\underline{v} + v^0)\right\}, p_1^* - c\right]$.

By iteratively applying the D1 criterion (Condition (13)), the only type that cannot be eliminated is $\mathbb{T}^{**}\left(Search\right) = \left\{T\right\}$.

In the second step, the equilibrium survives the D1 criterion iff

$$u^* \geq u\left(Search, p_2^*\left(\mathbb{T}^{**}\left(Search\right)\right), t\right)$$

As shown earlier, for $p_1^* \leq \frac{2-\mu}{1-\mu}T\mu(\overline{v} + \underline{v})$, given the rational belief that the deviation is made by the type T, firm 2's best response is to hold up the consumer by charging $p_2^* = p_1^* + T(\overline{v} - v^0)$. Then $u\left(Search, p_2^*\left(\mathbb{T}^{**}\left(Search\right)\right), t\right) = u^* - c < u^*$.

Case 2: $p_1^* > \frac{2-\mu}{1-\mu}T\mu(\overline{v} + \underline{v})$**.** In this case, $BR\left(f\left(t\right), Search\right) = \left[p_1^* - T(\underline{v} + v^0), p_1^*\right]$. Consumer t deviates iff

$$u\left(Search, p_2, t\right) = \mu\left(u^0 + t\overline{v} - p_2\right) + (1-\mu)\max\left\{u^0 - t\underline{v} - p_2, u^*(t)\right\} - c > u^*(t)$$

Similar to the first case, for $t \leq \frac{c}{\mu(\overline{v}+\underline{v})}$, consumer t deviates only when $p_2 < p_1^* - c$. Thus, $D(t, Search) \cup D^0(t, Search) = \left[p_1^* - T(\underline{v}+v^0), p_1^* - c\right]$. Next, for $\frac{c}{\mu(\overline{v}+\underline{v})} < t \leq \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$, the upper bound on p_2 that can make consumer t deviate $p_1^* + t(\overline{v} - v^0) - \frac{c}{\mu}$ is always smaller than p_1^*. Thus, $D(t, Search) \cup D^0(t, Search) = \left[p_1^* - T(\underline{v}+v^0), p_1^* + t(\overline{v} - v^0) - \frac{c}{\mu}\right]$, which increases in t. Lastly, for $t > \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$, $D(t, Search) = \left[p_1^* - T(\underline{v}+v^0), p_1^*\right]$.

By iteratively applying the D1 criterion, the only type that cannot be eliminated is $\mathbb{T}^{**}(Search) = \{T\}$ when $T \leq \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$ and $\mathbb{T}^{**}(Search) = \left[\frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}, T\right]$ when $T > \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$.

We proceed to the second step. When $T \leq \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$, given the rational belief that the deviation is made by the type T, firm 2 is better off with accommodating the consumer by charging $p_2^*(\mathbb{T}^{**}(Search)) = p_1^* - T(\underline{v}+v^0)$. The deviation payoff is larger than the equilibrium payoff iff $u(Search, p_2^*(\mathbb{T}^{**}(Search)), t) = u^0 + tv^0 - p_1^* + T(\underline{v}+v^0) - c > u^0 + tv^0 - p_1^* \Leftrightarrow T > \frac{c}{\mu(\overline{v}+\underline{v})}$. Conversely, for $T \leq \frac{c}{\mu(\overline{v}+\underline{v})}$, firm 1 is unable to commit as for any price firm 1 charges, the consumer will not deviate to visit firm 2.

When $T > \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$, the set of deviation types, $\mathbb{T}^{**}(Search) = \left[\frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}, T\right]$, is not a singleton. The consumer deviates if the worst deviation payoff is higher than the equilibrium payoff:

$$\min_{p_2 \in p_2^*(\mathbb{T}^{**}(Search))} u(Search, p_2, t) \geq u^* \Leftrightarrow$$

$$\mu\left(u^0 + t\overline{v} - p_1^* + \frac{c}{1-\mu}\right) + (1-\mu)\left(u^0 + tv^0 - p_1^*\right) - c \geq u^0 + tv^0 - p_1^* \Leftrightarrow$$

$$t \geq \frac{c}{(1-\mu)\,\mu(\overline{v}+\underline{v})} \cdot \frac{1-2\mu}{1-\mu}$$

In the above condition, the consumer obtains the worst payoff when $p_2^*(\mathbb{T}^{**}(Search)) = p_1^* - \frac{c}{1-\mu}$ and she considers purchasing firm 2 conditional on a positive draw. Since $\frac{1-2\mu}{1-\mu} < 1$ and $T > \frac{c}{(1-\mu)\mu(\overline{v}+\underline{v})}$, there exists a subset of consumer types who deviate.

To conclude, when $T > \frac{c}{\underline{v}+v^0}$, firm 1 is able to commit on the price $p_1^* = \frac{2-\mu}{1-\mu}T\mu(\overline{v}+\underline{v}) = \frac{2-\mu}{1-\mu}T(v^0+\underline{v})$. The search market exists (i.e., at least some consumer types will engage in searching) when $u^0 > \overline{u}^0 \equiv \frac{c}{\lambda} + p_1^* - Tv^0 = \frac{c}{\lambda} + \left(\frac{1}{1-\mu}v^0 + \frac{2-\mu}{1-\mu}\underline{v}\right)T$. The consumer searches once and purchases from firm 1. The equilibrium survives the D1 criterion.

Lastly, we show the uniqueness of the equilibrium by proving Lemma 1A. $\square$

Lemma A1. *For any consumer with $t > \tilde{t}$ who has searched in the first round, there does not exist an equilibrium in which a first-round searcher engages in the second-round search.*

Proof of Lemma A1.

Proof. $\{p_1, p_2, \mathbb{T} = [\tilde{t}, T]\}$ constitutes an equilibrium iff (1) upon firm 2 being visited, $\{p_1, p_2\}$ are sequential rational given the belief $t \in \mathbb{T} = [\tilde{t}, T]$; (2) the belief should be consistent such that all consumer types who search in the first round continue to search firm 2 given prices p_1 and p_2. We call this equilibrium as "all-searching equilibrium". Once the consumer has incurred the search cost c, firm 2's price p_2' will not take into account c in the pricing decision. Following the proof of Lemma 1, we analyze the following two cases.

Strategy 1 – Exploitation. In this case, $p_2' > p_1$ such that the consumer who receives $\overline{v}$ considers purchasing from firm 2. Firm 2 chooses p_2' by maximizing the following expected profit:

$$p_2' \in \arg\max_{p_2' > p_1} E\left[\Pi_2 \mid t \in \mathbb{T}\right] = p_2' \cdot \mu \cdot \min\left\{\max\left\{0, 1 - \frac{p_2' - p_1}{\left(T - \tilde{t}\right)\left(\overline{v} - v_0\right)}\right\}, 1\right\}$$

Thus, $p_2' = \max\left\{\frac{1}{2}\left(p_1 + (T - \tilde{t})(\overline{v} - v_0)\right), p_1\right\}$. For notational simplicity, from now on, we use $\tilde{T}$ to denote $T - \tilde{t}$. More specifically, if $p_1 \leq \tilde{T}(\overline{v} - v_0) = \tilde{T}(1 - \mu)(\overline{v} + \underline{v})$, $p_2' = \frac{1}{2}\left(p_1 + \tilde{T}(\overline{v} - v_0)\right)$; if $p_1 > \tilde{T}(1 - \mu)(\overline{v} + \underline{v})$, $p_2' = p_1$. The marginal consumer type who is indifferent between firm 1 and 2 is $\hat{t}' \equiv \frac{p_2' - p_1}{\overline{v} - v_0} = \max\left\{\frac{1}{2}\left(\tilde{T} - \frac{p_1}{\overline{v} - v_0}\right), 0\right\}$. Given $\overline{v}$, the searching consumer with type $t > \hat{t}'$ purchases firm 2 and the searching consumer with $t \leq \hat{t}'$ returns to firm 1. Given $-\underline{v}$, the consumer returns to firm 1.

Strategy 2 – Accommodation. In this case, $p_2' \leq p_1$ such that the consumer who receives $-\underline{v}$ considers purchasing from firm 2. Firm 2 chooses p_2' by maximizing the following expected profit:

$$p_2' \in \arg\max_{0 < p_2' \leq p_1} E\left[\Pi_2 \mid t \in \mathbb{T}\right] = p_2' \cdot \left[\mu + (1 - \mu)\min\left\{\frac{p_1 - p_2'}{\tilde{T}(\underline{v} + v_0)}, 1\right\}\right]$$

As a result, if $p_1 > \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v} + \underline{v})$, firm 2 optimally charges $p_2' = p_1 - \tilde{T}(\underline{v} + v_0)$ to captures the consumer with certainty. If firm 1 charges a price such that $\frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v} + \underline{v}) < p_1 \leq \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v} + \underline{v})$, firm 2 optimally charges $p'_2 = \frac{1}{2}\left(p_1 + \frac{\mu\tilde{T}(\underline{v}+v_0)}{1-\mu}\right)$ and captures the consumer who receives $\overline{v}$ or who receives $-\underline{v}$ but has a type of $t < \hat{t}' \equiv \frac{p_1 - p'_2}{\underline{v}+v_0} = \frac{p_1 - \frac{\mu\tilde{T}}{1-\mu}(\underline{v}+v_0)}{2(\underline{v}+v_0)}$. If $p_1 \leq \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$, firm 2 optimally charges $p'_2 = p_1$ and captures the consumer only when she receives a positive shock.

Hence, depending on p_1, firm 2's best response strategy differs. Notice that $\frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v}) > \tilde{T}(1-\mu)(\overline{v}+\underline{v})$ for all $0 < \mu < 1$ while $\frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v}) < \tilde{T}(1-\mu)(\overline{v}+\underline{v}) \Leftrightarrow \mu < \frac{1}{2}$.

We first discuss the case when $p_1 > \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$. Under the exploitation strategy, firm 2 charges p_1; while under the accommodation strategy, firm 2 charges $p_1 - \tilde{T}(\underline{v}+v_0)$ and keeps all consumer types with certainty. Firm 2 is better off with the accommodation pricing because $p_1 - \tilde{T}(\underline{v}+v_0) > p_1\mu \Rightarrow p_1 > \frac{\tilde{T}\mu(\overline{v}+\underline{v})}{1-\mu}$ and $\frac{\tilde{T}\mu(\overline{v}+\underline{v})}{1-\mu} < \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$. Nonetheless, firm 1 will never find profitable to charge such a high price because then the consumer never returns and firm 1 ends up with zero payoff. This path will never be reached.

Next, if $p_1 \leq \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$, it is straightforward to see that charging the exploitation price $p'_2 = \max\left\{\frac{1}{2}\left(p_1 + \tilde{T}(\overline{v}-\underline{v})\right), p_1\right\}$ weakly dominates charging the accommodation price $p'_2 = p_1$.

Lastly, if $\frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v}) < p_1 \leq \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$, firm 2 obtains a profit of $\frac{\left((1-\mu)p_1+\mu\tilde{T}(\underline{v}+v_0)\right)^2}{4(1-\mu)\tilde{T}(\underline{v}+v_0)}$ under the accommodation pricing strategy. When $\mu < \frac{1}{2}$, under the exploitation pricing, firm 2 obtains a profit of $\frac{\mu(p_1+\tilde{T}(\overline{v}-v_0))^2}{4\tilde{T}(\overline{v}-v_0)}$ for $p_1 \leq \tilde{T}(\overline{v}-v_0)$ and $p_1\mu$ for $p_1 > \tilde{T}(\overline{v}-v_0)$. For $p_1 \leq \tilde{T}(\overline{v}-v_0)$ and $\mu < \frac{1}{2}$, $\frac{\mu(p_1+\tilde{T}(\overline{v}-v_0))^2}{4\tilde{T}(\overline{v}-v_0)} \geq \frac{\left((1-\mu)p_1+\mu\tilde{T}(\underline{v}+v_0)\right)^2}{4(1-\mu)\tilde{T}(\underline{v}+v_0)} \Leftrightarrow \mu p_1 + \mu\tilde{T}(\overline{v}-v_0) \geq (1-\mu)p_1 + \mu\tilde{T}(\underline{v}+v_0) \Leftrightarrow (2\mu-1)p_1 \geq (2\mu-1)\tilde{T}\mu(\overline{v}+\underline{v}) \Leftrightarrow p_1 \leq \tilde{T}\mu(\overline{v}+\underline{v})$.

For $\mu < \frac{1}{2}$, firm 2 is better off with the exploitation pricing for $p_1 \leq \tilde{T}\mu(\overline{v}+\underline{v})$ and the accommodation pricing for $p_1 > \tilde{T}\mu(\overline{v}+\underline{v})$. For $p_1 > \tilde{T}(\overline{v}-v_0)$, $\frac{\left((1-\mu)p_1+\mu\tilde{T}(\underline{v}+v_0\right)^2}{4(1-\mu)\tilde{T}(\underline{v}+v_0)} > p_1\mu \Leftrightarrow \left((1-\mu)\,p_1 + \mu\tilde{T}(\underline{v}+v_0\right)^2 > 4\mu(1-\mu)p_1\tilde{T}(\underline{v}+v_0) \Leftrightarrow \left((1-\mu)p_1 - \mu\tilde{T}(\underline{v}+v_0)\right)^2 > 0$, which always holds. That is, firm 2 is better off with accommodation strategy. When $\mu \geq \frac{1}{2}$, $p_1 > \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$ implies that $p_1 > \tilde{T}(\overline{v}-v_0)$, the exploitation pricing strategy leads to a profit of $p_1\mu$. Again, $\frac{\left((1-\mu)p_1+\mu\tilde{T}(\underline{v}+v_0\right)^2}{4(1-\mu)\tilde{T}(\underline{v}+v_0)} > p_1\mu \Leftrightarrow \left((1-\mu)\,p_1 + \mu\tilde{T}(\underline{v}+v_0\right)^2 > 4\mu(1-\mu)p_1\tilde{T}(\underline{v}+v_0) \Leftrightarrow \left((1-\mu)p_1 - \mu\tilde{T}(\underline{v}+v_0)\right)^2 > 0$, which always holds. Hence, the accommodation strategy is always more profitable.

To summarize, when $p_1 \leq \max\left\{\tilde{T}\mu(\overline{v}+\underline{v}), \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})\right\}$, firm 2's best response is to exploit the searching consumer. Nonetheless, if firm 2 is better off with this exploitation pricing strategy, sequential rationality fails. When $\max\left\{\tilde{T}\mu(\overline{v}+\underline{v}), \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})\right\} < p_1 \leq \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$, firm 2's best response is to accommodate the searching consumer by charging $p_2' = \frac{1}{2}\left(p_1 + \frac{\mu \tilde{T}(\underline{v}+v_0)}{1-\mu}\right)$. And firm 2's best response is sequential rational given the belief of the consumer type $t \in \mathbb{T} = \left[\tilde{t}, T\right]$ and firm 1's price. Anticipating the firm 2's best response is $p_2' = \frac{1}{2}\left(p_1 + \frac{\mu}{1-\mu} \cdot \tilde{T}(\underline{v}+v_0)\right)$, the consumer with $t \leq \hat{t}'$ continues searching firm 2 iff the following condition holds:

$$\begin{aligned} &U(t) = u^0 + tv_0 - p_2' - c > u^0 + tv_0 - p_1 \\ \Rightarrow &p_1 - p_2' > c \\ \Rightarrow &p_1 > 2c + \frac{\mu}{1-\mu} \cdot \tilde{T}(\underline{v}+v_0) \end{aligned} \tag{14}$$

The consumer with $t > \hat{t}'$ continues searching firm 2 iff:

$$\begin{aligned} &U(t) = \mu\left(u^0 + t\overline{v} - p_2'\right) + (1-\mu)\left(u^0 + tv_0 - p_1\right) - c > u^0 + tv_0 - p_1 \\ \Rightarrow &t(\overline{v} - v_0) > p_2' - p_1 + \frac{c}{\mu} \end{aligned}$$

The above condition increases in t. If it holds for $t = \hat{t}'$, it holds for all $t > \hat{t}'$. To see this, $\frac{p_1 - p_2'}{\underline{v}+v_0} \cdot (\overline{v} - v_0) > p_2' - p_1 + \frac{c}{\mu} \Leftrightarrow (p_1 - p_2') \cdot \left(\frac{1-\mu}{\mu} + 1\right) > \frac{c}{\mu} \Leftrightarrow p_1 - p_2' > c$, which leads to the condition (14).

Therefore, the all-searching equilibrium can arise if the equilibrium price p_1 should satisfy: (1) $p_1 > 2c + \frac{\mu}{1-\mu} \cdot \tilde{T}(\underline{v}+v_0)$; and (2) $\max\left\{\tilde{T}\mu(\overline{v}+\underline{v}), \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})\right\} < p_1 \leq \frac{2-\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})$.

Then we turn to firm 1's pricing strategy and verify whether the optimal price satisfies the above two conditions. In the all-searching equilibrium, anticipating a consumer's searching strategy and firm 2's accommodation pricing strategy, firm 1 captures the returning demand and obtains the following expected profit,

$$E\left[\Pi_1 \mid t \in \mathbb{T}, p_2'\right] = p_1 \cdot (1-\mu)\left(1 - \frac{p_1 - p_2'}{\tilde{T}(\underline{v}+v_0)}\right).$$

Thus, firm 1's best response is $BR(p_2') = \frac{1}{2}\left(p_2' + \tilde{T}(\underline{v}+v_0)\right)$. Recall that firm 2's best response $BR(p_1) = p_2' = \frac{1}{2}\left(p_1 + \frac{\mu\tilde{T}(\underline{v}+v_0)}{1-\mu}\right)$. The mutual best responses lead to $p_1^* = \frac{2-\mu}{3(1-\mu)}\tilde{T}(\underline{v}+v_0) = \frac{2-\mu}{3(1-\mu)}\tilde{T}\mu(\overline{v}+\underline{v})$ and $p_2^* = \frac{1+\mu}{3(1-\mu)}\tilde{T}(\underline{v}+v_0) = \frac{1+\mu}{3(1-\mu)}\tilde{T}\mu(\overline{v}+\underline{v})$. However, $\frac{2-\mu}{3(1-\mu)}\tilde{T}\mu(\overline{v}+\underline{v}) > \max\left\{\tilde{T}\mu(\overline{v}+\underline{v}), \frac{\mu}{1-\mu}\tilde{T}\mu(\overline{v}+\underline{v})\right\}$ never holds, leading $p_2^* = \frac{1+\mu}{3(1-\mu)}\tilde{T}\mu(\overline{v}+\underline{v})$ to be sequential irrational. That is, as long as all consumer types who have searched in the first round continue to search firm 2, firm 2 is better off deviating to exploit the searching consumer given firm 1's optimal price. Anticipating firm 2's ex-post incentive, not every consumer type chooses to search it. □

Proof of Proposition 1.

Proof. We consider symmetric pricing under symmetric content provision. When both firms provide brand-neutral content, there are three equilibrium candidates to consider. First, consider a possible equilibrium in which a consumer searches both firms. As two firms are symmetric, the equilibrium prices should be symmetric. Otherwise, one firm can attract the consumer with certainty. However, anticipating symmetric prices, the consumer deviates to search once.

Second, consider a possible equilibrium in which a consumer searches once. If a consumer searches once, no price that induces searching is sequential rational once the consumer has searched the firm. To see this, when $v^0 > 0$, consumer t searches the firm iff $\lambda(u^0+tv^0-p_1) > c \Leftrightarrow t > \frac{\frac{c}{\lambda}+p-u^0}{v^0}$. Firm's ex-post optimal price, p', maximizes $\Pi' = p_1' \cdot \max\left\{1 - \frac{\frac{c}{\lambda}+p-u^0}{v^0T}, 1 - \frac{p'-u^0}{v^0T}\right\}$, which leads to $p' = \max\left\{\frac{c}{\lambda}+p, \frac{1}{2}(u^0+v^0T)\right\} > p$. When $v^0 = 0$, given the consumer has visited firm 1, firm 1 has an incentive to deviate to a profit-maximizing price u^0. However, expecting $p' = u^0$, no consumer has an incentive to search at the beginning as $\lambda(u^0 - p') = 0 < c$.

Hence, the only possible equilibrium is that no consumer searches and both firms expect zero payoff. When both firms provide brand-specific information, no search is induced. Both firms expect zero payoff. In conclusion, brand-neutral content marketing is a dominant strategy. □

Proof of Proposition 2

Proof. Under symmetric content provision when both are content marketers, with price commitment, undercutting the competitor's price by an infinitely small amount leads to a discrete jump in demand. Thus, the two firms engage in the Bertrand competition and end up with zero payoffs. The consumer searches iff $\lambda u^0 > c$, which always holds under the market-existing condition characterized in Lemma 1. Under symmetric content provision when both are brand advertisers, no search can be induced. Both firms expect zero payoffs.

Under asymmetric content provision, consider the type-dependent search in which $-c \leq \Delta p^* \leq (\overline{v} - v^0)T - \frac{c}{\mu}$. A consumer with $t \leq \hat{t}(\Delta p)$ does not search and buy brand 1. While a consumer with $t > \hat{t}(\Delta p)$ searches firm 2 and buy brand 2 if $v_2 = \overline{v}$ and buys 1 if otherwise. Later we will verify that the market is indeed fully covered under the market-existing condition characterized in Lemma 1. Then we can specify firms' expected payoff functions as follows,

$$\begin{aligned}\Pi_1 &= p_1 \cdot \left(1 - \mu\left(1 - \frac{\hat{t}(\Delta p)}{T}\right)\right) \\ \Pi_2 &= p_2 \cdot \mu\left(1 - \frac{\hat{t}(\Delta p)}{T}\right)\end{aligned}$$

The optimal interior solution is

$$p_1^* = \frac{(2-\mu)(1-\mu)(\overline{v}+\underline{v})T}{3\mu} + \frac{c}{3\mu}, p_2^* = \frac{(1-\mu)^2(\overline{v}+\underline{v})T}{3\mu} - \frac{c}{3\mu}. \tag{15}$$

To simply the notation, let us denote $\psi = \frac{(\overline{v}+\underline{v})T}{c}$. Equivalently, we have

$$p_1^* = \frac{(2-\mu)(1-\mu)\psi c}{3\mu} + \frac{c}{3\mu}, p_2^* = \frac{(1-\mu)^2\psi c}{3\mu} - \frac{c}{3\mu}. \tag{16}$$

In the online appendix, we show the condition for the type-dependent searching equilibrium to emerge is as follows: $\psi \in \left[\frac{1}{2}\left(\sqrt{5}+1\right), \frac{1}{3}\left(14\sqrt{7}+37\right)\right]$ and $\mu \in [\underline{\mu}^S, \overline{\mu}^S]$ or when $\psi > \frac{1}{3}\left(14\sqrt{7}+37\right)$, $\mu \in [\underline{\mu}^S, \overline{\mu}^S] \setminus \left(\underline{\mu}_1^{dev}, \overline{\mu}_1^{dev}\right)$.

Now we examine the first-round search decision. For a consumer with $t \leq \hat{t}$, she anticipates that she forgoes searching firm 2. She searches firm 1 iff $\lambda(u^0 + tv^0 - p_1^*) > c$, which always holds when $u^0 > \frac{c}{\lambda} + p_1^*$. For a consumer with $t > \hat{t}$, she anticipates that she searches both firms. She starts searching firm 1 iff $\lambda\left(\mu(u^0 + t\overline{v} - p_2^*) + (1-\mu)(u^0 + tv^0 - p_1^*) - c\right) > c$, which should hold for all $t > \hat{t}$. That is, $\mu\left(u^0 + \hat{t}\overline{v} - p_2^* - \left(u^0 + \hat{t}v^0 - p_1^*\right)\right) - c > \frac{c}{\lambda} - \left(u^0 + \hat{t}v^0 - p_1^*\right) \Leftrightarrow u^0 > \frac{c}{\lambda} + p_1^* - \hat{t}v^0$, which always hold under $u^0 > \frac{c}{\lambda} + p_1^*$. With a few algebraic steps, one can show that $u^0 > \frac{c}{\lambda} + p_1^*$ holds under the market-existing condition characterized in Lemma 1.

Lastly, we solve for the optimal content strategies. First, there exist two pure strategy equilibrium, i.e., $\{B, C\}$ and $\{C, B\}$. If firm i chooses brand advertising, B, firm $-i$'s best response, $BR(B) = C$, because $\Pi_2^* > 0$. If firm i chooses content marketing, C, firm $-i$'s best response, $BR(C) = B$, because $\Pi_1^* > 0$. Next, we solve for the mixed strategy. To make firm $-i$ indifferent between playing B and C, firm i chooses σ^* such that $\sigma^*\Pi_2^* = (1-\sigma^*)\Pi_1 \Leftrightarrow \sigma^* = \frac{\Pi_1^*}{\Pi_1^*+\Pi_2^*}$. $\square$

Proof of Proposition 3.

Proof. $Q_1^* - Q_2^* = 1 - \mu\left(1 - \frac{\hat{t}^*}{T}\right) - \mu\left(1 - \frac{\hat{t}^*}{T}\right) \geq 0 \Leftrightarrow p_1^* - p_2^* \leq \frac{2c+T(\overline{v}+\underline{v})(1-2\mu)(1-\mu)}{2\mu}$. Notice that $p_1^* - p_2^* = \frac{2c+T(\overline{v}+\underline{v})(1-2\mu)(1-\mu)}{3\mu}$, we can conclude that for $p_1^* - p_2^* \geq 0$, $p_1^* - p_2^* \leq \frac{2c+T(\overline{v}+\underline{v})(1-2\mu)(1-\mu)}{2\mu}$ holds and $Q_1^* - Q_2^* \geq 0$. In contrary, for $p_1^* - p_2^* < 0$, $p_1^* - p_2^* > \frac{2c+T(\overline{v}+\underline{v})(1-2\mu)(1-\mu)}{2\mu}$ holds and $Q_1^* - Q_2^* < 0$. Moreover, $p_1^* - p_2^* \leq 0 \Leftrightarrow \mu \in \left[\underline{\mu}^B, \overline{\mu}^B\right] \equiv \left[\frac{1}{4}\left(3 - \sqrt{1 - \frac{16}{\psi}}\right), \frac{1}{4}\left(3 + \sqrt{1 - \frac{16}{\psi}}\right)\right]$. The range arises when $\psi > 16$. Otherwise, $p_1^* - p_2^* > 0$. Next, we compare $\underline{\mu}^B(\psi)$ and $\underline{\mu}^S(\psi)$. As both cutoffs monotonically decrease in ψ, $\underline{\mu}^B(16) > \underline{\mu}^S(16)$ while $\lim_{\psi\to\infty} \underline{\mu}^S(\psi) > \lim_{\psi\to\infty} \underline{\mu}^B(\psi) = \frac{1}{2}$, we can find a cutoff $\tilde{\psi}$ such that when $\psi \leq \tilde{\psi}$, $\underline{\mu}^B \geq \underline{\mu}^S$; while $\psi > \tilde{\psi}$, $\underline{\mu}^B < \underline{\mu}^S$. □

Lemma A2. *Denote* $\hat{t}^{asy} \equiv \frac{1}{(1-\mu_1)(\overline{v}+\underline{v})}\left(\frac{c}{\mu_2} + \Delta p\right)$. *A consumer with* $t > \hat{t}^{asy}$ *continues searching while* $t \leq \hat{t}^{asy}$ *forgoes searching when* $-\frac{c\mu_1}{\mu_2} \leq \Delta p < (1-\mu_1)(\overline{v}+\underline{v})T - \frac{c}{\mu_2}$, *where* $\Delta p = p_2 - p_1$.

Proof of Lemma A2.

Proof. We first solve for the subgame under asymmetric content provision. Following the definition of the value of information in equation (2), we revise the equation with asymmetric firms as follows:

$$V(t) = \mu_2 \max\{t(\overline{v}_2 - v_1) - \Delta p, 0\} + (1-\mu_2)\max\{-t(\underline{v}_2 + v_1) - \Delta p, 0\}. \tag{17}$$

As $V(t)$ is heterogenous in consumer type t. Let us investigate search incentives for different consumer types. If there exists a consumer type t such that $t(\overline{v}_2 - v_1) - \Delta p < 0 \Leftrightarrow t < \frac{\Delta p}{\overline{v}_2 - v_1} = \frac{\Delta p}{(1-\mu_1)(\overline{v}+\underline{v})}$, her value of information from further searching is zero. Thus, she will forgo further search. The cutoff $\frac{\Delta p}{(1-\mu_1)(\overline{v}+\underline{v})}$ in the range $[0,T]$ when $0 \leq \Delta p \leq (1-\mu_1)(\overline{v}+\underline{v})T$.

If there exists a consumer type t such that $-t(\underline{v}_2 + v_1) - \Delta p \geq 0 \Leftrightarrow t \leq \frac{-\Delta p}{\underline{v}_2 + v_1} = \frac{-\Delta p}{\mu_1(\overline{v}+\underline{v})}$, her search benefit is $V(t) = t(v_2 - v_1) - \Delta p$. These types of consumer continue searching iff $\Delta p < t(v_2 - v_1) - c$. The cutoff $\frac{-\Delta p}{\mu_1(\overline{v}+\underline{v})}$ exists in the range $[0,T]$ when $-\mu_1(\overline{v}+\underline{v})T \leq \Delta p \leq 0$.

Lastly, if there exists a consumer type t such that $t(\overline{v}_2 - v_1) - \Delta p \geq 0 > -t(\underline{v}_2 + v_1) - \Delta p \Leftrightarrow t \geq \frac{\Delta p}{\overline{v}_2 - v_1}$ and $t > \frac{-\Delta p}{\underline{v}_2 + v_1}$, her search benefit is $V(t) = \mu_2(t(\overline{v}_2 - v_1) - \Delta p)$, which increases in t. A consumer type t continues searching iff $V(t) > c \Leftrightarrow t > \frac{1}{\overline{v}_2 - v_1}\left(\frac{c}{\mu_2} + \Delta p\right) = \frac{1}{(1-\mu_1)(\overline{v}+\underline{v})}\left(\frac{c}{\mu_2} + \Delta p\right) \equiv \hat{t}^{asy}$ and forgoes searching if $t \leq \hat{t}^{asy}$. The cutoff $\hat{t}^{asy}$ exists in the range $[0,T]$ when $-\frac{c}{\mu_2} \leq \Delta p \leq (1-\mu_1)(\overline{v}+\underline{v})T - \frac{c}{\mu_2}$.

Since $t \geq \frac{\Delta p}{\overline{v}_2 - v_1}$ and $t > \frac{-\Delta p}{\underline{v}_2 + v_1}$ are mutually exclusive. We break down the analysis by Δp.

Case 1: $\Delta p > 0$**.** Notice that the cutoff $\frac{\Delta p}{\overline{v}_2 - v_1} = \frac{\Delta p}{(1-\mu_1)(\overline{v}+\underline{v})}$ is always smaller than $\hat{t}^{asy}$. When $-\frac{c}{\mu_2} \leq \Delta p < (1-\mu_1)(\overline{v}+\underline{v})T - \frac{c}{\mu_2}$, the marginal consumer who is indifferent between searching further and no searching is determined by $\hat{t}^{asy}$. When $\Delta p \geq (1-\mu_1)(\overline{v}+\underline{v})T - \frac{c}{\mu_2} \Leftrightarrow \hat{t}^{asy} \geq T$, there exist no consumer type who continues searching. When $\Delta p < -\frac{c}{\mu_2} \Leftrightarrow \hat{t}^{asy} < 0$, all consumer types continue searching.

Case 2: $\Delta p \leq 0$**.** For any $t > \frac{-\Delta p}{\mu_1(\overline{v}+\underline{v})}$, the search decision is determined by $\hat{t}^{asy}$ when $0 \leq \frac{-\Delta p}{\mu_1(\overline{v}+\underline{v})} \leq \hat{t}^{asy} < T$ holds, which is equivalent to $-\frac{c\mu_1}{\mu_2} \leq \Delta p < (1-\mu_1)(\overline{v}+\underline{v})T - \frac{c}{\mu_2}$. The range exists when $c < \mu_2(\overline{v}+\underline{v})T$. Under the above condition, for any $t \leq \frac{-\Delta p}{\mu_1(\overline{v}+\underline{v})}$, her search benefit $V(t) = t(v_2 - v_1) - \Delta p \leq c$ for any $-\frac{c\mu_1}{\mu_2} \leq \Delta p$. □

Proof of Proposition 5

Proof. Asymmetric Content Provision. We first solve for the subgame given firms choose asymmetric content provision. In the $\{C, B\}$ equilibrium where the stronger firm becomes the content marketer while the weaker firm is the brand advertiser, $\mu_1 = \mu_s > \mu_w = \mu_2$. According to Lemma A2, if the equilibrium prices satisfy $\frac{c}{\mu_w} - (1-\mu_s)(\overline{v}+\underline{v})T < \Delta p \leq \frac{c\mu_s}{\mu_w}$, a consumer with $t \leq \hat{t}_1^{asy} = \frac{1}{(1-\mu_s)(\overline{v}+\underline{v})}\left(\frac{c}{\mu_w} - \Delta p\right)$ forgoes further searching and buys from the strong firm while a consumer with $t > \hat{t}_1^{asy}$ continues searching and buys from the weak firm only when $\overline{v}$ is realized. The type-dependent search rule leads to the following expected profit for the two firms: $\Pi_s^C = p_s \cdot \left(1 - \mu_w\left(1 - \frac{\hat{t}_1^{asy}(\Delta p)}{T}\right)\right)$ and $\Pi_w^B = p_w \cdot \mu_w\left(1 - \frac{\hat{t}_1^{asy}(\Delta p)}{T}\right)$, respectively. The profit-maximizing prices are $p_s^{C*} = \frac{(2-\mu_w)(1-\mu_s)(\overline{v}+\underline{v})T}{3\mu_w} + \frac{c}{3\mu_w}$ and $p_w^{B*} = \frac{(1+\mu_w)(1-\mu_s)(\overline{v}+\underline{v})T}{3\mu_w} - \frac{c}{3\mu_w}$.

In the $\{B, C\}$ equilibrium where the stronger firm becomes the brand advertiser while the weaker firm is the content marketer, $\mu_s = \mu_2 > \mu_w = \mu_1$. According to Lemma A2, if the equilibrium price satisfy $\frac{c}{\mu_s} - (1-\mu_w)(\overline{v}+\underline{v})T < \Delta p \leq c + (\mu_w - \mu_s)(\overline{v}+\underline{v})$, a consumer with $t \leq \hat{t}_2^{asy} = \frac{1}{(1-\mu_w)(\overline{v}+\underline{v})}\left(\frac{c}{\mu_s} - \Delta p\right)$ forgoes further searching and buys from the strong firm while a consumer with $t > \hat{t}_2^{asy}$ continues searching and buys from the weak firm only when $\overline{v}$ is realized. Such a search rule leads to the following expected profit for the two firms: $\Pi_s^B = p_s \cdot \mu_s\left(1 - \frac{\hat{t}_2^{asy}(\Delta p)}{T}\right)$ and $\Pi_w^C = p_w \cdot \left(1 - \mu_s\left(1 - \frac{\hat{t}_2^{asy}(\Delta p)}{T}\right)\right)$. The equilibrium prices $p_w^{C*} = \frac{(2-\mu_s)(1-\mu_w)(\overline{v}+\underline{v})T}{3\mu_s} + \frac{c}{3\mu_s}$ and

$p_s^{B*} = \frac{(1+\mu_s)(1-\mu_w)(\overline{v}+\underline{v})T}{3\mu_s} - \frac{c}{3\mu_s}$.

The necessary equilibrium conditions require the equilibrium prices to satisfy Lemma A2, that is, for any $T \geq 1$, when $\psi \equiv \frac{(\overline{v}+\underline{v})T}{c} > \frac{\sqrt{5}+1}{2}$, we can find the cutoffs $\underline{\mu}_s \equiv \frac{3}{4}\left(\frac{1}{\psi}+1\right) - \frac{1}{4}\sqrt{\frac{9}{\psi^2}+\frac{2}{\psi}+1}$ and $\overline{\mu}_s \equiv \sqrt{1-\frac{1}{\psi}}$ such that under the condition where $\mu_s \in \left[\underline{\mu}_s, \overline{\mu}_s\right)$ and $\Delta\mu \equiv \mu_s - \mu_w \leq \min\left\{\frac{3\mu_s-2}{2\psi(1-\mu_s)} + \mu_s - \frac{1}{2}, 1+\mu_s - \frac{1}{\psi(1-\mu_s)}\right\} \equiv \Delta\bar{\mu}$. The proceeding analysis focuses on the above parameter space. Without loss of generality, we normalize $c = 1$ and fix $\underline{v} + \underline{v} = 1$.

Now we examine the first-round search decision. In the $\{C, B\}$ equilibrium, for a consumer with $t \leq \hat{t}_1^{asy}$, she anticipates that she forgoes searching firm 2. She searches firm s iff $\lambda(u^0 + tv_s^0 - p_s^{C*}) > c \Leftrightarrow u^0 > \frac{c}{\lambda} + p_s^{C*}$. For a consumer with $t > \hat{t}_1^{asy}$, she anticipates that she searches both firms. She starts searching firm 1 iff $\lambda\left(\mu_w(u^0 + t\overline{v} - p_w^{B*}) + (1-\mu_w)(u^0 + tv_s^0 - p_s^{C*}) - c\right) > c$, which should hold for all $t > \hat{t}_1^{asy}$. That is, $\mu_w\left(u^0 + \hat{t}_1^{asy}\overline{v} - p_w^{B*} - \left(u^0 + \hat{t}_1^{asy}v_s^0 - p_s^{C*}\right)\right) - c > \frac{c}{\lambda} - \left(u^0 + \hat{t}_1^{asy}v_s^0 - p_s^{C*}\right) \Leftrightarrow u^0 > \frac{c}{\lambda} + p_s^{C*} - \hat{t}v_s^0$, which always hold under $u^0 > \frac{c}{\lambda} + p_s^{C*}$. Similarly, in the $\{B, C\}$ equilibrium, to induce all consumer types to search in the first round, $u^0 > \frac{c}{\lambda} + p_w^{C*}$ should hold.

Symmetric Content Provision. Given both firms choose content marketing, a consumer searches once and starts from the stronger firm iff $tv_s^0 - p_s \geq tv_w^0 - p_w \Leftrightarrow t \geq \frac{p_s - p_w}{v_s^0 - v_w^0} = \frac{p_s - p_w}{(\mu_s-\mu_w)(\overline{v}+\underline{v})}$. Consider $\overline{v} + \underline{v} = 1$, the stronger firm obtains a profit of $\Pi_s = p_s \cdot \left(1 - \frac{p_s - p_w}{(\mu_s - \mu_w)T}\right)$ while the weaker firm's profit is $\Pi_w = p_w \cdot \frac{p_s - p_w}{(\mu_s-\mu_w)T}$. It is straightforward to show that the equilibrium prices are $p_s^* = \frac{2\Delta\mu}{3}$ and $p_w^* = \frac{\Delta\mu}{3}$. Consequently, the equilibrium payoffs for the stronger and the weaker firms are $\Pi_s^{C*} = \frac{4T\Delta\mu}{9}$ and $\Pi_w^{C*} = \frac{T\Delta\mu}{9}$, respectively. To induce all consumer types to search in the first round, $\lambda(u^0 - p_w^*) > c \Leftrightarrow u^0 > \frac{c}{\lambda} + \frac{\Delta\mu}{3}$.

Hence, so long as u^0 is sufficiently large, all consumer types engages in searching and makes a purchase under all possible content provision.

Firms' incentives to become a content marketer. Since firms' best response $BR(B) = C$, the equilibrium outcome depends on firms' incentives to prefer content marketing over brand advertising i.e., $\mathcal{I}_i^C \equiv \Pi_i^{C*} - \Pi_i^{B*} \geq 0$ for $i = \{s, w\}$, given the competing firm chooses content marketing. The stronger firm's payoff as a brand advertiser is $\Pi_s^{B*} = \frac{(c-(\overline{v}+\underline{v})T(1+\mu_s)(1-\mu_w))^2}{9(\overline{v}+\underline{v})T\mu_s(1-\mu_w)}$ while the weaker firm's payoff as a brand advertising is $\Pi_w^{B*} = \frac{(c-(\overline{v}+\underline{v})T(1+\mu_w)(1-\mu_s))^2}{9(\overline{v}+\underline{v})T\mu_w(1-\mu_s)}$.

Notice that for all T and μ_w, both $\mathcal{I}_i^C$ increases in μ_s and $\mathcal{I}_i^C$ are negative as $\mu_s \to \frac{1}{2}$. Hence, we can find cutoffs $\underline{\mu}_i$ at which $\mathcal{I}_i^C(\underline{\mu}_i) = 0$ such that $\mathcal{I}_i^C \leq 0$ for all T and μ_w when $\mu_i < \underline{\mu}_i$. And when $\mu_s > \underline{\mu} \equiv \max\{\underline{\mu}_s, \underline{\mu}_w\}$, for any given $\mu_s > \underline{\mu}$, there exist some T and μ_w (or equivalently, $\Delta\mu = \mu_s - \mu_w$) so that $\mathcal{I}_i^C > 0$.

Next, we look for parameter space $\{T, \Delta\mu\}$ for any given $\mu_s > \underline{\mu}$ so that the equilibrium outcomes, $\{C,C\}$, $\{B,C\}$, $\{C,B\}$, can arise. First, when $\Delta\mu = 0$, $\mathcal{I}_s^C = \mathcal{I}_w^C < 0$. Second, both $\mathcal{I}_i^C$ is concave in $\Delta\mu$. We can find a cutoff $\Delta\mu_s(T,\mu_s)$ such that $\mathcal{I}_s^C > 0$ iff $\Delta\mu > \Delta\mu_s(T,\mu_s)$. The cutoff $\Delta\mu_s(T,\mu_s) = \frac{(1+\mu_s)-(1-\mu_s)\left(1+\mu_s^2\right)T-2\sqrt{\mu_s(\mu_s T - T + 1)(\mu_s^2 T - \mu_s T + 1)}}{(1-\mu_s)^2 T}$ exists in $0 < \Delta\mu \leq \Delta\bar{\mu}$ when $\max\left\{\frac{1}{1-\mu_s^2}, \frac{4+9\mu_s}{1+7\mu_s} - \frac{4\sqrt{\mu_s(9\mu_s^3-9\mu_s^2+7\mu_s-3)}}{(1-\mu_s)(1+7\mu_s)}\right\} \leq T \leq \frac{4+9\mu_s}{1+7\mu_s} + \frac{4\sqrt{\mu_s(9\mu_s^3-9\mu_s^2+7\mu_s-3)}}{(1-\mu_s)(1+7\mu_s)}$. Similarly, we can find a cutoff $\Delta\mu_w(T,\mu_s)$ such that $\mathcal{I}_w^C > 0$ iff $\Delta\mu > \Delta\mu_w(T,\mu_s)$. The cutoff $\Delta\mu_w(T,\mu_s) = \frac{\left(2-2\mu_s^2+\mu_s\right)T-2}{2(2-\mu_s)T} - \frac{\sqrt{T(4\mu_s+5\mu_s^2 T-4T+8)-\frac{4}{1-\mu_s}}}{2(2-\mu_s)T}$ exists in $0 < \Delta\mu \leq \Delta\bar{\mu}$ when $\mu_s < \frac{10}{11}$ and $\max\left\{\frac{1}{1-\mu_s^2}, \frac{2(6\mu_s-1)}{10-11\mu_s} - \frac{\sqrt{9(5\mu_s^4-4)+4\mu_s(27-17\mu_s-12\mu_s^2)}}{(1-\mu_s)(10-11\mu_s)}\right\} \leq T \leq \frac{2(6\mu_s-1)}{10-11\mu_s} + \frac{\sqrt{9(5\mu_s^4-4)+4\mu_s(27-17\mu_s-12\mu_s^2)}}{(1-\mu_s)(10-11\mu_s)}$ or $\mu_s \geq \frac{10}{11}$ and $T \geq \frac{1}{1-\mu_s^2}$. At $T = \frac{25-9\mu_s}{5(\mu_s^2-6\mu_s+5)}$, $\Delta\mu_s = \Delta\mu_w = \frac{9(1-\mu_s)\mu_s}{25-9\mu_s}$. Moreover, as both $\Delta\mu_i(T,\mu_s)$ increases in T, we can find an inverse function $T_i = \Delta\mu_i^{-1}(\Delta\mu,\mu_s)$ so that $\mathcal{I}_i^C > 0$ when $T < T_i(\Delta\mu,\mu_s)$. Hence, when $T < \min\{T_s, T_w\}$, $\mathcal{I}_s^C > 0$ and $\mathcal{I}_w^C > 0$ so that both firms choose content marketing as the dominant strategy. When $T \geq \max\{T_s, T_w\}$, $\mathcal{I}_s^C \leq 0$ and $\mathcal{I}_w^C \leq 0$ so that neither firms chooses content marketing as the dominant strategy. When $\Delta\mu \geq \widehat{\Delta\mu} = \frac{9(1-\mu_s)\mu_s}{25-9\mu_s}$, $T_s \leq T < T_w$, resulting in $\mathcal{I}_s^C \leq 0$ and $\mathcal{I}_w^C > 0$. In this case, $\{B,C\}$-equilibrium arises where the stronger firm opts for brand advertising while the weaker firm opts for content marketing. Lastly, when $\Delta\mu < \widehat{\Delta\mu}$, for $T_s \leq T < T_w$, $\{C,B\}$-equilibrium arises where the stronger firm opts for content marketing while the weaker firm opts for brand advertising. □

Proof of Corollary 2

Notice that Π_s^{B*} and Π_w^{B*} increase in T because both the price and demand increase in T as a brand advertiser. So $\mathcal{I}_s^C$ and $\mathcal{I}_w^C$ decrease in T. Moreover, due to steeper slopes of price and demand functions for the stronger firm, $\frac{\partial E[\Pi_s^{B*}]}{\partial T} > \frac{\partial E[\Pi_w^{B*}]}{\partial T} \Leftrightarrow \frac{\partial \mathcal{I}_s^C}{\partial T} < \frac{\partial \mathcal{I}_w^C}{\partial T}$.

Proof of Lemma 2.

Proof. First, we consider an equilibrium in which the role of brand advertising is to enable firms to extract match value, i.e., $p^* > u^0$. And a consumer only purchases the matched product. In equilibrium, a consumer searches iff $\mu\left(u^0 + t\overline{v} - p^*\right) > c \Leftrightarrow t > \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$, and purchases the product only if it is a matched one.

Consider a firm deviates from p^* once the consumer visits it. If a consumer receives $\overline{v}$ and observes a deviating price $\widehat{p}$, she continues searching iff

$$\mu\left(\max\left\{0, u^0+t\overline{v}-p^*\right\}-\max\left\{0, u^0+t\overline{v}-\widehat{p}\right\}\right)+(1-\mu)\left(0-\max\left\{0, u^0+t\overline{v}-\widehat{p}\right\}\right)>c$$

If the deviating price $\widehat{p}>p^*$, for $t<\frac{p^*-u^0}{\overline{v}}$, her utility $u^0+t\overline{v}-p^*<0$, and she does not purchase nor searches further. For a consumer with $t\in\left[\frac{p^*-u^0}{\overline{v}}, \frac{\widehat{p}-u^0}{\overline{v}}\right)$, she searches iff $\mu\left(u^0+t\overline{v}-p^*\right)>c \Leftrightarrow t>\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$. Notice that so long as $\frac{c}{\mu}+p^*-u^0>\widehat{p}-u^0 \Leftrightarrow \widehat{p}-p^* \leq \frac{c}{\mu}$, no consumer in the range $t\in\left[\frac{p^*-u^0}{\overline{v}}, \frac{\widehat{p}-u^0}{\overline{v}}\right)$ purchases nor continues to search. Otherwise, if $\widehat{p}-p^*>\frac{c}{\mu}$, a consumer with $t\in\left[\frac{p^*-u^0}{\overline{v}}, \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}\right]$ does not purchase nor continues to search while a consumer with $t\in\left(\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}, \frac{\widehat{p}-u^0}{\overline{v}}\right)$ continues to search. For a consumer with $t\geq\frac{\widehat{p}-u^0}{\overline{v}}$, she continues searching iff $\mu\left(\widehat{p}-p^*\right)-(1-\mu)\left(u^0+t\overline{v}-\widehat{p}\right)>c \Leftrightarrow t<\frac{\widehat{p}-u^0}{\overline{v}}+\frac{\mu(\widehat{p}-p^*)-c}{\overline{v}}$, which does not exist if $\widehat{p}-p^*\leq\frac{c}{\mu}$. Otherwise, when $\widehat{p}-p^*>\frac{c}{\mu}$, a consumer with $t\in\left(\frac{\widehat{p}-u^0}{\overline{v}}, \frac{\widehat{p}-u^0}{\overline{v}}+\frac{\mu(\widehat{p}-p^*)-c}{\overline{v}}\right]$ continues searching while a consumer with $t\in\left[\frac{\widehat{p}-u^0}{\overline{v}}+\frac{\mu(\widehat{p}-p^*)-c}{\overline{v}}, T\right]$ purchases immediately without search. If firm 1 deviates to $\widehat{p}<p^*$, a consumer immediately purchases the product without search iff $t\geq\frac{\widehat{p}-u^0}{\overline{v}}$.

If a consumer receives $-\underline{v}$ and observes price $\widehat{p}$, she continues searching iff

$$\mu\left(\max\left\{0, u^0+t\overline{v}-p^*\right\}-\max\left\{0, u^0-t\underline{v}-\widehat{p}\right\}\right)-(1-\mu)\max\left\{0, u^0-t\underline{v}-\widehat{p}\right\}>c$$

For a price deviation $\widehat{p}>p^*$ or $u^0\leq\widehat{p}<p^*$, the consumer searches iff $\mu\max\left\{0, u^0+t\overline{v}-p^*\right\}>c$. The search decision is independent of firm 1's price deviation.

To summarize, expecting p^* and upon observing a price deviation $\widehat{p}-p^*\in[u^0-p^*, \frac{c}{\mu}]$, a consumer's second-round search rule is specified as: (1) if a consumer receives a positive brand shock $\overline{v}$, the consumer with $t<\frac{\widehat{p}-u^0}{\overline{v}}$ does not purchase nor search while a consumer with $t\geq\frac{\widehat{p}-u^0}{\overline{v}}$ purchases immediately without further search; (2) if the consumer receives a negative brand shock $-\underline{v}$, $\widehat{p}$ has no influence on her search decision. Her search rule is the same as in the first round: She continues searching without returning iff $t>\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$; if $t\leq\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$, the consumer does not search nor purchase the product.

The above search rule suggests that a price deviation affects a consumer's second-round search decision only when she receives a positive brand shock. Moreover, if the deviating firm is visited by a second-round searcher, that consumer receives a negative draw from the first firm and will make the purchase at the deviation price so long as she receives a positive brand shock. This is because $\mu\left(u^0 + t\overline{v} - p^*\right) > c$ implies $u^0 + t\overline{v} - \widehat{p} > 0$ for any deviation such that $\widehat{p} - p^* < \frac{c}{\mu}$.

We then specify the deviating firm's payoff as follows,

$$\widehat{\Pi} = \frac{1}{2}\widehat{p}\left[\mu\left(1 - \frac{\widehat{p} - u^0}{\overline{v}T}\right) + (1-\mu)\,\mu\left(1 - \frac{\frac{c}{\mu} + p^* - u^0}{\overline{v}T}\right)\right]$$

To obtain the equilibrium price, we set $\frac{\partial\widehat{\Pi}}{\partial\widehat{p}}|_{\widehat{p}=p^*} = 0$, which leads to $p^* = \frac{(2-\mu)\mu\left(T\overline{v}+u^0\right)-c(1-\mu)}{(3-\mu)\mu}$. p^* can be an equilibrium iff $p^* > u^0$ and $\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}T} \in (0,1)$, which leads to the equilibrium condition $\max\{0, \frac{2c}{\mu} - \overline{v}T\} < u^0 < \overline{v}T(2-\mu) - \frac{c(1-\mu)}{\mu}$.

In equilibrium, a firm's expected payoff is given as

$$\begin{aligned}\Pi^* &= \frac{1}{2}p^*\left[\mu\left(1 - \frac{p^* - u^0}{\overline{v}T}\right) + (1-\mu)\,\mu\left(1 - \frac{\frac{c}{\mu} + p^* - u^0}{\overline{v}T}\right)\right] \\ &= \frac{\left((2-\mu)\,\mu\left(T\overline{v} + u^0\right) - c\,(1-\mu)\right)^2}{2\,(3-\mu)^2\mu T\overline{v}}\end{aligned}$$

And under symmetric adoption of brand advertising, the equilibrium outcomes remain the same with and without price commitment.

Second, we consider an equilibrium in which $p^* \le u^0$. There are two groups of consumers: (1) for $t > \frac{u^0-p^*}{\underline{v}} \Leftrightarrow u^0 - t\underline{v} - p^* < 0$, the consumer searches iff $\mu(u^0 + t\overline{v} - p^*) > c$; and (2) for $t \le \frac{u^0-p^*}{\underline{v}} \Leftrightarrow u^0 - t\underline{v} - p^* \ge 0$, the consumer searches iff $u^0 + tv^0 - p^* > c$. Notice that so long as $u^0 - p^* \le \frac{c}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$, a consumer searches iff $t > \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$, and purchases the product only if it is a matched one. To see this, for the first consumer group, $\mu(u^0 + t\overline{v} - p^*) > c \Leftrightarrow t > \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$. And $\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}} \ge \frac{u^0-p^*}{\underline{v}} \Leftrightarrow u^0 - p^* < \frac{c}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$, which implies that a consumer with $t > \frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}}$ searches and only purchases the matched product while the rest types of the first group do not search nor purchase. For the second consumer group, $u^0 + tv^0 - p^* > c \Leftrightarrow t > \frac{c+p^*-u^0}{v^0}$. And $\frac{c+p^*-u^0}{v^0} \ge \frac{u^0-p^*}{\underline{v}} \Leftrightarrow u^0 - p^* \le \frac{c}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$, which implies that the second group of consumer does not search nor purchase. Then we can apply the same analysis as the case of $p^* > u^0$. The equilibrium price $p^* = \frac{(2-\mu)\mu\left(T\overline{v}+u^0\right)-c(1-\mu)}{(3-\mu)\mu}$ need to satisfy $0 \le u^0 - p^* \le \frac{c}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$ and $\frac{\frac{c}{\mu}+p^*-u^0}{\overline{v}T} \in (0,1)$, which leads to the equilibrium condition $\max\{0, \frac{2c}{\mu} - \overline{v}T\} < u^0 < \overline{v}T(2-\mu) - \frac{c(1-\mu)}{\mu} + \frac{c(3-\mu)}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$.

Lastly, we consider an equilibrium in which $u^0 - p^* > \frac{c}{\mu}\frac{\underline{v}}{\overline{v}+\underline{v}}$. We show this cannot be an equilibrium. Following the analysis above, in equilibrium, a consumer with $t > \frac{c+p^*-u^0}{v^0}$ searches, and the searching consumer purchases the product, irrespective of the matching value. Then a firm can deviate to a price increase without losing any consumer. $\square$

Proof of Proposition 7

Proof. No Price Commitment. From Lemma 1, $Br(C) = C$. If $Br(B) = C$, then content marketing is the dominant strategy for both firms so that $\{C, C\}$ arises in equilibrium. Otherwise if $Br(B) = B$, both $\{B, B\}$ and $\{C, C\}$ can arise in equilibrium.

Let us denote a firm's incentive to become a content marketer given the competitor chooses brand advertising as $\mathcal{I}^C_{-i=B} \equiv \Pi^*_1(C, B) - \Pi^*(B, B)$, where $\Pi^*_1(C, B)$ is a firm's equilibrium payoff under the $\{C, B\}$-equilibrium as the content marketer and $\Pi^*(B, B)$ as the payoff in the $\{B, B\}$-equilibrium given $\chi = 1$. $\Pi^*_1(C, B) = p^*_1 = \frac{2-\mu}{1-\mu}\mu(\overline{v} + \underline{v})T$ when the market is fully covered under asymmetric content provision such that all consumer types engage in searching, i.e., $u^0 > \frac{c}{\lambda} + p^*_1$. When the market is partially covered, $t > \frac{c+\lambda\left(p^*_1-u^0\right)}{v^0\lambda}$ searches and purchases from the content marketer, resulting in $\Pi^*_1(C, B) = p^*_1\left(1 - \frac{c+\lambda\left(p^*_1-u^0\right)}{v^0\lambda T}\right)$. And $\Pi^*(B, B) = \frac{\left((2-\mu)\mu\left(T\overline{v}+u^0\right)-c(1-\mu)\right)^2}{2(3-\mu)^2\mu T\overline{v}}$ is given in Lemma 2.

We first analyze the full market coverage case. Notice that $\mathcal{I}^C_{-i=B}$ decreases in u^0 as $\Pi^*(B, B)$ increases in u^0. Then $\mathcal{I}^C_{-i=B} \leq 0 \Leftrightarrow u^0 \geq \frac{c(1-\mu)}{(2-\mu)\mu} + T\overline{v}\left((3-\mu)\sqrt{\frac{2(\underline{v}+\overline{v})}{\overline{v}(2-\mu)(1-\mu)}} - 1\right)$. In the $\{B, B\}$ equilibrium, u^0 should satisfy $\max\{0, \frac{2c}{\mu} - \overline{v}T\} < u^0 < \overline{v}T(2-\mu) - \frac{c(1-\mu)}{\mu} + \frac{c(3-\mu)}{\mu}\frac{\underline{v}}{\underline{v}+\overline{v}}$, which contradicts the above condition. Thus, $\mathcal{I}^C_{-i=B}$ is always positive, which implies that $BR(B) = C$ and content marketing is a dominant strategy.

In the partial coverage case, $\mathcal{I}^C_{-i=B}$ increases in u^0. At $u^0 \equiv \underline{u}^0 = \max\{0, \frac{2c}{\mu} - \overline{v}T\}$, the minimum value $\mathcal{I}^C_{-i=B}(\underline{u}^0)$ decreases in μ and T. Hence, we can find cutoff values $\tilde{\mu}$ and $\tilde{T}$ such that $\mathcal{I}^C_{-i=B}(\underline{u}^0, \tilde{\mu}) = 0$ and $\mathcal{I}^C_{-i=B}(\underline{u}^0, \tilde{T}) = 0$. And then for $\mu < \tilde{\mu}$ and $T < \tilde{T}$, $\mathcal{I}^C_{-i=B}$ is always positive. Otherwise, there exists u^0 so that $\mathcal{I}^C_{-i=B} < 0$.

Price Commitment. Recall from Proposition 2, the content marketer's equilibrium expected payoff is $\Pi^*_1(C, B) = \frac{\left(c+T(\overline{v}+\underline{v})(2-3\mu+\mu^2)\right)^2}{9T(\overline{v}+\underline{v})(1-\mu)\mu}$, which decreases in μ and independent of u^0. Applying the same analysis as in the partial coverage case, we can find cutoff values $\tilde{\mu}$ and $\tilde{T}$ such that $\mathcal{I}^C_{-i=B}(\underline{u}^0, \tilde{\mu}) = 0$ and $\mathcal{I}^C_{-i=B}(\underline{u}^0, \tilde{T}) = 0$. And then for $\mu < \tilde{\mu}$ and $T < \tilde{T}$, $\mathcal{I}^C_{-i=B}$ is always positive. Otherwise, there exists u^0 so that $\mathcal{I}^C_{-i=B} < 0$. $\square$

Proof of Proposition 8.

Proof. Suppose $T\left[\mu - M^{-1}\left(\frac{c}{T}\right)\right] < \Delta_p < c$. We first analyze demand for each firm in this case. Consumers with $t \leq \hat{t}\left(\Delta_p\right)$ donot search, so they constitute the "fresh demand" for firm 1:

$$\text{fresh demand for F1} = \Pr\left(t \leq \hat{t}\left(\Delta_p\right)\right) = \frac{\hat{t}\left(\Delta_p\right)}{T}.$$

Those consumers with $t > \hat{t}\left(\Delta_p\right)$ go further search. Some of them may buy from firm 2 if they receive some high idiosyncratic values for the non-focal brand, i.e., buying product 2 if $u^0 + t\mu - p_1 < u^0 + tv^b_{i2} - p_2 \Leftrightarrow v^b_{i2} > \mu - \frac{\Delta_p}{t}$. These constitute fresh demand for firm 2:

$$\text{fresh demand for F2} = \frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} \Pr\left(v^b_{i2} > \mu - \frac{\Delta_p}{t}\right) dt = \frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} \left[1 - F\left(\mu - \frac{\Delta_p}{t}\right)\right] dt.$$

All the other consumers go back to purchase product 1, which contribute to returning demand for firm 1:

$$\text{returning demand for F1} = \frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} \Pr\left(v^b_{i2} < \mu - \frac{\Delta_p}{t}\right) dt = \frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} F\left(\mu - \frac{\Delta_p}{t}\right) dt.$$

Putting together, the demand for each firm is given by

$$\begin{aligned} Q_1 &= \underbrace{\frac{\hat{t}\left(\Delta_p\right)}{T}}_{\text{fresh demand}} + \underbrace{\frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} F\left(\mu - \frac{\Delta_p}{t}\right) dt}_{\text{returning demand}}, \\ Q_2 &= \frac{1}{T}\int_{t=\hat{t}(\Delta_p)}^{T} \left[1 - F\left(\mu - \frac{\Delta_p}{t}\right)\right] dt. \end{aligned}$$

Profit optimizaztion for each firm is characterized, as usual, by maximizing their respective profit

$\pi_i = p_i Q_i$ while taking its rival's pricing as given. Hence, two FOCs yield

$$\max_{p_1} p_1 Q_1 \quad \Rightarrow \quad Q_1 + p_1 \frac{dQ_1}{d\Delta_p} = 0,$$
$$\max_{p_2} p_2 Q_2 \quad \Rightarrow \quad Q_2 - p_2 \frac{dQ_2}{d\Delta_p} = 0.$$

Since $Q_1 + Q_2 = 1$, the two first-order conditions can be rewritten as

$$\begin{aligned} 1 - Q_2 - p_1 \frac{dQ_2}{d\Delta_p} &= 0, \\ Q_2 - p_2 \frac{dQ_2}{d\Delta_p} &= 0. \end{aligned}$$

Taking difference yields

$$1 - 2Q_2 = \Delta_p \frac{dQ_2}{d\Delta_p}, \tag{18}$$

where

$$\frac{dQ_2}{d\Delta_p} = -\frac{1}{T} \left[1 - F\left(\mu - \frac{\Delta_p}{\hat{t}\left(\Delta_p\right)} \right) \right] \frac{d\widehat{t}\left(\Delta_p\right)}{d\Delta_p} + \frac{1}{T} \int_{t=\hat{t}(\Delta_p)}^{T} \frac{1}{t} f\left(\mu - \frac{\Delta_p}{t} \right) dt. \tag{19}$$

Recall that $\hat{t}\left(\Delta_p\right)$ is defined by

$$H\left(\hat{t}\left(\Delta_p\right)\right) = 0.$$

Differentiating wrt Δ_p:

$$H'\left(\hat{t}\left(\Delta_p\right)\right) \frac{d\hat{t}\left(\Delta_p\right)}{d\Delta_p} + \left. \frac{\partial H\left(t\right)}{\partial \Delta_p} \right|_{t=\hat{t}(\Delta_p)} = 0,$$

where

$$\begin{aligned}
H'\left(\hat{t}\left(\Delta_p\right)\right) &= \int_{\mu-\frac{\Delta_p}{\hat{t}(\Delta_p)}}^{\overline{v}} \left[1-F\left(x\right)\right]dx - \left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)}\right)\right]\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)} \\
&= \frac{H\left(\hat{t}\left(\Delta_p\right)\right)+c}{\hat{t}\left(\Delta_p\right)} - \left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)}\right)\right]\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)} \\
&= \frac{c}{\hat{t}\left(\Delta_p\right)} - \left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)}\right)\right]\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)} \\
&= \frac{1}{\hat{t}\left(\Delta_p\right)}\left\{c-\Delta_p\left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)}\right)\right]\right\}, \text{ and} \\
\left.\frac{\partial H\left(t\right)}{\partial \Delta_p}\right|_{t=\hat{t}(\Delta_p)} &= 1-F\left(\mu-\frac{\Delta_p}{\hat{t}\left(\Delta_p\right)}\right).
\end{aligned}$$

Therefore,

$$\frac{d\hat{t}\left(\Delta_p\right)}{d\Delta_p} = -\frac{\widehat{t}\left(\Delta_p\right)\left[1-F\left(\mu-\frac{\Delta_p}{\widehat{t}(\Delta_p)}\right)\right]}{c-\Delta_p\left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}(\Delta_p)}\right)\right]} = -\frac{(+)}{(+)} < 0,$$

where the denominator is strictly positive because $\Delta_p < c$.

Inserting back into (19) and rearranging,

$$\frac{dQ_2}{d\Delta_p} = \frac{1}{T}\frac{\hat{t}\left(\Delta_p\right)\left[1-F\left(\mu-\frac{\Delta_p}{\hat{t}(\Delta_p)}\right)\right]^2}{c-\Delta_p\left[1-F\left(\mu-\frac{\Delta}{\hat{t}(\Delta_p)}\right)\right]} + \frac{1}{T}\int_{t=\hat{t}(\Delta)}^{T}\frac{1}{t}f\left(\mu-\frac{\Delta_p}{t}\right)dt > 0. \tag{20}$$

Then, (18) implies that

$$1-2Q_2 = \Delta_p\frac{dQ_2}{d\Delta_p} \Rightarrow \begin{cases} Q_2 < \frac{1}{2} < Q_1 \text{ if } \Delta_p = p_1 - p_2 > 0 \\ Q_2 > \frac{1}{2} > Q_1 \text{ if } \Delta_p = p_1 - p_2 < 0 \end{cases}. \tag{21}$$

Now we explore when we can have $\Delta_p > 0$ ($p_1 > p_2$) in the interior solution. Denote the interior solution implied in (18) by Δ_p^*. We should expect both profit functions are concave in each firm's own price. Given this, $\frac{d\pi_1}{d\Delta_p} = Q_1 + p_1\frac{dQ_1}{d\Delta_p}$, as a function of Δ_p, must be decreasing in Δ_p. It follows that

$$\begin{aligned}
\left.\left(Q_1 + p_1 \frac{dQ_1}{d\Delta_p}\right)\right|_{\Delta_p=0} &> 0 \Leftrightarrow \Delta_p^* > 0, \\
\left.\left(Q_1 + p_1 \frac{dQ_1}{d\Delta_p}\right)\right|_{\Delta_p=0} &< 0 \Leftrightarrow \Delta_p^* < 0, \\
\left.\left(Q_2 - p_2 \frac{dQ_2}{d\Delta_p}\right)\right|_{\Delta_p=0} &< 0 \Leftrightarrow \Delta_p^* > 0, \\
\left.\left(Q_2 - p_2 \frac{dQ_2}{d\Delta_p}\right)\right|_{\Delta_p=0} &> 0 \Leftrightarrow \Delta_p^* < 0.
\end{aligned}$$

Therefore, $\Delta_p^* > 0$ iff

$$\left.\left(1 - 2Q_2 - \Delta_p \frac{dQ_2}{d\Delta_p}\right)\right|_{\Delta_p=0} = \left.(1-2Q_2)\right|_{\Delta_p=0} > 0 \Leftrightarrow Q_2\left(\Delta_p = 0\right) < \frac{1}{2}.$$

Here,

$$Q_2\left(\Delta_p = 0\right) = \frac{1}{T}\int_{t=\hat{t}(0)}^{T} \left[1 - F(\mu)\right] dt = \left[1 - F(\mu)\right]\left(1 - \frac{\hat{t}(0)}{T}\right),$$

where

$$\hat{t}(0) = \frac{c}{\int_{\mu}^{\overline{v}} \left[1 - F(x)\right] dx}.$$

Therefore, firm 1 can charge a higher price than firm 2 (i.e., $\Delta_p^* > 0$) if:

$$\begin{aligned}
Q_2\left(\Delta_p = 0\right) &< \frac{1}{2} \Leftrightarrow \left[1 - F(\mu)\right]\left(1 - \frac{c}{T}\frac{1}{\int_{\mu}^{\overline{v}} \left[1 - F(x)\right] dx}\right) < \frac{1}{2} \\
&\Leftrightarrow \frac{c}{T} > \left(1 - \frac{1}{2}\frac{1}{1 - F(\mu)}\right)\int_{\mu}^{\overline{v}} \left[1 - F(x)\right] dx = q^*.
\end{aligned}$$

First-stage Search If $T\left[\mu - M^{-1}\left(\frac{c}{T}\right)\right] < \Delta_p < c$, consumer with $t \leq \hat{t}\left(\Delta_p\right)$ forgo search and buy brand 1; whereas consumer with $t > \hat{t}\left(\Delta_p\right)$ search and return to buy brand 1 if $u^0 + t\mu - p_1 > u^0 + tv_{i2}^b - p_2 \Rightarrow v_{i2}^b < \mu - \frac{\Delta_p}{t}$. For consumers with $t \leq \hat{t}\left(\Delta_p\right)$, the condition to have them search firm 1 is the same as the first case, which is $p_1 \leq \left(u^0 - v_o\right) - \frac{c}{\lambda}$. For consumers with $t > \hat{t}\left(\Delta_p\right)$, their expected utility from searching firm 1 is given by

$$
\begin{aligned}
& \lambda\left[u^0+\mathbf{E}_{v_{i2}^b}\max\left\{t\mu-p_1,tv_{i2}^b-p_2\right\}-c\right]+(1-\lambda)\,v_o-c \\
= \quad & \lambda u^0+\lambda\left[(t\mu-p_1)\,F\left(\mu-\frac{\Delta_p}{t}\right)+\int_{\mu-\frac{\Delta_p}{t}}^{\overline{v}}\left(tv_{i2}^b-p_2\right)dF\left(v_{i2}^b\right)\right]-\lambda c+(1-\lambda)\,v_o-c \\
= \quad & \lambda u^0+\lambda\left[t\mu-p_1+t\int_{\mu-\frac{\Delta_p}{t}}^{\overline{v}}\left[1-F\left(x\right)\right]dx\right]-\lambda c+(1-\lambda)\,v_o-c \\
= \quad & \lambda u^0+\lambda\left[t\mu-p_1+H\left(t\right)\right]+(1-\lambda)\,v_o-c
\end{aligned}
$$

which should be no less than v_o, the utility from taking the status quo, for any $t>\hat{t}\left(\Delta_p\right)$. Hence, consumers with $t>\hat{t}\left(\Delta_p\right)$ search firm 1 if and only if $\lambda u^0+\lambda\left[t\mu-p_1+H\left(t\right)\right]+(1-\lambda)\,v_o-c>v_o$ hold for any $t>\hat{t}\left(\Delta_p\right)$, which can be further simplified to

$$
p_1<\left(u^0-v_o\right)-\frac{c}{\lambda}+\min_{t>\hat{t}(\Delta_p)}\left[t\mu+H\left(t\right)\right]. \tag{22}
$$

When $\Delta_p<0$, $H'\left(t\right)>0$ for any t. When $\Delta_p>0$, $H'\left(t\right)>0$ for $t>\hat{t}\left(\Delta_p\right)$. Therefore, $H\left(t\right)$ is strictly increasing in t for $t>\hat{t}\left(\Delta_p\right)$, so $\min_{t>\hat{t}(\Delta_p)}\left[t\mu+H\left(t\right)\right]=\hat{t}\left(\Delta_p\right)\mu+H\left(\hat{t}\left(\Delta_p\right)\right)=\hat{t}\left(\Delta_p\right)\mu$. As long as $p_1\leq\left(u^0-v_o\right)-\frac{c}{\lambda}$ holds, condition (22) is already guaranteed.

In summary, when equilibrium prices satisfy $\Delta_p\leq c$, all consumers engage in the first-round search so long as $u^0-v_o>p_1+\frac{c}{\lambda}$. first-round search so long as $u^0-v_o>p_2+\left(c+\frac{c}{\lambda}\right)$. $\square$

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