Online Advertising, Data Sharing, and Consumer Control*

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Abstract. We examine how competition between advertising exchanges influences the targeting options that these exchanges make available to advertisers. When advertisers have strong property rights over data regarding consumers' active purchase interests, competition between ad exchanges leads to too little sharing of data. This may harm consumers, who receive too few pertinent ads, and advertisers themselves can also be harmed due to a situation resembling a prisoner's dilemma. We find that giving consumers the right to opt out of tracking may also benefit consumers who allow tracking, by altering the incentives of ad exchanges to offer improved targeting options. In addition, we show that initiatives by Apple and Google to limit third-party tracking and to introduce alternative tracking systems such as Topics, might benefit consumers by weakening the data property rights of advertisers. Because more data is shared by default under such systems, this can be true even if these systems are less accurate than third-party tracking systems.

In this article we examine how competition between advertising exchanges influences the targeting options that these exchanges make available to advertisers. We are motivated by recent and ongoing market changes such as the introduction of the consumer privacy-rights legislation General Data Protection Regulation (GDPR) in the European Union, the deprecation of certain third-party tracking technologies by companies such as Apple and Google, and the introduction of new category-based tracking technologies such Google's Topics system.

Our analysis considers a particular type of online advertising, intent advertising. This refers to a situation in which a consumer has recently taken an action that indicates current intent to purchase in some category. In this context it is plausible that consumers prefer to see more rather than fewer relevant ads. For example, a consumer interested in buying athletic gear who has visited the Nike website may also benefit by receiving an advertisement from Adidas, even if the consumer did not visit the Adidas website. Yet we find that ad exchanges may limit the targeting tools available to advertisers in a way that consumers receive fewer such ads.

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However, we show that category-based advertising systems (such as Topics) may rectify the situation, ensuring consumers see additional relevant ads, while also potentially enhancing consumer privacy. Similarly, we show that giving consumers with strong privacy preferences the right to opt out of tracking can cause ad exchanges to offer further targeting options. This benefits even those consumers who continue to allow tracking, as they receive additional relevant ads.

Our results can be easily understood by taking a property rights perspective on data ownership, and by understanding the structural features inherent in current third-party tracking systems. Consider the example of a consumer interested in purchasing athletic gear who has visited the Nike website. Although this consumer may benefit from also seeing an Adidas ad, Nike itself may not wish Adidas to advertise to this consumer; Nike would like to limit the flow of information regarding this consumer's current purchase interests. If an ad exchange insists on facilitating Adidas' targeting of consumers who visited Nike's website, then Nike may refuse to share data with the ad exchange, and instead share data only with an exchange that does not allow such "cross-targeting." That is, to the extent that Nike has property rights over the data indicating a consumer's current purchase interests, it may refuse to share that data with an ad exchange, even if such sharing would benefit consumers. It is important to note that in reality ad exchanges do currently not enable cross-targeting, even though it is technically feasible.

In fact, under current third-party tracking systems advertisers do possess strong data property rights. For example, under the cookie system, ad exchanges that enable advertising on websites are able to track consumers only when websites consent to place tracking cookies into a consumer's browser on behalf of the ad exchange. Similarly, cross-app tracking on mobile devices requires that app developers use specific software development kits developed by ad exchanges.

Weakening the data property rights of advertisers can therefore benefit consumers if it induces ad exchanges to allow cross-targeting (that is, more widely disseminate data about a consumer's intent to purchase). As an application of this idea, consider Google's current proposal to replace the third-party cookie system (on Chrome) with the category-based system Topics, which uses on-browser machine-learning tools to assign consumers to different categories based on their browsing history. Google argues that this will increase consumer privacy because the different categories are large enough that it will not be possible for advertisers to know exactly which websites any individual consumer visited. Another important aspect of Topics is that it collects browsing data by default, rather than relying on individual opt-in decisions of websites. In other words, Google becomes the default owner of data indicating a consumer's current intent to purchase, and Google will benefit from letting both both Adidas and Nike target consumers based on that data.

Note that this beneficial outcome can emerge even if the Topics technology is less accurate than the current third-party cookie system, as advertisers often allege it will be. The reason is that, as already explained, under the current cookie system advertisers may withhold important data, whereas this cannot happen under Topics. This system may even benefit advertisers. The reason is that sometimes advertisers would jointly benefit if they all agreed to share information with ad exchanges that enabled cross-targeting, but due to a prisoner's dilemma type situation this does not happen in equilibrium. When this is the case, weakening the property rights of advertisers may eliminate the prisoner's dilemma, benefiting advertisers.

That said, there may be other costs associated with new advertising technologies. We show that ad exchanges controlling them may manipulate them to their own advantage. This is in line with the popular arguments that Google is eliminating third-party cookies to make Google's other advertising channels more attractive. Similarly, Apple's App Tracking Transparency policy makes cross-app tracking more difficult, which might benefit Apple's App Store ad business, or other ad businesses that Apple might develop in the future.

We now explain why giving consumers with strong privacy preferences the right to opt out of tracking can alter the data-sharing incentives of advertisers in a way that benefits consumers who do not opt out. When more consumers opt out of tracking, advertisers worry less about the downsides of cross-targeting. Instead, advertisers become more concerned with reaching "look-alike audiences." Look-alike audience targeting is extremely common online, and refers to a situation in which, for example, a consumer is targeted with ads for athletic gear by Nike or Adidas not because they have visited either company's website but rather because they exhibit online browsing habits more generally associated with consumers interested in athletic gear. When advertisers share more data, they are able to target a larger look-alike audience. Therefore, when consumers have veto rights over how their data is used for tracking, advertisers are more willing to share data, and consumers who do not opt out of tracking see additional relevant ads.

We emphasize that the main problem identified in our baseline model is not dependent on the use of third-party cookies as tracking tools a such. Rather, it is the default advertiser ownership of intent-to-purchase data, which gives advertisers some veto power over cross-targeting. Alternative tracking tools that are becoming pervasive, such as digital fingerprinting, pixel tracking, or the sign-in based technology Unified ID 2.0, maintain this ownership structure.

Our article is related to work on privacy, online advertising, and data intermediaries. Early work on privacy includes Taylor (2004) and Villas-Boas (2004), who study dynamic pricing when firms can track consumers over time. Montes, Sand-Zantman, and Valletti (2019) study price discrimination with endogenous privacy choices.[1] As in our study, privacy is often related to questions about data property and control rights, with early work from Hermalin and Katz (2006) and more recent contributions by Ichihashi (2020), Dosis and Sand-Zantman (2022) and Markovich and Yehezkel (2021). Choi, Jeon, and Kim (2019) examine privacy with data externalities.

Iyer, Soberman, and Villas-Boas (2005) argue that targeted advertising may lead to increased prices, but that it reduces wasted advertisements. Johnson (2013) and Bergemann and Bonatti (2011) both explore targeted advertising, examining the effect of changes in targeting accuracy. Athey, Calvano, and Gans (2018) identify how advertisers' desire to avoid reaching a given consumer too many times affects market outcomes; the possibility for duplicative or wasted ad impressions is an important part of our model. Goldfarb and Tucker (2011) empirically examine the effect of privacy regulations on the efficacy of online advertising. Villas-Boas and Yao (2021) consider dynamic retargeting.

Work on data intermediaries includes De Corniere (2016) and De Corniere and De Nijs (2016) who study search intermediaries. Hagiu and Jullien (2011) study search diversion. Bergemann and Bonatti (2015) study the pricing of information. Choi, Mela, Balseiro, and Leary (2020), Bergemann, Bonatti, and Gan (2022) and Ichihashi (2021) also investigate markets for data. The distinguishing feature of our work is our focus on practices specific to advertising markets. Other work that investigates the details of the modern advertising market includes Sayedi (2018), Kraemer, Schnurr, and Wohlfarth (2020), and D'Annunzio and Russo (2020).

1. Model

Here we present a model of information sharing and advertising. There are two advertisers, A_1 and A_2, and two advertising exchanges, ADX_1 and ADX_2. There is also a unit mass of consumers. We suppose that a mass $\alpha < 1/2$ consumers have recently and exclusively visited A_1's website, and α consumers have recently and exclusively visited A_2's website. The remaining $1 - 2\alpha > 0$ consumers have recently visited both websites.[2] Thus, a total of $1 - \alpha$ consumers have visited a given advertiser's website, where α of these consumers have exclusively visited this website.

The act of visiting one or more websites indicates that a consumer is actively interested in the product category served by both A_1 and A_2. However, even though consumers visit an advertiser

[1]Earlier work on targeting consumers and price discrimination that is not specific to an online environment includes Villas-Boas (1999), Fudenberg and Tirole (2000), and Chen and Iyer (2002).

[2]Although we use the term "website", our model is more general. The important assumption here is that some consumers have revealed their interest in a category to some advertiser(s) by their actions.

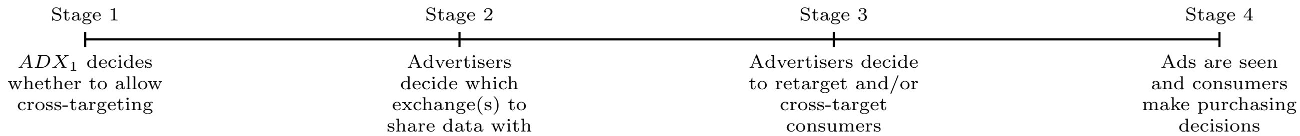

Figure 1. The Timeline

because they are interested in the product, we assume that they do not make a purchase immediately.[3] Completing a sale is only possible if the consumer is also displayed an advertisement on a third-party content publisher website.[4]

The ability of advertisers to reach consumers with advertisements depends on the information sharing decisions of ADX_1 and the advertisers. The timeline of these decisions is given in Figure 1. In the first stage, ADX_1 publicly commits to either allowing cross-targeting or not (explained just below), whereas ADX_2 by assumption does not allow cross-targeting. In the second stage, the two advertisers simultaneously and publicly decide which advertising exchange to (costlessly) share information with.[5] We assume each advertiser shares data with only one exchange but allowing it to share data with both has no effect on our results.

Advertising options in stage 3 are as follows. First, A_i can *retarget* those $1-\alpha$ consumers who have visited its website. It can do this on ADX_j if and only if it agreed to share data with ADX_j in stage 2. For example, perhaps A_i placed a tracking cookie on behalf of ADX_j when consumers visited A_i's website.[6] Second, A_i can *cross-target* those $1-\alpha$ consumers who have visited A_{-i}'s website. This option is only available if ADX_1 allowed cross-targeting in stage 1, and if A_i's rival A_{-i} shared information with ADX_1 in stage 2.

There is an additional advertising option available only through ADX_1. In particular, ADX_1 has the ability to reach an entirely separate group of consumers (who have not visited either the website of A_1 or A_2), namely "look-alike" consumers.[7] Look-alike consumers have similar online habits to

[3]The analysis is identical if consumers who purchase immediately can be identified and excluded from the set of consumers who receive ads. If this cannot be done, the analysis nonetheless remains similar: because these consumers will not be responsive to ads, the value of reaching a randomly selected consumer who visited a website (defined as C or M below) will be smaller. But as long as this value still exceeds the cost of an advertisement, the analysis goes through unchanged. In fact, data shows that visit-to-purchase conversion rates of online shoppers in the US are in fact below 3% (Statista, 2022). The consumers' reasons for not buying immediately, apart from not wanting the product, may involve time restrictions, distractions, forgetfulness, or fatigue (see Ursu, Zhang, and Honka, 2022, who find evidence of search fatigue).

[4]In Section 4, we consider an extension in which retargeting is not necessary to convert a website visit into a sale.

[5]Assuming public observability is for simplicity. We could dispense with this assumption so long as advertisers can indicate which consumers they want to target (if available) and that they are only charged if these consumers are actually targeted.

[6]As noted earlier, our analysis in much of this article applies to situations in which advertisers have some control over whether information they collect about consumers is shared with other parties. This is also true of some newer advertising initiatives such as tracking pixels or digital fingerprinting, not just existing cookie technology.

[7]Targeting users who are "like" one's own buyers is extremely common. To this end, many firms share lots of customer data with Google or Facebook.

those consumers that have visited one of the two advertiser's websites, and ADX_1 is more able to reach those consumers with advertisements whenever it has more data to work with.[8] Precisely, if one advertiser has chosen to share data with ADX_1, then an additional $\eta_1 > 0$ consumers can be targeted, but if both advertisers have chosen to share data with ADX_1 then an additional $\eta_2 > \eta_1$ consumers can be targeted. Both advertisers can target these consumers, irrespective of which exchange they share data with.[9]

Placing an ad costs an exogenously determined value $w > 0$, where this cost is the same for each consumer.[10] For advertisers that choose to both retarget and cross-target, there is an advantage to concentrating these advertisements on ADX_1. In particular, an advertiser that retargets on ADX_2 and cross-targets on ADX_1 reaches $1-\alpha$ consumers on each exchange, thus wasting $2(1-\alpha)-1 = 1-2\alpha$ impressions (we assume showing a consumer more than one ad from the same firm has no effect on the consumer's decision). These wasted impressions do not occur when an advertiser concentrates its advertisements on one ad exchange (ADX_1).[11]

In the fourth and final stage, each consumer makes purchasing decisions. Each consumer wishes to buy only one product, because the two products are substitutes and the incremental value of owning both is not high enough to justify buying both. Also, a product can only be purchased if a consumer saw an advertisement for it.[12] We assume that prices are exogenously fixed, and let M denote the "monopoly profit" that accrues to advertiser A_i from each consumer that exclusively sees an advertisement from A_i (gross of advertising fees). Let $C < M$ denote the "competition profit" that accrues to A_i from a consumer that sees an advertisement from both advertisers.[13] We suppose that consumers who see both ads are more likely to make a purchase and hence generate higher joint profits for A_1 and A_2, so that $2C > M$.[14]

To close the model, we specify payments to the advertising exchanges. Each exchange earns an exogenously given fraction $\phi \in (0,1)$ of the market price of an ad, which is exogenously set to w, for each advertisement that they facilitate.

[8]For example, an intermediary such as Google controls a content platform (Youtube), email services (Gmail), and even a smartphone operating system (Android), all of which provide extensive unique data. To identify look-alike audiences, Google can compare its unique data with the data of website visitors it collects from contracted advertisers who share data with Google.

[9]We could instead suppose that an advertiser can reach the look-alike audience only if it contracts with ADX_1. Then, the deviations we consider below entail a loss of look-alike consumers of size η_2 instead of $(\eta_2 - \eta_1)$, with little qualitative effect.

[10]In a later extension we consider the possibility that the advertising cost for a given consumer is higher if both advertisers are trying to reach that consumer with an ad.

[11]In the real world ad exchanges offer "frequency capping" services to limit how many times any one consumer receives the same ad from that particular exchange (see Buchbinder, Feldman, Ghosh, and Naor (2014)), thus avoiding wasted impressions.

[12]In an extension in Section 4.3 we relax this assumption.

[13]We are assuming it is either not profitable or not feasible for a single advertiser to purchase all ad slots for a given consumer so as to prevent that consumer from seeing a rival ad.

[14]This would be true in a symmetric discrete-choice model in which consumers value variety, prices are taken as given, and there is an outside option; aggregate demand would increase and hence raise firms' joint profits.

We are now ready to analyze the model described above. Our primary interest is identifying the circumstances under which cross-targeting emerges, which can only happen if ADX_1 allows it and if advertisers share information with ADX_1. There are thus two relevant subgames to consider, corresponding to whether ADX_1 allows cross-targeting or not.

1.1. ADX_1 **does not allow cross-targeting.** Consider the subgame in which ADX_1 (in stage 1) does not allow cross-targeting. This means that the only option for each advertiser is to retarget those consumers who have visited its website (and also to target any available look-alike consumers on ADX_1). By definition, if A_i wishes to retarget consumers on ADX_j, then it must agree to share data with (that is, contract with) ADX_j (in stage 2). Because both advertisers will always choose to target all look-alike consumers, each of these consumers is worth $C - w$ to a given advertiser.[15]

If both advertisers contract with ADX_1, then each advertiser has profits of

$$\alpha M + (1 - 2\alpha)C - (1 - \alpha)w + \eta_2(C - w). \tag{1}$$

The intuition for this expression is as follows. A_i is retargeting all α consumers who have exclusively visited its website, earning $M - w$ from each of them (factoring in the advertising cost w and the fact that A_i's rival cannot target these exclusive consumers when cross-targeting is not allowed). It will also retarget all consumers who have visited both websites, of which there are $1 - 2\alpha$, earning $C - w$ from each (given that these consumers are also being retargeted by A_i's rival). Finally, it will target all available look-alike consumers, of which there are η_2 given that both advertisers are sharing data with ADX_1.

The only available deviation is for A_i instead to contract with ADX_2. However, the only effect of this deviation on A_i's profits is a reduction in the targetable look-alike audience from η_2 to η_1, which is unprofitable. In fact, when neither ad exchange allows cross-targeting, it is a dominant strategy to contract with ADX_1 because doing so strictly expands the size of the look-alike audience. Thus, the unique equilibrium exhibits both advertisers contracting with ADX_1.

1.2. ADX_1 **allows cross-targeting.** Now consider the subgame in which cross-targeting is allowed by ADX_1. We first investigate the conditions required for the existence of an equilibrium in which each advertiser contracts with ADX_1 and cross-targets the other advertiser, that is an equilibrium in which there is "full cross-targeting." In such an equilibrium, A_i earns

$$(1 + \eta_2)(C - w). \tag{2}$$

[15]We relax this assumption in Corollary 1 below.

The reason is that each advertiser reaches the mass one of consumers that have visited at least one advertiser's website, and additionally reaches the η_2 look-alike consumers that are available to target through ADX_1. The full cross-targeting outcome occurs whenever both advertisers contract with ADX_1 because A_i's only available deviations in the advertising stage (stage 3) are to forgo targeting its own visitors, the exclusive visitors of A_{-i}, or the look-alike audiences. But since neither of these three activities leads to wasted impressions when A_i has contracted with ADX_1, it would never be profitable to quit any of them.

There are two possible deviations for A_i from such an equilibrium, each involving A_i contracting only with ADX_2; this ensures that A_i's rival A_{-i} cannot cross-target A_i. The deviating advertiser A_i must decide whether or not it will cross-target A_{-i}, which is possible because A_{-i} shares information with ADX_1 in the proposed equilibrium.

If A_i deviates but continues to cross-target A_{-i} ("leave and snipe"), it earns

$$\alpha M + (1-\alpha)C - 2(1-\alpha)w + \eta_1(C-w). \tag{3}$$

Comparing (3) to (2), we see that the deviation is not profitable if and only if

$$\alpha(M-C) \leq (1-2\alpha)w + (\eta_2-\eta_1)(C-w). \tag{No Leave \& Snipe}$$

The left-hand side of this inequality represents the benefit of preserving monopoly power on the α consumers that exclusively visit A_i's website. The right-hand side represents the two associated costs of leaving and sniping. First, there are $(1-2\alpha)w$ costs wasted on duplicative impressions, due to the fact that this advertiser retargets $1-\alpha$ on ADX_2 and cross-targets $1-\alpha$ consumers on ADX_1, but there are only a total of (mass) 1 unique website consumers. Second, by not sharing data with ADX_1, there are only η_1 rather than $\eta_2 > \eta_1$ look-alike consumers available.

Alternatively, A_i can contract with ADX_2 and not cross-target, giving it profits of

$$\alpha M + (1-2\alpha)C - (1-\alpha)w + \eta_1(C-w). \tag{4}$$

Comparing (4) to (2), this deviation to simply "leave" is not profitable if and only if

$$\alpha(M-C) \leq \alpha(C-w) + (\eta_2-\eta_1)(C-w). \tag{No Leave}$$

The left-hand side of this inequality represents the benefit of preserving monopoly power on the α consumers that exclusively visit A_i's website. The right-hand side represents the two associated costs. First, this advertiser can no longer cross-target its rival's exclusive customers, which are worth $\alpha(C - w)$. Second, by not sharing data with ADX_1, there are only η_1 rather than $\eta_2 > \eta_1$ look-alike consumers available.

We conclude that if ADX_1 allows cross-targeting, a subgame with full cross-targeting is an equilibrium if and only if both the "No Leave & Snipe" and "No Leave" conditions hold. In the Appendix we further show that if an equilibrium with full cross-targeting exists, then it is Pareto dominant in terms of the advertisers' payoffs.

1.3. **Overall Equilibrium.** We are now ready to describe what happens in the overall game. To rule out uninteresting situations, we make the following assumption for the case in which ADX_1 allows cross-targeting: if cross-targeting is an equilibrium in this subgame, then it is selected if and only if advertisers prefer it (which is also ADX_1's preferred outcome in this situation).

Assumption 1. *If a subgame has multiple equilibria, we remove any equilibrium which is strictly Pareto dominated by another equilibrium, in terms of the payoffs of the advertisers (who are the only players who take actions at this stage).*

The profits of ADX_1 are determined by how many advertisements are placed with it, recalling that it receives ϕw for each such advertisement (where the remaining $(1 - \phi)w$ is assumed to go to the publisher). Hence, the ideal outcome for ADX_1 is that both advertisers contract and do both their retargeting and cross-targeting with it; this also maximizes the size of the look-alike audience, ensuring A_1 and A_2 each place an additional η_2 ads with ADX_1.

However, from our earlier analysis we know that it may not be an equilibrium for both advertisers to indeed share data with ADX_1, if ADX_1 allows cross-targeting. Instead, one or both advertisers may choose to work with ADX_2 in such a situation, which not only directly reduces how many re- or cross-targeting ads are placed with ADX_1 but also limits the size of the look-alike audiences. This competitive pressure sometimes leads ADX_1 to not allow cross-targeting.

Proposition 1. *In equilibrium, ADX_1 allows cross-targeting if and only if* (No Leave & Snipe) *and* (No Leave) *both hold, in which case both advertisers cross-target. In addition, both advertisers always contract with ADX_1, retarget their exclusive website visitors, and target the look-alike audience of size η_2.*

Proposition 1 shows that ADX_1 can always ensure that both advertisers contract with it. Intuitively, this is due to its privileged ability to generate larger look-alike audiences when more advertisers contract with it. However, as expected we also see that securing this business may require it not to allow cross-targeting. As mentioned above, not allowing cross-targeting is not ideal for ADX_1 because it means such ads are never placed. But this sacrifice is worth it to ensure that neither advertiser uses ADX_2.

An implication of Proposition 1 is that cross-targeting may fail to emerge in equilibrium even when both A_1 and A_2 would prefer that it did.[16] To see this, note that advertiser profits if they cross-target each other are given in (2) while the profits if neither cross-targets (but both contract with ADX_1) are given by (1). From these expressions, advertisers prefer cross-targeting if and only if

$$M - C < C - w.$$

This condition says that the loss of monopoly power over an advertiser's own exclusive customers is made up for by the ability to reach the exclusive customers of its rival (where the mass of each group of consumers is α). If this condition holds, then (No Leave) also holds. However, (No Leave & Snipe) can still fail, and if it does then each advertiser is individually motivated to contract with ADX_2, so that it can cross-target its rival while its rival cannot cross-target. In particular, (No Leave & Snipe) tends to fail when w is smaller, whereas the condition for both advertisers to prefer cross-targeting, $M - C < C - w$, always holds for w sufficiently small, because we assumed earlier that $2C - M > 0$.[17] In this case, advertisers face a prisoner's dilemma.

More generally, an issue is that online advertising markets lack more sophisticated contracting options which make the eligibility to cross-target dependent on sharing data as well or paying a fee. We conjecture that this is because even if cross-targeting required that an advertiser share its own data, it would be easy to circumvent this restriction. For example, an advertiser could simply share data from any barely visited website it owns (or even create a new website solely for this purpose). In return, the advertiser would obtain the right to cross-target all other advertisers that contract with the same ad exchange without giving away access to the exclusive visitors of its main website. Alternatively, the monetary transfers that compensate the advertiser who shares data may be difficult to verify and monitor both for the advertiser that shares the data and the advertiser that pays the fee. It may even be be difficult to ascertain the correct price for the data given how complex the environment could be.[18]

[16] In those cases, too little data is shared, which is reminiscent of the potentially insufficient connectivity between asymmetric internet backbone providers (Crémer, Rey, and Tirole, 2000). However, we consider symmetric advertisers, and find that inefficiency arises due to the unique incentives to leave and snipe that we identify in online advertising markets.

[17] As noted earlier, $2C - M > 0$ would hold in any reasonable model of symmetrically differentiated products in which prices are given and there is an outside option, as it says in effect that total sales would be higher when consumers see both products.

[18] For example, some other advertisers interested in the data may sell substitute products as we have assumed but these substitution patterns might be unknown in detail, and depend on how effective the creative content of advertisers is. And other advertisers may be simply interested in selling unrelated products to the same consumers.

We now turn to the impact of ADX_1's decision on consumers. We begin by noting that there may be several effects on consumers that our analysis does not capture. For example, we do not consider the possibility that the cost of advertising is passed through to consumers. Also, we assume that the privacy costs borne by the consumer do not depend too much on whether they receive one targeted advertisement or more than one, but rather simply on whether or not they are being tracked. However, if instead some consumers suffer additional privacy-related costs when cross-targeting occurs, then this negative welfare effect must be factored in.

Under these maintained assumptions, there is a very clear consumer welfare prediction. In particular, consumers benefit from increased cross-targeting, because this provides them with more purchasing options. Therefore, consumers may be harmed by the equilibrium data sharing practices of ad exchanges in those cases where Proposition 1 predicts that cross-targeting does not emerge in equilibrium. In other words, there can be too little sharing of data.[19] The reason that too little data may be shared is that advertisers have property rights over the data regarding individual consumers' current intent to purchase, and that ad exchanges cater to the desires of the advertisers. In the following sections, we investigate both technological and policy measures that might encourage the sharing of more data so as to benefit consumers, while also giving consumers with strong privacy preferences the ability to avoid tracking.

1.4. **Comparative statics in the data advantage.** Here we investigate how ADX_1's ability to target look-alike audiences influences the equilibrium outcome. It is easy to see that both (No Leave & Snipe) and (No Leave) hold more easily as $\eta_2 - \eta_1$ increases. Intuitively, this is because a larger $\eta_2 - \eta_1$ makes deviating from the full cross-targeting equilibrium more costly as it implies a larger decrease in look-alike audiences that can be targeted. In addition, we have so far assumed that advertisers place the same value $C - w$ on a look-alike consumer as on other consumers. But whether this is true or not may well depend on how effective ADX_1's technology for identifying these look-alike consumers is. To study this effect, suppose instead look-alike consumers are valued at a possibly different amount π_L.

Corollary 1. *Improvements in look-alike audience targeting that increase $\eta_2 - \eta_1$ or the value π_L of the look-alike audience make it more likely that full cross-targeting emerges in equilibrium.*

[19]A potential caveat is that when consumers are exposed to more cross-targeting they may in reality also receive fewer other advertisements; we have for simplicity ignored these other ads, or equivalently assumed them to have zero value for consumers. However, if outside ads have substantial value for consumers, then a complete accounting of consumer welfare would need to recognize the prospect that consumers might receive fewer such ads when they receive more targeted ads from A_1 and A_2. We consider this possibility in Section 3.

In practice, improvements in targeting could either increase or decrease $\eta_2 - \eta_1$. If improvements primarily correspond to an expansion of the overall population that ADX_1 bases its look-alike targeting on, then it seems reasonable that both η_1 and η_2 would increase in a proportional manner, satisfying the assumption. On the other hand, improvements in such techniques might decrease $\eta_2 - \eta_1$ and, thus, make cross-targeting less likely. To see why, recall that the reason for assuming $\eta_2 > \eta_1$ was that ADX_1 has more data about website visitors if it contracts with both advertisers, which it relies on to identify look-alike audiences. Thus, if improving its technology means that ADX_1 obtains better predictions with less data, $\eta_2 - \eta_1$ might shrink.

2. Consumer Control

In this section, we study the effect of granting website visitors the right to opt out of tracking. Each website visitor has the option of completely blocking all tracking, which has the effect of ensuring that they receive no advertisements from A_1 or A_2.

We assume that a mass $q \in (0, 1)$ of website visitors value receiving relevant ads more than they mind being tracked, and so allow tracking. However, $1 - q$ website visitors value privacy more than they value receiving relevant ads, and so block tracking. From the viewpoint of advertisers, the consumers that block advertisements become irrelevant. Nonetheless, as we will show, the absence of these consumers may alter the equilibrium decision of ADX_1 to allow cross-targeting or not.

To derive the conditions under which a full cross-targeting equilibrium exists, we must again analyze the subgames that follow if ADX_1 allows cross-targeting or not. Due to the similarity with the previous section, we keep the analysis here short. If ADX_1 does not allow cross-targeting, then advertisers earn $\alpha q(M-w)+(1-2\alpha)q(C-w)+\eta_2(C-w)$ if both contract with ADX_1. This outcome remains the unique equilibrium in this case because whenever an advertiser does not contract with ADX_1, it could increase the targetable look-alike audience by deviating and contracting with ADX_1 since $\eta_2 > \eta_1 > 0$. (Note that η_1 and η_2 might well depend on how many consumers are blocking tracking, that is on q; we accommodate this possibility in our formal results below.)

On the other hand, if ADX_1 allows cross-targeting, then each advertiser earns $(q + \eta_2)(C - w)$ if both advertisers contract with ADX_1, which induces full cross-targeting. As before, there are two deviations available for A_i: either contract with ADX_2 and cease cross-targeting its rival, or contract with ADX_2 and continue cross-targeting its rival while bearing some wasted ad impressions. Deriving the corresponding conditions here for the case with consumer control is straightforward and yields the following two conditions as being individually necessary and jointly sufficient for an equilibrium with full cross-targeting to emerge (given that ADX_1 is allowing cross-targeting).

$$\alpha(M-C) \leq \alpha(C-w) + \frac{\eta_2 - \eta_1}{q}(C-w) \qquad \text{(C: No Leave)}$$

as well as

$$\alpha(M-C) \leq (1-2\alpha)w + \frac{\eta_2 - \eta_1}{q}(C-w). \qquad \text{(C: No Leave \& Snipe)}$$

Proposition 2. *In equilibrium,* ADX_1 *allows cross-targeting if and only if* (C: No Leave & Snipe) *and* (C: No Leave) *both hold, in which case both advertisers cross-target. In addition, both advertisers always contract with* ADX_1*, retarget their exclusive website visitors, and target the look-alike audience of size* η_2*.*

We now consider the effect of introducing consumer control on the overall equilibrium. Observe that the case in which consumers cannot block tracking is equivalent to the situation with $q = 1$, at least from the perspective of the advertisers and ad exchanges. Hence, to understand whether consumer control results in more or less cross-targeting in equilibrium, we wish to understand the impact of a reduction in q. Inspecting (C: No Leave) and (C: No Leave & Snipe), we see that each condition is more likely to hold for smaller q if

$$\frac{\eta_2 - \eta_1}{q}(C-w) \qquad (5)$$

is decreasing in q. If η_1 and η_2 did not depend on q, then it would be immediate that a reduction in q (that is, an increase in the number of consumers who block tracking) would make it easier to sustain an equilibrium with full cross-targeting. The reason is that as q becomes smaller, the size of the targetable group of website visitors relative to the size of the look-alike audience becomes smaller. Thus, advertisers care more about the look-alike audience as more consumers block, and in particular care more about increasing its size from η_1 to η_2, which is accomplished by sharing data with ADX_1, which also facilitates cross-targeting.

However, η_1 and η_2 are to likely depend on q. To explore the effect of consumer control in this case, we write $\eta_2 = \eta(q)$ and $\eta_1 = \eta(q(1-\alpha))$, where η is an increasing function. To understand these expressions, first consider η_2, the size of the look-alike audience when both advertisers contract with ADX_1. In this case, the data associated with all q consumers who do not block tracking is available to generate look-alike audiences, yielding $\eta_2 = \eta(q)$. But if only A_i contracts with ADX_1, then only the data of those consumers who do not block and who went to A_i's website is available to generate look-alike audiences. There are $q(1-\alpha)$ such consumers, yielding $\eta_1 = \eta(q(1-\alpha))$.

Proposition 3. *Consumer control makes it more likely that full cross-targeting emerges in equilibrium, if additional data has a decreasing marginal effect on the number of look-alike consumers that can be targeted (that is, if η is concave).*

As consumers receive control (or as q decreases), both η_2 and η_1, and possibly the difference between them, change. Concavity of η, however, guarantees that the difference between η_2 and η_1 does not shrink too much, so that the overall effect is that advertisers place relatively more weight on the look-alike audience compared to website visitors.

It is plausible that η is concave. Recall that η maps data from a pool of website visitors to a pool of look-alike users. Thus, the shape of η is determined by the returns to data, which are often considered to be decreasing (Agrawal, Gans, and Goldfarb, 2018).

We can now study the effect of consumer control on consumer welfare. For the q website consumers that do not block tracking, let $u_1 > 0$ be the value of receiving just one targeted ad and $u_2 > u_1$ the value of receiving two targeted ads. Note that we are assuming that the look-alike audience either cannot or does not block ads. This is reasonable so long as the technology used to generate look-alike audiences is not as invasive as the tracking used on website visitors, or if these consumers are both tracked and targeted on a single platform that is also owned by ADX_1, such as YouTube, Facebook, or an app store, where it is more difficult to block tracking.

The welfare effects on consumers can be understood as follows. First, for those $1-q$ website visitors who strongly dislike tracking, they are now able to avoid being tracked, which clearly benefits them. Second, those q website visitors who do not block ads may receive more ads than in the case without consumer control. This happens if the introduction of consumer control shifts the equilibrium to one with full cross-targeting, causing these consumers to receive two ads (and utility u_2) rather than possibly just one ad (and utility $u_1 < u_2$). Third, the total size of the look-alike audience is smaller (at least if η_1 and η_2 depend on how many consumers block as assumed above). In particular, in the absence of consumer control the data of all (mass one) website visitors is used by ADX_1 to generate a mass $\eta(1)$ of look-alike consumers. But, with consumer control there is only a mass q of consumers whose data can be used, giving a look-alike audience base of size $\eta(q)$.

Therefore we conclude that introducing consumer control weakly benefits all website visitors, but harms some look-alike audience members who no longer receive pertinent advertisements.

Proposition 4. *Giving consumers control (weakly) raises the surplus of all website visitors if additional data has a decreasing marginal effect on the number of look-alike consumers that can be targeted (that is, if η is concave). But, consumer control lowers the look-alike audience's welfare.*

As noted above, although the $1-q$ consumers who block ads always benefit from consumer control, those q consumers who do not block ads benefit only if the equilibrium switches to a full cross-targeting one. We can thus imagine even stronger data-control options for consumers that would ensure these q consumers benefit as well. In particular, this would be so if these consumers could costlessly share their data, causing both advertisers to target them with ads.

3. Category-Based Advertising

In 2020, Google first announced that it would discontinue the use of third-party tracking cookies on its industry-leading web browser Chrome. As an alternative, Google recently declared that it will offer firms the option to target consumers via "Topics," categories that represent consumer interests derived from their browsing history (techcrunch.com, 2022). These browsing histories are accessible to Google precisely because most consumers are using the Chrome browser, allowing Google to track consumers without the consent of advertisers that was required in the base model.[20]

Google argues that assigning consumers to Topics will better protect user privacy. One reason is that by definition Topics represent more aggregated data. Advertisers only know which Topics a consumer has been assigned to, compared to the current third-party cookie system which allows advertisers to infer personal information such as the browsing history of each targeted consumer. Another reason is that the data about an individual's browsing will only reside in the Chrome browser; the browser itself assigns the consumer to the appropriate Topics. Additionally, the Topics a consumer is assigned to will be refreshed every three weeks, and Google may even allow consumers to change their Topics themselves.[21]

The announcement of Google pivoting to a new tracking technology has created much consternation amongst advertisers (who argue that tracking via Topics will be inferior to the current system and therefore harm consumers and advertisers) and regulators (who welcome a higher level of consumer

[20]While other browsers differ in their support of third-party tracking—e.g., Apple's Safari now requires that users actively opt in and otherwise blocks third-party cookies by default—Google's dominant browser market share of about two thirds makes its actions by far the most important for the future of internet tracking technology.

[21]Google initially planned to replace third-party tracking using a technology called FLoC (Federated Learning of Cohorts). FLoC would have assigned consumers to a subset of as many as 30,000 different categories, using machine learning to analyze consumers' browsing histories (wired.com, 2022). The sheer amount of potential categories, including sensitive ones such as gender and race, however, raised privacy concerns. Google has since announced that it will replace FLoC with Topics, and that the number of Topics will (initially) be limited to 300; Topics will enhance privacy relative to FLoC because Topics is based on more-aggregated consumer data than FLoC. Both FLoC and Topics have been born out of Google's Sandbox initiative.

privacy but argue that the real goal of Google may be to provide further advantage to Google's advertising business). The advertising industry is developing and improving alternatives such as digital fingerprinting techniques that seek to replicate the individual-level tracking afforded by the third-party cookie system. In addition, a consumer sign-in system referred to as Unified ID 2.0 is being developed. These technologies may help advertisers maintain the targeting accuracy they are accustomed to, and may also help them retain control over the relevant data.

In this section, we build on our base model to allow for a look at some potential advantages and disadvantages of the Topics system, a tracking technique in essence closely related to look-alike audiences. We argue that the popular discussion has neglected a fundamental issue, which is that advertisers cannot opt out of the Topics or related systems, and thereby no longer have property rights over data corresponding to an individual's interest in a particular product category. More precisely, recall that under the third-party cookie tracking system any ad exchange that wishes to track consumers requires a contractual agreement with a particular advertiser under which that advertiser allows the ad exchange to place a cookie in the browser of consumers who visit the advertiser's website. In contrast, under the Topics system, data about a customer's browsing history is simply collected by the browser by default and used to assign the consumer to Topics.

As we will show, removing the property rights of advertisers leads readily to an equilibrium in which more information is shared about consumer's current purchase interests. We note that this consumer benefit is completely separate from any benefit that consumers may accrue from the greater privacy afforded by such a system.

Consistent with our earlier approach, we will continue to assume that $1 + \eta_2$ is the total number of consumers interested in a certain category. However, conceptually, all consumers are now identified using a category-based system like Topics. Due to the nature of the Topics system, advertisers cannot differentiate between consumers who previously visited their website and those who did not. An implication is that an advertiser who works with both exchanges cannot limit the consumers targeted on ADX_1 to exclude those targeted on ADX_2 (unlike in the base model where an advertiser could retarget on ADX_2 and then separately target η_2 distinct look-alike consumers on ADX_1).

Additionally, we assume that ADX_1 puts the Topics system in effect. The Topics system sometimes results in wasted impressions due to its imperfect nature. Precisely, we suppose that it takes $\tau > 1$ ad impressions to reach a mass one of consumers on ADX_1. Turning to ADX_2, we suppose it uses one of the alternatives to the Topics system to offer retargeting.[22] Because such systems may

[22] Note that buying ads on ADX_1 using the Topics system does not involve a contracting decision. In addition, advertisers do not mind contracting with ADX_2, as it does not support cross-targeting. We therefore omit a formal analysis of the contracting stage in this section.

require consumer opt-in (as Unified ID 2.0 does), we assume that an advertiser using this tracking technology on ADX_2 can reach a fraction $\sigma < 1$ of the $1 - \alpha$ consumers who recently visited its website. The timing is as follows. First, advertisers place bids. Then, ads are seen and consumers make purchasing decisions.

We now describe the possible outcomes of the Topics model analyzed in this section. One possibility is that both advertisers work with ADX_1, so that each advertiser is able to reach the entire market of $1 + \eta_2$ consumers, each of which is valued at C. However, due to the inaccuracy of the Topics system, this requires a total of $\tau(1 + \eta_2)$ ads, implying total profits per advertisers of

$$(1 + \eta_2)(C - \tau w).$$

Alternatively, if both advertisers retarget their website visitors on ADX_2, A_i has a profit of

$$\sigma [\alpha M + (1 - 2\alpha) C - (1 - \alpha) w].$$

Except for the scaling parameter σ, these are the same profits that advertisers would get in the base model if they both used only ADX_2.[23]

The third possibility is that A_1, say, uses ADX_1 while A_2 uses ADX_2. In this situation, A_1 earns

$$(1 + \eta_2) M - \sigma (1 - \alpha)(M - C) - (1 + \eta_2) \tau w.$$

The intuition is that A_1 reaches a total of $1 + \eta_2$ consumers, but faces competition only on those $\sigma(1 - \alpha)$ consumers that its rival A_2 also targets. The profit of A_2 in this case is

$$\sigma (1 - \alpha)(C - w),$$

which says that A_2 faces competition on all $\sigma(1 - \alpha)$ consumers it reaches. Note that, unlike in the base model, A_2 cannot avoid being cross-targeted by A_1 simply by doing its retargeting on ADX_2. Our first result identifies the condition under which both advertisers bid only on ADX_1.

Proposition 5. *In the equilibrium of the Topics model, both advertisers cross-target if the Topics system is not too inaccurate. In particular, both advertisers bid on ADX_1 if and only if*

$$\tau < \tilde{\tau} \equiv \frac{(1 + \eta_2) - \sigma (1 - \alpha)}{w} C + \frac{\sigma (1 - \alpha)}{1 + \eta_2}, \qquad (6)$$

where $\tilde{\tau} > 1$.

[23] Note that it cannot be optimal for an advertiser to use both exchanges because the set of consumers reached on ADX_2 is a strict subset of those reached on ADX_1.

Observe that cross-targeting is less efficient than in the base model because $\tau > 1$ (so that $\tau - 1$ impressions are wasted). Nonetheless, full cross-targeting is certain to emerge under the Topics system so long as this inefficiency is not too great, whereas without the Topics system cross-targeting might not occur. Intuitively, under the Topics system, the leave and snipe option is no longer available to advertisers. In terms of property rights, advertisers no longer have any exclusive rights to the data which consumers are interested in making a purchase; instead ADX_1 by default owns this data (for example, due to its control over the browser). That is, an advertiser can no longer prevent its exclusive website visitors from being cross-targeted by leaving ADX_1. Hence, leaving ADX_1 is less attractive than in the base model, making full cross-targeting with both advertisers using ADX_1 more likely (for τ close to 1). Note that this logic applies even if $\eta_2 = 0$.

We now turn to an assessment of consumer welfare under the Topics system. As just discussed, cross-targeting is more likely to arise under the Topics system as long as it is not too inaccurate. In turn, this suggests that consumers interested in the product category are better off under the Topics system. Another advantage of the Topics system is that some consumers may prefer the additional privacy associated with it. The reason for improved privacy is that the reduced tracking accuracy of the Topics system, as compared to third-party cookies, makes it more difficult for advertisers to track consumers across the web and identify them individually from their browsing behavior.

Nonetheless, the Topics system does carry an additional cost. In particular, because the Topics system is imperfect ($\tau > 1$) at identifying consumers who are interested in the product category, some consumers may receive ill-targeted advertisements. Suppose that these consumers incur a nuisance cost $\kappa > 0$ when receiving an ad from an advertiser in the product category.[24]

Recall that $u_1 > 0$ is the value of receiving one targeted ad, while $u_2 > u_1$ describes the value of receiving two targeted ads for a consumer interested in the product category. The following result characterizes the net welfare effect of adopting Topics if this leads to a full cross-targeting equilibrium ($\tau < \tilde{\tau}$) instead of one in which only retargeting occurs. (This result does not take into account the additional possible benefit of greater consumer privacy under Topics.)

Proposition 6. *Suppose that full cross-targeting arises if the Topics system is adopted by ADX_1, but that otherwise it does not. The net welfare effect of introducing the Topics system is positive if the system is sufficiently accurate, that is if*

$$\tau < 1 + \frac{\alpha}{1 + \eta_2} \frac{u_2 - u_1}{\kappa}.$$

[24] One might imagine that this cost might also arise (only) with the look-alike audience in the base model. We believe the issue is much more salient here because all consumers are targeted using a category-based system; it is as if all consumers are now look-alike consumers.

The net welfare benefit of adopting the Topics system is positive if the value of additional ads received by consumers interested in the product category exceeds the nuisance cost incurred by consumers not interested in the category. More precisely, the 2α consumers who exclusively visit a single website receive an additional ad under the Topics system, which they each value at $u_2 - u_1$. On the other hand, $(\tau - 1)(1 + \eta_2)$ consumers who are not interested in the category each now receive two ads for it, thus suffering a nuisance cost 2κ. Equating these costs and benefits and solving for the critical τ value gives the result above.

Although consumers incur the nuisance cost κ when being targeted with an irrelevant ad, we now show that ADX_1 may benefit from reducing the accuracy of the Topics system. Doing so leads to excess impressions and, hence, more ad sales for ADX_1. The only constraint is that the Topics system must not be too inaccurate because advertisers will not use it otherwise (Proposition 5).

Proposition 7. *In the region of Topics accuracy where both advertisers bid on ADX_1, i.e., $\tau < \tilde{\tau}$, (i) ADX_1's profits strictly increase as the Topics technology becomes less accurate, that is as τ increases. Moreover, (ii) the threshold $\hat{\tau}$ is larger if its rival ADX_2 is less efficient (σ is smaller).*

In summary, as long as both advertisers bid on its ad exchange, ADX_1 benefits from the Topics system being less accurate. At the same time, we see that ADX_1 may benefit from reducing the accuracy of its rival's system allowing it to reduce tracking accuracy further to sell even more ads. We have already identified that making third-party cookies less effective may be one means of accomplishing this. But consider an alternative context, that of in-app mobile advertising. Recently, Apple adopted its App Tracking Transparency program, which makes cross-app tracking of consumers more difficult and reduces the efficiency of existing in-app ad systems. Some commentators have suggested Apple's motivation for this initiative is less about privacy and more about giving an advantage to its own App Store ad system (vox.com, 2022).

Our final result concerns a potential regulatory constraint on τ. To the extent that higher τ values suggest consumer data being more disaggregated, regulators interested in privacy might impose a minimum level of τ. We now show that imposing too high a level of τ can have detrimental effects.

Corollary 2. *Suppose that there is an exogenously given requirement that the Topics system not be too accurate, that is, if ADX_1 uses Topics it must have $\tau > \bar{\tau}$. Then, if $\bar{\tau}$ is too large, ADX_1 is better off not introducing the Topics system.*

If τ is large, both advertisers opt to retarget their website visitors on ADX_2. As a consequence, ADX_1 is better off not adopting the Topics system. Note that this insight does not follow directly from Proposition 5. This is because τ may just exceed $\tilde{\tau}$, i.e., full-cross targeting does not obtain in equilibrium, but ADX_1 still prefers to adopt Topics. In fact, if τ just exceeds $\tilde{\tau}$, one advertiser bids on ADX_1, while the other advertiser retargets on ADX_2 in equilibrium. If the number of excess impressions is sufficiently large, prefers this outcome over advertisers retargeting only in the base model.

4. Extensions

In this section we consider several extensions of the base model considered earlier.

4.1. **Higher profits from consumers who visit both websites.** In this extension, we assume that consumers who visit both websites are more likely to make a purchase than consumers who visit one website only. We model this difference in behavior by assuming that expected profits from targeting such a consumer are $\hat{C} > C$ with competition and $\hat{M} > M$ without competition.

It turns out that this change has no affect on the decisions of advertisers or ad exchanges, and hence does not alter any equilibrium predictions (except for the level of advertiser profits). The reason is that the consumers who visit both websites are always targeted, regardless of the contracting choices. Hence, the gross profits associated with consumers who visited both websites, $(1-2\alpha)\hat{C}$, are always realized in any subgame, and hence do not factor into the attractiveness of any deviations.

To see this in a concrete example, consider an advertiser contemplating a deviation from a potential equilibrium with full cross-targeting. In the base model, we know that for this to be an equilibrium requires that (No Leave) and (No Leave & Snipe) both hold. For convenience we repeat the (No Leave & Snipe) condition here:

$$\alpha(M - C) \leq (1 - 2\alpha)w + (\eta_2 - \eta_1)(C - w).$$

The term $\alpha(M - C)$ represents the gain of avoiding competition for the consumers who have exclusively visited A_i, and hence does not change in this extension. The term $(1 - 2\alpha)w$ represents the costs of duplicative ads from operating on two ad exchanges, and does not change. Finally, $(\eta_2 - \eta_1)(C - w)$ represents profits from the look-alike audience, who have not visited either advertiser's website, and so this term does not change either.[25] Repeating this process for all potential deviations of advertisers in various subgames always leads to the conclusion that nothing has changed.

[25] One might ask whether changing the profits associated with look-alike consumers might affect the equilibrium. The answer is yes, as shown in Corollary 1.

4.2. **Higher cost of cross-targeting.** In this extension, we relax the assumption that the cost of targeting is independent of the number of advertisers targeting the same consumer. We assume that the cost of targeting a website visitor that is also targeted by one's rival is $\hat{w} > w$. We maintain the assumption that it is profitable to target all customers interested in the product category, so that $C > \hat{w}$. For consistency, we also assume that targeting look-alike audiences also costs $\hat{w}$, given that both advertisers target them.

This change does not affect the outcome when ADX_1 chooses not to allow cross-targeting. The reason is that the consumers who receive two ads are always the same whether advertisers work with ADX_1 or ADX_2. Only those $1 - 2\alpha$ consumers who visited both websites receive both ads. Hence, the fact that ADX_1 offers a look-alike audience that increases in size with more data ensures both advertisers contract with ADX_1.

But when ADX_1 allows cross-targeting, the decisions of advertisers may well change. Consider the potential equilibrium in which full cross-targeting occurs. As explained earlier, one potential deviation for A_i is to instead contract with ADX_2 and not cross-target its rival. In the base model, one effect of this is a loss of $\alpha(C-w)$ from not cross-targeting, and this loss now reduces to $\alpha(C-\hat{w})$. A second effect of this deviation in the base model is the gain of $\alpha(M - C)$ from protecting this advertiser's exclusive customers from its rival. In our extension, this gain still exists but there is an additional benefit which is that the costs of reaching these customers fall from $\hat{w}$ to w, exactly because the rival is now incapable of targeting these consumers. Overall, also factoring in changes in profits due to the look-alike audience, this deviation is not profitable if and only if

$$\alpha(M - C) + \alpha(\hat{w} - w) \leq \alpha(C - \hat{w}) + (\eta_2 - \eta_1)(C - \hat{w}). \quad (7)$$

Because the left-hand side is larger than in the base model and the right hand side is smaller, overall this deviation is attractive on a larger set of parameters than in the base model.

The leave and snipe deviation also changes. The additional effect is that the duplicative costs associated with this deviation are now higher, precisely because the cost of reaching a consumer with advertising is higher when both advertisers are targeting that consumer. Consequently, this deviation is not profitable if and only if

$$\alpha(M - C) + \alpha(\hat{w} - w) \leq (1 - 2\alpha)\hat{w} + (\eta_2 - \eta_1)(C - \hat{w}). \quad (8)$$

This condition may be either easier or harder to satisfy, depending on the parameters. In particular, the left-hand side becomes larger, but whether the right-hand side becomes smaller or larger depends on the parameters. Nonetheless, we obtain a result that resembles our earlier result: If ADX_1 is open, there is full cross-targeting if and only if both (7) and (8) hold.

The remainder of the equilibrium analysis from the base model goes through, except for one thing. In certain circumstances, allowing cross-targeting can be more profitable than not allowing it, even if the resulting subgame only exhibits one advertiser engaging in cross targeting. In particular, there may be an equilibrium in which A_1 shares data with ADX_1 and is cross-targeted by A_2, but A_2 only shares data with ADX_2 and so cannot be cross-targeted by A_1.

To see why, suppose that ADX_1 allows cross-targeting and it leads to this "one-sided cross-targeting". Compared to not allowing any cross-targeting, the loss is that each advertiser only spends $\hat{w}$ on η_1 rather than η_2 look-alike consumers, for a total loss (for ADX_1) of $2\hat{w}\phi(\eta_2 - \eta_1)$. The gains accrue from each advertiser: A_1 now pays an additional $\alpha(\hat{w} - w)$ because its exclusive customers are now cross-targeted, while A_2 also now pays an additional $\hat{w} - w$ on an α consumers.[26] Overall, this implies that allowing cross-targeting is profitable in this case if $\alpha(\hat{w} - w) \geq (\eta_2 - \eta_1)\hat{w}$.

However, if $\alpha(\hat{w} - w) < (\eta_2 - \eta_1)\hat{w}$, then ADX_1 does not favor the one-sided cross-targeting outcome. In this situation, as in the base model, it can be shown that cross-targeting prevails in equilibrium if and only if there is full cross-targeting when ADX_1 allows cross-targeting. Moreover, as in the base model, there are still circumstances in which advertisers prefer an outcome with full cross-targeting, yet it does not arise in equilibrium.

4.3. **Consumers buy without retargeting.** In this extension, we relax the assumption that advertisers must retarget their website visitors to sell to them. To examine the effect as starkly as possible, we make the extreme opposite assumption and suppose that consumers have perfect recall so that there is no value in an advertiser retargeting.[27]

As before, we assume that consumers do not buy immediately so that cross-targeting serves the same purpose as in the base model. That is, by cross-targeting, an advertiser may change the final purchase decision of a consumer. Precisely, if a consumer visits only one advertiser, this advertiser earns M if the rival does not cross-target and C otherwise. In addition, each advertiser can expect profits of C from a consumer who visits both websites (without retargeting). Lastly, cross-targeting on ADX_1 costs $(1 - \alpha)w$ if an advertiser does not contract with ADX_1 and αw otherwise. This is

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[26] Precisely, A_2 now pays $\hat{w}$ on all $1 - \alpha$ customers it cross-targets on ADX_1 rather than only paying $\hat{w}$ on the $1 - 2\alpha$ consumers who visited both websites. As $1 - \alpha - (1 - 2\alpha) = \alpha$, its total increase in outlay is $\alpha\hat{w}$.

[27] It would be straightforward to model the case that consumers buy without being retargeted with a probability r less than one. If $rC > w$, sellers would continue to retarget all visitors and the results are similar to the base model. If $rC < w$, it is more profitable not to retarget, implying that the analysis is similar to the case we present here.

consistent with the base model, where we assumed that contracting with ADX_1 allows advertisers to avoid wasted impressions since the exchange has access to the advertisers' data.

This model is very similar to the base model. Intuitively, in the base model advertisers used retargeting both in equilibrium and when deviating. Hence, in effect removing retargeting as in this extension has no effect on the advertisers' behavior. To see this formally, consider the potential equilibrium with full cross-targeting. This exhibits profits for an advertiser of $(1+\eta_2)C-(\alpha+\eta_2)w$. In comparison, the deviation to leave and snipe yields

$$\alpha M + (1-\alpha+\eta_1)C - (1-\alpha+\eta_1)w$$

whereas the deviation to leave without cross-targeting yields

$$\alpha M + (1-2\alpha+\eta_1)C - \eta_1 w.$$

Comparing these terms yields necessary and sufficient conditions for cross-targeting (if ADX_1 allows cross-targeting) that are equivalent to (No Leave & Snipe) and (No Leave) from the base model.

Although the decisions of advertisers do not change, it turns out that ADX_1 may wish to act differently. In particular, ADX_1 might choose to allow cross-targeting even it does not lead to full cross-targeting but instead leads to only one advertiser cross-targeting the other. This is because not allowing cross-targeting only leads to a total number of ads on ADX_1 equal to $2\eta_2$ due to the lack of both retargeting and cross-targeting. By contrast, if one advertiser does not contract with ADX_1 but cross-targets on ADX_1, then ADX_1 sells $1-\alpha+2\eta_1$ ads. Thus, ADX_1 prefers the outcome with "one-sided cross-targeting" over not allowing cross-targeting if

$$1-\alpha > 2(\eta_2-\eta_1). \tag{9}$$

Intuitively, if allowing cross-targeting leads to one-sided cross-targeting, ADX_1 must weigh the benefit of guaranteeing a look-alike audience of size η_2 instead of η_1 against the profits from having at least one advertiser cross-target. If (9) holds, then the profits from enabling cross-targeting dominate and ADX_1 will allow cross-targeting even if the resulting subgame exhibits only one-sided cross-targeting. Of course, ADX_1 also chooses to allow cross-targeting if full-cross-targeting is an equilibrium since this guarantees a look-alike audience of size η_2 and leads to cross-targeting, thus strictly dominating not allowing cross-targeting, just like in the base model.

Overall, there is weakly more cross-targeting in equilibrium if advertisers do not need retargeting. This change is solely driven by the possibility of one-sided cross-targeting being more profitable for

ADX_1 than not allowing cross-targeting, which does not happen in the base model because of the presence of retargeting.

5. Conclusion

In this paper, we build a model to analyze the economics of a particular type of targeted online advertising. Consumers who have shown intent to purchase (for instance by visiting a website) in some product category are targeted by advertisers. In equilibrium, ad exchanges may adopt data-sharing policies that give rise to a fundamental inefficiency: too little data about the consumers' purchase intentions is shared and, as a consequence, mutually beneficial market transactions do not take place. This prediction is consistent with real-world observations that ad exchanges do not facilitate cross-targeting, that is do not allow advertisers to directly target visitors of a rival advertiser's website.

We argue that the current industry model grants excessive data property rights to advertisers since the ability of ad exchanges to track consumers generally requires consent by website owners. An ad exchange may respond to this by designing restrictive data-sharing policies, so as to entice advertisers to work with it. The advertisers' strong property rights are not confined to the present third-party cookie system but are also inherent to new third-party tracking systems that continue to require advertiser consent such as digital fingerprinting, pixel tracking, or Unified 2.0. We show that providing consumers with the ability to opt out of third-party tracking can mitigate the issue. Likewise, moves by companies such as Apple and Google that circumvent the third-party tracking system—widely criticized as a power grab by advertisers—may actually benefit consumers by weakening advertisers' property rights. On the other hand, category-based tracking (such as Google's Topics system) may also increase the number of excess impressions that do not benefit consumers or advertisers, but nonetheless generate revenue for the ad exchange.

Appendix: Omitted Proofs

To simplify exposition in the proofs below, we sometimes use the term "open" to mean that ADX_1 has allowed cross-targeting, and "closed" to mean that it has not. The proofs of the next two lemmas follow from arguments in the main text.

Lemma 1. *If ADX_1 is closed, both advertisers contract with ADX_1 in the unique equilibrium.*

Lemma 2. *Suppose ADX_1 is open. Then, full cross-targeting is an equilibrium if and only if both* (No Leave) *and* (No Leave & Snipe) *holds.*

Lemma 3. *Suppose ADX_1 is open. If full cross-targeting is an equilibrium, there is no other equilibrium in which at least one advertiser contracts with ADX_1.*

Proof of Lemma 3: As shown in the main text, if both advertisers contract with ADX_1 in stage 2, then it always leads to full cross-targeting in stage 3. Thus, to prove the result, we can restrict attention to ruling out equilibria in which exactly one advertiser, say A_i, contracts with ADX_1. Suppose that A_{-i} contracts with ADX_2 and cross-targets A_i's visitors. Then, A_{-i} earns profits of $\alpha M + (1-\alpha)C - 2(1-\alpha)w + \eta_1(C-w)$. By deviating and contracting with ADX_1, A_{-i} would induce full cross-targeting and earn profits of $(1+\eta_2)(C-w)$. Thus, deviating is profitable if $\alpha M + (1-\alpha)C - 2(1-\alpha)w + \eta_1(C-w) \leq (1+\eta_2)(C-w)$ or, equivalently, if (No Leave & Snipe) holds. Since full cross-targeting is an equilibrium by assumption, Lemma 2 implies that (No Leave & Snipe) does hold, ruling out that A_{-i} cross-targets in equilibrium.

Suppose next that A_{-i} contracts with ADX_2 but does not cross-target A_i's visitors. That is, A_{-i} only retargets on ADX_2 and targets look-alike audiences on ADX_1, implying profits of $\alpha M + (1-2\alpha)C - (1-\alpha)w + \eta_1(C-w)$. By deviating and contracting with ADX_1, A_{-i} would induce full cross-targeting, leading to profits of $(1+\eta_2)(C-w)$. Thus, the deviation is profitable if $\alpha M + (1-2\alpha)C - (1-\alpha)w + \eta_1(C-w) \leq (1+\eta_2)(C-w)$ or, equivalently, if (No Leave) holds. Since full cross-targeting is an equilibrium by assumption, Lemma 2 implies that (No Leave) does hold, ruling out that this can be an equilibrium. ∎

Lemma 4. *Suppose ADX_1 is open. If full cross-targeting is an equilibrium, then it strictly Pareto-dominates any other equilibrium for advertisers.*

Proof of Lemma 4: If full cross-targeting is an equilibrium, (No Leave) implies that

$$(1+\eta_2)(C-w) \geq \alpha M + (1-2\alpha)C - (1-\alpha)w + \eta_1(C-w), \tag{10}$$

where the left-hand side represents each A_i's profits in the full cross-targeting equilibrium. By Lemma 3, both advertisers must contract with ADX_2 in the only alternative equilibrium that may exist. In this equilibrium, each A_i earns $\alpha M + (1-2\alpha)C - (1-\alpha)w$, which involves no look-alike audience targeting since look-alike audiences cannot be identified if no A_i contracts with ADX_1. The profits are thus strictly less than the expression on the right-hand side of (10) and, thus, less than profits with full cross-targeting. ∎

Lemma 5. *If ADX_1 is open, a there is always an equilibrium in pure strategies*

Proof of Lemma 5: We first establish two new conditions relevant to the subgame in which one advertiser, A_{-i} say, contracts with ADX_1 and A_i contracts with ADX_2. We assume that ADX_1 is open throughout as stated in the premise of the lemma.

First, A_i strictly prefers cross-targeting A_{-i}'s visitors over not doing so if and only if

$$\begin{aligned} \alpha M + (1-2\alpha)C - (1-\alpha)w + \eta_1(C-w) \geq& \alpha M + (1-\alpha)C - 2(1-\alpha)w + \eta_1(C-w) \\ \iff \quad \alpha C \leq& (1-\alpha)w \qquad \text{(No Sniping)} \end{aligned}$$

fails. Second, A_{-i} strictly prefers being cross-targeted by A_i over contracting with ADX_2 to escape cross-targeting if and only if the following holds

$$\begin{aligned} (1-\alpha+\eta_1)(C-w) >& \alpha(M-w) + (1-2\alpha)(C-w) \\ \iff \quad \eta_1(C-w) >& \alpha(M-C). \qquad \text{(Being Sniped)} \end{aligned}$$

We claim that both advertisers contracting with ADX_2 is an equilibrium if and only if (No Sniping) and (Being Sniped) fail. If: Suppose A_i deviates and contracts with ADX_1 instead. Then, A_{-i} would cross-target A_i since (No Sniping) fails, which makes A_i worse off than the equilibrium strategy since (Being Sniped) fails as well. Only if: Suppose at least one condition holds and that A_i deviates and contracts with ADX_1 instead. Then, at least one of the following must be true: A_{-i} does not cross-target (No Sniping) or A_i prefers contracting with ADX_1 even if A_{-i} cross-targets (Being Sniped). While the deviation is clearly profitable in the second case, it also is in the first. That is because if A_{-i} does not cross-target, the deviation allows A_i (and A_{-i} in fact) to reach η_1 consumers via look-alike audience targeting while not incurring any disadvantage. Thus, both advertisers contracting with ADX_2 is not an equilibrium if (No Sniping) or (Being Sniped) holds.

It remains to show that some equilibrium exists if an equilibrium does not exist in which both contract with ADX_2 (meaning (No Sniping) or (Being Sniped) hold) and an equilibrium does not exist in which both contract with ADX_1 (meaning (No Leave) or (No Leave & Snipe) fail). Alternatively, that an equilibrium exists if (No Sniping) or (Being Sniped) hold and (No Leave) or (No Leave & Snipe) fail.

First suppose that (No Sniping) holds. Then, A_{-i} contracting with ADX_1 and A_i contracting with ADX_2 without any cross-targeting is an equilibrium, in which the advertisers' profits are given by (4). To see why, note that A_{-i} can only deviate by contracting with ADX_2, which is unprofitable because it reduces profits by $\eta_1(C-w)$. Consider A_i next. Since (No Sniping) holds, A_i does not want to deviate by cross-targeting on ADX_1. Thus, A_i's only relevant deviation is to contract with ADX_1, which causes full cross-targeting and profits equal to (2). It is therefore profitable if and only if (No Leave) holds with strict inequality. Notably, (No Leave) also implies (No Leave & Snipe) in this case since

$$\alpha(M-C) \leq \alpha(C-w) + (\eta_2-\eta_1)(C-w) \leq (1-2\alpha)w + (\eta_2-\eta_1)(C-w), \tag{11}$$

which holds because $\alpha C \leq (1-\alpha)w$ due to (No Sniping). That is, if (No Leave) holds (so that this deviation is profitable), (No Leave & Snipe) holds as well. But if both (No Leave) and (No Leave & Snipe) hold, full cross-targeting is an equilibrium, contradicting the assumption that it is not. Consequently, (No Leave) must fail so that A_i has no profitable deviation.

Second suppose that (No Sniping) fails, in which case (Being Sniped) must hold or both advertisers contracting with ADX_2 would be an equilibrium. Then, A_{-i} contracting with ADX_1 and A_i contracting with ADX_2 while cross-targeting on ADX_1 is an equilibrium. To see why, observe that A_{-i} can only deviate by contracting with ADX_2, which is unprofitable because (Being Sniped) holds. In addition, the failure of (No Sniping) implies that A_i does not want to deviate by stopping cross-targeting. Thus, the only viable deviation is for A_i to contract with ADX_1 instead, leading to full cross-targeting and profits equal to (2). This deviation is profitable if (No Leave & Snipe) holds. However, if (No Leave & Snipe) holds, it also implies (No Leave) in this case since

$$\alpha(M-C) \leq (1-2\alpha)w + (\eta_2-\eta_1)(C-w) < \alpha(C-w) + (\eta_2-\eta_1)(C-w), \tag{12}$$

which holds because $\alpha C > (1-\alpha)w$ due to the failure of (No Sniping). It follows then that (No Leave & Snipe) must fail, or else full cross-targeting is an equilibrium, contradicting the assumption that it is not. If (No Leave & Snipe) fails, A_i has no profitable deviation. ■

Proof of Proposition 1: By Lemma 1, both advertisers contract with ADX_1 if ADX_1 is closed. Total profits of ADX_1 are thus

$$2w\phi(1-\alpha+\eta_2). \tag{13}$$

Next, consider profits if ADX_1 is open and assume that both (No Leave & Snipe) and (No Leave) hold. By Lemma 2, both advertisers contracting with ADX_1 is an equilibrium in this case. By Lemma 4, this equilibrium strictly Pareto-dominates (for advertisers) any other equilibrium that may exist. Thus, Assumption 1 allows us to to restrict attention to the full cross-targeting equilibrium, in which total profits for ADX_1 are equal to

$$2w\phi(1+\eta_2). \tag{14}$$

Since this is strictly higher than (13), ADX_1 allows cross-targeting if both (No Leave & Snipe) and (No Leave) hold, proving "if" part.

For the "only if" part, assume that ADX_1 is open but that (No Leave & Snipe) or (No Leave) fail. Lemma 5 guarantees that a pure strategy equilibrium still exists, though at most one advertiser may contract with ADX_1 or else there would be full cross-targeting, contradicting Lemma 2. Thus, profits of ADX_1 are bounded from above by

$$2w\phi(1-\alpha+\eta_1), \tag{15}$$

which equals ADX_1's profits if one advertiser contracts with ADX_1, the other cross-targets on ADX_1, and both advertisers target η_1 look-alike consumers. This is less than the profits from being closed as given in (13) because $\eta_2 > \eta_1$. Thus, ADX_1 chooses closed if (No Leave & Snipe) or (No Leave) fail, proving the "only if" part of the Proposition.

The remainder of the proposition follows from the discussion in the main text and the proof above. If ADX_1 allows cross-targeting as (No Leave & Snipe) and (No Leave) hold, both advertisers contract with ADX_1 by Lemma 2, implying a look-alike audience size of η_2. Moreover, if ADX_1 does not allow cross-targeting, both advertisers contract with ADX_1 as well according to Lemma 1, again implying a look-alike audience size of η_2. ■

Proof of Proposition 2: Given the similarity to Proposition 1, we keep this proof short. Note that Lemma 4 and Lemma 5 readily extend to consumers having control over tracking. Consequently, Assumption 1 implies that being open always leads to full cross-targeting if and only if both (C: No Leave) and (C: No Leave & Snipe) hold. Thus, choosing open if both (C: No Leave) and (C: No Leave & Snipe) hold leads to profits for ADX_1 of

$$2w\phi(q+\eta_2). \tag{16}$$

Next, we must consider the outcome if ADX_1 chooses open and full cross-targeting is not an equilibrium. This subgame has a pure strategy equilibrium since the logic of Lemma 5 still holds. Furthermore, the best possible outcome for ADX_1 if it chooses open but full cross-targeting is not an equilibrium involves A_i contracting with and retargeting on ADX_1, A_{-i} cross-targeting, and both targeting η_1 look-alike consumers. In this case, allowing cross-targeting leads to profits of

$$2w\phi((1-\alpha)q+\eta_1). \tag{17}$$

Lastly, as argued in the main text, both advertisers contract with ADX_1 if it chooses closed, implying profits of

$$2w\phi((1-\alpha)q+\eta_2). \tag{18}$$

As in the base model, it is easy to verify that the profits of ADX_1 if it allows cross-targeting and if full cross-targeting is an equilibrium exceed its profits if it is closed, which, in turn, exceed its profits if ADX_1 is open but full cross-targeting is not an equilibrium. Analogously to the proof of Proposition 1, this is is sufficient to establish that ADX_1 allows cross-targeting if and only if (C: No Leave) and (C: No Leave & Snipe) hold, in which case it leads to full cross-targeting. ■

Proof of Proposition 3: The result follows if the term in (5) decreases in q upon substituting $\eta_2 = \eta(q)$ and $\eta_1 = \eta(q(1-\alpha))$. Let $\eta'(x)$ denote the derivative of η with respect to x. Then, the derivative of $\dfrac{\eta(q)-\eta(q(1-\alpha))}{q}(C-w)$ with respect to q equals

$$\frac{[\eta'(q)-(1-\alpha)\eta'((1-\alpha)q)]\,q-[\eta(q)-\eta((1-\alpha)q)]}{q^2}(C-w), \tag{19}$$

which is negative if and only if

$$\eta'(q)q-\eta(q)<(1-\alpha)q\eta'((1-\alpha)q)-\eta((1-\alpha)q). \tag{20}$$

Since $q>(1-\alpha)q$, the inequality above is implied if $\eta'(x)x-\eta(x)$ decreases in x. Taking the derivative shows that $\eta'(x)x-\eta(x)$ decreases in x if and only if $\eta''(x)<0$ or, equivalently, if η is concave. ■

Proof of Proposition 4 : Proposition 4 directly follows from the discussion on page 14. ■

Proof of Proposition 5: Only if: If both advertisers bid on ADX_1, their respective payoffs equal $(1+\eta_2)(C-\tau w)$, while a unilateral deviation to the ADX_2 accrues $\sigma(1-\alpha)(C-w)$. It follows that both advertisers bidding on ADX_1 constitutes an equilibrium if $w<w_1\equiv\frac{(1+\eta_2)-\sigma(1-\alpha)}{\tau(1+\eta_2)-\sigma(1-\alpha)}C \Leftrightarrow \tau<\frac{(1+\eta_2)-\sigma(1-\alpha)}{w}C+\frac{\sigma(1-\alpha)}{1+\eta_2}$. If: Furthermore, note that if both advertisers bid on the ADX_2, their respective payoffs equal $\sigma[\alpha M+(1-2\alpha)C-(1-\alpha)w]$. When playing asymmetric strategies, the advertiser bidding on ADX_1 earns $(1+\eta_2)M-\sigma(1-\alpha)(M-C)-(1+\eta_2)\tau w$, while the advertiser bidding on the ADX_2 earns $\sigma(1-\alpha)(C-w)$. It follows that both advertisers bidding on the

ADX_2 constitutes an equilibrium only if $w > w_2 \equiv \frac{[(1+\eta_2)-\sigma]M+\alpha\sigma C}{\tau(1+\eta_2)-\sigma(1-\alpha)}$, while asymmetric strategies do not exhibit a profitable unilateral deviation if $w_1 \leq w \leq w_2$. Since $w_2 = \frac{[(1+\eta_2)-\sigma]M+\alpha\sigma C}{\tau(1+\eta_2)-\sigma(1-\alpha)} = \frac{(1+\eta_2)-\sigma(1-\alpha)}{\tau(1+\eta_2)-\sigma(1-\alpha)} + \frac{1+\eta_2-\sigma}{\tau(1+\eta_2)-\sigma(1-\alpha)}(M-C) > w_1$, it follows that if $\tau < \frac{(1+\eta_2)-\sigma(1-\alpha)}{w}C + \frac{\sigma(1-\alpha)}{1+\eta_2}$ the unique equilibrium has both advertisers bid on ADX_1. Finally, using the fact that $C < w$ and $\sigma < 1$, simple algebra shows that $\tilde{\tau} > 1$ if $\eta_2 > \dfrac{\eta_2}{1+\eta_2}(1-\alpha)\sigma$, which holds true for all $\eta_2 > 0$. ■

Proof of Proposition 6: Recall from Proposition 1 that if full-cross targeting does not obtain in the base model, both advertisers contract with ADX_1, retarget their own website visitors and target the look-alike audience of size η_2. As such, $1-2\alpha+\eta_2$ consumers receive two targeted ads while the 2α exclusive website visitors receive a single targeted ad. Under full cross-targeting, on the other hand, everyone receives two targeted ads. Thus, the total benefit of consumers interested in the product category when moving from an overall retargeting to an overall full cross-targeting equilibrium is $2\alpha(u_2-u_1)$. Under full cross-targeting in the Topics model $\tau(1+\eta_2)$ consumers receive both ads, a fraction $\tau-1$ of which is not interested in the product category. It follows that the total cost of introducing the Topics system is $(\tau-1)(1+\eta_2)2\kappa$. The claim follows from

$$2\alpha(u_2-u_1) > (\tau-1)(1+\eta_2)2\kappa \Leftrightarrow \tau < 1 + \frac{\alpha}{1+\eta_2}\frac{u_2-u_1}{\kappa}.$$

■

Proof of Proposition 7: (i) There are $\tau(1+\eta_2)$ consumers under consideration. In the region where both advertisers bid on ADX_1, ADX_1 is selling two targeted ads for the $1+\eta_2$ consumers interested in the product category, and for the $(\tau-1)(1+\eta_2)$ who are not. It follows that ADX_1 earns $\tau(1+\eta_2)2\phi w$ from selling targeted ads, which is clearly increasing in τ. (ii) Note that

$$\frac{\partial\tilde{\tau}}{\partial\sigma} = (1-\alpha)\left(\frac{1}{1+\eta_2} - \frac{C}{w}\right) < 0,$$

which proves the claim. ■

Proof of Corollary 2: Let

$$w > w_2 = \frac{[(1+\eta_2)-\sigma]M+\alpha\sigma C}{\tau(1+\eta_2)-\sigma(1-\alpha)} \Leftrightarrow \tau > \frac{[(1+\eta_2)-\sigma]M+\alpha\sigma C}{w(1+\eta_2)} + \frac{\sigma(1-\alpha)}{1+\eta_2},$$

and recall from the proof of Proposition 5 that this condition implies the equilibrium under the Topics system to have advertisers retargeting on ADX_2, which means ADX_1 earns zero profits. By Proposition 1, the worst possible outcome under the system without Topics is for ADX_1 to not allow cross-targeting, which nonetheless ensures that both advertisers retarget their own website visitors, as well as the η_2 look-alike consumers, on ADX_1. As such, if τ is large enough such that $w > w_2$, ADX_1 prefers not to adopt the Topics system. ■

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