Who Benefits from Platform Entry if Multi-Agent Prices Signal Product Quality?

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Abstract

Merchants are wary of a platform entering as a competitor and adversely affecting sales and margins. A platform wanting to maximize profits is interested also in gaining consumer trust and confidence by providing credible quality information to consumers. As a practical matter, could a platform benefit by entering, by using multi-sender prices as signals to alleviate consumers' uncertainty of quality? Further, can a merchant also benefit from platform entry? We answer these questions by analyzing strategic pricing by a platform and merchant under quality uncertainty.

We innovate by modeling platform-merchant competition in a leader-follower framework. We invoke what we label Perfect Bayesian-consistent beliefs to characterize the sub-game perfect pricing strategy that is also PBE with consumers resolving quality uncertainty using both platform and merchant prices.

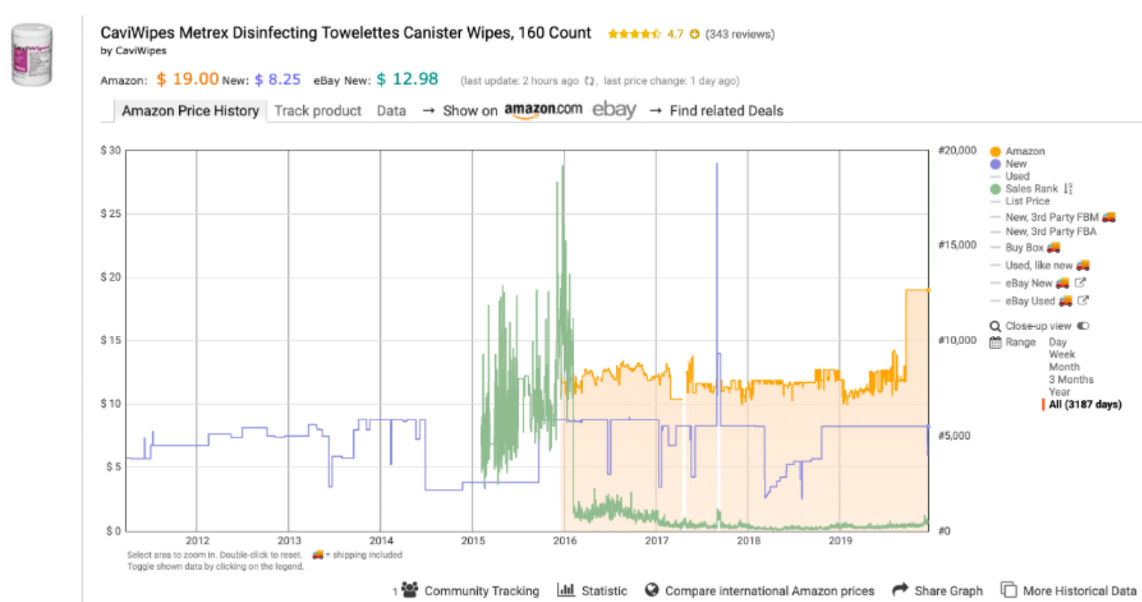
A substantive finding is that platform entry can raise merchant profits when the quality is high: by moving the equilibrium from pooling to separating, and supporting higher prices. We find that platform entry and resulting multi-agent signaling can also help to inform consumers of quality. Consumers can benefit by lower prices and increased market coverage due to seller competition. An important result is that conditions exist for an equilibrium outcome identical to complete information.

Keywords: E-commerce, Online Selling, Merchant Profits, Platform Entry, Multi-Agent Signaling, Product Quality, Pricing Strategy, Game Theory

1 Introduction

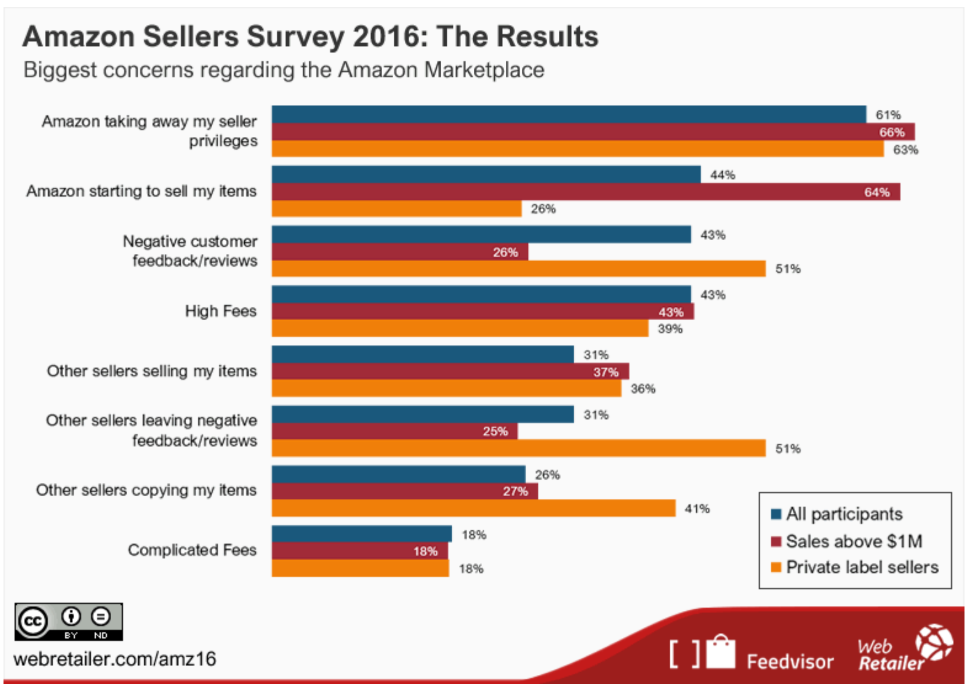
What can happen when a platform such as Amazon enters by selling directly a product that a merchant carries? Decreased sales and lower margins for the merchant, lower commission revenues for the platform, lower prices for consumers, and even possible exit by the merchant are some obvious consequences. Another effect of platform entry could be significantly higher sales due to a larger consumer base that Zhu and Liu (2018) found in their empirical study of Amazon's entry. An example of this phenomenon is displayed in figure 1. Is it possible then that platform's entry results in more complete market coverage? We provide an answer to this by showing that under conditions of quality uncertainty, a separating equilibrium with incomplete market coverage sustained by the merchant could be replaced by one that results in complete coverage because of platform entry. Our finding is based on analysis of the strategic implications of platform entry. A novel aspect of our approach that represents a contribution to the literature is signaling quality through prices by multiple agents who are part of a marketing channel.

Figure 1: An Example of Decrease Sales Rank after Amazon Entry



Another interesting and intriguing result we obtain is that higher prices for a high quality product as a result of revelation could actually lead to higher profits for the merchant. In other words, platform entry can help the merchant. This is important because a concern on the minds of merchants is the loss of sales and revenues as a result of platform entry. For example, in a 2016 survey of Amazon sellers conducted by Webretailer, the second highest concern expressed by merchants is "... Amazon starting to sell my items ...", as displayed in figure 2. In this paper we identify conditions under which platform's entry has a positive effect on merchant's profits. Indeed, somewhat counter to intuition, platform entry can result in a win-win for both sellers.

Figure 2: Amazon Sellers Survey 2016



Quality uncertainty naturally raises the question of whether a platform can adopt strategies that lead to revelation of quality. This would benefit consumers, and also in the long-run promote greater consumer confidence and trust in the platform. Our concern in this paper is with the effect of entry strategy on revelation. Obviously, if the platform's entry is conditional on quality, then entry can itself be a signal of quality. What we show is that even in those cases in which entry is not driven by quality, prices can lead to revealing quality. We show that indeed, if the competition between the platform and the merchant is neither too intense nor too subdued, multi-agent pricing leads to revelation. What we find interesting is that competition that is too intense may result in quality not being revealed though obviously prices would be lower. What we find interesting is that competition that is too intense may result in quality not being revealed though obviously prices would be lower.

Our focus in this paper is on what happens following platform entry. In addition to the potential for platform's entry to have a positive effect on merchant's profits, a few of our novel findings are worth highlighting. In the case of multi-agent signaling, the outcome under uncertainty can be identical to that under complete information, thus eliminating any inefficiency due to quality uncertainty, and this occurs only due to platform entry. This is an important finding. More generally, if consumers can use two (price) messages to learn the private (quality) information, then there exist both separating and pooling equilibria that don't exist were the platform not to enter.

The rest of the paper is organized as follows. Section 2 places our work relative to extant work and reviews the literature.

2 Review of Literature

Our paper is closely related to two streams of literature. The first one focuses on the competition between a platform and third-party sellers when the platform decides to carry the product. The second one is on signaling.

The relative anonymity of sellers and the very large number of such sellers in a marketplace make informational issues relevant. Past research that has focused on this includes Jiang, Jerath and Srinivasan (2011) who model the uncertainty faced by the platform. In their model, the merchant and consumers are assumed to know the true quality of the seller's product but the platform is uninformed. They also assume that the platform can strategically choose the sales commission depending on the degree of uncertainty of quality. They then study the platform's entry strategy in the second period after learning the quality in the first period, assuming that entry would displace the seller. In our model, it is the consumers who are uncertain of the product quality while both the merchant and the platform are informed of product quality. Zhu and Liu (2018) examine empirically Amazon's decision to carry third-party sellers' products. They find that Amazon sells products that are ranked higher in sales and also rated higher by consumers. Interestingly, they find additionally that Amazon's entry causes the demand for the product to increase. Our work focuses on uncertainty of quality, exploring the effect of platform entry on resolution of this uncertainty. Indeed, platform entry by giving rise to multi-sender signaling renders prices as a credible signal. Thus, we add to prior work.

There is also research that focuses on channel relationships rather than asymmetric information. Mantin, Krishnan and Dhar (2014) shed light on how third-party sellers who may carry a used product or lower-quality product impact manufacturer and first-party sellers who sell the new product. Ryan, Sun and Zhao (2012) focus on channel conflict. The marketplace firm, such as Amazon, can choose whether to offer marketplace service to an online retailer and whether to sell the product itself. The online retailer, on the other hand, can choose whether to sell through the marketplace firm if the service is offered. Chen and Guo (2014) study how a platform can maximize its profit by allowing small sellers to sell on it to leverage lower adverting cost.

The second relevant literature pertains to signaling. It is now well established that in certain situations a seller can profitably reveal private information to consumers: Nelson (1974), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986) and Bagwell and Riordan (1991), to list a few. In practice, the exact mechanism used to signal information depends on the marketing situation. In different situations, a seller may want to signal prices, demand, cost or quality to consumers or a downstream firm. The seller is usually assumed to be a monopolist having some private information. Prior work in marketing has examined various instruments as signals: prices, advertising, assortment, branding, warranty and so on. Anderson and Simester (1998, 2001) argue that sale signs can convey valuable information about a seller's current and future prices to consumers, and further offer managerial guidelines on how many sale signs may be best employed. Desai (2000) proposes a mix of advertising and slotting allowance for signaling high demand by a manufacturer to an uninformed retailer. Shin (2005) argues that depending on a retailer's selling cost an informative or vague advertising message of price can serve as a signal. Stock and Balachander (2008) demonstrate scarcity as a way to signal quality. They show that it is the high-quality seller that may have the incentive to adopt such a strategy. Kalra and Li (2008) find that a firm can specialize in one category and forego the potential profit in another category to signal quality in homogeneous markets. Moorthy (2012) shows how brand extension can be used to signal product quality given that it costs less than a new brand. Miklós-Thal and Zhang (2013) argue that demarketing can be used as a signaling device to affect perception of quality based on observed sales. Yu, Kapuscinski and Ahn (2015) show how capacity rationing can be a signal in combination with advance selling. Kremer and Debo (2015) show how consumers can use wait time as a signal of quality. This in turn leads to observational learning in which consumers prefer products even if there is a long wait time for it. Subramanian and Rao (2016) show that by displaying deal sales, daily deal sites can leverage experienced consumers who are informed to convey information through observational learning to uninformed, new consumers and thus help merchants to acquire new consumers.

Some of the more recent work has studied situations in which private information may be held by multiple firms that compete. Then the question is: how is information revelation affected by the possibility of multi-agent signaling? Our work falls into this category since in our model both the platform and the merchant are assumed to know the quality of the product. Moreover, they both compete by choosing prices, and so consumers could potentially use both prices to infer quality. Hertzendorf and Overgaard (2001) analyze a similar situation in which vertically differentiated firms compete by choosing prices and advertising and what is unknown is the identity of the high quality seller, though one is known to be of high quality and the other low quality. Thus, both firms are fully informed, a feature of our model as well. However, in our model, the platform and the merchant are not vertically differentiated and moreover they are not ex-ante identical. Also, the sellers in our model are part of a channel and have ongoing contractual relationships. Their main result turns on quality revelation being facilitated by advertising when prices can't. Also, in their model, revelation is accompanied by some loss of efficiency. Yehezkel (2008) extends Hertzendorf and Overgaard with a fraction of the market informed and the rest uninformed. Daughety and Reinganum (2007, 2008) and Janssen and Roy (2010) model competition among firms who don't know the other firms' type. Daughety and Reinganum model a duopoly that is also horizontally differentiated while Janssen and Roy study an oligopoly that is not horizontally differentiated. In other words, consumers cannot infer other firms' type from the focal firm's signals. In our model too the platform is differentiated from the merchant through branding, as we will see, but in contrast, the platform and the merchant both sell product of identical quality of which they are informed. The main finding of Daugherty and Reinganum (2007, 2008) is that (upward) price distortion to reveal quality moderates price competition since prices are strategic complements in their model. In Jansen and Roy in the fully revealing equilibrium the low-type firm adopts a mixed strategy and has market power through its relatively low production cost. However, the high-type firm only realizes positive sales if the other firm is of the same type. Revelation is an issue also in cheap talk and Battaglini (2002) addresses the question of how the presence of multiple senders affects it. A fully revealing equilibrium in his model depends essentially on the presence of "opposed biases" of the two senders. In our model too the fact that platform and merchant prices can serve as signals leads to new ways for revelation to occur but unlike the cheap talk case we must take into account the fact that the price messages affect the profits of the senders directly. Quality affects consumers' willingness-to-pay by raising it, and along with prices also the purchase decision. In other words, the quality state is mapped into consumers' utility in two different ways. In our model, the two senders have opposed biases because of the second mapping. The magnitude of opposed biases, moreover, is contingent on the true state that makes possible a fully revealing equilibrium without loss in efficiency.

We contribute to prior work on platform-merchant strategic interaction by explicitly modeling the role of prices in resolving consumer uncertainty of product quality. This has marketing implications, especially for the platform. In this way our work yields new insights into an aspect of online marketing strategy. We use a novel approach, of multi-agent signaling, to analyze our model. Our work departs from prior work on multi-sender signaling in two ways. The senders in our model sell product of identical quality. Second, we use a leader follower framework that results in a class of separating equilibria that prior work does not comprehend. These innovations potentially expand the scope of applications.[1] Thus, our addition to the growing literature on signaling by competing firms would be of interest to game theoretic modelers.

3 Model of Platform-Merchant Pricing

We first elaborate on the equilibrium concept we invoke to explore how pricing by a platform such as Amazon, and a merchant that sells on the platform affects equilibrium outcomes if consumers do not know product quality but sellers do.[2] We proceed by addressing, in 3.1, the incomplete information Platform-Merchant game in a very general way to elaborate on the equilibrium concept we invoke because of its novelty. We then describe, in 3.2, a more specific Platform-Merchant Model that incorporates the characteristics of the marketplace. In 3.3, we use the equilibrium concept to analyze our model and obtain results.

[1] Kuksov and Lin (2017) model two competing firms that are differentiated on quality but only one quality is uncertain and so essentially only one firm can signal margin through assortment. Rao and Turut (2019) also model two competing firms but only one is endowed with a higher future quality that it must choose to pre-announce and thus signal its private information.

[2] A common feature of a platform, for example Amazon, is that it offers a product for sale that a merchant also carries.

3.1 The Platform-Merchant Game

We want to examine pricing of a product sold by both a platform P and a merchant M on the platform. The platform and merchant choose prices p_P and p_M respectively to maximize their profits. We model the pricing problem in a leader-follower framework: the platform chooses price first, followed by the merchant. Sellers' profits are denoted by $\Pi_M(p_M; p_P)$ for the merchant and $\Pi_P(p_P, R_M(p_P))$ for the platform. Note that the merchant takes platform's price p_P as given, while the platform anticipates the merchant's reaction $R_M(p_P)$ to p_P. Thus, $R_M(p_P)$ maximizes the merchant' profits in the sub-game induced by p_P, denoted by $g_{p_P}(p_M; p_P)$ while p_P maximizes platform's profits in the game denoted by $G(p_P, p_M)$, with $p_M = R_M(p_P)$.

We focus on a market in which the product's quality is known to the sellers but not to consumers. This gives prices a dual role: sellers' choice of prices affects consumers' choice of seller on the one hand, and their willingness-to-pay depending on quality, on the other. There are two innovations in our model: first, consumers' belief about quality is informed by prices of both the platform and the merchant, and so, two prices chosen independently by two agents, act as signals of quality; second, the leader-follower structure lends a natural way to model the resulting multi-agent signaling problem. We want to identify the conditions under which prices lead to quality revelation. We assume that quality q of product i can be high or low, $q = q_i$, $i \in \{H, L\}$. Further, consumers' prior belief about product quality (type) is: $\Pr(i = H) = \theta$. Define $\overline{q} \triangleq \theta q_H + (1-\theta) q_L$.

Since sellers' prices can affect consumers' posterior belief $\hat{\theta}(p_P, p_M) \triangleq \Pr(i = H | p_P, p_M)$, the game must account for it. Keeping in mind this as well as the fact that profits would depend on quality $i \in \{H, L\}$, denote the profits of the merchant and platform respectively by, $\Pi^i_M(p_M, \hat{\theta}(p_P, p_M); p_P)$ and $\Pi^i_P(p_P, R^i_M(p_P, \tilde{\theta}(p_P, R^i_M(p_P))), \tilde{\theta}(p_P, R^i_M(p_P)))$. Therefore, consumers' posterior belief that the merchant must take into account in the sub-game $g_{p_P}(p_M, \hat{\theta}(p_P, p_M); p_P, i)$ is different from the one the platform must account for in the game $G(p_P, R^i_M(p_P, \tilde{\theta}(p_P, R^i_M(p_P))), \tilde{\theta}(p_P, R^i_M(p_P)); i)$. Consequently, equilibrium must be a PBE that is also sub-game perfect.

3.1.1 Sub-game Perfect PBE

A Perfect Bayesian Equilibrium (PBE) consists of firms' prices $(\hat{p}_P^i, \hat{p}_M^i)$ and consumers' posterior beliefs $\hat{\theta}(\hat{p}_P^i, \hat{p}_M^i)$. The beliefs inform consumer purchase decisions, affecting profits; prices maximize firms' profits and are sub-game perfect; and finally, beliefs are Bayes-consistent. We address each in turn.

Consumer Purchases and Sales: Consumers' purchase decision would depend on their valuation of quality, and the two seller prices. For example, willingness-to-pay for quality denoted by $\hat{W}$ could be:

$$\hat{W}\left(\hat{\theta}(p_P, p_M)\right) = \begin{cases} W_H, & \text{if } \hat{\theta} = 1 \\ W_L, & \text{if } \hat{\theta} = 0 \\ \overline{W}, & \text{if } \hat{\theta} = \theta \end{cases} \tag{1}$$

In turn, $\hat{W}$ would affect sellers' profits.

Merchant: Merchant's choice, depending on quality q_i, must solve a suitable maximization problem:

$$\hat{p}_M^i = \arg\max_{p_M} \Pi_M^i(p_M, \hat{\theta}(\hat{p}_P, p_M); \hat{p}_P) \tag{2}$$

Platform: Platform must anticipate the merchant's reaction to its price. Then, the platform's choice is given by:

$$\hat{p}_P^i = \arg\max_{p_P} \Pi_P^i(p_P, R_M^i(p_P, \tilde{\theta}(p_P, R_M^i(p_P))), \tilde{\theta}(p_P, R_M^i(p_P))) \tag{3}$$

where $R_M^i(p_P, \tilde{\theta}(p_P, p_M)) = \arg\max_{p_M} \Pi_M^i(p_M, \tilde{\theta}(p_P, p_M); p_P)$ is the merchant's best response and $\tilde{\theta}(p_P, R_M^i(p_P, \tilde{\theta}(p_P, p_M)))$ is the posterior that the platform anticipates. What should be clear is that the necessary condition for platform's choice in equilibrium requires us to specify not only on-equilibrium beliefs but also certain off-equilibrium beliefs. Specifically, we need to specify $\tilde{\theta}(p_P, R^i_M(p_P, \tilde{\theta}(p_P, p_M)))$, $p_M = R^i_M$, for all p_P in order to solve for the sub-game perfect equilibrium prices. We invoke what we label perfect Bayes-consistent beliefs to meet this challenge. This is a consequence of multi-agent signaling. Two points are worth highlighting. First, the platform's equilibrium choice $\hat{p}^i_P$ is informed not only by the on-equilibrium merchant response but also by the off-equilibrium responses and resulting Bayes-consistent consumer beliefs. Second, consumer beliefs must rationalize both platform's choice and merchant's choice.[3] The leader-follower framework plays a critical role in arriving at Bayes-consistent beliefs that addresses both these challenges since it imposes the sub-game perfectness criterion on equilibrium. That permits a way to accommodate outcomes in which only the platform's choice or the merchant's is on equilibrium.

On-Equilibrium Bayes-consistent beliefs: These should satisfy

$$\hat{\theta}(p_P, p_M) = \begin{cases} \begin{cases} 1, & \text{if } (p_P, p_M) = (\hat{p}^H_P, \hat{p}^H_M) \\ 0, & \text{if } (p_P, p_M) = (\hat{p}^L_P, \hat{p}^L_M) \end{cases}, & \text{if } (\hat{p}^H_P, \hat{p}^H_M) \neq (\hat{p}^L_P, \hat{p}^L_M) \\ \theta, & \text{if } (\hat{p}^H_P, \hat{p}^H_M) = (\hat{p}^L_P, \hat{p}^L_M) \end{cases} \tag{4}$$

PBE is sustained by (1)-(4) as well as Perfect Bayes-consistent beliefs that we next characterize.

3.1.2 Perfect Bayes-consistent beliefs

We construct Perfect Bayes-consistent beliefs $\hat{\theta}(p_P)$ for all platform prices $p_P \in \Omega$, where Ω is the set of feasible platform prices, by identifying subsets of Ω, denoted by Ω_j, $j = 1, 2$. We then characterize the necessary conditions for each of them. As we will see, Ω_1 contains only outcomes with quality revelation while prices in Ω_2 lead to outcomes with or without

[3] A similar challenge can arise in a single-agent signaling problem if the signal is multi-dimensional (Ramey, 1996). In that case, a receiver must rationalize the elements of the vector that are on-equilibrium while assuming the off-equilibrium elements as arising with zero probability. In our model, sub-game perfectness offers a precise way of meeting the challenge.

revelation. The intuition behind our approach is that prices in Ω_1 must satisfy platform's IC constraints while prices in Ω_2 must satisfy merchant's IC constraints for revelation. Consumer beliefs then rationalize platform's choice if $p_P \in \Omega_1$, and rationalize both merchant's and platform's choices if $p_P \in \Omega_2$.

First, we examine PBE in the single-agent incomplete information game between platform and consumer subject to the constraint that in any equilibrium the merchant is at best response, denoted by $R^*_M(p_P, \hat{\theta}(p_P))$, which is conditioned on p_P and $\hat{\theta}(p_P)$.[4] Note that since the merchant is at a best response, it satisfies the sub-game perfectness condition. If $\hat{\theta}(p_P) \in \{0, 1\}$, then we say $p_P \in \Omega_1$; moreover, there is revelation, and so the perfect Bayes-consistent belief is:[5]

$$\hat{\theta}(p_P) = \begin{cases} 1, & \text{if } q = q_H \\ 0, & \text{if } q = q_L \end{cases} \qquad p_P \in \Omega_1$$

It is obvious that if $p_P \in \Omega_1$ is equilibrium outcome in G, then $\hat{\theta}(p_P) = \hat{\theta}(p_P, R^*_M(p_P, \hat{\theta}(p_P))) =$ 0 (or 1) if quality is low (or high). The reason why also $\hat{\theta}(p_P) = \hat{\theta}(p_P, p_M) \forall p_M$ is as follows. Essentially, consumers rationalize platform price. Any merchant prices that are off-equilibrium are viewed as unintentional deviations that occur with probability zero. So they are not in need of rationalization. Thus, Perfect Bayes consistent beliefs account for all off-equilibrium prices of the merchant. As a result, if $p_P \in \Omega_1$, merchant price has no force in revelation, and the *platform price is both informative and sufficient* for Bayes-consistent beliefs.

Second, $\forall p_P \in \overline{\Omega}_1 \subset \Omega$ we can characterize the PBE in the single-agent incomplete information sub-game induced by p_P denoted $g_{p_P}(p_M, \hat{\theta}(p_M; p_P); p_P)$. We must consider the possibility of both SE and PE existing. If there are multiple equilibria, the minimality criterion in the leader-follower framework implies that the unique equilibrium in the sub-

[4]In the presence of multiple PBE, we can use the intuitive criterion to arrive at the unique PBE.
[5]The intuitive criterion is sufficient to rule out multiple separating equilibria as well as pooling equilibrium.

game that survives an appropriate refinement, is also what determines the best responses.[6] We then say that the platform's assessment of the posterior belief for $p_P \in \Omega_2$ is

$$\hat{\theta}(p_P) = \begin{cases} \begin{cases} 1, & \text{if } q = q_H \\ 0, & \text{if } q = q_L \end{cases}, & \text{if } R_M^H(p_P; 1) \neq R_M^L(p_P; 0) \\ \theta, & \text{if } R_M^H(p_P; 1) = R_M^L(p_P; 0) \end{cases}$$

Now it is easy to state the necessary conditions for the platform price in each subset. Suppose a pair of prices p_P^H, p_P^L satisfy the following platform IC constraints:

$$\Pi_P^H \left(p_P^H, R_M^H(p_P, 1), 1\right) > \Pi_P^H \left(p_P^L, R_M^L(p_P, 0), 0\right) \tag{5}$$

$$\Pi_P^L \left(p_P^H, R_M^H(p_P, 1), 1\right) < \Pi_P^L \left(p_P^L, R_M^L(p_P, 0), 0\right) \tag{6}$$

Then, p_P^H, $p_P^L \in \Omega_1$ and moreover, $\hat{\theta}(p_P^H) = 1$ and $\hat{\theta}(p_P^L) = 0$. Constraint (5) ensures platform rationality and (6) is the no mimic constraint.

Next, suppose a price $p_P \notin \Omega_1$ satisfies the following merchant IC constraints:

$$\Pi_M^H \left(R_M^H(p_P, 1); p_P, 1\right) > \Pi_M^H \left(R_M^L(p_P, 0); p_P, 0\right) \tag{7}$$

$$\Pi_M^L \left(R_M^H(p_P, 1); p_P, 1\right) < \Pi_M^L \left(R_M^L(p_P, 0); p_P, 0\right) \tag{8}$$

Then, $p_P \in \Omega_2$, and moreover, p_P admits the possibility that $\hat{\theta}(p_P) = 1$. However, for $\hat{\theta}(p_P) = 1$, a possible PE in the sub-game induced by p_P must be ruled out so that SE is the best response for the merchant. Only in that case, $\hat{\theta}(p_P) = 1$. Constraint (7) ensures merchant rationality and (8) is the no mimic constraint. Note that the IC constraints are satisfied only if $R_M^H(p_P; 1) \neq R_M^L(p_P; 0)$. If the IC constraints are not satisfied, we say $p_P \in \Omega_2$, and moreover, $\hat{\theta}(p_P) = \theta$.

We can now see what aspects of our model allow us to use the proposed equilibrium

[6] With two quality levels the intuitive criterion suffices as a refinement in our model.

concept. Obviously, it makes sense only in a leader-follower framework. In this framework, the platform's problem is well behaved if the merchant's best response price is unique. This also guarantees that we can assign $p_P \in \Omega_1$ and ensures that for $p_P \in \Omega_2$, in the PBE in the sub-game induced by p_P among all SEs (PEs) there must be a unique one.

3.2 Model of Platform-Merchant

In this section we describe the main features of our model.

Firms (Sellers): In our model, the product is assumed to meet a minimum level of performance of a base product with probability 1 and q is the probability that it meets the maximum level of performance that we label a "perfect" product. The constant marginal cost of the product is $K(q)$. The platform's profits derive from its own sales and a fraction $c \geq 0$ of merchant sales.[7] The merchant's profits are derived from its net revenues. We assume that $K(q) < (1-c)q$, so that $\exists$ a merchant price yielding non-negative profits.

Consumers and Demand: The market consists of a unit of consumers. All consumers have a willingness-to-pay $\$w$ for a perfect product and $\$v$ for the base perfect. Without loss of generality, let $w = 1$. Then, for a product of quality q consumers' willingness-to-pay, assuming them to be risk neutral, is $W(q) = q + v(1-q)$. Seller $n \in \{P, M\}$ is then in the choice set only if $p_n \leq W(q)$. Three possibilities arise. First, consumers' choice set is empty if $\min\{p_P, p_M\} > W(q)$ in which case they forgo purchase. Second, if $\max\{p_P, p_M\} > W(q) \geq \min\{p_P, p_M\}$, their choice set contains a single item and so they buy from the lower priced seller. Finally, if $\max p_P, p_M \leq W(q)$, their choice set contains both sellers and so they must decide who to buy from. How might consumers decide whether to buy from the platform or the merchant? We incorporate findings of past research on consumer choice for example, Erdem, Swait and Louviere (2002), that brands with lower credibility must rely more on price to attract consumers. They found that "...impact of price on consumer utility may be moderated by brand credibility... our results strongly

[7]The commission reflects channel structure that we wish to model. In section 3.3, there is no loss of generality if we were to set $c = 0$.

suggest that more credible brands generate a number of consumer benefits that are then rewarded, as it were, by decreased price sensitivity...". Keeping our application in mind, we incorporate in our model the reality that the platform's offering is more credible than the merchant's by assuming that all consumers prefer to buy from the platform if $p_M > p_P$. Switching from platform to merchant can be induced by the merchant with a sufficiently lower price. A useful way to model consumer utility follows Narasimhan (1988). Some consumers are extremely price sensitive, the switching segment in Narasimhan's framework, who would prefer to buy from the merchant if the discount offered by the merchant relative to the platform $\delta = p_P - p_M \geq 0$. Keep in mind that there are two segments: the price sensitive segment and platform loyal segment. Then consumer's utility u is given by,

$$u(p_P, p_M; \delta) = \begin{cases} q - \min\{p_P, p_M\}, & \text{price sensitive consumer} \\ q - \min\{p_P, p_M - \delta I\}, I = \begin{cases} 0, & \text{if } q < p_P \\ 1, & \text{else} \end{cases} & \text{platform loyal consumer} \end{cases}$$

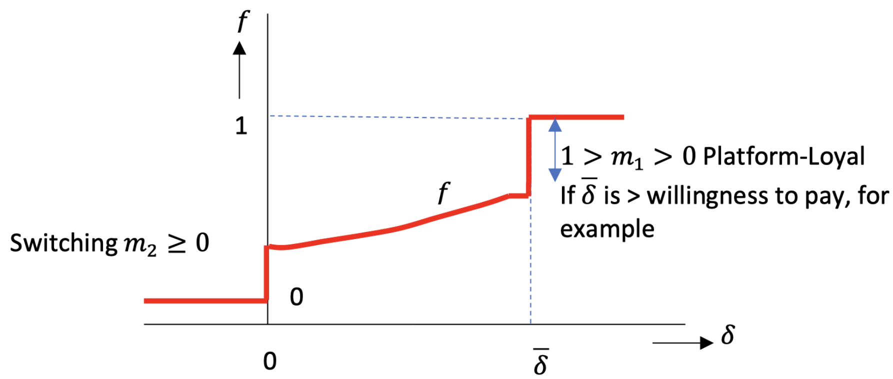
We should expect that with higher δ more consumers would switch from the platform to the merchant. So, it is reasonable to suppose that $f(\delta)$ is increasing in δ, and if δ is large enough, say $\overline{\delta}$, then all consumers would buy from the merchant. We assume that f is continuous in $\delta \in (0, \overline{\delta})$, as illustrated in figure 3. Additional switching at $\delta > 0$ and a finite $\overline{\delta}$ generalize the extreme loyalty in Narasimhan's model.[8] The platform and merchant sales S_P and S_M are then given by:

$$S_P(p_P, p_M) = \begin{cases} 0, & \text{if } p_P > W \\ 1, & \text{if } p_M > \min\{W, p_P\} = p_P \\ 1 - m_2 - \int_0^{p_P - p_M} f(\delta) d\delta, & \text{else} \end{cases} \tag{9}$$

[8]Narasimhan's model of perfect brand loyalty would apply in our model if $\overline{\delta} > q_H$, for example. Our model of seller preference is closer to Rao's (1990) model of national brand-private label competition, but with the added feature of a possible jump at $\overline{\delta}$. If the jump m_1 were to occur at $\underline{\delta} > 0$, the model would be similar to Raju, Srinivasan and Lal (1990).

$$S_M(p_P, p_M) = \begin{cases} 0, & \text{if } p_M > \min\{W, p_P\} \\ 1, & \text{if } p_P > W \geq p_M \\ m_2 + \displaystyle\int_0^{p_P - p_M} f(\delta) d\delta, & \text{else} \end{cases} \quad (10)$$

Figure 3: Model of Platform to Merchant switching



Game Structure and Sellers' Profits: We invoke the game structure described in 3.1 with the pricing game following a leader-follower framework.[9] Making the platform the leader is attractive for several reasons. We could of course treat the order as endogenous, determined in a prior stage. It is easy to demonstrate that both the platform and the merchant are better off with the platform moving first, since $f(\delta) = 0$, $\delta < 0$. Then, if order were endogenous, indeed in the equilibrium outcome the platform would be the leader.[10] Among two additional important and relevant reasons are: it is more tractable; also, our interactions with numerous merchants confirmed to us that for example, merchants follow and react to Amazon closely but they don't see Amazon reacting to merchants. Less compelling, but favoring our model is also that price data, even after adjusting for shipping suggests a modest price umbrella

[9] The sequential choice has the advantage that it allows us to focus on pure strategy pricing.

[10] The argument here is based on when there is no uncertainty in q, however, we think this would be true even after incorporating quality uncertainty but formally analyzing that would introduce greater complexity without commensurate reward.

provided by Amazon that would be consistent with the merchant being the follower.[11]

$$\Pi_M(p_M; p_P) = S_M(p_P, p_M)\left((1-c)p_M - K\right)$$
$$\Pi_P(p_P, R_M(p_P)) = S_P(p_P, R_M(p_P))\,(p_P - K) + cS_M(p_P, R_M(p_P))R_M(p_P)$$

The platform's problem is well defined if $R_M(p_P)$ is unique. One way to ensure this is, for example, to select from among the multiple R_Ms the one that maximizes channel sales. Another way is to make suitable assumptions on $f(\delta)$ that guarantees uniqueness of R_M. In the application we have in mind, we will see that a meaningful specification of $f(\delta)$ results in uniqueness of R_M.

Quality Uncertainty: As noted in 3.1, we assume that both sellers are informed of realized quality but consumers are not. While user generated reviews and ratings can resolve some of the uncertainty, online evaluations pose challenges, are noisy and also user generated content is subject to manipulation.[12] As a result, consumers cannot avoid some level of quality uncertainty. Let K_i denote $K(q_i)$, $i \in \{H, L\}$.

Consumer Uncertainty and Sales: Consumers' purchase decision would depend on their willingness-to-pay for quality, denoted by $\hat{W}$ that in turn depends on their belief:

$$\hat{W}\left(\hat{\theta}(p_P, p_M)\right) = \begin{cases} \hat{W}_P\left(\hat{\theta}\right) = \begin{cases} q_H, & \text{if } \hat{\theta} = 1 \\ q_L, & \text{if } \hat{\theta} = 0 \\ \overline{q}, & \text{if } \hat{\theta} \in (0,1) \end{cases} & \text{for platform's offering} \\ \hat{W}_M\left(\hat{\theta}\right) = \begin{cases} \hat{W}_P\left(\hat{\theta}\right) - \delta, & \text{if } p_P \leq q_L \\ \hat{W}_P\left(\hat{\theta}\right) - \hat{\theta}\delta, & \text{if } p_P \in (q_L, q_H] \\ \hat{W}_P\left(\hat{\theta}\right), & \text{if } p_P > q_H \end{cases} & \text{for merchant's offering} \end{cases} \tag{11}$$

[11] While the price umbrella may well be a result of other factors, its presence renders our assumption acceptable.

[12] (2020) "Amazon and Google are being investigated for failing to remove fake product reviews" https://edition.cnn.com/2021/06/25/tech/amazon-google-fake-reviews/index.html

We see from (11) that platform-loyal consumers have a lower WTP for the merchant's product, depending on their degree of loyalty captured by δ. Therefore, for quality $i \in \{H, L\}$, sales of platform and merchant become $S_P(p_P, R_M^i(p_P); \hat{\theta}(p_P, R_M^i(p_P)))$ and $S_M(p_P, p_M; \hat{\theta}(p_P, p_M))$ with $\hat{W}$ invoking $\hat{W}_P(\hat{\theta}; \delta)$ and $\hat{W}_M(\hat{\theta}; \delta)$ suitably in (9) and (10).

3.3 Multi-Agent Signaling Equilibrium

In this section, we solve for the equilibrium prices and illustrate revelation resulting from multi-agent signaling in the simple case of consumers homogeneous in their preference for quality. This serves to illustrate

1. our equilibrium concept in a sequential multi-agent game

2. the force of multi-agent signaling on revelation

In section 4, we extend the simple model by incorporating consumer heterogeneity in quality valuation to capture another dimension of reality and obtain insights into firm profits and consumer welfare in equilibrium. To retain focus on multi-agent signaling, we consider the simplest situation by making the following assumptions.

Assumption 1: WTP for base product $v = 0$ so that consumer willingness to pay $W(q) = q$ under complete information.

Assumption 2: Commission rate $c = 0$.

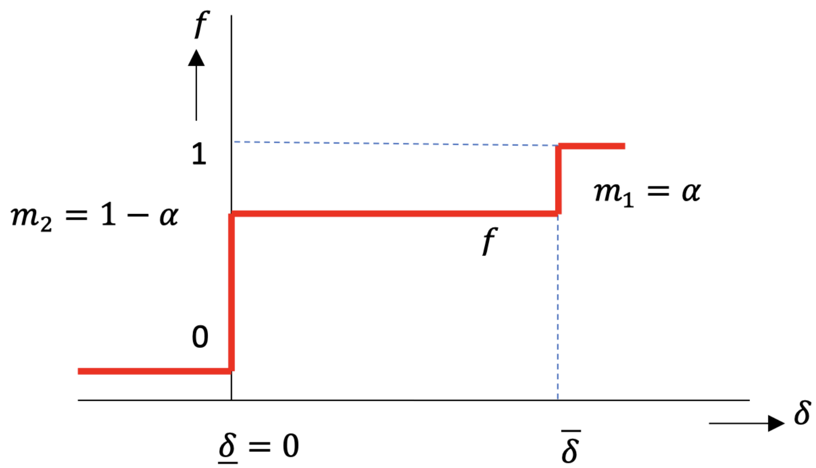
Assumption 3: Switching fraction $f(\delta)$ consists of two step functions, or masses of consumers:

$$f(\delta) = \begin{cases} 0, & \text{if } \delta < 0 \\ 1 - \alpha, & \text{if } 0 \leq \delta < \overline{\delta} \\ 1, & \text{else} \end{cases}$$

Figure 4 displays $f(\delta)$ in this case.[13]

[13]It is reasonable to suppose that at $\delta = 0$, consumers are indifferent to platform and merchant, and so they can split across the two firms in an infinite number of ways. However, since the game has a sequential structure, and so the merchant can always choose a price arbitrarily close to the platform's but below it, it

Figure 4: Fraction switching from Platform to Merchant



Assumption 4: Platform loyalty is significant. In particular we assume that $\delta > \alpha \max\{q_i - K_i\}$. This ensures that platform sales is non-zero under complete information.[14]

Assumption 5: Cost of high quality product is not too high. Specifically, $K_H < q_L$. This is for ex-positional simplicity. Thus, $K_L < K_H < q_L < q_H$.

3.3.1 Quality Revelation

It is straightforward to show that in our model, neither the merchant nor the platform is able to signal the product quality through its price acting alone. This is because it is impossible for the high-type seller to prevent mimicking from low-type seller when charging high price for the high-quality product. Hence, in our model, if only one seller exists in market, no separation can be forced in equilibrium.

Next, we examine the equilibrium outcome if both the platform and the merchant sell the product. Denote the equilibrium prices as $\hat{p}^i = [\hat{p}^i_P \ \hat{p}^i_M]$, $i \in \{H, L\}$. Of-course, $\hat{p}^i_M = R^i_M(\hat{p}^i_P)$. We know that $p_P \in \Omega_1$ implies a price in a separating equilibrium. So in any

is correct to assume that if $p_M = p_P \Rightarrow \delta = 0$, the sales shares are $S_M = 1 - \alpha$, $S_P = \alpha$. Similarly, if $\delta = \overline{\delta}$, the merchant captures all consumers leaving the platform with zero sales. Also this specification of f makes merchant's best response unique.

[14]This helps to highlight multi-agent signaling. We will follow that with a discussion of how other values of $\overline{\delta}$ affects equilibrium outcomes.

equilibrium $q_H \geq \hat{p}_P^H > q_L$. Also if $p_P \in \Omega_1$, merchant's price does not affect consumer belief. So, in light of assumptions 4 and 5, $R_M^i(\hat{p}_P^i) = \hat{p}_P^i$. In turn this implies that platform IC constraints cannot be satisfied in our model of consumers homogeneous in valuation of quality, and so the set Ω_1 is empty as shown in lemma 1.

Lemma 1. *The set Ω_1 is empty.*

Proof (by contradiction):

Suppose $\exists$ a $p_P^H \in \Omega_1$. We know the merchant's best response then is $R_M^H(p_P^H; 1) = p_P^H$. It is easy to see that the profit maximizing price for the platform when quality is low is $p_P^L = q_L$, followed by merchant response $R_M^L(p_P^L; 0) = p_P^L = q_L$. Thus, the platform's IC constraints are:

$$\Pi_P^H \left(p_P^H, p_P^H, 1\right) = \alpha(p_P^H - K_H) > \Pi_P^H \left(q_L, q_L, 0\right) = \alpha(q_L - K_H) \tag{12}$$

$$\Pi_P^L \left(p_P^H, p_P^H, 1\right) = \alpha(p_P^H - K_L) < \Pi_P^L \left(q_L, q_L, 0\right) = \alpha(q_L - K_L) \tag{13}$$

It is easy to see that (12) and (13) contradict. Thus, $\nexists$ any $p_P \in \Omega_1$.

$\square$

From lemma 1, we see that quality revelation can occur only if $p_P \in \Omega_2$. We can understand the intuitive meaning of lemma 1 as follows. The platform is unable to signal quality using only its price as a message. This is because of two forces. First, the merchant can do no better than match the platform price given the assumption on δ. Second, with consumer homogeneity in price sensitivity, demand depends on quality but not on price and so price alone cannot act as a signal.

Revelation can occur only if $p_P \in (q_L, q_H]$. We will characterize $\hat{\theta}(p_P)$ for these prices. For the sake of completeness we examine $\hat{\theta}(p_P)$ for other platform prices also in lemma 2, with proof in technical appendix.[15]

[15] If $p_P \in [K_L, K_H)$, it is straightforward to show that only the low-quality product will be sold in market

Lemma 2. *If platform price*

a. $p_P \in [K_H, q_L)$ *then* $p_p \in \Omega_2$ *and* $\hat{\theta}(p_P) = \theta$.

b. $p_P \in (q_H, 1]$ *then* $p_p \in \Omega_2$ *and* $\hat{\theta}(p_P) = \theta$.

Proof: (See technical appendix.)

Lemma 3 (necessary condition for separating equilibrium). *Define* $p_M^{**} = \frac{q_L - \alpha K_H}{1-\alpha}$. *Then in a sub-game induced by* $p_P \in (q_L, q_H]$, *SE exists only if* $p_P \in (p_M^{**}, q_H]$.

Proof (by construction):

We first investigate SE by invoking the merchant's IC constraints to show that the merchant's best response given high quality, $R_M^H(p_P, 1)$, lies in the interval $(p_M^{**}, p_M^*(p_P)]$ where $p_M^*(p_P) \triangleq \min\left\{\frac{q_L - \alpha K_L}{1-\alpha}, p_P\right\}$. Consider the following merchant's best responses for any $p_P \in (q_L, q_H]$:

$$R_M^H(p_P, 1) \neq R_M^L(p_P, 0) = q_L \tag{14}$$

Following (7) and (8), these result in the merchant's IC constraints:

$$\Pi_M^H\left(R_M^H(p_P, 1); p_P, 1\right) = (1-\alpha)(R_M^H(p_P, 1) - K_H) > \Pi_M^H(q_L; p_P, 0) = q_L - K_H \tag{15}$$

$$\Pi_M^L\left(R_M^H(p_P, 1); p_P, 1\right) = (1-\alpha)(R_M^H(p_P, 1) - K_L) \leq \Pi_M^L(q_L; p_P, 0) = q_L - K_L \tag{16}$$

(15) leads to $R_M^H(p_P, 1) > \frac{q_L - \alpha K_H}{1-\alpha} \triangleq p_M^{**}$ while (16) leads to $R_M^H(p_P, 1) \leq \frac{q_L - \alpha K_L}{1-\alpha}$. Since any $p_M > p_P$ leads to zero sales for the merchant, it must be $R_M^H(p_P, 1) \leq \min\left\{\frac{q_L - \alpha K_L}{1-\alpha}, p_P\right\} \triangleq p_M^*$. Note that since $K_H < q_L$, $p_M^{**} > q_L$. Moreover, if $p_P \leq p_M^{**}$, the first constraint is violated. Hence, a SE exists only in sub-game induced by $p_P \in (p_M^{**}, q_H]$.

□

since the price is lower than the cost of the high-quality product. Thus, we consider $p_P \in [K_H, 1)$ in the rest of discussion.

Lemma 3 is critical to understanding how revelation comes about taking into account the merchant's price. Note that the platform price must be sufficiently high. The high price offers an incentive to the merchant to not cut price with a view to to gaining sales when the quality and the cost are high. Cutting price would squeeze margins. On the other hand, if the quality and cost are low, it pays the merchant to cut price since margin is higher. From lemmas 1 and 3, we see that a separating equilibrium can occur only in sub-game induced by $p_P \in (p_M^{**}, q_H]$.

Lemma 4 (pooling equilibrium). *$\exists$ a PE in sub-game induced by any $p_P \in (q_L, q_H]$.*

Proof:

It will be useful to examine, in turn, $p_P \in (q_L, \overline{q}]$ and $p_P \in (\overline{q}, q_H]$. In the first interval, the pooling price is p_P unless it is profitable to attract platform-loyal consumers by offering them a higher utility. The needed reduction in price, if $\hat{\theta} = \theta$, is $\theta\delta$ since platform offering is in the choice set with probability θ. Note that pooling at $p_P - \theta\delta$ is a profitable strategy for the merchant only if $p_P - \theta\delta - K_i \geq q_L - K_i$, equivalently $p_P \geq q_L + \theta\delta$. Then, it is readily seen that

$$R_M^i(p_P, \theta) = \begin{cases} p_P - \theta\delta, & \text{if } \max\left\{q_L + \theta\delta, K_H + \dfrac{\theta\delta}{\alpha}\right\} \leq p_P \leq \overline{q} \\ p_P, & \text{if } p_M^{**} < p_P < K_H + \dfrac{\theta\delta}{\alpha} \\ q_L, & \text{if } q_L < p_P \leq \min\left\{p_M^{**}, q_L + \theta\delta\right\} \end{cases}$$

Thus, $\exists$ a PE if $p_P \in (q_L, \overline{q}]$.[16] Turning to the second interval, if $\hat{\theta} = \theta$ no consumers would buy from the platform. Consumers do have a choice: whether to buy from merchant, or forgo purchase. Price sensitive consumers would buy from merchant if $p_M \leq \overline{q}$, and platform-loyal consumers if $p_M \leq \overline{q} - \theta\delta$. Thus, merchant price does not depend on platform price or

[16] The existence of PE does not imply that it would survive a refinement such as the intuitive criterion, as we will see in lemma 5.

quality. Therefore, we can see that

$$R_M^i(p_P,\theta) = \begin{cases} \overline{q}-\theta\delta, & \text{if } \overline{q} \geq \max\left\{q_L+\theta\delta, K_H+\dfrac{\theta\delta}{\alpha}\right\} \\ \overline{q}, & \text{if } p_M^{**} < \overline{q} < K_H+\dfrac{\theta\delta}{\alpha} \\ q_L, & \text{if } \overline{q} \leq \min\left\{p_M^{**}, q_L+\theta\delta\right\} \end{cases}$$

□

Lemma 4 is interesting because it says that in any sub-game induced by $p_P \in (q_L, q_H]$, there exists a pooling equilibrium. If platform price is higher than $\overline{q}$, the merchant could send a price message that is independent of quality by choosing a price below $\overline{q}$ that leaves consumers' posterior the same as the prior belief. If the platform price is at or below $\overline{q}$, any merchant response that is best for high quality turns out to be also best for low quality whether the price leaves the posterior identical to the prior or the posterior is that the quality is low, thereby gaining sales.

Lemmas 3 and 4 imply that either PE is unique or both PE and SE exist. In the latter instance we can appeal to a refinement to identify the unique equilibrium. Since in our model, the merchant is the follower, minimality refinement is to go with what rationalizes merchant price. Thus, combining lemmas 2 - 4, we have the perfect Bayes-consistent beliefs $p_P \in (K_L, q_H]$:

$$\hat{\theta}(p_P) = \begin{cases} \theta, & \text{if } K_L < p_P < q_L \\ \theta, & \text{if } q_L < p_P \leq p_M^{**} \\ \begin{cases} 1, & \text{if } \Pi_M^H\left(R_M^H(p_P,1);p_P,1\right) > \Pi_M^H\left(R_M^H(p_P,\theta);p_P,\theta\right) \\ \theta, & \text{if } \Pi_M^H\left(R_M^H(p_P,1);p_P,1\right) \leq \Pi_M^H\left(R_M^H(p_P,\theta);p_P,\theta\right) \end{cases}, & \text{if } p_M^{**} < p_P \leq q_H \end{cases}$$

These beliefs imply that a necessary condition for SE to be the unique equilibrium in the sub-game induced by p_P is $p_P \in (p_M^{**}, q_H]$. The sufficient condition obviously is $\Pi_M^H\left(R_M^H(p_P,1);p_P,1\right) > \Pi_M^H\left(R_M^H(p_P,\theta);p_P,\theta\right)$. Lemma 5 identifies the conditions for this.

Lemma 5 (sufficient condition for separating equilibrium to be unique equilibrium). *Define* $p_P^* = \frac{\overline{q}-\theta\delta-\alpha K_H}{1-\alpha}$. *Then SE in the sub-game is the unique equilibrium,* $p_P \in \Omega_2$ *and* $\hat{\theta}(p_P) = 1$,

1. *if* $p_P \in \left(p_M^{**}, \min\left\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}, p_M^*\right\}\right]$. *Moreover,* $\hat{\theta}(p_P) = 1$ *and* $R_M^H(p_P, 1) = p_P \neq R_M^L(p_P, 0) = q_L$.

2. ***(Key Results)*** *if* $p_M^* > p_P^*$ *and* $p_P \in (\max\{p_M^{**}, p_P^*\}, q_H]$. *Moreover,* $\hat{\theta}(p_P) = 1$ *and* $R_M^H(p_P, 1) = \min\{p_M^*, p_P\} \neq R_M^L(p_P, 0) = q_L$.

Proof:

In lemma 3, we show that there exists a SE for $p_P \in (p_M^{**}, q_H]$. Moreover, $R_M^H(p_P, 1) \neq R_M^L(p_P, 0) = q_L$. A sufficient condition for such SE to be the unique equilibrium is that the merchant must prefer SE over PE given same p_P chosen in the first stage. Following lemma 4, we discuss two cases: $p_P \in (p_M^{**}, \overline{q}]$, $p_M^{**} < \overline{q}$ and $p_P \in (\overline{q}, q_H]$. First consider $p_P \in (p_M^{**}, \overline{q}]$, $p_M^{**} < \overline{q}$.

1. $\overline{q} < p_M^*$: The unique equilibrium is SE if $p_P \in \left(p_M^{**}, \min\left\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\right\}\right]$. We can see that $R_M^i(p_P, \hat{\theta} = \theta) = p_P$ if $p_M^{**} < p_P < K_H + \frac{\theta\delta}{\alpha}$. In this case, the merchant's profit in PE when selling a low-type product is $(1-\alpha)(p_P - K_L)$, while in a SE, the merchant's profit is given by $q_L - K_L$. Since $p_P < p_M^*$, a deviation to q_L by the low type merchant in the PE is optimal, thus failing the intuitive criterion. Therefore, the unique equilibrium is SE.

2. $p_M^* \leq \overline{q}$: The unique equilibrium is SE if $p_P \in \left(p_M^{**}, \min\left\{K_H + \frac{\theta\delta}{\alpha}, p_M^*\right\}\right]$, following the arguments as in 1.

Next consider $p_P \in (\overline{q}, q_H]$. We can ignore $\overline{q} < p_P \leq p_M^{**}$ since in that cases there is no SE.

1. $p_M^* \leq p_P^*$: The SE is never the unique equilibrium. If $p_P \in (\max\{p_M^{**}, \overline{q}\}, q_H]$, the merchant's profit in SE with $\hat{\theta} = 1$, is $(1-\alpha)(\min\{p_M^*, p_P\} - K_H) < \overline{q} - \theta\delta - \alpha K_H$, the profit from a deviation to $\overline{q} - \theta\delta$, and so the SE fails intuitive criterion.

2. $p_M^* > p_P^*$: If $p_P \in (\max\{p_M^{**}, \overline{q}\}, p_P^*]$, the merchant's profit in SE with $\hat{\theta} = 1$, is $(1-\alpha)(p_P^* - K_H) < \overline{q} - \theta\delta - \alpha K_H$, the profit from a deviation to $\overline{q} - \theta\delta$, and so the SE fails intuitive criterion. If $p_P \in (\max\{p_M^{**}, p_P^*\}, q_H]$, the merchant's profit in SE with $\hat{\theta} = 1$, is $(1-\alpha)(\min\{p_M^*, p_P\} - K_H) > \overline{q} - \theta\delta - \alpha K_H$, the maximum profit in a PE. Thus, in this case SE is the unique equilibrium.

□

We know from lemma 4 that in any sub-game there exists a PE. Therefore, the only way a SE can come about is if it satisfies the condition in lemma 3 and it survives the intuitive criterion in the sub-game. That is precisely what lemma 5 accomplishes. For uniqueness of SE, the lower bound for the platform price is $\max\{p_M^{**}, p_P^*\}$. The role of p_P^* can be intuitively understood as follows. If the platform price is not sufficiently high, the merchant could choose to send an uninformative message by choosing a price at or below $\overline{q}$. With platform's price higher than $\overline{q}$, the merchant can potentially gain sales by sacrificing margin. To prevent this, the platform must choose a high enough price so that the merchant sends an informative message. Also in needs of emphasis is the fact that the platform's no-mimic condition is enforced by the merchant's best response. Were the platform to lie by setting price higher than p_M^{**}, the merchant would deviate to $R_M^L(p_P, 1) = q_L$. In other words, the merchant's price validates the platform's choice of price for the consumer. Since p_M^{**} is the lower bound for platform's price that can possibly lead to a SE in the sub-game, it could well be the case that there exists a SE with platform price at q_H. In such a SE the price corresponds to the full information outcome and there is no price distortion in equilibrium. Lemmas 1 - 5 can be combined to understand the conditions for revelation. We identify the conditions in proposition 1, stated without proof.

Proposition 1. *If assumptions 1-5 hold, then equilibrium prices $\hat{p}^i = [\hat{p}^i_P\ \hat{p}^i_M]$, $i \in \{H, L\}$, are:*

1. ***(SE)*** $\hat{p}^H = [q_H\ p^*_M]$ *and* $\hat{p}^L = [q_L\ q_L]$ *if* $p^*_M > \max\{p^*_P, \overline{q}\}$.

2. ***(SE)*** $\hat{p}^H = \left[\min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\}\ \ \min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\}\right]$ *and* $\hat{p}^L = [q_L\ q_L]$ *if* $p^*_M \leq \max\{p^*_P, \overline{q}\}$ *and* $p^{**}_M < \min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\} \leq p^*_M$.

3. ***(PE)*** $\hat{p}^H = \hat{p}^L = \left[\min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\}\ \ \min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\}\right]$ *if* $p^*_M \leq \max\{p^*_P, \overline{q}\}$ *and* $\min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\} > p^*_M$.

4. ***(PE)*** $\hat{p}^H = \hat{p}^L = [q_L\ q_L]$ *if* $p^*_M \leq \max\{p^*_P, \overline{q}\}$ *and* $\min\{K_H + \frac{\theta\delta}{\alpha}, \overline{q}\} \leq p^{**}_M$.

Proof: Follows from lemmas 1 - 5.

3.3.2 Example Illustrating Revelation

A numerical example illustrates how the platform also selling the product allows the merchant's price to reveal quality. Consider the following example: In this case, $\overline{q} = 0.75$,

q_H	q_L	α	K_H	K_L	θ	δ	c
0.95	0.7	0.3	0.6	0.4	0.2	0.12	0

$p^{**}_M = 0.743 < 0.78 = p^*_P < p^*_M = 0.829 < q_H$. Therefore, invoking proposition 1, the equilibrium is

$$\hat{p}^H_P = 0.95, \hat{p}^H_M = p^*_M = 0.829 \neq q_L = 0.7 = \hat{p}^L_P = \hat{p}^L_M$$

Without platform entry, the equilibrium is pooling, and the price of the merchant is $\overline{q} = 0.75$. The profits of the platform and the merchant with and without platform entry are:

We see that revelation is profitable for the channel. As it turns out, it is profitable for the merchant despite platform entry! It is also worth emphasizing that the platform price is at the willingness-to-pay for quality. What sustains revelation is the merchant's willingness to reduce its price below that if quality is high. We could say that the merchant bears

Platform Enters	Entity	Quality q_H	Quality q_L
Yes	Platform	0.105	0.09
	Merchant	0.16	0.21
	Channel	0.265	0.3
No	Platform	0	0
	Merchant	0.15	0.35
	Channel	0.15	0.35

the cost of revelation that the separating equilibrium effects. The downward distortion of merchant price is somewhat different from the more common upward distortion that reduces demand in the single-agent price signaling problem. In this example, demand is unaffected by platform entry but prices, profits and of-course revelation are all affected.

In the next section we will see the effect of the commission rate c, the probability of high quality θ, and a possible entry cost F for the platform.

4 Relaxing Assumptions and Extensions

We now wish to see how relaxing some of the assumptions affect equilibrium outcomes and also a model extension incorporating consumer heterogeneity in valuation of quality.

4.1 Relaxing Assumptions

First we relax assumption 2 by assuming $c > 0$. Reflecting practice we will assume $c = 0.15$ and keeping all other parameters as in the example in 3.3.2. In this case, $p_M^{**} = 0.697 < 0.735 = p_P^* < p_M^* = 0.798 < q_H$. Therefore, invoking proposition 1, the equilibrium is we find the equilibrium is:

$$\hat{p}_P^H = 0.95, \hat{p}_M^H = p_M^* = 0.798 \neq q_L = 0.7 = \hat{p}_P^L = \hat{p}_M^L$$

We then display the profits in the table below:

Platform Enters	Entity	Quality q_H	Quality q_L
Yes	Platform	0.189	0.1635
	Merchant	0.055	0.1365
	Channel	0.244	0.3
No	Platform	0.1125	0.1125
	Merchant	0.0375	0.2375
	Channel	0.15	0.35

What we see is that while the numbers change, the platform entry is indeed profitable not only for the platform but also for the merchant. In other words, equilibrium outcome leads to revelation while also being a win-win situation for both platform and merchant. The effect of a non-zero commission rate is to merely effect a transfer to the platform but the force of signaling is not changed. Next we ask whether this would hold if there were a fixed cost $F > 0$ for the platform to enter.

Suppose $F = 0.07$, and we retain $c = 0.15$. The, clearly, a high quality product would want to enter since the platform's profit with entry would be $0.189 - 0.07 = 0.119 > 0.1125$. This is beneficial for the merchant also. A natural question is: would it make sense to enter only high quality but not low quality? The next table illuminates the situation. We see that it pays to enter only if the quality is high. Moreover, this is better for both the platform and the merchant than platform entering regardless of quality. However, this does not imply that consumers can infer quality from platform's entry decision. Such a belief would not be Bayes-consistent because that would invite platform entry. In other words, it is the prices that can effect revelation. Thus, though empirically we may see entry and quality to be positively correlated, it would be inappropriate to interpret entry as a signal of quality. Such an outcome is favored because of the non-zero commission rate and the entry cost.

Finally, we ask what would happen if the likelihood of high quality were high, say $\theta = 0.8$ instead of $\theta = 0.2$ as in the original example. We retain $c = 0.15$ and $F = 0.07$. Then, $\overline{q} = 0.90$, $p_M^{**} = 0.697 < 0.798 = p_M^* < p_P^* = 0.846 < q_H$. Therefore, invoking proposition 1,

Platform Enters	Entity	Quality q_H	Quality q_L
Yes	Platform	0.119	0.0935
	Merchant	0.055	0.1365
	Channel	0.174	0.23
Only High Quality	Platform	0.119	0.105
	Merchant	0.055	0.195
	Channel	0.174	0.3
No	Platform	0.1125	0.1125
	Merchant	0.0375	0.2375
	Channel	0.15	0.35

the equilibrium is

$$\hat{p}_P^H = \hat{p}_M^H = \overline{q} = 0.9 = \hat{p}_P^L = \hat{p}_M^L$$

We compare the profits from entering both qualities or not entering at all.[17] The profits are displayed in the next table.

Platform Enters	Entity	Quality q_H	Quality q_L
Yes	Platform	0.1145	0.1745
	Merchant	0.1155	0.2555
	Channel	0.23	0.43
No	Platform	0.135	0.135
	Merchant	0.165	0.365
	Channel	0.3	0.5

In this case, since entry is profitable for low quality, entry would always take place but there would be no revelation. We can ask what market conditions would lead to revelation and win-win following platform entry. If there is small likelihood that the product is of a significantly higher quality, such a condition makes platform entry desirable not only for the platform but also for consumers and the merchant. We know that merchants on platforms like Amazon offer hundreds of thousands of products and only in most of these instances

[17]If only low quality were entered, the price would still correspond to the separating price, and so we can ignore that possibility.

it is reasonable to suppose that quality would be high enough but possibly fall short of a perfect product. In the less likely case of a truly very high quality product, the platform's entry would lead to revelation. In other cases platform entry may be determined more by market size.

4.2 Consumer Heterogeneity in Valuation of Quality

In this section, we extend the model in section 3 by assuming that consumers have a heterogeneous preference for quality. Specifically, we assume that there are two segments of consumers in market: $v = 1$ and $v = 0$. The segment with $v = 1$, referred as quality-insensitive consumers, has a willingness-to-pay $W(q_L) = W(q_H) = 1$. In other words, these consumers' purchase decision won't be affected by the perceived product quality $\hat{\theta}$. The other segment with $v = 0$, referred as quality-sensitive consumers, has a willingness-to-pay $W(q_i) = q_i$. For these consumers, it is important to consider both prices and quality before making purchase decision as described in section 3. We assume that a proportion m of consumers are quality insensitive. The rest $1 - m$ consumers are quality sensitive. We further assume $\overline{\delta} \geq \max\left\{1 - q_i + \alpha m\left(q_i - \frac{K_i}{1-c}\right)\right\}$. This guarantees that in any equilibrium, the platform's sales are strictly positive. In the rest of the section, we impose the condition $m \leq \min\left\{\frac{(1-c)q_i - K_i}{1-K_i-cq_i}\right\}$ such that in the case of complete information, we have $\hat{p}_P^i = \hat{p}_M^i = q_i$, $i \in \{H, L\}$. We then show that with a heterogeneous preference over quality, Ω_1 is no longer empty.

Lemma 6. *Define* $\underline{p}_P = \frac{c((1-\alpha)q_L-(1-\alpha m)q_H)+\alpha(q_L-(1-m)K_H)}{\alpha m}$ *and* $\overline{p}_P = \frac{c((1-\alpha)q_L-(1-\alpha m)q_H)+\alpha(q_L-(1-m)K_L)}{\alpha m}$. *If* $K_H \geq \frac{\alpha((1-c)q_L-m(1-cq_H))-c(q_H-q_L)}{\alpha(1-m)}$ *and* $K_L < \frac{\alpha((1-c)q_L-m(q_H-cq_H))-c(q_H-q_L)}{\alpha(1-m)}$, *we have* $\Omega_1 = \left(\max\left\{q_H, \underline{p}_P\right\}, \min\left\{1, \overline{p}_P\right\}\right] \neq \emptyset$.

Proof (by construction):

We first establish $p_P \in (q_L, q_H] \not\subset \Omega_1$ by contradiction. Suppose there exists a SE with $p_P^H \in (q_L, q_H]$ and $p_P^L = q_L$ that satisfies (5) and (6). Since $R_M^*(p_P^H, 1) = p_P^H$ and $R_M^*(q_L, 0) = q_L$.

(5) and (6) can be rewritten as:

$$\Pi_P^H\left(p_P^H, p_P^H, 1\right) = \alpha(p_P^H - K_H) + c(1-\alpha)p_P^H > \Pi_P^H\left(q_L, q_L, 0\right) = \alpha(q_L - K_H) + c(1-\alpha)q_L \tag{17}$$

$$\Pi_P^L\left(p_P^H, p_P^H, 1\right) = \alpha(p_P^H - K_L) + c(1-\alpha)p_P^H < \Pi_P^L\left(q_L, q_L, 0\right) = \alpha(q_L - K_L) + c(1-\alpha)q_L \tag{18}$$

It is straightforward to show that (17) and (18) contradicts. Thus, $p_P \in (q_L, q_H] \not\subset \Omega_1$.

We next ask whether $p_P \in (q_H, q_1]$ might belong to Ω_1. We prove by construction. Suppose there exists a SE with $p_P^H \in (q_H, q_1]$ and $p_P^L = q_L$ that satisfies (5) and (6). Note that in this case, $R_M^*(p_P^H, 1) = q_H$[18] and $R_M^*(q_L, 0) = q_L$. Thus, (5) and (6) can be rewritten as:

$$\Pi_P^H\left(p_P^H, q_H, 1\right) = \alpha m(p_P^H - K_H) + c((1-\alpha)m + 1 - m)q_H > \alpha(q_L - K_H) + c(1-\alpha)q_L \tag{19}$$

$$\Pi_P^L\left(p_P^H, q_H, 1\right) = \alpha m(p_P^H - K_L) + c((1-\alpha)m + 1 - m)q_H \leq \alpha(q_L - K_L) + c(1-\alpha)q_L \tag{20}$$

Inequalities (19) and (20) lead to $K_L \leq \frac{\alpha\left((1-c)q_L - m(p_P^H - cq_H)\right) - c(q_H - q_L)}{\alpha(1-m)} \triangleq h(p_P^H) < K_H$. Define $\underline{p}_P = \frac{c((1-\alpha)q_L - (1-\alpha m)q_H) + \alpha(q_L - (1-m)K_H)}{\alpha m}$ and $\overline{p}_P = \frac{c((1-\alpha)q_L - (1-\alpha m)q_H) + \alpha(q_L - (1-m)K_L)}{\alpha m}$. Since $h(p_P^H)$ is decreasing in p_P^H, we get $\Omega_1 = \left(\max\left\{q_H, \underline{p}_P\right\}, \min\left\{1, \overline{p}_P\right\}\right]$. Then, the necessary condition for Ω_1 to be not empty is $\underline{p}_P \leq 1$, equivalently $K_H \geq \frac{\alpha((1-c)q_L - m(1-cq_H)) - c(q_H - q_L)}{\alpha(1-m)}$ and $\overline{p}_P > q_H$, equivalently $K_L < \frac{\alpha((1-c)q_L - m(q_H - cq_H)) - c(q_H - q_L)}{\alpha(1-m)}$.

Therefore, if $K_H \geq \frac{\alpha((1-c)q_L - m(1-cq_H)) - c(q_H - q_L)}{\alpha(1-m)}$ and $K_L < \frac{\alpha((1-c)q_L - m(q_H - cq_H)) - c(q_H - q_L)}{\alpha(1-m)}$, $\left(\max\left\{q_H, \underline{p}_P\right\}, \min\left\{1, \overline{p}_P\right\}\right] = \Omega_1$.

□

In Ω_1, it is the platform that distort its price upward to effect separation in equilibrium.

[18] If $m \leq \frac{(1-c)q_L - K_L}{1 - K_L - cq_L}$, it must be $((1-\alpha)m + 1 - m)((1-c)q_H - K_H) \geq (1-\alpha)m((1-c) - K_H)$.

The merchant's price remains identical to that under complete information. The no mimic condition is enforced by the platform who bears the cost of separation.

5 Managerial Implications and Conclusions

The important managerial takeaway from our analysis is that a platform's entry can lead to consumers being better informed of quality. This is obviously desirable since it enhances the reputation of the platform. In other words, in the larger scheme of things, the higher profits that a platform obtains by cannibalizing merchant sales could well be a less important consequence of platform entry than the higher consumer trust and confidence that results from consumers being informed of quality. While a merchant's profits could decrease because of platform entry, the merchant could benefit from platform entry because of higher prices for the high quality product. To the extent that market conditions map into product categories, a platform may also be able to design policies and specify metrics governing entry in different product categories. In this way, a platform can ensure good relations with merchants so that they don't view the platform as a predator.

One practical issue that platforms like Amazon must contend with is maintaining good relations with third party merchants. So the real question for the merchant is on the benefits of being on the platform versus the possibility of the platform also selling the product. A potential benefit is communicating product quality to consumers. Consumers who are better informed of quality are likely to be more satisfied, and so trust the platform more.

From a theoretical perspective, our work contributes to work on multi-agent signaling in a marketing channel. The strategic interaction of channel members comprehends both competition and cooperation. We extend the idea of co-opetition beyond pricing to informational issues under quality uncertainty.

Finally, our work adds to the literature on the strategic consequences of a platform directly selling to consumers. In practice, not only does the merchant not exit after the platform decides to carry the product but it also enjoys a price umbrella provided by the platform. Our analysis shows that under some conditions, platform entry leads to higher profits to both the platform and the merchant. This win-win outcome occurs because when both platform and merchant sell, consumers can be informed of quality and so pricing is more efficient. Consumers benefit from not only lower prices but also being better informed.

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Appendices

A Technical Appendix

Proof of lemma 2: Consider the first interval $p_P \in [K_H, q_L)$. Note that the merchant's IC constraints can be written as follows:

$$q = q_H : (1-\alpha)(R_M^H(p_P, 1) - K_H) > (1-\alpha)(R_M^L(p_P, 0) - K_H) \tag{21}$$

$$q = q_L : (1-\alpha)(R_M^H(p_P, 1) - K_L) < (1-\alpha)(R_M^L(p_P, 0) - K_L) \tag{22}$$

It is straightforward to show that (21) and (22) contradict. This is because at $p_P \in [K_H, q_L)$, even if the merchant chooses $R_M^L(p_P, 0)$ which leads to $\hat{\theta} = 0$, it cannot gain sales by attracting the platform's loyal segment since $p_P < q_L$. Therefore, if $p_P \in [K_H, q_L)$ then $p_P \in \Omega_2$ and $\hat{\theta}(p_P) = \theta$.

Consider the second interval $p_P \in (q_H, 1]$. In this case, regardless of the true quality, none of the consumers will buy from the platform. Thus, the merchant's IC constraints can be written as follows:

$$q = q_H : R_M^H(p_P, 1) - K_H > R_M^L(p_P, 0) - K_H \tag{23}$$

$$q = q_L : R_M^H(p_P, 1) - K_L < R_M^L(p_P, 0) - K_L \tag{24}$$

It is straightforward to show that (23) and (24) contradict. Therefore, if $p_P \in [K_H, q_L)$, then $p_P \in \Omega_2$ and $\hat{\theta}(p_P) = \theta$.

□

B Online Appendix

Following proposition 1, we delineate additional findings of interest in the multi-agent signaling game.

Quality revelation without price distortion. In proposition 1, if $p_M^* = \min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, p_P\right\} = p_P = q_H > \max\{p_P^*, \overline{q}\}$, equivalently $\frac{q_H-q_L}{q_H-K_L} \leq \alpha < \frac{q_H-\overline{q}+\theta\delta}{q_H-K_H}$, the unique equilibrium in the multi-agent signaling game is a SE with $\hat{p}^H = \{q_H\ q_H\}$ and $\hat{p}^L = \{q_L\ q_L\}$. Note that this pricing outcome is identical to that of complete information. Even more interesting, quality signaling occurs with no price distortion.

Win-win if $q = q_H$. If the equilibrium outcome is a SE with $\hat{p}^H = \{q_H\ p_M^*\}$ and $\hat{p}^L = \{q_L\ q_L\}$, there is a chance that the merchant benefits from the platform entry due to the raised margin despite a reduced sales volume. We explore this possibility in proposition 2.

Proposition 2 (win-win if $q = q_H$). *If $\theta < \frac{K_H-K_L}{q_H-K_L}$ and $\frac{\theta(q_H-q_L)}{K_H-K_L} < \alpha < \frac{(1-\theta)(q_H-q_L)}{q_H-K_H}$, the high-type merchant benefits from the platform entry.*

Proof:

Without the platform entry, the PBE is a PE with $p_M^H = p_M^L = \overline{q}$ since the merchant is unable to signaling the quality through its price and $q_H > q_L > K_H > K_L$. In this case, $\pi_M^H = \overline{q} - K_H$.

Upon the platform entry, from proposition 1, we know that if $p_M^* = \min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, q_H\right\} > \max\{p_P^*, \overline{q}\}$, the equilibrium outcome is a SE with $\hat{p}^H = \{q_H\ p_M^*\}$ and $\hat{p}^L = \{q_L\ q_L\}$. In this case, we have $\pi_M^H = (1-\alpha)(p_M^* - K_H)$. Thus, if $\min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, q_H\right\} > \max\{p_P^* = \frac{\overline{q}-\theta\delta-\alpha K_H}{1-\alpha}, \overline{q}\}$ and $\min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, q_H\right\} > \frac{\overline{q}-\alpha K_H}{1-\alpha}$, we have a win-win outcome in which the high-type merchant benefits from the platform entering the market. Note that since $\frac{\overline{q}-\alpha K_H}{1-\alpha} > \frac{\overline{q}-\theta\delta-\alpha K_H}{1-\alpha}$, the conditions become $\min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, q_H\right\} > \overline{q}$ and $\min\left\{\frac{q_L-\alpha K_L}{1-\alpha}, q_H\right\} > \frac{\overline{q}-\alpha K_H}{1-\alpha}$, equivalently $\theta < \frac{K_H-K_L}{q_H-K_L}$ and $\frac{\theta(q_H-q_L)}{K_H-K_L} < \alpha < \frac{(1-\theta)(q_H-q_L)}{q_H-K_H}$.

$\square$

Platform entry decision. Suppose that there is a fixed cost $F > 0$ for the platform enter. We first establish that entry by itself cannot force a separation in equilibrium. Denote the platform's entry decision by $\kappa \in \{1, 0\}$, where 1 stands for entry.

Lemma 7. *If $\alpha(q_L - K_H) < F \leq \alpha(q_L - K_L)$, there $\exists$ a SE in which platform entry signals $q = q_L$. Moreover, there $\nexists$ a SE in which platform entry signals $q = q_H$.*

Proof:

We proceed by construction. We consider two cases: (1) platform entry signals $q = q_H$; (2) platform entry signals $q = q_L$.

We first consider the case in which $\hat{\theta}(\kappa = 1) = 1$ and $\hat{\theta}(\kappa = 0) = 0$. In other word, the platform enters if and only if $q = q_H$. In this case, the platform's IC constraints are as follows:

$$q = q_H : \alpha(q_H - K_H) - F \geq 0 \tag{25}$$

$$q = q_L : \alpha(q_H - K_L) - F < 0 \tag{26}$$

Since $K_H > K_L$, (25) and (26) contradicts. Thus, there $\nexists$ a SE in which platform entry signals the $q = q_H$.

We next consider the case in which $\hat{\theta}(\kappa = 1) = 0$ and $\hat{\theta}(\kappa = 0) = 1$. In other word, the platform enters if and only if $q = q_L$. In this case, the platform's IC constraints are as follows:

$$q = q_H : \alpha(q_L - K_H) - F < 0 \tag{27}$$

$$q = q_L : \alpha(q_L - K_L) - F \geq 0 \tag{28}$$

From (27)) and (28), we get $\alpha(q_L - K_H) < F \leq \alpha(q_L - K_L)$. Thus, if $\alpha(q_L - K_H) < F \leq \alpha(q_L - K_L)$, there $\exists$ a SE in which platform entry signals $q = q_L$.

$\square$

Lemma 7 shows that if the platform entry itself acts as a signal, there exists only one type of SE in which the platform enters only if $q = q_L$. This result is against our daily observations. More importantly, it is against the findings in Zhu and Liu (2018). Our multi-agent signaling game, however, offers another explanation behind why we often observe that the platform picks a relatively good product to sell.

Proposition 3 (platform entry decision)**.** *If $\alpha(q_L - K_L) < F \leq \alpha(q_H - K_H)$, the platform enters if and only if $q = q_H$.*

Proof:

In proposition 1, we know that if the equilibrium outcome is a SE with $\hat{p}^H = \{q_H \ p_M^*\}$ and $\hat{p}^L = \{q_L \ q_L\}$ in the multi-agent signaling game, the platform's profits are given by $\pi_P^H = \alpha(q_H - K_H) - F$ and $\pi_P^L = \alpha(q_L - K_L) - F$ if it sells both types of products. On the contrary, if the platform stays out of the market for both types, its profits are given by $\pi_P^H = \pi_P^L = 0$. Thus, if $\alpha(q_H - K_H) - F \geq 0$ and $\alpha(q_L - K_L) - F < 0$, equivalently $\alpha(q_L - K_L) < F \leq \alpha(q_H - K_H)$, the platform enters if and only if $q = q_H$. Moreover, in this case, although the platform sells the high-type product only, it is a result of the multi-agent signaling game rather than a strategy to signal the quality through entry.

□