When to Target Customers? Retention Management using Constrained Dynamic Off-Policy Policy Learning

Ryuya Ko[†] Kosuke Uetake[‡] Kohei Yata[§] Ryosuke Okada[¶]

Abstract

We propose a method to learn personalized customer retention management strategies when customers' intentions to purchase evolve over time. Working with a Japanese online platform, we first implement a large-scale randomized experiment, in which coupons are randomly sent to first-time buyers at different times. The experimental data allow us to estimate personalized dynamic retention policies using off-policy policy learning methods. We extend the existing methods by allowing non-Markovian strategies and by considering inter-temporal budget constraints. Our offline evaluation results show that the optimal dynamic policy is more cost-effective than baseline policies. Finally, we test the optimal policy online to confirm its performance.

1 Introduction

Managing customer retention is a central part of customer relationship management (CRM). In particular, it is well-known in both academia and practice that the attrition rate at the early stage of the customer life cycle is quite high (e.g., Fader and Hardie, 2007, 2010; Kim, 2022),

First draft: June 18, 2021. This version: March 15, 2023. We thank Tat Chan, Dean Eckles, Arun Gopalakrishnan, Xueming Luo, Puneet Manchanda, Harikesh Nair, Shohei Sakaguchi, Jiwoong Shin, K Sudhir, Raph Thomadsen, and Duncan Simester for helpful comments. We also thank the seminar and conference participants at AI Conference at Harvard Business School, CODE, Johns Hopkins, KDD, Marketing Science Conference, Marketing Dynamics Conference, RecSys, Temple, Washington University of Saint Louis. The paper's results are our own and do not represent the company's views.

[†]University of Texas at Austin ryuya.ko@utexas.edu
[‡]Yale School of Management. kosuke.uetake@yale.edu
[§]University of Wisconsin-Madison. yata@wisc.edu
[¶]ZOZO Inc.

but attrition tends to be smaller as they repeat purchases over time. Hence, it is essential to increase the retention of first-time buyers to increase the overall customer lifetime value.[1]

To improve the retention of first-time buyers, companies usually make special treatment for those first-time buyers. For example, as many popular marketing strategy books suggest, it is common for companies to send a special thank-you message to first-time buyers. It is also commonly observed that companies send them a coupon so that they are going to make another purchase with the coupon.[2]

In the era of Big Data, data-driven CRM strategies can improve the retention of first-time buyers through targeting and personalization. Many e-commerce companies collect not only basic demographic or socioeconomic information about customers but also a large number of variables on customer behavior prior to and after the first purchase such as browsing history and installing an app. These massive, fine-grained data allow companies to design personalized targeting policies for sending messages or coupons, which enhance customer retention and facilitate overall CRM performance.

Although the availability of large-scale detailed data has helped develop policy learning methods for personalized data-driven marketing strategies on how and whom to target, there are some challenges. The first challenge is that retention management inherently involves dynamics in that customers' behavior and interest may change over time. For example, it is well-known that the retention probability tends to decline as the length of time since the customer's first-time purchase increases, as known as a "recency trap" (Neslin, Taylor, Grantham, and McNeil, 2013). Given the empirical pattern, it may be too early to send retention incentives right after the first purchase, or it can be too late to do so a month after the purchase. Thus, timing matters. Retention management strategies in the existing papers, however, mainly focus on a static setting where a company sees customers at one point in time and decides how to treat customers right away. It is crucial to incorporate dynamics in policy learning for retention management.

The second challenge is that many marketing campaigns have a budget ceiling as compa-

[1]Throughout the paper, we use the "retention" instead of "attrition" even though our application is not contractual setting as subscription businesses. Our method can be applied to both contractual and non-contractual settings.

[2]There are a lot of blog posts and articles online on how to send appreciation emails to first-time buyers. For example, https://www.drip.com/blog/customer-appreciation-emails.

nies do not want to waste their resource too much. A common rule of thumb is that the marketing budget for B2B companies is between 2 and 5% of their revenue and between 5 and 10% for B2C companies.[3] Hence, marketing managers need to efficiently select marketing strategies within the budget. Even if there are no explicit budget constraints, companies may have other types of constraints due to privacy concerns, for example. Such fairness constraints restrict what kind of policies companies can implement in practice.[4] When the resource is limited such that only a subset of customers can receive retention incentives, it is even more important to determine a "cost"-effective personalized targeting strategy.

In this paper, we aim for overcoming these challenges. Specifically, we propose an empirical framework to create dynamic personalized targeting strategies for retention management when there exist budget constraints (or other constraints that limit what fraction of the customers can be treated). We then apply the methodology to the experimental data from a leading Japanese e-commerce company, which contain detailed customer information for personalization. Since the company serves more than 100,000 first-time buyers every month, even a small improvement in the cost-effectiveness of a personalized retention strategy can have large impacts on the company's profits.

Our method follows the recent literature on the dynamic treatment regime (DTR) used in statistics, medicine, computer science, and economics. Dynamic treatment regimes, also called adaptive treatment strategies, are sequential decision rules that adapt over time to the changing status of each customer. A DTR, for example, determines whether or not to offer a coupon as a function of state variables such as past purchase history, past browsing history, past responses to emails, etc. This dynamic nature makes the estimation of DTRs challenging because the treatment assignment today should have both direct effects on current outcomes and indirect effects on future treatment assignments and outcomes by affecting future state variables. We extend the methodologies to develop DTRs by explicitly incorporating budget constraints, which makes the dynamic optimization problem even more complicated because the current treatment assignment affects future treatment assignments and outcomes not only through dynamic customer behavior but also through inter-temporal budget constraints. In-

[3] See, e.g., https://www.bdc.ca/en/articles-tools/marketing-sales-export/marketing/what-average-marketing-budget-for-small-business

[4] See, e.g., Kallus and Zhou (2021) for refefence.

corporating dynamics and constraints, however, makes the targeting policy more practical and allows us to develop cost-effective dynamic targeting policies.

Our approach to estimating the optimal DTR given the budget constraint builds on both Q-learning (e.g., Murphy, 2003) and Backward Outcome Weighted Learning (BOWL) (e.g., Zhao, Zeng, Laber, and Kosorok, 2015). Q-learning is an approximate dynamic programming procedure that estimates the optimal DTR by maximizing the conditional expectation of the cumulative sum of the current and future payoffs given the current state and action, known as a Q-function. We model the Q-function by various machine learning methods such as Random Forest, Gradient Boosting Machine (GBM), and LASSO. Those machine learning methods are attractive as they allow one to avoid fully parameterizing underlying data-generating processes. We then maximize the Q-functions to obtain the optimal DTRs. By contrast, the BOWL approach reframes the estimation of an optimal DTR as a sequential weighted classification problem, starting from the very end period. This reformulation is helpful as one can use existing classification algorithms such as Support Vector Machine (SVM) and Logistic Regression, which are readily available in popular programming languages. Hence, BOWL is a direct approach to learning the optimal DTR. We provide the characterization of the optimal DTR given the budget constraint and propose an algorithm to derive the optimal policy.

In this paper, we consider two models. The baseline model has two actions, where a customer receives an appreciation email only or a coupon in addition to the email. The baseline model allows us to estimate the value of financial incentives for retention. The extended model has more than two actions, where a customer now may not receive an appreciation email. This model can separately estimate the effect of sending appreciation messages and financial incentives.

To estimate the optimal DTR, we use experimental data from a large e-commerce platform in Japan. Our randomized experimental design guarantees the "sequential ignorability" condition, meaning treatment assignments are independent of potential future outcomes, conditional on the history up to the current period. The experimental data allow us to consistently estimate the optimal DTR.

Applying the method to the experimental data, we find that the optimal DTRs can achieve significantly higher retention than the company's status-quo policy of just sending apprecia-

tion emails, and are also more cost-effective than alternative policies. For our baseline model, the return on advertising (coupon) spending (ROAS), the company's main KPI (Key Performance Indicator), can be as high as 430% with our constrained BOWL, which is significantly higher than other policies. For the extension model, the constrained BOWL achieves almost 900% in ROAS and outperforms the non-targeted policies by a lot. Moreover, due to personalization, our results reveal that it is not always optimal to send incentives right after the first purchase. For some users, it may be more beneficial to send incentives later.

Lastly, based on our offline evaluation results, the company tested our optimal DTR online. The online results confirm our offline evaluation results. For the baseline model, the constrained BOWL earns 550% in ROAS, and the extension earns 306% in ROAS and hence achieves better performance than other compared models. Now the company implements our algorithm for their retention management.

Our approach is practically important for many marketing managers. For companies with limited resources such as start-ups, they may not have enough marketing resources to give coupons or financial incentives to many customers. Since we leverage one experimental dataset to learn optimal DTRs under different scenarios, even start-ups can set up the data and derive dynamic personalized policies. Also, our approach is useful for marketing managers in bigger institutions. That is because our cost-effective approach allows the companies to save a significant amount of resources for other marketing campaigns than retention management. Moreover, since our approach derives the optimal strategies within the budget constraint, it is much easier for the company to manage expenses. As Ascarza, Ross, and Hardie (2021) point out, many companies do not effectively use their customer data to achieve the companies' goals within their budget. Our method can contribute to those companies' objectives.

The rest of the paper is organized as follows. In Section 2, we review the related literature in marketing, economics, and computer science. Section 3 introduces the background of the setup we study and Section 4 describes the experimental design and provides some descriptive statistics with a special emphasis on consumer heterogeneity. Section 5 explains the model and our solution approaches. In Section 6, we discuss the results of the off-policy policy evaluation and show the online evaluation results. Lastly, Section 7 concludes.

2 Related Literature

Our paper is related to the marketing literature on proactive churn/attrition management.[5] Churn management is one of the key priorities for most businesses as customer retention is a major component of customer lifetime value (CLV) and hence a cornerstone of successful CRM (Ascarza, Neslin, Netzer, et al., 2018; Ascarza, Ross, and Hardie, 2021). As summarized by Neslin, Taylor, Grantham, and McNeil (2013), a popular industry practice for data-driven retention management is to flag risky customers who are likely to churn using behavioral and demographic variables. By predicting customer attrition before they decide, firms can proactively communicate with those who are at risk of churning to convince them to stay (e.g., Neslin, Gupta, Kamakura, Lu, and Mason, 2006). Recently, a few papers go beyond estimating the churn prediction model and targeting customers with the highest risk. Ascarza (2018) points out that it is not effective to target customers with a higher chance of predicted retention as they may not be responsive to marketing interventions and propose to determine targeting based on uplift. Lemmens and Gupta (2020) note that it is crucial to take the financial impact of a retention intervention based on CLV into account. Our approach adds to the literature by developing a method that estimates a (counterfactual) dynamic targeting policy that maximizes retention given budget constraints.[6]

In marketing, a growing number of papers propose methods for learning optimal personalized policies.[7] Hitsch and Misra (2018) propose a policy learning method based on the estimation of conditional average treatment effect (CATE) using k-nearest neighbors. Simester, Timoshenko, and Zoumpoulis (2020) consider an efficient policy evaluation method when existing

[5] Although "churn" is used in the context of contractural settings such as subsciption business and "attrition" is used in non-contractural settings, we use those words almost interchangeably as our method can be used in both cases.

[6] For a review of the literature, see, e.g., Ascarza, Neslin, Netzer, et al. (2018).

[7] In economics, there is a strand of papers on policy learning. Athey and Wager (2021), for example, develop methods for policy learning that can work with observational data. Their method can be used to optimize various types of treatment allocation such as binary treatments and infinitesimal changes in continuous treatments. Kitagawa and Tetenov (2018) study policy learning in a nonparametric setting and obtain regret bounds for the Empirical Welfare Maximization (EWM) method. Bhattacharya and Dupas (2012) and Sun (2021) study how to incorporate a certain policy constraint in a static setting of policy learning. Sakaguchi (2022) extends the EWM method to a dynamic setting with inter-temporal constraints. Our method is different from Sakaguchi (2022) in that our approach transforms the constrained problem into a sequence of unconstrained problems, which allows us to use the existing approaches such as Q-learning and BOWL. Moreover, our method is computationally light and can accommodate a large number of state variables.

policies and new policies are compared. Yoganarasimhan, Barzegary, and Pani (2022) estimate CATE with different machine learning models and compare the performance of targeting policies constructed based on these models. Yang, Eckles, Dhillon, and Aral (2022) consider how to derive targeting policies when an outcome of interest is observed only in the long term. To do so, they use the idea of statistical surrogacy (Athey, Chetty, Imbens, and Kang, 2019) and optimal policy learning.

Recently, a few papers in marketing explicitly examine dynamic personalized policies. The most closely related paper is Liu (2022). Liu (2022) develops a dynamic reinforcement learning algorithm for dynamic pricing in e-commerce and finds that the dynamic reference price effect plays an important role. Also, Wang, Li, Luo, and Wang (2022) propose a sequential targeting strategy using deep reinforcement learning, and apply it to the experimental data from a mobile app promotion campaign. In a different context, Kar, Swaminathan, and Albuquerque (2015) and Rafieian (2022) examine dynamic ad targeting and show the importance of the intertemporal externalities that the dynamic targeting ad allocation policy should take into account. We add to this brand-new literature by proposing a method to estimate dynamic cost-effective policies, which explicitly take constraints into account. Moreover, we differ from those papers as we consider a non-Markov dynamic setup.[8] Finally, our paper investigates a substantively different topic on retention management.

3 Background

The company we work with is one of the largest e-commerce platforms in Japan (about $2,300 million transaction volume in 2021) that mainly sells apparel products for young female customers. There are more than 1,500 retailers on the platform and more than 8 million active users. Those users purchase products from the retailers through the platform's website or through the mobile app. Those apparel brands and retailers basically delegate marketing activities to the platform company that also manages inventories at the company's warehouse and handles shipping directly to consumers. The platform charges a certain fee to retailers for each trans-

[8] In the literature of personalized medicine, DTRs are developed in non-Markov settings to adaptively select clinical treatments in response to the factors emerging over time (e.g., Murphy, 2003; Murphy, Lynch, Oslin, McKay, and TenHave, 2007; Zhao, Kosorok, and Zeng, 2009; Zhang, Tsiatis, Laber, and Davidian, 2013, to name a few). Another related paper is Nie, Brunskill, and Wager (2021), which study when to start treatment and learn the optimal policy.

action, but retailers do not have to bear any costs for keeping their products in the platform's inventory warehouse.

The company is interested in increasing the retention of the customers who have just made their first purchase as those first-time buyers' retention rate is lower than other customers who have experienced multiple purchases before. Also, based on the company's examination, the contribution of the second purchase to the customer lifetime value (LTV) is much larger than that of the third/fourth/fifth purchases. Hence, it is critical to have first-time buyers make a second purchase for improving the overall retention rate and LTV of customers. Before we started the project, as suggested by many popular marketing strategy books, the company sends "thank you" messages to first-time buyers to show appreciation and to provide them with some useful information about the platform such as rankings and the company's app. The purpose of such emails is to convince them to buy another item and also to collect customer behavior information.

Although the company knows that the current appreciation emails to first-time buyers have some positive impacts on retention, the company would like to increase the retention rate even further by providing financial incentives.

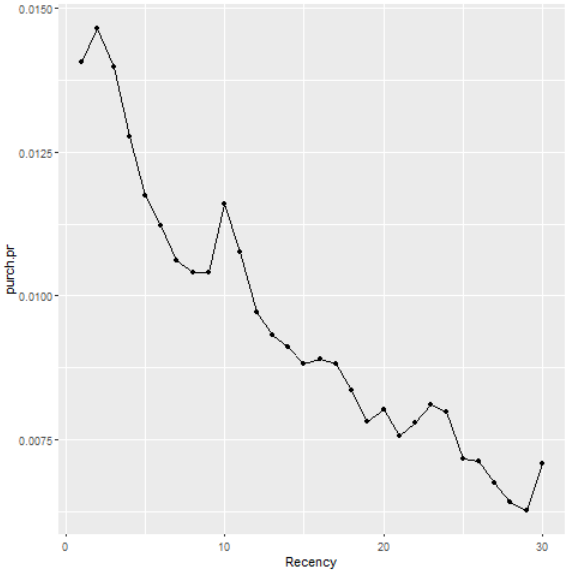
As to providing incentives, the timing can be crucial. It has been well-known in marketing that there is a recency trap as pointed out by Neslin, Taylor, Grantham, and McNeil (2013), i.e., the retention probability becomes smaller as time passes since the last purchase increases. Figure 1 shows the relationship between the days since the last purchase and the average purchase probability in our application. As in the previous papers, the average purchase probability generally declines as recency increases.[9]

The declining pattern in the figure has implications on when to target customers. It may imply that waiting too long after the first purchase may not be optimal as the intention to buy is already too low, while sending incentives right away may not be optimal either, as they may purchase for the second time even without such incentives.

Moreover, the company has a practical constraint in its marketing strategy. While the company's general objective function is to maximize the retention rate (and hence LTV), in particular for first-time buyers, the company hesitates to distribute too generous incentives. That

[9] Previous papers consider the recency trap in longer terms than ours. Although we are not allowed to disclose the average purchase intervals, customers in the platform we study buy more frequently than other prior studies.

Figure 1: Recency and Retention Probability



Note: The graph shows the time (days) since the first purchase on the horizontal axis and purchase probabilities, i.e., retention probabilities on the vertical axis. The purchase probabilities are calculated from the experimental data we use for the estimation of the optimal DTR. The company sends incentives 2 days, 10 days, and 30 days after the customer's first purchase, giving hikes around those days.

is because some customers purchase other items even without coupons and some customers may use coupons to buy low-margin items. This concern leads us to study how to design cost-effective personalized strategies.

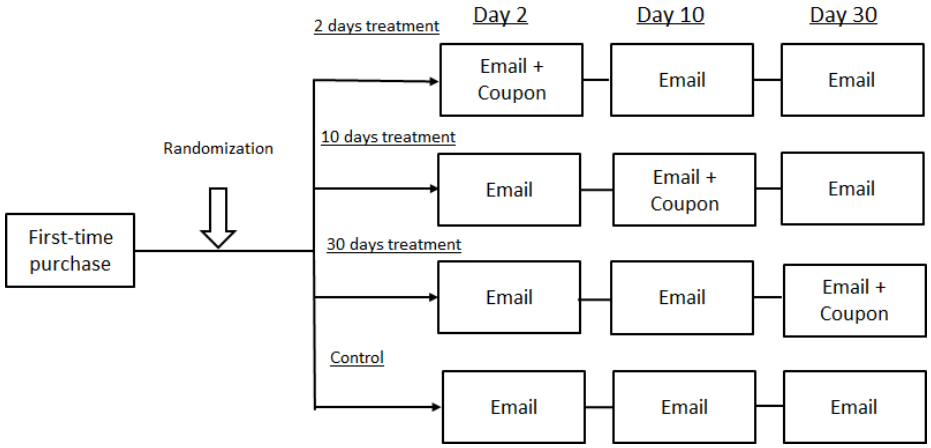
4 Experimental Design and Data

4.1 Experimental Design

In this section, we describe the design of the randomized experiment that we will use for estimating the optimal DTR later. The company conducted two experiments: one for the baseline model we will explain in Section 5.2, and another for the extension in Section 5.4. Below, we discuss the first experiment to save space. In Online Appendix C.1, we discuss the experimental design of the second experiment, which is an extension of the first experiment.

The company conducted the first experiment from September 2020 to December 2020. As explained above, the company's focus is on the customers who have just made their first pur-

Figure 2: Experimental Design



Note: The figure shows the experimental design of the first experiment where there are two actions, email only and email and coupon.

chase, and there are about 150,000-200,000 first-time buyers per month during the experiment period.[10]First-time buyers need to provide demographic information and contact information when they make a purchase. We randomly pick about 70% of those first-time buyers for the experiment.

In the first experiment, customers in the control group receive only appreciation emails, each of which contains information about how to use the platform or a generic ranking of the items sold on the platform. They receive appreciation emails at three different times: 2 days, 10 days, and 30 days after the first purchase. The customers in the treatment group receive the financial incentive of 1,000 points (about $10) along with the appreciation emails.[11] There is no expiration date for those points.[12] Since providing coupons for first-time buyers was never implemented on the platform before, users did not know if they could receive coupons before they made a purchase.

In the treatment group, there are three sub-groups depending on when the customers receive the incentive (Figure 2). The customers in the 2-day treatment group receive the incentive 2 days after the first purchase, and they receive only the appreciation emails 10 days and 30 days after the first purchase. The 10-day treatment and the 30-day treatment groups are similarly

[10]The NDA does not allow us to report the exact number of customers involved in the experiment.

[11]The company was not willing to offer %-off coupons because such a coupon can be very costly for the platform if users purchase expensive items. The design of coupons is beyond our research question.

[12]For an impact of coupon expiration dates, see, e.g., Inman and McAlister (1994).

Table 1: Treatment Allocation

	Percent
Control	39.76%
2 day	19.15%
10 day	20.50%
30 day	20.59%

Note: The table reports the fraction of each group including three treatment groups and the control group. Due to the NDA, we cannot report the exact number of observations in each group.

defined. Hence, each first-time buyer in the treatment groups receives the incentive at most once during the experimental period. Note that in Section 5, we consider the optimal DTR in this class of the strategy. Hence, our experimental design covers all possible combinations of treatments to estimate the optimal DTR.

The randomization is done at the user level. We randomly assign each user right after their first purchase to one of the four groups: the control, the 2-day treatment, the 10-day treatment, or the 30-day treatment. Since we randomize the assignment right after each first-time purchase, we do not have to re-randomize at each of the three possible times. Table 1 reports the fraction of first-time buyers allocated to each of the treatment conditions and the control group. We have about 40% of users for the control and 20% of users for each treatment condition.

Table 2 reports the summary statistics of the subset of the variables we use. In the first two columns, we show the mean and standard deviation of each variable for the customers in the 2-day treatment. Similarly, the third and fourth columns are for the customers in the 10-day treatment, the fifth and sixth columns for the customers in the 30-day treatment, and the seventh and eighth columns for the customers in the control group.

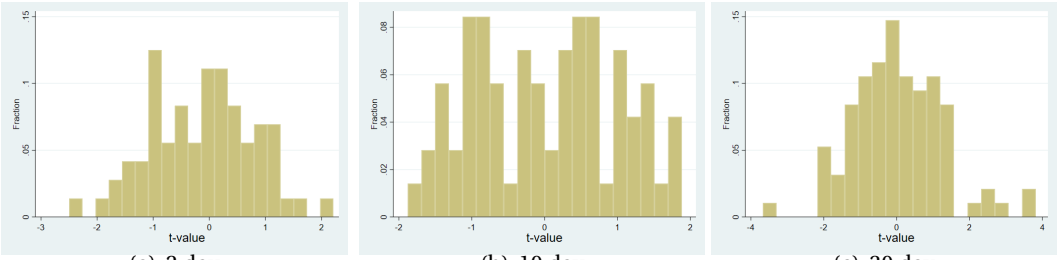
For user demographics, about 63% of users are female and the average age of users is 30. Also, we do not see any significant difference across conditions. The number of items purchased (quantity) and total spending (sales) on the user's first purchase occasion are on average 1.7 and 6,200 JPY, respectively, across conditions. Also, users apply about 555 points for the

Table 2: Summary Statistics

Variable	2 day		10 day		30 day		Control	
	mean	sd	mean	sd	mean	sd	mean	sd
Female	0.631	0.483	0.631	0.483	0.629	0.483	0.634	0.482
Age	30.24	12.69	30.31	12.76	30.23	12.68	30.25	12.67
Quantity: first buy	1.699	1.520	1.697	1.482	1.691	1.588	1.697	1.493
Sales: first buy	8065.3	8213.6	8127.3	8405.5	8102.7	8361.3	8089.4	8157.9
Points used: first buy	557.5	756.9	556.5	760.2	556.5	762.0	560.3	860.9
# of sessions/day (pre 1st buy)	0.697	2.016	0.698	2.032	0.703	2.049	0.704	2.047
# of PVs/day (pre delivery)	15.08	31.78	15.38	33.02	15.24	32.73	15.23	32.64
# of favorites/day (2-10 day)	0.207	1.036	0.178	1.119	0.179	1.047	0.180	1.018
# of messages (10-30 day)	32.21	37.81	32.31	37.16	31.60	37.28	31.61	37.29

Note: The first six columns report the mean and standard deviation of each variable for each of the three treatment groups. The last two columns are for the control group. The number of observations in each treatment is not reported due to the non-disclosure agreement (NDA).

Figure 3: Balance Check



(a) 2 day (b) 10 day (c) 30 day

Note: Each figure plots the histogram of t-values for a mean-comparison test between each treatment and the control group.

first purchase across all conditions.[13]

The last four rows in the table contain a subset of the user behavior variables. Since we use more than 100 behavioral variables based on the user access data (sessions, page views (PVs), favorites, messages sent, etc.), it is not possible to report the summary statistics of all of those variables. Hence, we *randomly* choose four variables and show their summary statistics in the table.[14]

Next, we check the balance between the treatment groups and the control group to make sure if the randomization is done accurately. Since we use more than 100 variables for the estimation of DTRs, we do not report the mean comparison of each variable in a table. Rather,

[13]Most users receive 500 to 2000 points when they sign up for the platform. They can use them immediately, especially for their first purchase.

[14]Note that we do not mean those reported variables are more important than other variables. We simply choose those variables at random. This is required by the company for confidentiality issues.

in Figure 3, we show the histograms of the t-values for the mean comparison test between the treatment and control groups for each variable used in the estimation of DTRs. As the graph shows, there is no significant difference in mean between the control and treatment groups as the t-value is located between -2 and 2 for most of the variables.

In terms of the outcome, we consider the second-time purchase (i.e., retention) within 8 weeks after the first purchase or within 12 weeks, and we estimate the optimal DTR to maximize them as the objective function as it is the company's main KPI. We also investigate the effects of the derived optimal DTRs on long-term outcomes to see if there are any inter-temporal substitution effects by simply shifting the timing of purchases.

4.2 Average Treatment Effect

Before we explain the estimation of DTR with the experiment data, to understand the data in more detail, we examine the average treatment effects in this section. Specifically, we estimate the following simple linear regression:

$$y_{it} = \beta_{0t} + \beta_{1t} D_{it} + \varepsilon_{it},$$

where y_{it} is the outcome of interest, D_{it} is the dummy for the treatment, and $t = 2, 10, 30$. We run three separate regressions for each treatment group. For each regression, we use the treatment group which receives a coupon on day t and the control group which receives no financial incentives at any time. As the outcome variable, we consider three variables: whether or not user i makes the second purchase (retention), the total purchase (sales) in JPY, and the number of items purchased (quantity).[15]

We estimate the treatment effects with the outcomes measured 8 weeks and 12 weeks after the first purchase (not after each treatment). As Panel (A) of Table 3 shows, the 2-day treatment increases the retention rate by 2.4%, the 10-day treatment by 1.4%, and the 30-day treatment by 1.5%. Panel (B) reports similar treatment effects for the outcomes within 12 weeks. Note that the 10-day treatment effect is 1.1% for the 12-week retention, which is smaller than the one for

[15]The estimation sample includes the users who make the second purchase before they receive incentives. We run another set of regressions where we remove all users who make a purchase before they receive coupons. The results are virtually the same as Table 3.

Table 3: Average Treatment Effect

	Retention	Sales	Quantity
Panel (A): 8 Week Outcomes			
2 day	0.024***	229.6**	0.084***
	(0.002)	(101.8)	(0.022)
10 day	0.014***	-33.0	0.025
	(0.002)	(84.7)	(0.018)
30 day	0.015***	149.9	0.045**
	(0.002)	(94.7)	(0.019)
Panel (B): 12 Week Outcomes			
2 day	0.024***	385.0***	0.056***
	(0.002)	(157.0)	(0.010)
10 day	0.011***	-90.3	0.012***
	(0.002)	(102.8)	(0.009)
30 day	0.015***	144.0	0.024***
	(0.002)	(95.6)	(0.007)

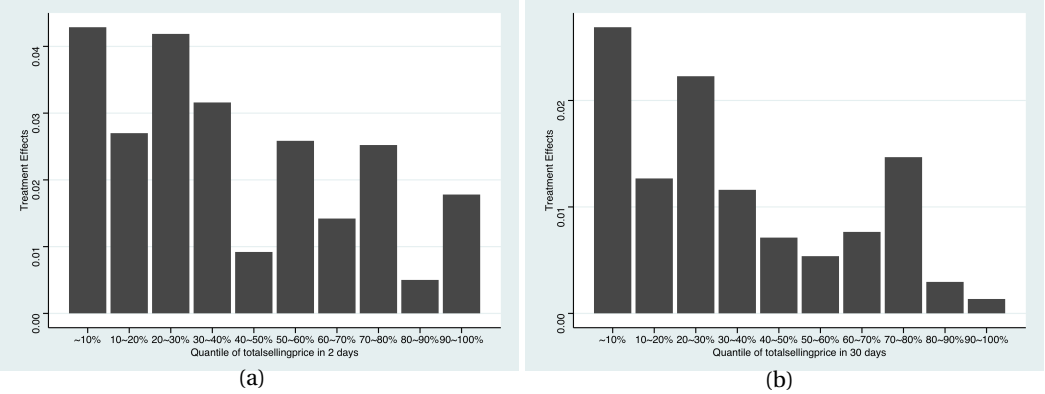
Note: The first column reports the treatment effects on whether a customer makes any purchases within 8 weeks (Panel (A)) or 12 weeks (Panel (B)) since their first purchase. The second column reports the treatment effects on total sales and the third column reports the treatment effects on the number of items purchased. The table does not report the constants as the constants reveal the baseline retention rates, baseline sales, and baseline quantity, which is prohibited due to the NDA.

the 8-week retention. This is because the users in the control group eventually increase their purchases over time and hence the treatment effect could decrease. This suggests that there is an inter-temporal substitution within a user.

In terms of total spending, the 2-day treatment increases the total spending by 230 JPY ($2.3) in 8 weeks, while the 10-day and 30-day treatment effects on total spending are not statistically significant. The third column reports the treatment effects on the number of items purchased. We find that the treatment effects are mostly positive for the 2-, 10-, and 30-day treatments. Even for the 30-day treatment, the effect is positive and statistically significant. In sum, the optimal uniform strategy is to send incentives 2 days after the first purchase. We consider it as the baseline strategy.

Although the average treatment effects are informative to see which treatment is more effective than others on average, it may not necessarily be the case that the 2-day treatment is

Figure 4: Heterogeneous Treatment Effects



(a) (b)

Note: The figures show the average treatment effects (2-day treatment for Panel (a) and 30-day treatment for Panel (b)) against the total spending for the first purchase. The total spending is split into deciles.

optimal for all users. To see how much heterogeneity exists, we now estimate the heterogeneous treatment effects by estimating interaction effects.[16]. Note that our main purpose is not to investigate the mechanism behind heterogeneity, but to merely show heterogeneity in treatment effects across different customer segments. More specifically, we estimate the following regression with interaction terms

$$y_{it} = \beta_{0t} + \beta_{1t} D_{it} + \beta_{2t} X_{it} + \beta_{3t}(D_{it} \times X_{it}) + \varepsilon_{it},$$

where X_{it} is a variable indicating customer segments. Figure 4 shows the bar charts of the average treatment effects of the 2-day treatment (Panel (a)) and 30-day treatment (Panel (b)) for each decile of the total spending for the first purchase. For both treatments, it seems that the average treatment effects generally decrease as the total spending increases. Although our main objective is not to identify the mechanism behind the decreasing patterns, we want to emphasize that treatment effects vary a lot across users. Significant heterogeneity of the treatment effects can be found for other conditioning variables.

One might wonder if those who receive the 2-day treatment get other marketing interventions triggered by the 2-day treatment, and hence the estimated average treatment effects are confounded by the impacts of other marketing campaigns. The company confirmed that there

[16]We report more results in Online Appendix B. We examined more variables to see heterogeneous effects. For most of the variables, there is significant heterogeneity.

are no major campaigns that target the first-time buyers in our experimental sample. Moreover, in our optimization in the next section, the state variables include those marketing campaigns. Therefore, our final results do not suffer from this issue.

Lastly, in Online Appendix C.2, we report the average treatment effects for the second experiment, where users may not receive an appreciation email. Note that the second experiment allows us to separately estimate the effect of a coupon and the one of an appreciation email. We find that both the coupon and the email have positive treatment effects, while the coupon has a larger impact on outcomes.

5 Dynamic Treatment Regime with Constraints

In this section, we propose our strategy for estimating the optimal dynamic treatment regime (DTR) when there exist inter-temporal budget constraints. We start with discussing the model setup and then explain how to estimate the optimal DTR.

5.1 Setup

We study the following dynamic environment. In our model, there are three periods ($t = 1, 2, 3$) and $X_t \in \mathscr{X}_t$ denotes the state variables in period t. Note that our methodology can be extended to the case with more than three periods. The state variables include the user demographic information, past purchase information, browsing information, responses to past marketing activities, etc. Our methodology can easily accommodate a large number of state variables. In our application, we use more than 100 variables for X_t. The company can choose an action every period from the action set $A_t \in \mathscr{A}_t$. In our baseline application, the company either sends an incentive or not in addition to the appreciation message, and we denote $\mathscr{A}_t = \{0, 1\}$, where option 1 is sending the incentive. Later, we will extend the model to the case where the action set includes more than two options. Lastly, the final outcome is $Y \in \mathbb{R}$, which will realize after period 3. In our case, we consider customer retention within two or three months after the first purchase.[17]

We introduce the history $H_t \in \mathscr{H}_t$ to describe the summary of the events up to period t.

[17] It is straightforward to apply our method to the case where the outcome is the discounted cumulative reward, e.g., $\sum_t \beta^t Y_t$, where Y_t is the period-t outcome and β is the discount factor.

More precisely, we define $H_1 = X_1$, $H_2 = (X_1, A_1, X_2)$, and $H_3 = (X_1, A_1, X_2, A_2, X_3)$. Note that the history includes not only the state variables but also the actions taken up to that period. The initial state distribution and state transition distributions are denoted by $P_{X_1}(x_1)$, $P_{X_2}(x_2|h_1, a_1)$, and $P_{X_3}(x_3|h_2, a_2)$. The final outcome is then determined by the entire history up to period 3, h_3, and the action taken in period 3, a_3, i.e., $P_Y(y|h_3, a_3)$. Thus, the model is non-Markov and hence the state transition depends not only on the previous state but also on the entire history. This implies that we may not be able to use off-the-shelf reinforcement learning algorithms that typically assume a Markov environment.

With this dynamic environment, a *dynamic treatment regime* (DTR) is a sequence of decision rules $\mathbf{d} = (d_1, d_2, d_3)$, where $d_t : \mathscr{H}_t \to \Delta(\mathscr{A}_t)$ is a function that maps the history up to time t into a probability distribution over actions. We use $d_t(a_t|h_t)$ to denote the probability of choosing action a_t given history h_t. If the decision rule is deterministic and chooses an action with probability one, then we use $d_t(h_t)$ to denote the action.

When a DTR $\mathbf{d}$ is applied to the above dynamic environment, the trajectory $H = (X_1, A_1, X_2, A_2, X_3, A_3, Y)$ is generated by the following process.

1. $X_1 \sim P_{X_1}$, $A_1 \sim d_1(\cdot|H_1)$
2. $X_2 \sim P_{X_2}(\cdot|H_1, A_1)$, $A_2 \sim d_2(\cdot|H_2)$
3. $X_3 \sim P_{X_3}(\cdot|H_2, A_2)$, $A_3 \sim d_3(\cdot|H_3)$
4. $Y \sim P_Y(\cdot|H_3, A_3)$

That is, the DTR generates the state variables and actions for each period, which determine the final outcome of interest Y. The resulting distribution of H is given by

$$P_{X_1}(x_1)d_1(a_1|h_1)P_{X_2}(x_2|h_1, a_1)d_2(a_2|h_2)P_{X_3}(x_3|h_2, a_2)d_3(a_3|H_3)P_Y(y|h_3, a_3).$$

We denote the distribution by $P_{\mathbf{d}}$ and the expectation with respect to $P_{\mathbf{d}}$ by $E_{\mathbf{d}}$.

Suppose that we observe data $\{H^{(i)}\}_{i=1}^n$ of n individuals, where trajectories $H^{(1)}, ..., H^{(n)}$ are independently and identically generated by the above process with some DTR $\mathbf{d}^0$. In our empirical setting, $\mathbf{d}^0$ is the random assignment policy used by the experiment and is known to

us.[18] We denote the distribution of the observed trajectory $H^{(i)}$ by P and the expectation with respect to P by E.

Our learning objective is to use the data $\{H^{(i)}\}_{i=1}^n$ to choose a DTR that maximizes the expected value of Y over a (possibly constrained) class of DTRs $\mathscr{D}$:

$$\mathbf{d}^* \in \arg\max_{\mathbf{d}\in\mathscr{D}} E_{\mathbf{d}}[Y].$$

In order to identify the value $E_{\mathbf{d}}[Y]$ for every DTR $\mathbf{d} \in \mathscr{D}$, we assume the following overlap condition.

Assumption 1. *For all* $\mathbf{d} \in \mathscr{D}$, t, *and* $(a_t, h_t) \in \mathscr{A}_t \times \mathscr{H}_t$, *if* $d_t(a_t|h_t) > 0$, *then* $d_t^0(a_t|h_t) > 0$.

This assumption ensures that in the data generated by DTR $\mathbf{d}^0$, we observe every trajectory $(x_1, a_1, x_2, a_2, x_3, a_3, y)$ that can be realized under candidate DTRs $\mathbf{d} \in \mathscr{D}$.

5.2 Estimation Approaches without Constraints

We consider two approaches to estimating the optimal dynamic treatment strategy: Q-learning and Backward Outcome Weighted Learning (BOWL). We first discuss the two approaches without any constraints and then extend to the case with constraints in the next subsection.

5.2.1 Q-Learning

The first approach to estimating the optimal DTR is to use Q-learning (e.g., Murphy, 2003; Murphy, Lynch, Oslin, McKay, and TenHave, 2007). This approach first models the Q-function, which is the conditional mean function of the outcome given the history and current action, by a parametric, semiparametric, or nonparametric function, and then derives the optimal DTR by maximizing the estimated Q-function over actions. In our application, we try several machine learning models such as LASSO, Random Forest, and GBM to estimate Q-functions.[19]

[18] Note that our experiment randomly divided the customers into four groups who would receive different action profiles (A_1, A_2, A_3) prior to the first treatment. Implementing this randomization design is equivalent to implementing a sequential randomization design (or a DTR) that randomly determines each customer's treatment prior to each period based on the past treatment profile.

[19] In principle, one can apply deep reinforcement learning (DRL) methods to estimate the Q-function. There are a few challenges in our application. First, typically DRL is applied to the case with a longer time horizon. Since our application has only three periods, the benefits of using DRL may be limited. Second, typical applications of DRL consider a Markov situation, while our application considers a non-Markov situation. Hence, widely-used DRL algorithms may not work.

In the dynamic setup we consider, we can define the Q-function sequentially as follows:

$$\begin{aligned}
Q_3(h_3, a_3) &= E[Y|H_3 = h_3, A_3 = a_3], \\
Q_2(h_2, a_2; d_3) &= E_{d_3}[Y|H_2 = h_2, A_2 = a_2] \\
&= E\left[\sum_{a_3 \in \mathscr{A}_3} d_3(a_3|H_3) Q_3(H_3, a_3)|H_2 = h_2, A_2 = a_2\right], \\
Q_1(h_1, a_1; d_2, d_3) &= E_{d_2, d_3}[Y|H_1 = h_1, A_1 = a_1] \\
&= E\left[\sum_{a_2 \in \mathscr{A}_2} d_2(a_2|H_2) Q_2(H_2, a_2; d_3)|H_1 = h_1, A_1 = a_1\right].
\end{aligned}$$

Letting $\mathscr{D}$ be the set of all DTRs, a (deterministic) optimal DTR $\mathbf{d}^*$ that maximizes $E_{\mathbf{d}}[Y]$ can be solved backward:

$$\begin{aligned}
d_3^*(h_3) &\in \arg\max_{a_3 \in \mathscr{A}_3} Q_3(h_3, a_3), \\
d_2^*(h_2) &\in \arg\max_{a_2 \in \mathscr{A}_2} Q_2(h_2, a_2; d_3^*), \\
d_1^*(h_1) &\in \arg\max_{a_1 \in \mathscr{A}_1} Q_1(h_1, a_1; d_2^*, d_3^*),
\end{aligned}$$

and the optimal Q-function (Q_1^*, Q_2^*, Q_3^*) is given by

$$Q_3^*(h_3, a_3) = Q_3(h_3, a_3), \;\; Q_2^*(h_2, a_2) = Q_2(h_2, a_2; d_3^*), \;\; Q_1^*(h_1, a_1) = Q_1(h_1, a_1; d_2^*, d_3^*).$$

We can solve this problem by the following Q-learning algorithm that estimates Q-functions backward and obtains the period-t decision rule maximizing the period-t Q-function.

Algorithm 1 (Q-Learning)**.**

1. Conduct a (possibly nonparametric or semiparametric) regression of Y on H_3 and A_3. Let $\hat{Q}_3$ be the estimated function, and let $\hat{d}_3(h_3) \in \arg\max_{a_3 \in \mathscr{A}_3} \hat{Q}_3(h_3, a_3)$.

2. Conduct a regression of $\hat{Q}_3(H_3, \hat{d}_3(H_3))$ on H_2 and A_2. Let $\hat{Q}_2$ be the estimated function, and let $\hat{d}_2(h_2) \in \arg\max_{a_2 \in \mathscr{A}_2} \hat{Q}_2(h_2, a_2)$.

3. Conduct a regression of $\hat{Q}_2(H_2, \hat{d}_2(H_2))$ on H_1 and A_1. Let $\hat{Q}_1$ be the estimated function, and let $\hat{d}_1(h_1) \in \arg\max_{a_1 \in \mathcal{A}_1} \hat{Q}_1(h_1, a_1)$.

Steps 2 and 3 are motivated by the following observation:

$$Q_2(H_2, A_2; \hat{d}_3) = E[Q_3(H_3, \hat{d}_3(H_3))|H_2, A_2], \;\; Q_1(H_1, A_1; \hat{d}_2, \hat{d}_3) = E[Q_2(H_2, \hat{d}_2(H_2); \hat{d}_3)|H_1, A_1].$$

Since $Q_3(\cdot,\cdot)$ and $Q_2(\cdot,\cdot;\hat{d}_3)$ are unobserved in data, Steps 2 and 3 use the estimated values from the previous step $\hat{Q}_3(\cdot,\cdot)$ and $\hat{Q}_2(\cdot,\cdot)$, respectively, as the outcome variable in the regressions.[20] Since the standard Q-learning chooses the action with the largest estimated value at each step, it may result in the overestimation of the Q-value for the chosen action, i.e., $\hat{Q}_t(H_t, \hat{d}_t(H_t))$ (e.g., Lan, Pan, Fyshe, and White, 2020), which affects the optimization in the next step. To reduce the bias, we do sample splitting for Q-learning. That is, at each step we split the sample into two subsamples, use one subsample to choose the action, and use the other to estimate the Q-value for the chosen action.[21]

5.2.2 Backward Outcome Weighted Learning (BOWL)

Another approach to estimating the optimal DTR we consider is the Outcome Weighted Learning (OWL) algorithm proposed by Zhao, Zeng, Rush, and Kosorok (2012) for a static setting. A key observation of Zhao, Zeng, Rush, and Kosorok (2012) is to formulate optimal policy learning as a weighted classification problem. This transformation allows one to use existing classification algorithms to learn the optimal DTR. Zhao, Zeng, Laber, and Kosorok (2015) extend the idea of OWL to dynamic settings, called the Backward Outcome Weighted Learning (BOWL) algorithm. In BOWL, a classification problem is sequentially solved from the last period going backward toward the first period. Since BOWL directly maximizes over a class of DTRs a nonparametric estimator of the expected long-term outcome, it is different than regression-based methods such as Q-learning, which indirectly attempts such maximization and relies on the correct model specification.

[20] An alternative approach is to use the importance-weighted outcomes $\frac{Y\mathbf{1}[A_3=\hat{d}_3(H_3)]}{d_3^0(A_3|H_3)}$ and $\frac{Y\mathbf{1}[(A_2,A_3)=(\hat{d}_2(H_2),\hat{d}_3(H_3))]}{d_2^0(A_2|H_2)d_3^0(A_3|H_3)}$ instead of the estimated Q-values $\hat{Q}_3(H_3, \hat{d}_3(H_3))$ and $\hat{Q}_2(H_2, \hat{d}_2(H_2))$ in Steps 2 and 3, respectively.

[21] This method is similar to the idea of the "honest" approach proposed by Athey and Imbens (2016).

To see how it works, observe first that if $\mathbf{d}$ is deterministic, we can write

$$E_{d_3}[Y|H_3 = h_3] = E\left[\frac{Y\mathbf{1}[A_3 = d_3(H_3)]}{d_3^0(A_3|H_3)}|H_3 = h_3\right].$$

Again, $\mathbf{d^0} = (d_1^0, d_2^0, d_3^0)$ in our empirical setting is the random allocation rule of the treatments, which is known. Note that the expectation on the right-hand side is taken with respect to the distribution under $\mathbf{d^0}$.[22] This way of adjusting distributions is called the inverse probability weighting or importance sampling technique.

If there are no constraints in the class of DTRs, the optimal deterministic rule in period 3 then satisfies $d_3^*(h_3) \in \arg\max_{a_3 \in \mathscr{A}_3} E\left[\frac{Y\mathbf{1}[A_3=a_3]}{d_3^0(A_3|H_3)}|H_3 = h_3\right]$ for every $h_3 \in \mathscr{H}_3$. Therefore, the optimal rule d_3^* is a solution to the following maximization problem:

$$d_3^* \in \arg\max_{d_3} E\left[\frac{Y\mathbf{1}[A_3 = d_3(H_3)]}{d_3^0(A_3|H_3)}\right].$$

We can similarly obtain the optimal rules d_2^* and d_1^* for periods 2 and 1. Since the action is binary and hence $\mathbf{1}[A_3 = d_3(H_3)] = 1 - \mathbf{1}[A_3 \neq d_3(H_3)]$, the above maximization problem is equivalent to the following *minimization* problem:

$$d_3^* \in \arg\min_{d_3} E\left[\frac{Y\mathbf{1}[A_3 \neq d_3(H_3)]}{d_3^0(A_3|H_3)}\right].$$

The objective function in this problem can be viewed as a weighted misclassification error. We can define similar minimization problems for $t = 2$ and 1.[23] Hence, we can obtain the optimal DTR by sequentially applying any classification algorithms, which have been extensively

[22] For $t = 1$ and 2, we can write as follows:

$$E_{d_2,d_3}[Y|H_2 = h_2] = E\left[\frac{Y\mathbf{1}[(A_2,A_3) = (d_2(H_2), d_3(H_3))]}{d_2^0(A_2|H_2)d_3^0(A_3|H_3)}|H_2 = h_2\right]$$
$$E_{\mathbf{d}}[Y|H_1 = h_1] = E\left[\frac{Y\mathbf{1}[(A_1,A_2,A_3) = (d_1(H_1), d_2(H_2), d_3(H_3))]}{d_1^0(A_1|H_1)d_2^0(A_2|H_2)d_3^0(A_3|H_3)}|H_1 = h_1\right].$$

[23] The minimization problems for period 2 and period 1 are as follows:

$$d_2^* \in \arg\min_{d_2} E\left[\frac{Y\mathbf{1}[A_3 = d_3^*(H_3)]}{d_2^0(A_2|H_2)d_3^0(A_3|H_3)}\mathbf{1}[A_2 \neq d_2(H_2)]\right],$$
$$d_1^* \in \arg\min_{d_1} E\left[\frac{Y\mathbf{1}[(A_2,A_3) = (d_2^*(H_2), d_3^*(H_3))]}{d_1^0(A_1|H_1)d_2^0(A_2|H_2)d_3^0(A_3|H_3)}\mathbf{1}[A_1 \neq d_1(H_1)]\right].$$

studied in machine learning. Thus, we can easily apply existing machine learning algorithms to estimate the optimal DTR.[24]

BOWL estimates the solution by minimizing the sample analog of the objective function (plus the penalty) backward for $t = 3, 2, 1$, which is basically solving a classification problem sequentially. Zhao, Zeng, Laber, and Kosorok (2015) show that the obtained DTRs are consistent, and provide finite sample bounds for the errors using the estimated rules. Their simulation results suggest that BOWL outperforms Q-learning.

5.3 Estimation Approaches with Constraints

In this subsection, we extend the estimation of the optimal DTR to the case where constraints are imposed. For expositional simplicity, we start with a single-period problem and then extend to a multi-period case.

5.3.1 Single-period Optimization

To get the basic idea of how we deal with constraints, we first consider the single-period constrained maximization problem:

$$\max_d E_d[Y] \text{ s.t. } E_d[C] \leq B,$$

where C is the cost variable, which is generated together with the outcome Y, and B is the per-person budget. In our empirical context, C is the coupon amount that the customer uses, and the company has a budget ceiling of B on how much can be spent for each customer. We can also consider capacity constraints. For example, if the company has a limited number of coupons, we can denote the constraint as $E_d[A] \leq B$, where A is a dummy variable indicating whether the customer receives a coupon, and $B \in [0,1]$ is the capacity on the fraction of customers who can receive a coupon.

Under certain conditions, it is straightforward to show that an optimal rule is a threshold strategy. That is, letting $\beta(h)$ and $\gamma(h)$ denote the conditional average treatment effects (CATEs)

[24] Since the function is non-convex and discontinuous, Zhao, Zeng, Laber, and Kosorok (2015) propose to replace the 0–1 loss $\mathbf{1}[A_t \neq d_t(H_t)]$ with a hinge loss $\phi(A_t f_t(H_t)) = \max(1 - A_t f_t(H_t), 0)$, where $f_t : \mathscr{H}_t \to \mathbb{R}$ is the decision function so that $d_t(h_t) = \text{sign}(f_t(h_t))$. Zhao, Zeng, Laber, and Kosorok (2015) show that the change in the loss function does not change the solution to the minimization problems.

on the outcome and cost, respectively, i.e., $\beta(h) = E[Y|H = h, A = 1] - E[Y|H = h, A = 0]$ and $\gamma(h) = E[C|H = h, A = 1] - E[C|H = h, A = 0]$ for $h \in \mathscr{H}$, the following rule is optimal:

$$d^*(h) = \mathbf{1}[\beta(h) \geq \lambda^* \gamma(h)], \tag{5.1}$$

where λ^* satisfies $E_{d^*}[C] = B$ (see Online Appendix A.1). If $\gamma(h) > 0$ for all h, the rule is written as $d^*(h) = \mathbf{1}[\beta(h)/\gamma(h) \geq \lambda^*]$. In other words, the optimal rule assigns the treatment to individuals from those with the highest ratios between the CATEs on the outcome and cost until the capacity is reached.

Now, we consider how to solve the constrained optimization problem. The key idea to solve the constrained optimization problem is to transform the problem into an unconstrained problem by introducing a shadow price as follows.

$$d_\lambda \in \arg\max_d E_d[Y - \lambda C]$$

for given $\lambda \geq 0$. It is straightforward to see that an optimal policy targets a customer if and only if $E[Y - \lambda C|H = h, A = 1] - E[Y - \lambda C|H = h, A = 0] \geq 0$. The solution is

$$\begin{aligned} d_\lambda(h) &= \mathbf{1}[E[Y - \lambda C|H = h, A = 1] - E[Y - \lambda C|H = h, A = 0] \geq 0] \\ &= \mathbf{1}[\beta(h) \geq \lambda \gamma(h)]. \end{aligned} \tag{5.2}$$

Comparing equation (5.1) with equation (5.2), observe that $d^* = d_{\lambda^*}$. In other words, the solution to the unconstrained problem with $\lambda = \lambda^*$ indeed can yield the optimal constrained rule. Note that $E_{d_\lambda}[C] = E[E[C|H, A = 0] + \mathbf{1}[\beta(H) \geq \lambda \gamma(H)]\gamma(H)]$ is decreasing in λ. Using this property, it is easy to obtain the optimal constrained rule by the following modified OWL algorithm.

Algorithm 2 (Constrained OWL)**.**

1. For given $\lambda \geq 0$, apply a single-period OWL with the outcome set to $Y - \lambda C$ to obtain d_λ.
2. Find λ^* such that $E_{d_{\lambda^*}}[C] = B$.

For the second step, we need to evaluate the expected cost under the policy d_λ. We can use existing off-policy policy evaluation (OPE) methods to estimate $E_{d_\lambda}[C]$. We will explain our

evaluation method in detail in Section 5.6.

It is straightforward to find λ^* in the second step thanks to the monotonicity in λ. Thus, we can effectively convert the original constrained optimization problem to a loop through unconstrained optimization problems.

Although the algorithm above uses OWL for deriving the optimal policy, we can also use Q-learning to estimate it. To do so, we can simply use the Q-learning approach in the first step to obtain d_λ.

5.3.2 Multi-period Optimization

We extend the above idea to the problem with multiple periods. For convenience, we describe the method in a two-period setup, but the model can be easily extended to more than two periods, accommodating our empirical application with three periods. We consider the following constrained maximization problem:

$$\max_{(d_1,d_2)} E_{d_1,d_2}[Y] \text{ s.t. } E_{d_1,d_2}[C] \leq B, \tag{5.3}$$

where B is the per-person budget. Note that the budget constraint is not period-specific, but an inter-temporal one. Similarly to the static problem, we can show the following proposition.

Proposition 1. *Under suitable conditions, there exists* $(\lambda_1^*, \lambda_2^*)$ *such that the following threshold strategy is a solution to the problem (5.3):*

$$d_2^*(h_2) = \mathbf{1}[\beta_2(h_2) \geq \lambda_2^* \gamma_2(h_2)], \tag{5.4}$$

$$d_1^*(h_1) = \mathbf{1}[\beta_1(h_1; d_2^*) \geq \lambda_1^* \gamma_1(h_1; d_2^*)], \tag{5.5}$$

where $\beta_2(h_2) = Q_2(h_2, 1) - Q_2(h_2, 0)$, $\beta_1(h_1; d_2) = Q_1(h_1, 1; d_2) - Q_1(h_1, 0; d_2)$, $Q_2(h_2, a_2) = E[Y|H_2 = h_2, A_2 = a_2]$, *and* $Q_1(h_1, a_1; d_2) = E_{d_2}[Y|H_1 = h_1, A_1 = a_1]$. $\gamma_2(h_2)$ *and* $\gamma_1(h_1; d_2)$ *are analogously defined for the cost* C.

Proof. See Online Appendix A.2. ■

Again, as in the static optimization problem, we can solve the constrained problem by transforming the problem into an inter-temporal unconstrained problem by introducing shadow

values $\lambda = (\lambda_1, \lambda_2)$. To do so, we start by considering the following period-2 problem:

$$d_{2,\lambda_2} \in \arg\max_{d_2} E_{d_1^0, d_2}[Y - \lambda_2 C],$$

where d_1^0 is the data-generating rule. The solution to this problem is the same as the static one.

$$\begin{aligned} d_{2,\lambda_2}(h_2) &= \mathbf{1}[E[Y - \lambda_2 C | H_2 = h_2, A_2 = 1] \geq E[Y - \lambda_2 C | H_2 = h_2, A_2 = 0]] \\ &= \mathbf{1}[\beta_2(h_2) \geq \lambda_2 \gamma_2(h_2)]. \end{aligned} \tag{5.6}$$

Now, given d_{2,λ_2}, consider the following period-1 problem:

$$d_{1,\lambda_1,\lambda_2} \in \arg\max_{d_1} E_{d_1, d_{2,\lambda_2}}[Y - \lambda_1 C].$$

Since $E_{d_1, d_{2,\lambda_2}}[Y - \lambda_1 C] = E[E_{d_{2,\lambda_2}}[Y - \lambda_1 C | H_1, A_1 = d_1(H_1)]]$, the solution to this problem is given by

$$\begin{aligned} d_{1,\lambda_1,\lambda_2}(h_1) &= \mathbf{1}[E_{d_{2,\lambda_2}}[Y - \lambda_1 C | H_1 = h_1, A_1 = 1]] \geq E_{d_{2,\lambda_2}}[Y - \lambda_1 C | H_1 = h_1, A_1 = 0]] \\ &= \mathbf{1}[\beta_1(h_1; d_{2,\lambda_2}) \geq \lambda_1 \gamma_1(h_1; d_{2,\lambda_2})] \end{aligned} \tag{5.7}$$

Comparing equations (5.4)–(5.5) with equations (5.6)–(5.7), observe that $(d_1^*, d_2^*) = (d_{1,\lambda_1^*,\lambda_2^*}, d_{2,\lambda_2^*})$. Thus, the optimal threshold strategy can be obtained by correctly specifying the shadow costs λ_1 and λ_2. This motivates us to use the following modified BOWL algorithm.

Algorithm 3 (Constrained BOWL)**.**

1. For given $(\lambda_1, \lambda_2) \in \mathbb{R}_+^2$, apply BOWL with the outcomes for periods 1 and 2 set to $Y - \lambda_1 C$ and $Y - \lambda_2 C$ to obtain $\mathbf{d}_{\lambda_1,\lambda_2} = (d_{1,\lambda_1,\lambda_2}, d_{2,\lambda_2})$.

2. Find (λ_1, λ_2) that maximizes $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[Y]$ subject to the constraint that $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[C] \leq B$.

As in the static case, for the second step, we can use existing OPE methods to estimate $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[Y]$ and $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[C]$ and we will explain the evaluation methods in Section 5.6.

There are a few remarks in order. First, the algorithm can also be applied to Q-learning. In the first step, for a given pair of λ_1 and λ_2, we estimate the Q-functions for the outcome

and cost and plug the estimates into (5.6)–(5.7) to obtain $\mathbf{d}_{\lambda_1,\lambda_2} = (d_{1,\lambda_1,\lambda_2}, d_{2,\lambda_2})$. Then in the second step, we can search for the optimal shadow values. Second, unlike the static model, the monotonicity of $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[C]$ with respect to λ_1 and λ_2 may not hold. Hence, we have to use a grid search to find the optimal λ_1 and λ_2.

5.4 Extension: Multiple Actions

In the previous section, we consider the model where the platform's action set is a binary one, i.e., sending a message with or without a coupon. In this section, we extend the model to the case where the action space includes more than two options.

Note that it is straightforward to extend Q-learning to the case with multiple actions. We can estimate the Q-function and choose the action with the largest Q-value. Hence, we focus on how to extend BOWL. This extension is important for both methodological and substantive purposes. Methodologically, since BOWL is based on the idea of the sequential binary classification problem, it is not straightforward to extend the model to multiple action cases. Substantively, our extension considers three options: (i) sending an appreciation email with a coupon, (ii) sending an appreciation email only, and (iii) no emails. By comparing the effect of the first option with the effect of the second option, one can decompose the effect of incentives and the mere effect of messages.

We propose a method to extend the baseline BOWL approach. Intuitively, our algorithm identifies the optimal policy by running a series of round-robin tournaments among actions to determine the optimal DTR.

Algorithm 4 (Constrained BOWL with multiple actions)**.**

1. For given $(\lambda_1, \lambda_2) \in \mathbb{R}^2_+$, apply the following procedure backward for $t = 2, 1$:

   (a) For each pair $(a_t, a'_t) \in \mathscr{A}_t \times \mathscr{A}_t$ with $a_t \neq a'_t$, apply the binary single-period OWL to the subsample with $A_t \in \{a_t, a'_t\}$ to solve

$$\min_{d_2:\mathscr{H}_2 \to \{a_2, a'_2\}} E\left[\frac{(Y - \lambda_2 C)\mathbf{1}[A_2 \neq d_2(H_2)]}{d_2^0(A_2|H_2)} | A_2 \in \{a_2, a'_2\}\right]$$

when $t=2$ and

$$\min_{d_1:\mathcal{H}_1\to\{a_1,a_1'\}} E\left[\frac{(Y-\lambda_1 C)\mathbf{1}[A_2=d_{2,\lambda_2}(H_2)]}{d_1^0(A_1|H_1)d_2^0(A_2|H_2)}\mathbf{1}[A_1\neq d_1(H_1)]|A_1\in\{a_1,a_1'\}\right]$$

when $t=1$, where d_{2,λ_2} is the majority rule obtained in Step (b). This determines whether a_t is better than a_t' given each history h_t.

(b) Construct a rule that uses the majority rule to choose the optimal action among $\mathcal{A}_t$ for each history h_t. That is, we choose the action with the highest winning probability.[25]

Let $\mathbf{d}_{\lambda_1,\lambda_2}=(d_{1,\lambda_1,\lambda_2},d_{2,\lambda_2})$ denote the resulting DTR.

2. Find (λ_1,λ_2) that maximizes $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[Y]$ subject to the constraint that $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[C]\leq B$.

As in the baseline model with two actions, for the second step, we can use existing OPE methods to estimate $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[Y]$ and $E_{\mathbf{d}_{\lambda_1,\lambda_2}}[C]$.

5.5 Discussion

Before we show the results in the next section, we discuss some modeling considerations.

Comparison of Two Approaches Q-learning is an indirect approach in the sense that one needs to estimate the Q-function first and then derive the optimal strategy. By contrast, BOWL is a direct approach because one can derive the optimal strategy by solving the classification error minimization problem. Hence, there is no interim step.

The performance of the Q-learning approach depends on how well Q-functions are approximated. The approximation of the Q-function depends on how well the data cover the action-state space. In other words, if some states never arise in the data, then its value is extrapolated based on the functional form. Therefore, one may need sufficiently large experimental data.

The performance of the BOWL approach relies on classification accuracy. Although nonlinear classification algorithms such as deep neural networks have high classification accuracy,

[25] One may ask why we do not simply apply multi-class classification for the multiple action case. First, the transformation from the maximization problem to the minimization problem does not generally work for the multi-class case. Second, in many computer software packages available in R or Python, the multi-class classification is actually implemented by reducing the problem to multiple binary classification problems as we do.

they may take a long time when the state space is large. Hence, one may need to rely on a linear classification algorithm.

Dynamic Model One may wonder whether a dynamic model using dynamic programming is necessary given that our application is basically a three-period model. In other words, it may be possible to estimate the optimal strategy in period 1.

First, our proposed model can be easily extended to more than three periods. For problems with longer periods, it easily becomes untractable to derive the optimal policy without dynamic programming due to too many possible combinations of actions over time. Second, in our model, two components, not just one, create dynamics. The policy in the current period affects both the users' behaviors and the remaining budget, which in turn changes the optimal policy in the next period. It makes learning the static optimal policy with a budget constraint challenging. That is because a static problem needs to be provided a certain fixed budget ex-ante, but it is not straightforward to determine how to allocate a budget across time. Hence, it is appropriate to consider a dynamic model.

More precisely, a naive, static approach is to construct a policy d_t only using the data on (H_t, A_t, Y) separately for each period. This implicitly assumes that if the estimated policy $\hat{d}_t$ is actually implemented, the actions in the future periods would be determined by the experimental policy, not by the estimated policy $(\hat{d}_{t+1}, ..., \hat{d}_T)$. That is, it ignores the policy in future periods. Furthermore, it is hard to satisfy the inter-temporal budget constraint with this approach, since the policy in each period is separately optimized.

Another static approach is to construct a policy that determines the action profile $(A_1, ..., A_T)$ based on the initial state X_1, using the data on $(X_1, A_1, ..., A_T, Y)$. This approach wastes information on the updated states $(H_2, ..., H_T)$, potentially leading to worse performance than a dynamic approach.

Deep Reinforcement Learning This paper proposes two methods of estimating DTR with inter-temporal budget constraints using Q-learning and BOWL, but we do not consider deep reinforcement learning to estimate DTR. One reason that we do not do so is that there is, as long as we are aware, no explicit reinforcement learning algorithm that can accommodate inter-temporal constraints easily. Also, another reason is that there seem to be no established

(off-policy, offline) deep reinforcement learning algorithms that can be used for learning the optimal dynamic treatment regime under a non-Markov decision process. Since our setup of retention management for first-time buyers necessitates a non-Markov strategy, it is important to consider a non-stationary dynamic programming problem.

5.6 Evaluation Methods

To see if the proposed methods work, we use the method of "off-policy policy evaluation" (OPE). OPE evaluates the performance of hypothetical policies leveraging only offline log data, i.e., the experiment data in our case. To do so, we use the inverse probability weighting (IPW) approach (see, e.g., Precup, Sutton, and Singh, 2000).[26] Let $\mathbf{d} = (d_1, d_2, d_3)$ be a deterministic DTR that we are interested in evaluating (e.g., the estimated policy in the previous section). When there are three periods, IPW estimates the value of the DTR as follows:

$$\hat{V}_{\mathrm{IPW}} = \frac{1}{n}\sum_{i=1}^{n} \frac{\mathbf{1}[A_1^{(i)} = d_1(H_1^{(i)})]\mathbf{1}[A_2^{(i)} = d_2(H_2^{(i)})]\mathbf{1}[A_3^{(i)} = d_3(H_3^{(i)})]}{d_1^0(A_1^{(i)}|H_1^{(i)})d_2^0(A_2^{(i)}|H_2^{(i)})d_3^0(A_3^{(i)}|H_3^{(i)})} R^{(i)}, \tag{5.8}$$

where $R^{(i)}$ is either the outcome $Y^{(i)}$ or cost $C^{(i)}$. Hence, we can evaluate both $E_{\mathbf{d}}[Y]$ and $E_{\mathbf{d}}[C]$, which we need for Step 2 of Algorithms 2–4. The IPW-based evaluation has been used in existing papers such as Hitsch and Misra (2018) and Yoganarasimhan, Barzegary, and Pani (2022). When the treatment assignment probabilities are known, the IPW-based OPE is unbiased and consistent, but its variance tends to be large due to extreme weights.

6 Results

6.1 Results of the Baseline Model

We use the first experimental data to derive the optimal DTR under the budget constraint for the baseline model. Remember that the baseline model has two actions, sending a coupon with the appreciation email and sending only the email.

[26] We also tried the doubly robust (DR) approach as in Jiang and Li (2016), which may lead to lower variance. The results are qualitatively similar.

6.1.1 Comparison of Regression and Classification Algorithms

For Q-learning, estimation of Q-functions can be done with various machine learning algorithms such as LASSO, Random Forest, LightGBM (Light Gradient Boosting Machine), or deep learning.[27] For BOWL, any classification algorithm can be used to solve the weighted classification error minimization problem. For example, one can use SVM (Support Vector Machine), Logistic Regression, Random Forest, and SGDC (Stochastic Gradient Descent Classifier).[28]

To see which algorithm works better, we compare the performance of different algorithms without imposing any budget constraints. Based on the results, we choose to use SGDC for BOWL, and LASSO for Q-learning as those algorithms provide sufficiently high retention rates with smaller costs per user.[29] Those algorithms are attractive also in terms of computational time.

6.1.2 Offline Policy Evaluation Results

Table 4 summarizes the results of the OPE for BOWL, Q-learning, and the baseline policy. We consider three different levels of costs including 20, 30 JPY or ∞ (no constraint). The company selected those budgets. We learn policies subject to the budget constraint as well as a practical constraint that does not allow one to send coupons to users who have already made the second purchase or received coupons at earlier times.[30] Our baseline policy is the "Two-days-after" policy, which sends coupons to all users 2 days after the first purchase unless the user has not made the second purchase. Note that the average treatment effects we estimate in Section 4.2 indicate that it is optimal to send incentives 2 days after the first purchase if we do not personalize.[31]

For outcomes, we display the uplift in retention probabilities of the second and third pur-

[27] LightGBM is a tree-based algorithm using distributed gradient boosting framework. LightGBM runs faster than XGBoost, a well-known gradient boosting machine, while maintaining a high level of prediction accuracy.

[28] SGDC is a linear classifier including SVM, optimized by the stochastic gradient descent. We try using the L1 norm (as LASSO), the L2 norm (as Ridge regression), and combinations of them (the elastic net) for regularization.

[29] We try LASSO, Random Forest, and LightGBM for Q-learning and SGDC and Random Forest for BOWL. The results are available upon request.

[30] To incorporate the latter constraint into policy learning, we exclude those users from the data when estimating the optimal decision rule for each period.

[31] We also tried "static" targeting policies without budgets as discussed in Section 5.5. Since the results are similar to the baseline policy, we omit them here.

Table 4: Offline Uplift Performance (Baseline Model)

	2nd	3rd	Cost	ROAS
BOWL: SGDC, cost=20	1.16%	0.30%	29.97	401%
BOWL: SGDC, cost=30	1.46%	0.32%	35.81	388%
BOWL: SGDC, cost=∞	1.73%	0.32%	47.96	314%
Q-Learning: LASSO, cost=20	0.59%	0.32%	17.97	431%
Q-Learning: LASSO, cost=30	0.76%	0.05%	28.24	213%
Q-Learning: LASSO, cost=∞	1.86%	0.40%	48.95	342%
Two Days After	1.86%	0.40%	48.95	342%

Note: The table reports the uplift in the second and third purchases within three months after the first purchase, compared to the case with no incentives. Costs are measured in JPY and ROAS is the return on advertising spending.

chases relative to the no-incentive policy within three months after the first purchase.[32] Although we maximize the second-time purchase as the objective function, we also examine the effect on the third-time purchase. That is because if customers simply shift their purchase timing due to the incentive, the third-time purchase probability may go down. Such inter-temporal substitution may harm overall lifetime values. The table also reports the total cost of each policy and the return on advertising spending (ROAS), which is the company's key KPI as it is easy to compare the performance across different settings.[33]

The baseline policy of sending coupons 2 days after the purchase achieves an uplift of 1.86% for the second purchase, and the baseline policy costs 48.95 JPY. Without any budget constraint (i.e., setting cost = ∞), we find that the uplift in the second purchase is 1.73% for BOWL and 1.86% for Q-learning. In terms of costs, BOWL spends 47.96 JPY per user, which is 1 JPY less than Q-learning and Two-days-after. Taking both retention and cost into account, the ROAS of Q-learning and Two-days-after is 342%, while the ROAS of BOWL is 314%. Hence, in our application, without budget constraints, dynamic targeting policies based on BOWL and Q-learning do not outperform the baseline policy.

Next, when we impose budget constraints (20 or 30 JPY), both BOWL and Q-learning have lower retention rates than the baseline policy, but costs are much smaller. For BOWL, the up-

[32] Due to the NDA between the company and us, we are not allowed to disclose the baseline retention probabilities but are allowed to report only uplift probabilities.

[33] ROAS is computed as follows. For each of the second-time to fifth-time purchases, we compute the uplift in the purchase probability and the average spending conditional on the purchase. We then take the sum of the uplift times the conditional average spending over the second-time to fifth-time purchases. Finally, we divide it by the cost.

Table 5: Offline Treatment Allocation (Baseline Model)

	2 day	10 day	30 day
BOWL: SGDC, cost=20	30.61%	0.00%	46.23%
BOWL: SGDC, cost=30	47.74%	0.01%	32.36%
BOWL: SGDC, cost= ∞	87.80%	0.01%	0.74%
Q-Learning: LASSO, cost=20	29.46%	17.05%	14.07%
Q-Learning: LASSO, cost=30	36.44%	6.81%	20.63%
Q-Learning: LASSO, cost=∞	88.99%	0.00%	0.00%
Two Days After	88.99%	0.00%	0.00%

Note: The table reports the fraction of customers who receive the incentive 2 days, 10 days, or 30 days after their first purchase.

lift in the retention rate is 1.46% when the budget constraint is 30 JPY. The estimated cost of the optimal DTR is 35.8 JPY.[34] By contrast, for Q-learning, the uplift is just 0.76% and the cost is 28.2 JPY when the budget is set at 30 JPY. For the third-time purchase, BOWL achieves an uplift of 0.32%. Hence, the uplift is slightly lower for BOWL than the baseline (0.4%). Overall, BOWL policies generate greater ROAS than Q-learning (401% or 388% vs. 342%) and the baseline (342%). Thus, when there is a budget constraint, the constrained optimal DTR policies are more cost-effective than other policies.

In Table 5, we show how each policy allocates incentives across days. First of all, the baseline Two-days-after policy sends coupons to 89% of the first-time buyers on day 2 because 11% of users have already purchased before they receive coupons. We find that the optimal DTRs under BOWL and Q-learning send most of the coupons on day 2 when there is no budget ceiling. When there is a budget constraint, it is no longer optimal to send all coupons on day 2. Rather, the optimal policies, especially BOWL-based policies, send more coupons on day 30.

In sum, we find that the optimal DTR from BOWL under the budget constraint gives cost-effective personalized retention strategies compared to other approaches.

6.2 Results of Extension

Next, we report the results of the offline policy evaluation for the dynamic model with the option of not sending the message. This extension allows us to separately estimate the effect of

[34] The reason why the estimated costs are sometimes bigger than the budget is that we use a holdout sample for OPE.

Table 6: Offline Uplift Performance (Extension)

	2nd	3rd	Total Cost	ROAS
Constrained BOWL	8.04%	4.83%	116.21	877%
Two Days After	7.40%	4.61%	129.27	741%

Note: The table reports the uplift in the second and third purchases within three months after the first purchase. Total costs are measured in JPY and ROAS is the return on advertising spending.

sending appreciation emails and the effect of providing incentives on top of the email.

Since the extension of Q-learning to a multi-action setup is straightforward, and also BOWL mostly performs better than Q-learning in the baseline model, we compare the constrained BOWL with the baseline Two-days-after policy. The budget for the constrained BOWL is set at 130 JPY.[35]

Table 6 summarizes the OPE results. The first two columns report the uplift in retention probabilities of the second and third purchases relative to the no-email policy within the three months after the first purchase. The third column reports the per-user marketing cost including the cost of coupons and the cost of sending messages. The last column reports ROAS.

We find that the constrained BOWL achieves a greater chance of retention. In terms of the second purchase, the constrained BOWL leads to an uplift of 8.04% in the retention rate, which is higher than the baseline Two-days-after policy. We also investigate longer-term effects and find that even for the third purchase, the constrained BOWL achieves higher retention.

For the financial implications, the constrained BOWL leads to better performance. The total marketing cost of the constrained BOWL is 116.21 JPY, while that of the Two-days-after policy is 129.27 JPY, 11.2% higher. The cost difference translates into a huge difference in ROAS; ROAS for the constrained BOWL is 136% greater than ROAS for the Two-days-after policy. Thus, our approach generates more cost-effective targeting strategies to improve customer retention not only in the short run but also in the long run.

In Table 7, we describe how the optimal policy sends incentives at different timings. The constrained BOWL sends the incentives to 62.40% of users 2 days after the first purchase, 17.34% 10 days after, and 6.28% 30 days after. Hence, the optimal strategy assigns incentives to

[35]The budget for the second experiment is bigger than the first one because, after the first experiment, the platform tried to encourage users to spend more points.

Table 7: Offline Treatment Allocation (Extension)

	2 day	10 day	30 day
	Email with incentives		
Constrained BOWL	62.40%	17.34%	6.28%
Two Days After	90.54%	0.00%	0.00%
	Email without incentives		
Constrained BOWL	20.29%	50.60%	49.36%
Two Days After	9.46%	100.00%	100.00%

Note: The table reports the fraction of customers who receive an email with or without incentives 2 days, 10 days or 30 days after their first purchase.

later days. For emails without incentives, the constrained BOWL sends 50% and 49.4% of users 10 days and 30 days after. Hence, it is not necessary to send emails to all users, which can save the cost of targeting through messages. Moreover, the optimal policy can save money because it does not send incentives for those who would have purchased even without incentives.[36]

6.3 Online Evaluation

Finally, the company tested the optimal policies that we developed for both the baseline model and the extension model. The company runs an A/B test for each model by randomly allocating first-time buyers to each candidate policy to see its effects on retention. More precisely, for the baseline model, the test considers four candidates: BOWL (without constraints), Q-learning (without constraints), static OWL, and BOWL with constraints as well as the control group with no offers. For the extended model, the test considers only the constrained BOWL (due to the company's policy). The first test was implemented in the winter of 2022, and the second one was implemented in the fall of 2022. The duration of each test is a few months.

Table 8 reports the uplift in the retention rate and ROAS for each policy for the baseline model. Overall, the online test results show that all targeting policies perform better than the offline test results even though the control group (no offer) sees slightly lower retention rates during the online test period. Among the policies without any budget constraints, BOWL

[36]Note that the Two-days-after policy does not send a coupon to all users. That is because some customers make their second purchase before receiving the coupon. The Two-days-after policy does not send coupons to those customers.

Table 8: Online Uplift Performance (Baseline Model)

	2nd	3rd	Cost	ROAS
BOWL (no constraint)	5.87%	2.23%	120.9	401%
Q-Learning (no constraint)	5.16%	1.84%	107.2	310%
Constrained BOWL	4.55%	1.76%	94.4	550%
Two Days After	5.97%	2.16%	122.8	409%

Note: The table reports the results of the online evaluation.

Table 9: Online Uplift Performance (Extension)

	2nd	3rd	Cost	ROAS
Constrained BOWL	3.41%	2.00%	94.0	306%

Note: The table reports the results of the online evaluation of the extension model.

achieves a higher retention rate and better ROAS than Q-learning. Our proposed constrained BOWL can also achieve a high retention rate, and importantly, much better ROAS than any other strategies. Therefore, we confirm that our approach works for both offline and online settings.

Lastly, Table 9 reports the online test results of the extension model. For this online test, the company was willing to test only the constrained BOWL against the control group who received neither emails nor incentives. The online test was conducted from July 2022 to October 2022. The uplift in the second purchase is 3.41% and ROAS is 306%, which are smaller than the offline evaluation results. We suspect this is because the Japanese economy went back to normal during this period and hence users tend to spend less.

7 Conclusion

This paper proposes a method to infer the optimal dynamic targeting policy for customer retention management when there is a budget constraint. Dynamic policies are crucial for customer retention management as the states of consumers such as an intention for future purchases inherently evolve over time. Moreover, since most marketing campaigns have certain budget ceilings, it is practically important to consider a cost-efficient way to target consumers.

To do so, we extend the existing methods of estimating dynamic treatment regimes to ac-

count for inter-temporal budget constraints. In particular, we examine Q-learning and Backward Outcome Weighted Learning methods, which we incorporate into constrained optimization problems. We provide the algorithms to find the optimal DTR under budget constraints for both Q-learning and BOWL.

Our empirical application is a large online e-commerce platform in Japan. The company sends "thank you" messages to those who make their first purchase to urge second purchases, and we personalize it by adding coupons. The company runs a series of large-scale randomized experiments with more than 100,000 monthly new buyers. The experimental data allow us to estimate the optimal DTR in the offline setting before the company actually implements the dynamic personalized policies for the entire population.

The results show that the estimated DTRs are highly effective. With budget constraints, we can derive cost-effective optimal policies with almost the same level of customer retention relative to non-constrained policies. Hence, the return on advertising spending, the company's main KPI, can be as high as 800%, which is much higher than typical marketing campaigns.

Funding and Competing Interests

This project was done when one of the authors was working for the company that provided us with the data. All of the authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript. The authors have no funding to report.

References

ASCARZA, E. (2018): "Retention Futility: Targeting High-Risk Customers Might be Ineffective," *Journal of Marketing Research*, 55(1), 80–98.

ASCARZA, E., S. NESLIN, O. NETZER, ET AL. (2018): "In Pursuit of Enhanced Customer Retention Management: Review, Key Issues, and Future Directions," *Customer Needs and Solution*, 5, 65–81.

ASCARZA, E., M. ROSS, AND B. HARDIE (2021): "Why You Aren't Getting More from Your Marketing AI," *Harvard Business Review*, 99(4), 48–54.

ATHEY, S., R. CHETTY, G. W. IMBENS, AND H. KANG (2019): "The Surrogate Index: Combining Short-Term Proxies to Estimate Long-Term Treatment Effects More Rapidly and Precisely," *NBER Working Paper No. 26463*.

ATHEY, S., AND G. IMBENS (2016): "Recursive Partitioning for Heterogeneous Causal Effects," *Proceedings of the National Academy of Sciences*, 113(27), 7353–7360.

ATHEY, S., AND S. WAGER (2021): "Policy Learning with Observational Data," *Econometrica*, 89(1), 133–161.

BHATTACHARYA, D., AND P. DUPAS (2012): "Inferring Welfare Maximizing Treatment Assignment under Budget Constraints," *Journal of Econometrics*, 167(1), 168–196.

FADER, P. S., AND B. G. HARDIE (2007): "How to Project Customer Retention," *Journal of Interactive Marketing*, 21(1), 76–90.

FADER, P. S., AND B. G. S. HARDIE (2010): "Customer-Base Valuation in a Contractual Setting: The Perils of Ignoring Heterogeneity," *Marketing Science*, 29(1), 85–93.

HITSCH, G. J., AND S. MISRA (2018): "Heterogeneous Treatment Effects and Optimal Targeting Policy Evaluation," *Available at SSRN*.

INMAN, J. J., AND L. MCALISTER (1994): "Do Coupon Expiration Dates Affect Consumer Behavior?," *Journal of Marketing Research*, 31(3), 423–428.

JIANG, N., AND L. LI (2016): "Doubly Robust Off-Policy Value Evaluation for Reinforcement Learning," in *Proceedings of the 33rd International Conference on International Conference on Machine Learning*, ICML'16, pp. 652–661. JMLR.org.

KALLUS, N., AND A. ZHOU (2021): "Fairness, Welfare, and Equity in Personalized Pricing," in *Proceedings of the 2021 ACM Conference on Fairness, Accountability, and Transparency*, FAccT '21, pp. 296–314, New York, NY, USA. Association for Computing Machinery.

KAR, W., V. SWAMINATHAN, AND P. ALBUQUERQUE (2015): "Selection and Ordering of Linear Online Video Ads," in *Proceedings of the 9th ACM Conference on Recommender Systems*, RecSys '15, pp. 203–210, New York, NY, USA. Association for Computing Machinery.

KIM, Y. (2022): "Customer Retention under Imperfect Information," *Available at SSRN*.

KITAGAWA, T., AND A. TETENOV (2018): "Who Should Be Treated? Empirical Welfare Maximization Methods for Treatment Choice," *Econometrica*, 86(2), 591–616.

LAN, Q., Y. PAN, A. FYSHE, AND M. WHITE (2020): "Maxmin Q-learning: Controlling the Estimation Bias of Q-learning," in *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net.

LEMMENS, A., AND S. GUPTA (2020): "Managing Churn to Maximize Profits," *Marketing Science*, 39(5), 956–973.

LIU, X. (2022): "Dynamic Coupon Targeting Using Batch Deep Reinforcement Learning: An Application to Livestream Shopping," *Marketing Science*.

MURPHY, S. A. (2003): "Optimal Dynamic Treatment Regimes," *Journal of the Royal Statistical Society, Series B (Statistical Methodology)*, 65(2), 331–355.

MURPHY, S. A., K. G. LYNCH, D. OSLIN, J. R. MCKAY, AND T. TENHAVE (2007): "Developing Adaptive Treatment Strategies in Substance Abuse Research," *Drug Alcohol Dependence*, 88(Suppl 2), S24–30.

NESLIN, S. A., S. GUPTA, W. KAMAKURA, J. LU, AND C. H. MASON (2006): "Defection Detection: Measuring and Understanding the Predictive Accuracy of Customer Churn Models," *Journal of Marketing Research*, 43(2), 204–211.

NESLIN, S. A., G. A. TAYLOR, K. D. GRANTHAM, AND K. R. MCNEIL (2013): "Overcoming the "Recency Trap" in Customer Relationship Management," *Journal of the Academy of Marketing Science*, 41(3), 320–337.

NIE, X., E. BRUNSKILL, AND S. WAGER (2021): "Learning When-to-Treat Policies," *Journal American Statistical Association*, 116(533), 392–409.

PRECUP, D., R. S. SUTTON, AND S. P. SINGH (2000): "Eligibility Traces for Off-Policy Policy Evaluation," in *Proceedings of the Seventeenth International Conference on Machine Learning*, ICML '00, pp. 759–766, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

RAFIEIAN, O. (2022): "Optimizing User Engagement Through Adaptive Ad Sequencing," *Marketing Science*.

SAKAGUCHI, S. (2022): "Estimation of Optimal Dynamic Treatment Assignment Rules under Policy Constraints," *arXiv:2106.05031*.

SIMESTER, D., A. TIMOSHENKO, AND S. I. ZOUMPOULIS (2020): "Efficiently Evaluating Targeting Policies: Improving on Champion vs. Challenger Experiments," *Management Science*, 66(8), 3412–3424.

SUN, L. (2021): "Empirical Welfare Maximization with Constraints," *arXiv:2103.15298*.

WANG, W., B. LI, X. LUO, AND X. WANG (2022): "Deep Reinforcement Learning for Sequential Targeting," *Management Science*.

YANG, J., D. ECKLES, P. S. DHILLON, AND S. ARAL (2022): "Targeting for Long-term Outcomes," *arXiv:2010.15835*.

YOGANARASIMHAN, H., E. BARZEGARY, AND A. PANI (2022): "Design and Evaluation of Personalized Free Trials," *Management Science*.

ZHANG, B., A. A. TSIATIS, E. B. LABER, AND M. DAVIDIAN (2013): "Robust Estimation of Optimal Dynamic Treatment Regimes for Sequential Treatment Decisions," *Biometrika*, 100(3), 681–694.

ZHAO, Y., M. R. KOSOROK, AND D. ZENG (2009): "Reinforcement Learning Design for Cancer Clinical Trials," *Statistics in Medicine*, 28(26), 3294–3315.

ZHAO, Y., D. ZENG, A. J. RUSH, AND M. R. KOSOROK (2012): "Estimating Individualized Treatment Rules Using Outcome Weighted Learning," *Journal of the American Statistical Association*, 107(449), 1106–1118.

ZHAO, Y.-Q., D. ZENG, E. B. LABER, AND M. R. KOSOROK (2015): "New Statistical Learning Methods for Estimating Optimal Dynamic Treatment Regimes," *Journal of the American Statistical Association*, 110(510), 583–598.

Online Appendix

A Mathematical Appendix

In this section, we provide a detailed discussion on the solution to the constrained optimization problem in Section 5.3.1 and the proof of Proposition 1 in Section 5.3.2.

A.1 Single-period Optimization

Consider the single-period constrained maximization problem:

$$\max_d E_d[Y] \text{ s.t. } E_d[C] \leq B, \tag{A.1}$$

where C is the cost variable and B is the budget per person.

Let $\beta(h) = E[Y|H=h, A=1] - E[Y|H=h, A=0]$ and $\gamma(h) = E[C|H=h, A=1] - E[C|H=h, A=0]$. That is, $\beta(h)$ and $\gamma(h)$ are the conditional average treatment effects of action A on the outcome Y and cost C, respectively, given $H=h$. Now, define DTR $d(\cdot;\lambda)$ by

$$d(h;\lambda) = \mathbf{1}[\beta(h) \geq \lambda\gamma(h)], \quad h \in \mathscr{H}.$$

We show the following proposition.

Proposition 2. *Suppose that a deterministic solution to the maximization problem (A.1) exists. Suppose also that there exists $\lambda^* \geq 0$ such that $E_{d(\cdot;\lambda^*)}[C] = B$. Then $d(\cdot;\lambda^*)$ solves (A.1).*

Proof. Observe that for any DTR d,

$$E_d[Y] = E[E[Y|H, A=0]] + E[d(H)\beta(H)], \;\; E_d[C] = E[E[C|H, A=0]] + E[d(H)\gamma(H)].$$

The problem (A.1) is then equivalent to

$$\max_d E[d(H)\beta(H)] \text{ s.t. } E[d(H)\gamma(H)] \leq \bar{B},$$

where $\bar{B} = B - E[E[C|H, A=0]]$. Let d^* be a deterministic solution to (A.1). Also, let

$$S_1 = \{h \in \mathscr{H} : d(h;\lambda^*) = 0, d^*(h) = 1\}, \;\; S_0 = \{h \in \mathscr{H} : d(h;\lambda^*) = 1, d^*(h) = 0\}.$$

Observe that

$$E[d^*(H)\gamma(H)] = E[d(H;\lambda^*)\gamma(H)] + E[\mathbf{1}[H \in S_1]\gamma(H)] - E[\mathbf{1}[H \in S_0]\gamma(H)].$$

Since $E[d^*(H)\gamma(H)] \leq \bar{B}$ and $E[d(H;\lambda^*)\gamma(H)] = \bar{B}$, it follows that $E[\mathbf{1}[H \in S_1]\gamma(H)] \leq E[\mathbf{1}[H \in S_0]\gamma(H)]$. We then obtain that

$$\begin{aligned} E[d^*(H)\beta(H)] &= E[d(H;\lambda^*)\beta(H)] + E[\mathbf{1}[H \in S_1]\beta(H)] - E[\mathbf{1}[H \in S_0]\beta(H)] \\ &\leq E[d(H;\lambda^*)\beta(H)] + \lambda^*(E[\mathbf{1}[H \in S_1]\gamma(H)] - E[\mathbf{1}[H \in S_0]\gamma(H)]) \\ &\leq E[d(H;\lambda^*)\beta(H)], \end{aligned}$$

where the second line holds since $d(H;\lambda^*) = \mathbf{1}[\beta(H) \geq \lambda^*\gamma(H)]$. Therefore, $d(\cdot;\lambda^*)$ is a solution to (A.1). ■

Note that the condition about the existence of * is not a strong assumption. When C and B are both continuous, there always exists such λ.

A.2 Multi-period Optimization

Next, we consider Proposition 1 in Section 5.3.2. We consider a more general model than the two-period model in the main text. Consider the T-period constrained maximization problem:

$$\max_{\mathbf{d}} E_{\mathbf{d}}[Y] \text{ s.t. } E_{\mathbf{d}}[C] \leq B, \tag{A.2}$$

where $\mathbf{d} = (d_1, ..., d_T)$, C is the cost variable and B is the budget per person.

We introduce some notation. Given a DTR $\mathbf{d}$, let $\underline{\mathbf{d}}_t = (d_1, ..., d_t)$ and $\bar{\mathbf{d}}_t = (d_t, ..., d_T)$. Also, let $Q_T(h_T, a_T) = E[Y|H_T = h_T, A_T = a_T]$, $Q_T^C(h_T, a_T) = E[C|H_T = h_T, A_T = a_T]$, $\beta_T(h_T) = Q_T(h_T, 1) - Q_T(h_T, 0)$, and $\gamma_T(h_T) = Q_T^C(h_T, 1) - Q_T^C(h_T, 0)$. In addition, for $t = 1, ..., T-1$, let $Q_t(h_t, a_t; \bar{\mathbf{d}}_{t+1}) = E_{\bar{\mathbf{d}}_{t+1}}[Y|H_t = h_t, A_t = a_t]$, $Q_t^C(h_t, a_t; \bar{\mathbf{d}}_{t+1}) = E_{\bar{\mathbf{d}}_{t+1}}[C|H_t = h_t, A_t = a_t]$,

$\beta_t(h_t;\bar{\mathbf{d}}_{t+1}) = Q_t(h_t,1;\bar{\mathbf{d}}_{t+1}) - Q_t(h_t,0;\bar{\mathbf{d}}_{t+1})$, and $\gamma_t(h_t;\bar{\mathbf{d}}_{t+1}) = Q_t^C(h_t,1;\bar{\mathbf{d}}_{t+1}) - Q_t^C(h_t,0;\bar{\mathbf{d}}_{t+1})$. Lastly, for each $\boldsymbol{\lambda} \in \mathbb{R}_+^T$, let $\bar{\boldsymbol{\lambda}}_t = (\lambda_t, ..., \lambda_T)$ and define DTR $\mathbf{d}(\boldsymbol{\lambda}) = (d_t(\cdot;\bar{\boldsymbol{\lambda}}_t))_{t=1}^T$ recursively as

$$d_T(h_T;\lambda_T) = \mathbf{1}\left[\beta_T(h_T) \geq \lambda_T \gamma_T(h_T)\right], \quad h_T \in \mathscr{H}_T$$

and for $t = T-1, ..., 1$,

$$d_t(h_t;\bar{\boldsymbol{\lambda}}_t) = \mathbf{1}\left[\beta_t(h_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1})) \geq \lambda_t \gamma_t(h_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}))\right], \quad h_t \in \mathscr{H}_t.$$

Proposition 3. *Suppose that a deterministic solution to the maximization problem (A.2) exists, and let* $\mathbf{d}^*$ *denote a solution. Suppose also that there exists* $\boldsymbol{\lambda}^* \in \mathbb{R}_+^T$ *such that* $E_{\mathbf{d}(\boldsymbol{\lambda}^*)}[C] = B$ *and that* $E_{\underline{\mathbf{d}}_t^*, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)}[C] = B$ *for all* $t = 1, ..., T-1$. *Then* $\mathbf{d}(\boldsymbol{\lambda}^*)$ *solves (A.2).*

Proof. Given the optimal DTR $\mathbf{d}^*$, we show by induction that $\mathbf{d}(\boldsymbol{\lambda}^*)$ solves (A.2).

First, consider period T. Since $\mathbf{d}^*$ is optimal, d_T^* solves

$$\max_{d_T} E_{\underline{\mathbf{d}}_{T-1}^*, d_T}[Y] \text{ s.t. } E_{\underline{\mathbf{d}}_{T-1}^*, d_T}[C] \leq B. \tag{A.3}$$

Since $E_{\underline{\mathbf{d}}_{T-1}^*, d_T(\cdot;\lambda_T^*)}[C] = B$, using the argument in the proof of Proposition 2 shows that $d_T(\cdot;\lambda_T^*)$ solves (A.3). Therefore, $(\underline{\mathbf{d}}_{T-1}^*, d_T(\cdot;\lambda_T^*))$ solves (A.2).

Now consider period $t \leq T-1$, and suppose that $(\underline{\mathbf{d}}_t^*, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*))$ solves the problem (A.2). We show that $(\underline{\mathbf{d}}_{t-1}^*, \bar{\mathbf{d}}_t(\bar{\boldsymbol{\lambda}}_t^*))$ solves the problem (A.2), where we interpret $(\underline{\mathbf{d}}_{t-1}, \bar{\mathbf{d}}_t)$ as $\mathbf{d}$ when $t = 1$. Since $(\underline{\mathbf{d}}_t^*, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*))$ is optimal, d_t^* solves

$$\max_{d_t} E_{\underline{\mathbf{d}}_{t-1}^*, d_t, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)}[Y] \text{ s.t. } E_{\underline{\mathbf{d}}_{t-1}^*, d_t, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)}[C] \leq B. \tag{A.4}$$

Observe that for any d_t,

$$\begin{aligned}
E_{\underline{\mathbf{d}}_{t-1}^*, d_t, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)}[Y] &= E_{\underline{\mathbf{d}}_{t-1}^*}[Q_t(H_t, 0; \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)) + d_t(H_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*))], \\
E_{\underline{\mathbf{d}}_{t-1}^*, d_t, \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)}[C] &= E_{\underline{\mathbf{d}}_{t-1}^*}[Q_t^C(H_t, 0; \bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*)) + d_t(H_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}_{t+1}^*))].
\end{aligned}$$

The problem (A.4) is then equivalent to

$$\max_{d_t} E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \text{ s.t. } E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \le \bar{B},$$

where $\bar{B} = B - E_{\underline{\mathbf{d}}^*_{t-1}}[Q^C_t(H_t,0;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))]$. Let

$$S_1 = \{h_t \in \mathscr{H}_t : d_t(h_t;\bar{\boldsymbol{\lambda}}^*_t) = 0, d^*_t(h_t) = 1\},$$
$$S_0 = \{h_t \in \mathscr{H}_t : d_t(h_t;\bar{\boldsymbol{\lambda}}^*_t) = 1, d^*_t(h_t) = 0\}.$$

Observe that

$$\begin{aligned}
&E_{\underline{\mathbf{d}}^*_{t-1}}[d^*_t(H_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1})] \\
&= E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] + E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_1]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \\
&\quad - E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_0]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))].
\end{aligned}$$

Since $E_{\underline{\mathbf{d}}^*_{t-1}}[d^*_t(H_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1})] \le \bar{B}$ and $E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t)\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] = \bar{B}$, it follows that $E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_1]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \le E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_0]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))]$. We then obtain that

$$\begin{aligned}
&E_{\underline{\mathbf{d}}^*_{t-1}}[d^*_t(H_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1})] \\
&= E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] + E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_1]\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \\
&\quad - E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H_t \in S_0]\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \\
&\le E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] + \lambda^*_t(E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H \in S_1]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))] \\
&\quad - E_{\underline{\mathbf{d}}^*_{t-1}}[\mathbf{1}[H \in S_0]\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))]) \\
&\le E_{\underline{\mathbf{d}}^*_{t-1}}[d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t)\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))],
\end{aligned}$$

where the first inequality holds since $d_t(H_t;\bar{\boldsymbol{\lambda}}^*_t) = \mathbf{1}\left[\beta_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1})) \ge \lambda^*_t\gamma_t(H_t;\bar{\mathbf{d}}_{t+1}(\bar{\boldsymbol{\lambda}}^*_{t+1}))\right]$. Therefore, $(\underline{\mathbf{d}}^*_{t-1}, \bar{\mathbf{d}}_t(\bar{\boldsymbol{\lambda}}^*_t))$ solves the problem (A.2).

By induction, $\mathbf{d}(\boldsymbol{\lambda}^*)$ solves (A.2). ■

B Supplemental Figures on Heterogeneity in Treatment Effects

This section provides supplemental figures on the heterogeneity in the treatment effects. In Section 4.2, we report in Figure 4 some evidence of heterogeneity in treatment effects of financial incentives. In this subsection, we provide more detailed results in Figure 5.

In the figure, we provide treatment effects across deciles of two variables, the customer's total spending and the number of messages they receive before the delivery of their first purchase. The figure shows a similar pattern of heterogeneity in the treatment effects of three types of treatment. In the left panels, the treatment effects largely decline as the user's spending increases. In the right panels, the treatment effects vary as the number of messages changes, where the effects are highest at the 90-100 percentile range.

C Details of the Second Experiment

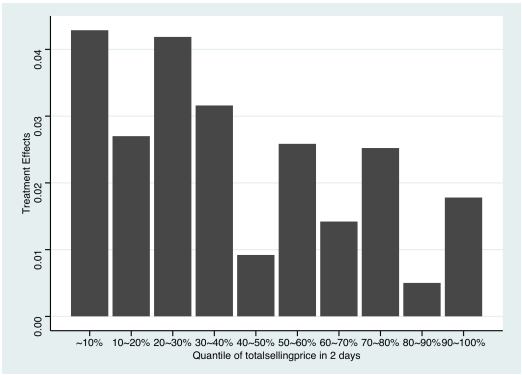
C.1 Experimental Design

In addition to the experiment we discuss in the main text, the company conducted another experiment to learn the optimal DTR of the model in Section 5.4, which includes not sending incentives. The experimental design is similar to the original experiment and looks like Figure 6. Other features of the experimental design, including the timing and the number of treatments, the amount of financial incentive, and the user-level randomization, are the same as the first experiment.

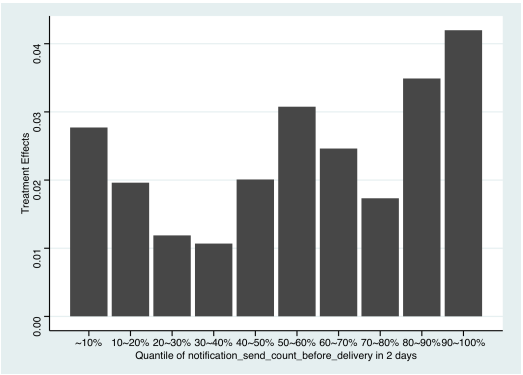
Table 10 reports the summary statistics of the subset of the variables we use, computed from the data of the second experiment. Note that there are two types of incentives: coupon and appreciation email. To save space, we do not report the summary statistics for all possible treatment conditions, but we report the summary statistics only for the conditions where coupons are sent. The control group includes the users who do not receive incentives or emails.

For both demographic and behavioral variables, we do not find any significant differences across conditions. Also, we do not find any significant difference between Table 10 and Table 2.

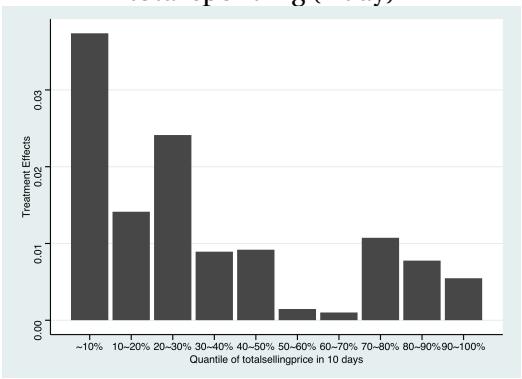
Figure 5: Heterogeneous Treatment Effects of 2, 10, and 30-day treatment



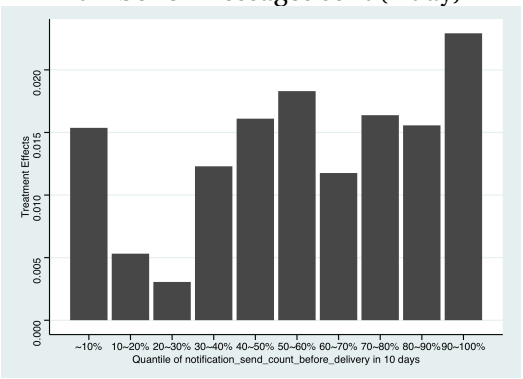
Treatment effects across deciles of the total spending (2 day)



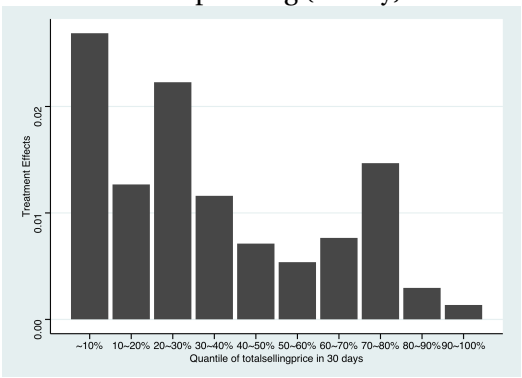
Treatment effects across deciles of the number of messages sent (2 day)



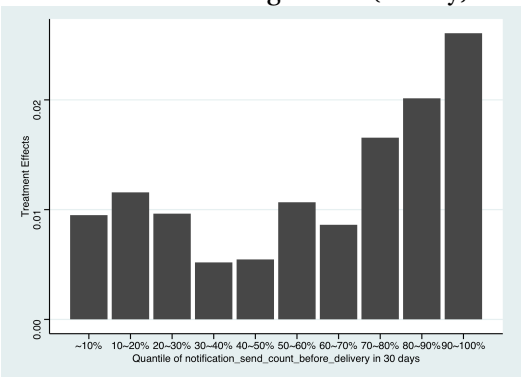
Treatment effects across deciles of the total spending (10 day)



Treatment effects across deciles of the number of messages sent (10 day)



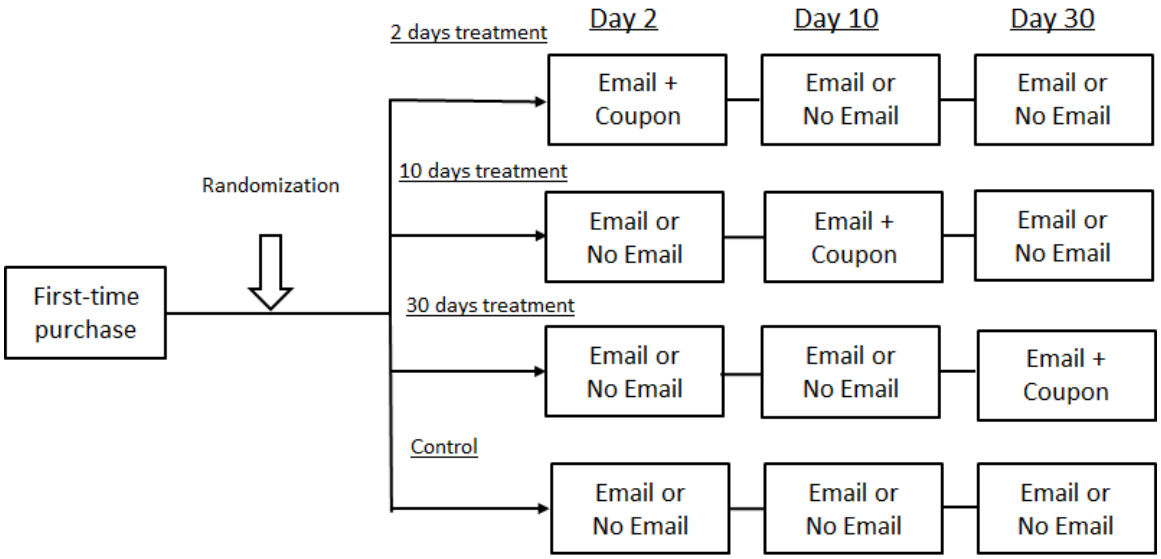
Treatment effects across deciles of the total spending (30 day)



Treatment effects across deciles of the number of messages sent (30 day)

Note: The figures show the average treatment effects of the 2, 10, and 30-day treatments against the total spending for the first purchase (left panels) and the number of messages sent since the delivery of the first item (right panels). The total spending and the number of messages are split into deciles.

Figure 6: Experimental Design: Extension



Note: The figure shows the experimental design of the second experiment where there are three actions, no email, email only, and email with coupon.

Table 10: Summary Statistics (Second Experiment)

Variable	**2 day**		**10 day**		**30 day**		**Control**	
	mean	**sd**	**mean**	**sd**	**mean**	**sd**	**mean**	**sd**
Female	0.632	0.482	0.631	0.483	0.628	0.483	0.632	0.482
Age	32.18	18.23	32.21	18.25	32.05	18.21	31.97	17.98
Quantity: first buy	1.013	0.179	1.013	0.172	1.014	0.221	1.015	0.252
Sales: first buy	4057	4187	4090	4298	4063	4082	4070	4090
# of sessions/day (pre 1st buy)	0.777	2.108	0.769	2.067	0.779	2.129	0.759	2.032
# of PVs/day (pre delivery)	12.81	28.34	12.75	29.30	12.96	29.38	12.60	27.27
# of favorites/day (2-10 day)	0.155	0.805	0.128	0.911	0.135	0.824	0.119	0.640
# of messages sent (10-30 day)	5.145	17.389	5.167	17.941	5.192	18.177	5.260	19.651

Note: The first six columns report the mean and standard deviation of each variable for each of the three treatment groups who receive incentives 2, 10, and 30 days after their first purchase. The last two columns are for the control group with no emails.

C.2 Average Treatment Effects

Next, we estimate the average treatment effects with the second experiment data. Notice that there are two treatments, i.e., emails with coupons and emails without coupons. To save space, we report the results of the regression with the outcome measured for 8 weeks.

Table 11 reports the results. The estimation results show that the appreciation email without coupons can increase retention. The 2-day treatment effect of emails on retention is 2.2%

Table 11: Average Treatment Effect (Second Experiment)

	Retention	Sales	Order
Panel (A): Financial incentive			
2 day	0.066***	364.865**	0.160***
	(0.002)	(57.363)	(0.017)
10 day	0.055***	255.669	0.117***
	(0.002)	(42.484)	(0.013)
30 day	0.046***	204.958	0.095**
	(0.002)	(28.086)	(0.011)
Panel (B): Only appreciation email			
2 day	0.022***	260.748**	0.108***
	(0.004)	(106.041)	(0.033)
10 day	0.024***	110.355	0.076***
	(0.004)	(72.875)	(0.025)
30 day	0.003	45.196	0.011
	(0.003)	(48.195)	(0.014)

Note: The first column reports the treatment effects on whether a customer makes any purchases within 8 weeks since their first purchase. The second column reports the treatment effects on total sales and the third column reports the treatment effects on the number of items purchased. The table does not report the constants as the constants reveal the baseline retention rates, sales, and orders, but the NDA does not allow us to do so.

and the 10-day treatment effect is 2.4%. Sales also increase by 260 JPY by emails for the 2-day treatment, but not for 10 days or 30 days.

The results also reveal that the coupons in addition to the appreciation emails further increase retention, sales, and the number of items purchased. Hence, by comparing the treatment effects in Panel (A) and the ones in Panel (B), we can back out the pure effect of coupons. For the effect on retention, the coupons themselves increase the retention by 4.4% ($=6.6\%-2.2\%$) if users receive them after 2 days, and sales increase by 100 JPY.