# Self-Preferencing in E-commerce Marketplaces:

Role of Sponsored Advertising and Private Labels

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**Abstract**

Traditionally, e-commerce marketplaces have enabled third-party sellers to sell to potential consumers and have earned commission from the sales. In recent years, e-commerce platforms have begun to leverage private label and sponsored advertising to generate additional revenue. This raises the question of when and why a platform may seek to give preference to its private label in sponsored advertising, and what the implications of this are for consumers and third-party sellers. To examine this issue, we build a model where two horizontally-differentiated third-party sellers and one private label compete for a prominent ad slot to increase their respective demand. The platform can concede the ad slot to third-party sellers, generate ad revenue, and increase its commission from third-party sales. Alternatively, the platform can contest for the ad slot, place its private label in a prominent position, increase private label sales, and thus show self-preference. Counter to our intuition, we find that self-preferencing hurts consumers even though the platform offers the private label at a price lower than the price of third-party sellers. Furthermore, such self-preferencing on the part of the platform can improve the profits of some third-party sellers. We also find that it is not always optimal for the platform to self-preference its private label in sponsored advertising and place it in a prominent position. Specifically, it is optimal for the platform to concede the ad slot to third-party sellers when sponsored advertising is more effective in boosting demand or when the commission rate is large. If the commission rate is endogenous and if the third-party sellers' outside option is small, the platform concedes the ad slot to third-party sellers. Moreover, the private label and sponsored advertising are not two independent sources of profits. They can function as complements or substitutes in improving the platform's profits if the commission rate is exogenous, but become complements if the commission rate is endogenous.

**Keywords: E-commerce marketplace, hybrid platforms, private label, sponsored advertising, self-preferencing**

# 1 Introduction

When visiting Amazon, we see brands such as *Amazon Basics*, *Amazon Essential*, *Amazon Baby*, *Amazon Elements*, *Pinzon*, *Goodthreads*, *Nature's Wonder*, and *Presto!* in a wide range of product categories. It is hard to miss these private labels, which generated $8.1 billion in the United States in 2020 and have grown 50% over the previous year (Garcia 2021, Medium 2020). These private labels provide consumers more variety and are often sold at a price lower than that of a comparable product from third-party sellers, implying that private labels are inherently designed to benefit consumers (Iain and Jessica 2020, Creswell 2018). Moreover, private labels generate higher margins for the platform than the commissions from third-party sellers (Waters 2021). This creates tension as the platform may hurt third-party sellers in its pursuit to increase sales of private labels.

Traditionally, third-party sellers use sponsored advertising to increase the prominence of their products and raise sales.[1] More recently, Amazon has reportedly placed its private label in the most prominent sponsored advertising slots (under the heading "featured from our brands") even without bidding (Dudley 2020, Zaerr 2020). This is alleged to give Amazon's private label an unfair advantage over third-party sellers because it does not have to pay itself for the prominent placement. This practice of self-preferencing on the part of the platform is at the center of antitrust and regulatory debate against Amazon and other tech platforms (Khan 2016, Morrison 2021).[2] Regulators are evaluating proposals to bar platforms from engaging in such self-preferencing (Mattioli and Tracy 2021) and are considering even legally separating the marketplace and the private label into different entities (e.g., the Ending Platform Monopolies Act proposed in the U.S. in 2021).

Though private labels exist in physical stores, they pose a far more potent challenge on e-commerce platforms. This is because the placement of private labels has a more pronounced impact on online shopping. In brick-and-mortar stores, customers can easily navigate competing products even if the offline retailer features its private label in a premium shelf space. In e-commerce platforms, however,

[1] https://sell.amazon.com/advertising

[2] More broadly, an e-commerce marketplace can favor its private label in several ways. First, it can give its private label a higher rank in organic search results, making it hard for third-party sellers to compete. For instance, Amazon is reported to dedicate prime positions to its private label in product listings (Dudley 2020, Zaerr 2020, Kaziukenas 2020, Jeffries and Yin 2021, Waters 2021, Kalra and Stecklow 2021). However, Amazon has disputed this claim, saying that its ranking models do not purposely prioritize the private label over third-party sellers (McCabe 2021). Second, the e-commerce platform can leverage the sales data on third-party sellers' products and information on the customers of third-party sellers to launch its own private label (Jiang et al. 2011). Third, the platform can also favor its private label in how it manages the availability and delivery of products (Etro 2021b, Bodoni et al. 2021). Our focus is on examining the strategic implications of self-preferencing the private label in sponsored advertising.

consumers rarely consider products beyond the top few search results (Waters 2021). Indeed, when Amazon places its private label in the first sponsored ad slot, it is making the most visible ad slot unavailable for third-party sellers. This significantly affects mobile product search, where only one sponsored product placement shows up followed by the list of organic results (Kaziukenas 2020).

In choosing to give prominence to its private label in sponsored advertising, the platform needs to make a nuanced trade-off among its revenue streams. The platform has the opportunity to monetize the prominent ad slot through a search advertising auction. Thus, when Amazon places its private label in some of the most valuable ad slots, it is boosting private label sales but sacrificing third-party commissions and advertising revenue from those slots (Dudley 2020). According to industry research (Kaziukenas 2020), Amazon concedes the ad slot to third-party sellers in 60% search queries, but in the case of 20% of the search queries it contests for the corresponding ad slot and places its private label in a prominent position.[3] This raises several interesting research questions. Can it be profitable for the platform to concede the ad slot to third-party sellers? When and why does a platform engage in self-preferencing the private label in sponsored advertising? Does self-preferencing on the part of the platform hurt both third-party sellers and consumers? What is the nature of the relationship between the private label and sponsored advertising in improving the platform's profits? How does consumer valuation influence the platform's choice of strategy?

To theoretically examine these issues, we consider a marketplace with two horizontally-differentiated third-party sellers. The platform has three potential sources of revenue. First, it can earn a percentage of third-party sales as commission. Second, the platform can introduce a private label brand, which increases the variety of products in the market and yields a higher margin than the commission from third-party sellers. Third, the top position in the platform's webpage is prominent in the eyes of consumers, and they are likely to consider purchasing the product in such a prominent position. This presents an opportunity for the platform to create a sponsored ad slot and auction it based on consumer search. Note that when the platform concedes the ad slot to third-party sellers, it earns ad revenue from the auction. But if the platform contests for the ad slot and places its private label in the sponsored slot, it forgoes the ad revenue. Third-party sellers may have an opportunity cost for selling on the platform or may incur a fixed cost to participate in the marketplace, and we capture such costs with a fixed outside option. Thus, third-party sellers participate in the marketplace only if the expected profits are at least

[3]In the case of the remaining 20% of search queries, we see no sponsored ad slot and a third-party seller shows up in the first organic slot.

equal to the outside option profits. The third-party sellers participating in the marketplace need to decide on how much to bid for the ad slot and the price for their products. Based on the prices and fit, consumers then decide to buy one of the products in their consideration set or nothing. Following Chen and Riordan [2007] and Amaldoss and He [2013], we incorporate the notions of prominence and product variety in a horizontally differentiated market. Initially, we focus on products of moderate valuation so that there is scope for the market to expand depending on price. Later we relax this assumption and consider products of high valuation.

Our intuition may suggest that the platform's best strategy is to contest for the ad slot and make its private label prominent, thus showing self-preference. Note that the private label yields a higher margin than the commission from third-party sellers. Hence, placing the private label in the ad slot should be more profitable for the platform than conceding the ad slot to a a third-party seller. In other words, the ad revenue earned from conceding the ad slot to third-party sellers is unlikely to match the additional private label sales accruing from placing the private label in a prominent position. Our intuition may also suggest that the platform will have an even more significant incentive to contest for the ad slot if the top ad slot makes the product more prominent and raises demand. Contrary to these intuitions, we find that it is not always optimal for the platform to contest for the ad slot and evince self-preference in the placement of the private label. Interestingly, it is optimal for the platform to contest for the ad slot when the ad slot does not add much prominence to a product, but to concede the ad slot to third-party sellers if the ad slot adds more prominence to a product.

To follow the intuition for these findings, let us begin by understanding when and why it is in the self interest of the platform to concede the ad slot to third-party sellers (instead of self-preferencing the private label in sponsored advertising). Note that the platform earns sales commission from third-party sellers and hence it is less motivated to compete with them on price. Third-party sellers do not have such a consideration, and hence are more motivated to compete on price. Consequently, if the platform concedes the ad slot to a third-party seller, the third-party seller competes more aggressively on price, the market expands by attracting some of the more price sensitive consumers, and this raises the total surplus. A larger total pie translates into more profits for the platform, all else being equal. The downside of the platform conceding the ad slot to third-party sellers is that a third-party seller's product gets featured more prominently rather than the higher-margin private label. Consequently, the platform loses some private label sales. Thus, if the sponsored ad slot is more prominent and effective in increasing consumer demand, it is profitable for the platform to concede the ad slot to third-party sellers. This is because it is more profitable for the platform to raise the total surplus rather than prioritize private label sales. However, when the ad is less effective in increasing consumer demand, conceding the ad slot does not improve the total surplus much. Hence, the the platform shifts to prioritizing private label sales, choosing to contest the ad slot and engaging in self-preferencing the private label in sponsored advertising.

As discussed earlier, self-preferencing the private label in product listing has drawn the attention of regulators. The fear is that it hurts third-party sellers in particular and consumers in general. Amazon disputes the charge by arguing that the private label is often offered at a lower price, implying it causes no harm to consumers (Iain and Jessica 2020, Creswell 2018). Our analysis shows that to examine the welfare implications of self-preferencing by an e-commerce marketplace, it is important to take into account both the *price* and *placement* of products. It is clear that a private label is beneficial to consumers as it offers them more product variety (relative to that in the absence of the private label). However, placing a private label in a prominent ad slot need not be beneficial to consumers even if the private label is priced lower *because of* its prominent position. To see this, note that the top slot motivates a seller to reduce its price and cater to more consumers. Now if the platform concedes the ad slot and a third-party seller takes the prominent position, the downward pressure on the price of the third-party seller is more pronounced and it benefits consumers even more. Furthermore, contrary to our intuition, if the private label takes the prominent placement, it may actually benefit some third-party sellers. This is because a prominent private label is priced higher than a prominent third-party seller and it attracts fewer consumers from the losing third-party seller. It is useful to note that a prominent private label has an asymmetric effect on the two third-party sellers: it benefits one seller but hurts the other seller. Specifically, it benefits the third-party seller who would have lost the ad auction if the platform concedes, whereas it hurts the seller who would have won the auction.

We may be naturally inclined to think that the private label and sponsored advertising are independent instruments at the disposal of the platform to improve its profits. The private label can help the platform increase the variety of products offered in the marketplace and attract some consumers from third-party sellers. The sponsored ad slot can aid the platform to extract more surplus from third-party sellers by making them compete for the ad slot. We find that these two instruments can play a complementary role in improving the platform's profits. Note than sponsored advertising increases price competition by forcing the prominent seller to reduce its price, hurting the platform's commission revenue from third-party sellers. The platform can counterbalance this negative effect by using the private label to soften the competition between the third-party sellers. We observe such a complementary relationship only when the third-party commission rate is above a threshold, sponsored advertising is moderately effective in raising demand, and product valuation is moderate. Otherwise, the two instruments function as substitutes in improving the platform's profits.

Though the platform's commission rate may be fixed in the short run to conform to product category norms, the platform can optimize it in the long run (by ensuring that third-party sellers earn their outside option profits and participate in the marketplace). One may intuit that when the third-party sellers have a large outside option, the platform should give up the ad slot to raise the profits of third-party sellers so that they are motivated to participate in the marketplace. Counter-intuitively, we find that the platform contests for the ad slot when third-party sellers' outside option profits are large, but concedes the ad slot when the outside option profits are small. To understand why, note that the equilibrium commission rate that ensures the participation of third-party sellers decreases with the sellers' outside option. When the third-party sellers have a larger outside option, the optimal commission rate becomes smaller. As the platform takes a smaller percentage of sales from third-party transactions, it is more motivated to prioritize private label sales and contest for the ad slot. If the commission rate is not fixed, the platform adjusts the commission rate so that sponsored advertising and private label function in a complementary way to improve its profits. When consumer valuation is high, the two instruments play a complementary role irrespective of whether the commission rate is exogenous or endogenous to the model.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 presents the main model, and Section 4 analyzes each player's payoff for different configurations of the platform. Section 5 presents the optimal strategy of the platform when the commission rate is exogenous, discusses the welfare implications and examines the incremental value of the private label and sponsored advertising. Later this section presents the results when the platform endogenously decides the commission rate. Section 6 extends the model to consider products of high valuation. Section 7 concludes the paper by outlining directions for further research.

## 2 Relevant Literature

Our work is closely related to the emerging literature on hybrid platforms that perform the dual role of being both a marketplace for third-party sellers and a seller in its own right (Anderson and Bedre-Defolie 2021, Hagiu et al. 2020, Etro 2021b, Zennyo 2021, Jiang et al. 2011). Anderson and Bedre-Defolie 2021 show that a hybrid platform charges a high commission rate to reduce the number of third-party sellers and thereby improve the sales of the private label. Furthermore, as the platform forgoes commissions from third-party sales when it sells its own private label, it is priced higher. This particular result is consistent with our finding in the presence of the private label and absence of sponsored advertising. Hagiu et al. [2020] allow the third-party seller to directly sell to consumers through its own website in addition to selling on the marketplace. Such a hybrid platform is more beneficial to consumers than a pure marketplace because the platform's private label increases price competition in the marketplace and reduces third-party sellers' prices. Etro [2021b] investigates a hybrid platform where only the promoted product has access to consumers (e.g., Amazon's Buy-box) and finds that the hybrid platform will promote the private label to curb the power of third-party sellers. Zennyo [2021] compares a fair platform with a biased platform. The fair platform ranks products randomly, whereas a biased platform is more likely to include its own product in the organic search result. As the bias makes it difficult for third-party sellers to sell their products, the platform reduces the commission rate to increase sellers' participation in the platform. The reduced commission, in turn, leads to lower prices. This body of work on hybrid platforms has focused attention on the price of the private label and commission rate, but does not consider sponsored advertising. We examine how a hybrid platform can strategically use sponsored advertising in its competition with third-party sellers.

Our paper contributes to the literature on e-commerce platform monetization (Choi and Mela 2019, Zhong 2020, Ke et al. 2020, Long et al. 2021). The platform can glean private information about the quality of third-party sellers' products from their bids. Long et al. [2021] show that the platform can strategically use this information to rearrange organic search results. Although we do not consider such asymmetric information and rearrangement of organic search results, we allow the platform to offer its own private label, and to also engage in self-preferencing the private label in sponsored search advertising. This brings our work closer to the literature on platform bias, which studies how the platform may steer consumer search to increase platform's profits (Hagiu and Jullien 2011, Cornière and Taylor 2014, De Corniere and Taylor 2019, Teh and Wright 2020).

Our work builds on prior literature on prominence (Amaldoss and He 2013, Armstrong et al. 2009, Armstrong and Zhou 2011, Wang et al. 2021). Using a Spokes model, Amaldoss and He [2013] show that the prototypical product is priced lower than a non-prototypical product when consumers' base evaluation for a product is moderate. This is because the prototypical product is more prominent in consumers' minds and thus is included more often in consumers' consideration, thereby influencing the relative price sensitivity of consumers. With the aid of a sequential consumer search model, Armstrong et al. [2009] show that a prominent firm will charge a lower price than its less prominent rivals to take advantage of its first-search advantage and dissuade consumers from searching further. In contrast to this body of work, we examine how a platform can monetize the prominent ad slot by strategically allocating it either to the third-party sellers or to its own private label.

Our work also contributes to the extensive literature on sponsored search advertising (Edelman et al. 2007, Varian 2007, Chen and He 2011, Athey and Ellison 2011, Jerath et al. 2011, Zhu and Wilbur 2011, Gomes and Sweeney 2014, Sayedi et al. 2014, Sayedi et al. 2018). Prior theoretical literature has examined the role of sponsored advertising in increasing sellers' prominence (e.g., Chen and Riordan 2007, Athey and Ellison 2011), and we see evidence of it in the empirical literature (e.g., Narayanan and Kalyanam 2015, Yang and Ghose 2010). Slotting allowance plays a similar role in brick-and-mortar retailing. Specifically, manufacturers pay a fixed lump sum to retailers to obtain prime shelf space at the brick-and-mortar retail store (Chu 1992, Lariviere and Padmanabhan 1997, Desai 2000, Sudhir and Rao 2006, Shaffer 1991, Kuksov and Pazgal 2007, Kim and Staelin 1999 and Gu and Liu 2013). Our paper investigates how the value of the prominent slot depends on whether the private label or the third-party seller takes the slot and examines how it affects the third-party sellers and consumers.

Finally, our work contributes to the research on private labels. The private label can increase the retailer's power over national brands, and enable the retailer to obtain lower wholesale prices from national brands (Mills 1995, Narasimhan and Wilcox 1998, Lal 1990, Raju et al. 1995, Sayman et al. 2002, Chintagunta et al. 2002, Meza and Sudhir 2010). The retailer earns a higher margin from selling the private instead of the national brand (Hoch and Banerji 1993, Mills 1995, Ailawadi and Harlam 2004), Pauwels and Srinivasan 2004, Meza and Sudhir 2010) and builds higher store loyalty (Ailawadi et al. 2008, Corstjens and Lal 2000). Moreover, a low-quality private label can expand the market because of its lower price (Hoch and Banerji 1993, Pauwels and Srinivasan 2004, Amaldoss and Shin 2015). It is useful to note that in an e-commerce marketplace, it is the manufacturer, not the platform, who decides the retail price. Thus, while double marginalization is a problem in brick-and-mortar retail stores, it is not an issue in online marketplaces (see Abhishek et al. 2016). Yet the retailer wields power over third-party sellers by setting the commission rate, which indirectly affects the retail price. We see horizontally differentiated private labels in hybrid platforms (Hagiu 2009, Anderson and Bedre-Defolie 2021, Soberman and Parker 2006) as well as in brick-and-mortar stores (Kumar et al. 2007, Soberman and Parker 2006). In keeping with this literature, the private label adds variety in our formulation. Unlike this literature, we examine the relationship between the private label and sponsored advertising in improving the platform's profits.

# 3 Model

Consider an e-commerce platform that hosts two third-party sellers, $i \in \{1, 2\}$. Each third-party seller offers a product. In addition to these two products, the platform could offer its own private label. Let $l$ denote the private label. The products are horizontally differentiated and the marginal cost of producing them is normalized to zero. Thus, we abstract away from potential vertical differentiation between the private label and third-party sellers (e.g, Hagiu 2009, Anderson and Bedre-Defolie 2021, Soberman and Parker 2006; see also Kumar et al. 2007, Soberman and Parker 2006). Next we discuss the three strategic players in this marketplace: consumers, the platform, and third-party sellers.

**Consumers.** Consumers interested in the product category use the search box on the platform to quickly find information about the products in the marketplace. For each keyword search, the platform presents a list of organic results. Furthermore, the platform could place an ad slot on top of the organic listings and auction the slot through a second-price auction. The order in which the sellers appear on the platform's webpage affects consumer choice. The seller in the top position is more prominent. Prominence increases the the probability of a seller being included in consumers' consideration sets. Consumers click only on sellers in their consideration set (e.g., Amaldoss and He [2013]). The seller in the top position could be the one in the first organic slot in the absence of sponsored advertising, or the one in the ad slot in the presence of sponsored advertising. For simplicity, we assume that only the top position is prominent. Our results remain qualitatively the same if we allow each position to be more prominent than the one below it.

We use the Spokes framework to model the horizontally differentiated market (Chen and Riordan

2007). Consumers have diverse tastes and seek $N \geq 3$ different varieties in the product category. Each of the $N$ varieties is represented by a spoke on a plane, and the distance between each pair of spokes is 1, implying that the length of each spoke is $\frac{1}{2}$. A unit mass of consumers is uniformly distributed on the $N$ spokes. All the different varieties sought by consumers may not be available. Let $n$ be the number of products in the market. If the platform does not offer a private label, we have a product from each of the two third-party sellers, and hence $n = 2$. However, if the platform offers a private label brand, we have a total of three products in the marketplace and $n = 3$.

Following prior literature on the Spokes model, we assume that each consumer considers at most two products. Furthermore, the product in the top position is included in consumers' consideration set with a higher probability because of its prominence (e.g., Amaldoss and He [2013]). A consumer's first-preferred variety is the local variety on the spoke where the consumer resides. Therefore, each variety is a consumer's first-preferred product with a probability $\frac{1}{N}$. The product in the top position, denoted by $z$, is more likely to be the second-preferred product. In particular, if the consumer's first-preferred product is not $z$, then her second-preferred product is $z$ with a probability of $\alpha$, whereas one of the other varieties is her second-preferred product with probability $\frac{1-\alpha}{N-1}$.

A consumer's base valuation for product $i$ in her consideration set is $v \geq 1$ (whereas the base valuation for a product not in her consideration set is 0). The net utility from purchasing product $i$ is $v - x - p_i$, where $x$ is the distance between the consumer and product $i$, and $p_i$ is the product's price. If the consumer exits the market without purchasing, she derives a utility of zero from the outside option. Of the varieties in her consideration set, the consumer clicks and evaluates those that are available. The consumer purchases the product which gives the highest net utility if and only if the net utility from purchasing is greater than zero; otherwise, the consumer exits the market without purchasing a product. Note that in our formulation, prominence influences a seller's chance of being included in a consumer's consideration set without affecting her valuation of the seller's product.

**The platform.** The platform has three potential sources of revenue: a) commission from third-party sellers, b) private label sales, and c) sponsored advertising. While the platform collects only $\phi$ ($0 < \phi < 1$) proportion of the third-party sellers' sales revenue as its commission, it retains the entire sales revenue from selling a private label.

We seek to understand if and when the platform has an incentive to place its private label in a favorable position. To incorporate the notion of self-preferencing, we let the platform choose between two options. On the one hand, the platform can concede the ad slot to third-party sellers. Then the platform's private label brand does not participate in the ad auction, leaving the two third-party sellers to compete head-to-head in the auction. Consequently, the platform earns advertising revenue from third-party sellers if it chooses to concede the ad slot. On the other hand, the platform can contest for the ad slot. In this case the platform can outbid the third-party sellers and appropriate the ad slot. Alternatively, the platform can place its private label in the prominent ad slot without running the ad auction. As both ways yield the same profits for the platform, without loss of generality, we assume that the platform runs the ad auctions and outbids third-party sellers when contesting for the ad slot. Thus, the platform can engage in preferential allocation of ad slot for the private label. When the platform evinces such self-preference and places the private label in the top position, it forgoes potential advertising revenue from third-party sellers.

The platform makes three key decisions: a) it sets the commission rate $\phi$ charged to a third-party seller, b) decides whether or not to introduce a private label $l$ to compete with the third-party sellers and, if so, it sets the price for the private label, and c) chooses whether or not to place an ad slot on the top of the organic listing. If the platform places an ad slot, it further decides whether to concede the ad slot to third-party sellers or to contest for the ad slot and display the private label in the top position. In making these decisions, the platform maximizes the total revenue from third-party commission, the private label, and advertising.

**Third-party sellers.** If a third-party seller decides to sell its product on the platform, it sets a price $p_i$ and chooses the bid $b_i$ for the ad auction. Note that if the seller does not want to participate in the ad auction, it can bid $b_i = 0$. However, as we show later, sellers bid a positive amount. We assume that each third-party seller has an outside option of $\pi_0$. The outside option reflects the opportunity cost of selling on the platform, which could be the profits a third-party seller can gain by selling on another marketplace or its own website. Consequently, a third party seller participates in the marketplace only if it expects a net profit (i.e., sales revenue minus the advertising cost) no less than the outside option, $\pi_0$. We let the seller's outside option be exogenous to the model so that we can focus on the platform's decisions regarding sponsored advertising and the private label and abstract away from the competition between marketplaces.

**Timing of the game.** The game unfolds in the following sequence. At $T = 0$, the platform sets the commission rate $\phi$ it charges third-party sellers. Furthermore, it decides whether to introduce its

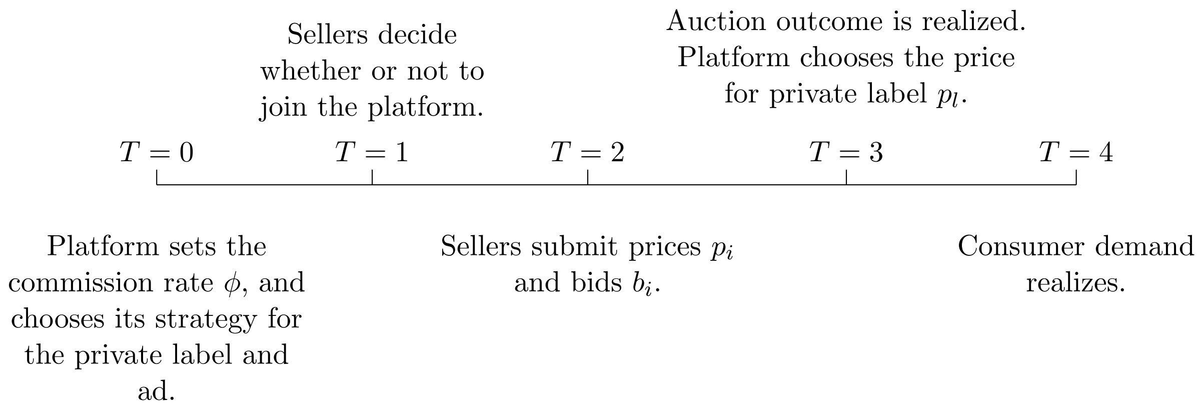


Figure 1: Timing of the game.

private label and whether to place a sponsored ad slot on top of the organic listing. If the platform adds a sponsored ad slot, it needs to decide whether to concede the ad slot to third-party sellers or to contest for the ad slot and place the private label on the top position. At $T = 1$, each third-party seller decides whether to sell its product on the marketplace by evaluating the net profits against the outside option $\pi_0$. At $T = 2$, if a third-party seller joins the marketplace, then the third-party seller simultaneously sets its selling price $p_i$ and chooses its bid $b_i$ for the ad auction (if there is an ad slot). To keep the main analysis simple, we focus on a pay-per-impression second-price auction. However, as we show later, the main results remain qualitatively and quantitatively the same if the platform uses a pay-per-click ad auction. At $T = 3$, the auction outcome is realized. If there is a private label, the platform chooses the price for its private label $p_l$. At $T = 4$, consumers observe the list of products and make their purchase decisions.

In our setting, the third-party sellers and the platform have maximal flexibility in setting prices. The third-party seller can optimize its price simultaneously with its bid to maximize the net profit. The platform can choose its private label price after third-party sellers make their pricing and bidding decisions. Our results remain the same if the platform decides its private label price before or at the same time when third-party sellers choose their prices. It is also useful to note that the platform announces the configuration of the marketplace (i.e., the commission rate, the existence of a private label, and the inclusion of sponsored advertising) before third-party sellers make their strategic decisions (i.e., participation, bidding, and pricing). This decision sequence captures the reality that the configuration of the marketplace is a long-term strategic decision of the platform, whereas third-party sellers' decisions (whether or not to join the platform, bid amount for the ad slot, and price for its product) are more flexible and easier to modify than the platform's configuration.

### 3.1 Demand model

To facilitate the analysis, we first derive a seller's demand conditional on whether the seller obtains the top position. Let "$z$" denote the seller in the top position, and "$k$" denote a seller in one of the other (non-top) positions. The top position in the webpage is taken by the ad slot in the presence of sponsored advertising, but by the first organic slot in the absence of sponsored advertising. Lemma 1 summarizes a seller's demand (see Appendix A1 for a detailed proof).[4]

**Lemma 1.** *The demand for seller z in the top position is given by:*

$$\underbrace{\sum_{k\in\{1\ldots n\}}^{k\neq z}\begin{cases} Q^1_{ao}(p_z,p_k), & \text{if } 0 < p_z - p_k \leq 1, \\ Q^2_{ao}(p_z,p_k), & \text{if } -1 < p_z - p_k \leq 0, \\ \overline{Q^2_{ao}}, & \text{if } p_z - p_k \leq -1, \end{cases}}_{\text{Demand from the competitive segment}} + \underbrace{\begin{cases} (N-n)Q^1_m(p_z), & \text{if } v - p_z \leq \frac{1}{2}, \\ (N-n)Q^2_m(p_z), & \text{if } \frac{1}{2} < v - p_z \leq 1, \\ (N-n)\overline{Q^2_m}, & \text{if } v - p_z > 1. \end{cases}}_{\text{Demand from the monopoly segment}}$$

*The demand for seller k in a non-top position is as follows:*

$$\underbrace{\begin{cases} \overline{q^1_{ao}}, & \text{if } p_z - p_k > 1, \\ q^1_{ao}(p_z,p_k), & \text{if } 0 < p_z - p_k \leq 1, \\ q^2_{ao}(p_z,p_k), & \text{if } -1 < p_z - p_k \leq 0, \end{cases} + \sum_{j\in\{1\ldots n\}}^{j\neq z, j\neq k}\begin{cases} \overline{q_{oo}}, & \text{if } p_j - p_k > 1, \\ q_{oo}(p_j,p_k), & \text{if } -1 < p_j - p_k \leq 1, \end{cases}}_{\text{Competitive segment}} + \underbrace{\begin{cases} (N-n)q_m(p_k), & \text{if } v - p_k \leq 1, \\ (N-n)\overline{q_m}, & \text{if } v - p_k > 1, \end{cases}}_{\text{Monopoly segment}}$$

In Lemma 1, we use the subscript "ao" to denote consumers in the competitive segment, for whom both of the preferred products are available with one in the top position (ad slot) and the other in a non-top position (organic slot). The subscript "oo" denotes consumers in the competitive segment, for whom

[4]In Lemma 1, $Q^1_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1}(1+p_k-p_z)$, $Q^2_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)(p_k-p_z)$, $\overline{Q^2_{ao}} = \left(\frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\right)$, $Q^1_m(p_z) \equiv \frac{2}{N}\frac{1}{N-1}(v-p_z)$, $Q^2_m(p_z) \equiv \frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)(2(v-p_z)-1)$, $\overline{Q^2_m} \equiv \frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)$, $\overline{q^1_{ao}} = \frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)$, $q^1_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1}(p_z-p_k)+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)$, $q^2_{ao}(p_z,p_k) \equiv \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)(1+p_z-p_k)$, $\overline{q_{oo}} = (1-\alpha)\frac{1}{N}\frac{2}{N-1}$, $q_m(p_k) \equiv (1-\alpha)\,2\frac{1}{N}\frac{1}{N-1}(v-p_k)$, $\overline{q_m} \equiv (1-\alpha)\,2\frac{1}{N}\frac{1}{N-1}$.

both the preferred products are available and they are displayed in non-top positions (organic slots). Finally, "m" denotes the consumers in the monopoly segment, for whom only one of the two-preferred varieties is available.

## 4 Analysis

We solve the game through backward induction. To begin with, we keep the platform's commission rate fixed, assume that the third-party sellers' participate in the marketplace, and then examine the sellers' pricing and advertising decisions in the following four subgames:

1) Both sponsored advertising and the private label are absent. This case serves as a benchmark (Case "BM"),

2) Sponsored advertising is present but the private label is absent (Case "Ad"),

3) Sponsored advertising is absent but private label is present (Case "PL"),

4) Both sponsored advertising and the private label are present (Case "APL"). In this subgame, we separately consider two subcases:

   4a) The platform concedes the ad slot to a third-party party ("Subcase Concede"), and

   4b) The platform contests for the ad slot (Subcase "Contest").

Initially, we focus on the more interesting parameter space where product valuation $v$ is moderate so that none of the products is sold at the monopoly price, implying $v - 1 \leq p^* \leq v - \frac{1}{2}$ with $p^*$ denoting the equilibrium price. Product valuation is moderate when

$$\underline{v} < v < \overline{v}, \tag{1}$$

where $\underline{v} = \max\{\underline{v}^{BM}, \underline{v}^{PL}, \underline{v}^{Ad}, \underline{v}^{Concede}, \underline{v}^{Contest}\}$ and $\overline{v} = \min\{\overline{v}^{BM}, \overline{v}^{PL}, \overline{v}^{Ad}, \overline{v}^{Concede}, \overline{v}^{Contest}\}$ are given in Appendix A2. Note that Condition (1) is a subset of $1 < v < 2$. For the rest of the analysis in this section, Condition (1) applies. Later in Section 5, we extend the model to examine the case when product valuation is high, implying $v - 1 > p^*$.

### 4.1 In the absence of both sponsored advertising and the private label (Case "BM")

In this benchmark case, denoted "BM," the platform neither features an ad slot nor offers a private label. As the third-party sellers are symmetric, the platform randomly assigns the first organic slot to one of two third-party sellers, implying that either seller could assume the top position with probability $\frac{1}{2}$.

*Pricing decision:* We focus on the symmetric pricing equilibrium where both the third-party sellers set their price at $p_i^*$. When the other seller prices at $p_i^*$, if seller $i$ deviates to price $p_i$, its expected profits are $(1-\theta)p_i D_i(p_i, p^*)$, where $1-\phi$ is the margin and $D_i(p_i, p^*)$ is the demand. To make sure that a seller has no incentive to deviate from $p_i^*$ if the other seller sets the price at $p_i^*$, we need $\frac{\partial((1-\phi)p_i D_i(p_i,p^*))}{\partial p_i}\Big|_{p_i=p_i^*} = 0$. Then, the optimal price of a third-party seller is:

$$p_i^* = p_i^{BM} \equiv \frac{\alpha(N-3)(N-2)(2v-1)+4(N-2)v+2}{22\alpha+N(\alpha(4N-19)+8)-14}.$$

The details of the derivation are presented in Appendix A2.1. We initially assume that $v-1 < p_i < v-\frac{1}{2}$ and solve for the optimal price. In Appendix A2.1, we verify that $p_i^{BM}$ satisfies the condition $v-1 < p_i^{BM} < v-\frac{1}{2}$ if product valuation is moderate ( $\underline{v}^{BM} < v < \overline{v}^{BM}$). Furthermore, if the other third-party seller sets the price at $p_i^{BM}$, then seller $i$ has no incentive to deviate globally to price $p_i < v-1$ or $p_i > v-\frac{1}{2}$. Thus, when product valuation is moderate, $p_i^{BM}$ is indeed the equilibrium price in the benchmark case (Case "BM").

*Players' Payoffs:* We can calculate the total surplus by considering whether a given consumer's preferred varieties are available.

$$\begin{aligned} TS^{BM} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-2)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_{1+p_i^{BM}-v}^{\frac{1}{2}}\big(v-(1-x)\big)2dx \\ &+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_0^{\frac{1}{2}}(v-x)2dx + (N-2)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_i^{BM}}(v-x)dx. \end{aligned}$$

The first term represents the total surplus from consumers located on the spoke of third-party seller $z$ displayed in the first organic slot. Consumers are on the local spoke of seller $z$ with probability $\frac{1}{N}$ and located at a distance $x \sim U[0, \frac{1}{2}]$ from seller $z$. These consumers purchase their most preferred variety $z$ because $v - p_i^{BM} > \frac{1}{2}$, generating a total surplus of $v - x$. The second term represents the surplus from consumers whose most preferred variety is not available and their second-preferred variety is $z$

with probability $(N-2)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)$. These consumers are at a distance $x \sim U[0, \frac{1}{2}]$ from their first-preferred seller. Consumers in this segment purchase $z$ and generate a total surplus of $v-(1-x)$ if and only if it is larger than $1+p_i^{BM}-v$. The third term represents the surplus from consumers whose first-preferred variety is $k$ which appears in the second organic slot, and their second-preferred variety is $z$ with probability $\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)$. All the consumers in this segment purchase from seller $k$ as $p_k = p_z = p_i^{BM}$. The fourth term represents the surplus from consumers whose only available variety is $k$ with probability $(N-2)(1-\alpha)\frac{2}{N}\frac{1}{N-1}$. These consumers buy from seller $k$ if and only if their distance from the seller, $x$, is smaller than $v-p_i^{BM}$, where $x \sim U[0,1]$.

Below, we present the platform's revenue $\Pi_p^{BM}$, each third-party seller's profit $\pi_i^{BM}$, and the consumer surplus $CS^{BM}$:

$$\begin{cases} \Pi_p^{BM} = 2\ \phi\ p_i^{BM}\ D_i, \\ \pi_i^{BM} = (1-\phi)\ p_i^{BM}\ D_i, \\ CS^{BM} = TS^{BM} - \Pi_p^{BM} - \pi_i^{BM}. \end{cases}$$

Note that in the absence of both the private label and sponsored advertising, the platform's only source of revenue is the commission it earns from the two third-party sellers.

## 4.2 In the presence of sponsored advertising and the absence of the private label (Case "Ad")

Now the platform auctions the top ad slot through a second-price auction but does not offer a private label to compete with third-party sellers (Case "Ad"). We show that there exists an asymmetric equilibrium where one third-party seller prices lower, bids higher and wins the auction, whereas the other third-party seller prices higher, bids lower and loses the auction.

Let $p_z^*$ and $b_z^*$ denote the equilibrium price and bid of the third-party seller $z$ who wins the ad auction. Let $p_k^*$ and $b_k^*$ denote the equilibrium price and bid of the third-party seller $k$ who loses the ad auction. Recall that the two sellers choose prices and bids simultaneously. In order for $(p_z^*, b_z^*, p_k^*, b_k^*)$ to be in equilibrium, it needs to satisfy the following three conditions: (1) Given the two sellers' equilibrium bids $b_z^*$ and $b_k^*$ and seller $k$'s price $p_k^*$, seller $z$ has no incentive to deviate from pricing at $p_z^*$. (2) Given the two sellers' equilibrium bids $b_z^*$ and $b_k^*$ and seller $z$'s price $p_z^*$, seller $k$ has no incentive to deviate from pricing at $p_k^*$. (3) Given the equilibrium prices $p_z^*$ and $p_k^*$, seller $z$ indeed overbids seller $k$, that is, $b_z^* > b_k^*$.

*Pricing decision:* We first analyze seller $z$'s optimal price, anticipating that seller $k$ prices at $p_k^*$ and the two sellers bid $b_z^* > b_k^*$ so that seller $z$ wins the ad auction. As in the benchmark case, we can write down the demand of third-party seller $z$ when deviating to price $p_z$, namely $D_z^{win}(p_z, p_k^*)$. Expecting itself to win the ad auction, seller $z$'s net profit upon pricing at $p_z$ is equal to

$$\pi_z(p_z, p_k^*) = (1-\phi)\ p_z\ D_z^{win}(p_z, p_k^*) - b_k^*.$$

In the above expression, seller $z$ expects a sales revenue of $(1-\phi)p_z D_z^{win}(p_z, p_k^*)$ and pays the losing seller's bid $b_k^*$.

Similarly, for seller $k$, given $b_z^* > b_k^*$, seller $k$ is expected to lose the ad auction. The demand of seller $k$ upon deviating to price $p_k$ is $D_k^{lose}(p_k, p_z^*)$, and its net profits are:

$$\pi_k(p_k, p_z^*) = (1-\phi)p_k D_k^{lose}(p_k, p_z^*).$$

Note that when a third-party seller chooses its price, it expects to pay either zero advertising cost (if it loses the ad auction) or pay the competitor's bid that does not vary with its product price (if it wins the advertising auction). Hence, when the two third-party sellers simultaneously choose their prices to maximize their net revenue, it is equivalent to choosing their prices to maximize the sales revenue without considering the advertising cost (which does not vary with the seller's price).

Noting that $\frac{\partial \pi_z(p_z, p_k^*)}{p_z}\Big|_{p_z=p_z^*} = 0$ and $\frac{\partial \pi_k(p_k, p_z^*)}{p_k}\Big|_{p_k=p_k^*} = 0$, we find that the equilibrium prices of the two third-party sellers are given by:

$$\begin{cases} p_z^* = p_z^{Ad} \equiv \frac{\alpha^2(N-2)^2(2N-3)(2v-1)-2\alpha(N-2)(N(N(4v-2)-15v+7)+13v-6)-(4N-5)(2(N-2)v+1)}{(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)(\alpha(N-2)+1)}, \\ p_k^* = p_k^{Ad} \equiv \frac{\alpha(N-2)(3N-4)(2v-1)-(4N-5)(2(N-2)v+1)}{22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35}. \end{cases}$$

The above prices satisfy the condition $v - 1 < p_z^{Ad} < p_k^{Ad} < v - \frac{1}{2}$ when $\underline{v}^{Ad} < v < \overline{v}^{Ad}$. We further show in Appendix A2.2 that if product valuation is moderate, then $p_z^{Ad} > p_k^{Ad}$ cannot be sustained in equilibrium. This implies that the only pure strategy equilibrium is $p_z^{Ad} < p_k^{Ad}$. The following lemma summarizes these results.

**Lemma 2** (Equilibrium Price under Case "Ad")**.** *If product valuation is moderate (* $\underline{v}^{Ad} < v < \overline{v}^{Ad}$*), then in the presence of sponsored advertising and the absence of a private label, the third-party seller* $z$

*who wins the ad auction charges a lower price than the price of the third-party seller $k$ in the organic slot, implying $p_z^{Ad} < p_k^{Ad}$.*

To follow the rationale for the result, let us first take a look at the demand. The demand of seller $z$ in the ad slot and that of seller $k$ in the organic slot are given by:

$$\begin{cases} D_z^{win}(p_z, p_k) & = \underbrace{\frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)(p_k - p_z)}_{\text{Competitive segment: } Q_{ao}^2(p_z, p_k)} + \underbrace{(N-2)\left[\frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\left(\frac{v - p_z - \frac{1}{2}}{\frac{1}{2}}\right)\right]}_{\text{Monopoly segment: } Q_m^2}, \\ D_k^{lose}(p_k, p_z) & = \underbrace{\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)(1 + p_z - p_k)}_{\text{Competitive segment: } q_{ao}^2(p_z, p_k)} + \underbrace{(N-2)\left[(1-\alpha)\ \frac{1}{N}\frac{1}{N-1}\frac{v - p_k}{\frac{1}{2}}\right]}_{\text{Monopoly segment: } q_m}. \end{cases}$$

It is easy to see that third-party seller $z$ (compared to seller $k$) sells to more consumers in its monopoly segment. That is, $\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) > (1-\alpha)\ \frac{1}{N}\frac{1}{N-1}$. Intuitively, when a consumer's first-preferred product is not available, third-party seller $z$ has a higher chance to be the consumer's second-preferred product than seller $k$ due to its prominent placement, and such consumers are in the seller's monopoly segment. Furthermore, the competitive segment comprised of consumers who consider both sellers $z$ and $k$ is of the same size for the two sellers (i.e., $\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)$). Therefore, seller $z$ (compared to seller $k$) caters to more consumers in the monopoly segment.

Second, when product valuation is moderate, the monopoly segment is more price-sensitive than the competitive segment. This is because the outside option for consumers in the monopoly segment is zero, and the seller can cater to more consumers by reducing its price. In contrast, for consumers in the competitive segment, the firm needs to wean them away from the other preferred seller. Thus, the winning third-party seller $z$ caters to a higher fraction of price-sensitive consumers than the losing third-party seller $k$. Consequently, third-party seller $z$ can reduce its price and increase the demand from the monopoly segment (more than the demand from the competitive segment). Therefore, third-party seller $z$ lowers its price, takes advantage of its prominent position, and sells to more consumers. It is useful to note that in the special case when $\alpha = 0$, the price under Case "Ad" is the same as the price under Case "BM" (that is, $p_z^{Ad} = p_k^{Ad} = p_i^{BM}$). This is because a prominent position no longer increases a seller's demand. This observation is an important interim step toward analyzing the platform's optimal strategy.

*Advertising decision:* Based on the equilibrium prices, we derive the equilibrium bids. Specifically, we use the locally envy-free equilibrium selection criterion of Edelman et al. [2007] and obtain the highest possible equilibrium bid for a seller. Under the envy-free equilibrium, each seller's pay-per-impression equilibrium bid is equal to the incremental value of winning the ad auction relative to losing it. In equilibrium, each third-party seller's bid is

$$\begin{cases} b_z^* = b_z^{Ad} \equiv (1-\phi)\; p_z^{Ad}\; \Big( D_z^{win}(p_z^{Ad}, p_k^{Ad}) - D_z^{lose}(p_z^{Ad}, p_k^{Ad}) \Big), \\ b_k^* = b_k^{Ad} \equiv (1-\phi)\; p_k^{Ad}\; \Big( D_k^{win}(p_k^{Ad}, p_z^{Ad}) - D_k^{lose}(p_k^{Ad}, p_z^{Ad}) \Big). \end{cases}$$

In the above expression, $D_z^{lose}(p_k^{Ad}, p_z^{Ad})$ is the expected demand of seller $z$ if it loses the auction and $D_k^{win}(p_k^{Ad}, p_z^{Ad})$ represents seller $k$'s demand if it wins the ad auction (given that the two sellers price optimally).

Appendix A2.2 presents the detailed solution of $b_z^{Ad}$ and $b_k^{Ad}$. We can verify that $b_z^{Ad} > b_k^{Ad} > 0$ given $1 < v < 2$, implying that the lower priced seller $z$ indeed wins the ad auction, as we have assumed at the beginning of the calculation. Intuitively, the incremental value of winning the ad slot is larger for the lower-priced seller than the higher-priced seller because featuring in the ad slot raises more demand for the former more than it does for the latter.

*Players' Payoffs:* The third-party seller who wins the ad auction expects a sales revenue of $(1-\phi)\; p_z^{Ad}\; D_z^{win}(p_z^{Ad}, p_k^{Ad})$ and pays the losing seller's bid, $b_k^{Ad}$. The third-party seller who loses the ad auction expects a sales revenue of $(1-\phi)\; p_k^{Ad}\; D_k^{lose}(p_z^{Ad}, p_k^{Ad})$. The expected profits of the two third-party sellers (net of the advertising cost) are given by:

$$\begin{cases} \pi_z^{Ad} = (1-\phi)\; p_z^{Ad}\; D_z^{win} - b_k^{Ad}, \\ \pi_k^{Ad} = (1-\phi)\; p_k^{Ad}\; D_k^{lose}. \end{cases}$$

We find that $\pi_z^{Ad} > \pi_k^{Ad}$. In other words, obtaining the top position improves a third-party seller's sales revenue, even though winning the ad auction pushes its price lower and makes it pay for the placement. This is because the gain in demand more than offsets the reduction in price and the advertising cost.

The platform's total revenue is as follows:

$$\Pi_p^{Ad} = \underbrace{\phi\Big(\; p_z^{Ad}\; D_z^{win} + p_k^{Ad}\; D_k^{lose}\Big)}_{\text{Commission revenue}} + \underbrace{\;\; b_k^{Ad} \;\;}_{\text{Ad revenue}} \quad .$$

In the above expression, the platform takes $\phi$ fraction of third-party sales as its commission revenue. It also earns the ad revenue $b_k^{Ad}$. The consumer surplus $CS^{Ad}$ and the total surplus $TS^{Ad}$ can be derived as in the benchmark case, and we relegate the details to Appendix A2.2.

Finally, we would like to clarify that the results remain qualitatively the same under pay-per-click ad auctions. Note that consumers only click on product varieties in their consideration set if a seller offers such a product. Let $c$ denote the number of clicks on the prominent seller $z$, which is equal to the probability of seller $z$ entering a consumer's consideration set. Equation A1 in Appendix A1 shows that $c$ is a function of the product varieties that consumers seek $N$, the number of sellers in the market $n$, and the effectiveness of advertising $\alpha$. Therefore, if the platform adopts a pay-per-click auction, it proportionally inflates each seller's bid without affecting the ad revenue. To see this, a seller's pay-per-click bid is equal to the pay-per-impression bid divided by $c$, implying $b_i^{Pay-per-click} = \frac{b_i^{Pay-per-impression}}{c}, i \in \{z, k\}$. Seller $z$ still wins the ad auction by paying for seller $k$'s bid. Upon winning the ad slot, it pays for $c$ number of clicks. Then the platform's ad revenue becomes $b_k^{Pay-per-click} * c = \frac{b_k^{Pay-per-impression}}{c} * c = b_i^{Pay-per-impression}$. In other words, the platform expects the same ad revenue whether it adopts a pay-per-click auction or pay-per-impression auction to sell the ad slot.

## 4.3 In the absence of sponsored advertising and presence of the private label brand (Case "PL")

Here we consider the case where the platform offers a private label brand but does not host sponsored advertising. Note that there are three products in the marketplace: one from each of the two third-party sellers and the platform's private label. We focus on the situation where the platform is not biased towards its private label, and hence randomly selects one of the three products in the marketplace to be in the top organic slot. We assume that the platform does not manipulate the organic search results in favor of the private label.[5] However, later we examine the implications of the platform giving its private label preferential treatment in sponsored advertising (see §4.4).

*Pricing decision:* Denote the equilibrium price of a third-party seller as $p_i^*$, and denote the equilibrium price of the private label as $p_l^*$. To solve the equilibrium private label price, we use the notion of rational expectation. That is, at $T = 2$, if the two third-party sellers make pricing decisions expecting that the platform sets $p_l = p_l^*$ at $T = 3$, then the platform does not have incentive to deviate from $p_l^*$ at $T = 3$.[6]

[5] For example, when Amazon features its private label brand in the top slot (under the "Featured from our brands" label), it is a consequence of the advertising system rather than manipulation of the organic search ranking algorithm. See https://www.marketplacepulse.com/articles/amazon-giving-up-advertising-revenue-to-promote-its-brands

[6] Our results remain the same if the platform chooses $p_l$ before the third-party sellers choose $p_i$ (i.e., after $T = 1$ and

We find that the equilibrium price is $p_i^* = p_i^{PL}$ and $p_l^* = p_l^{PL}$, where $p_i^{PL}$ and $p_l^{PL}$ are given in Appendix A2.3. Upon comparing the two equilibrium prices, we have following finding.

**Lemma 3** (Equilibrium Price under Case "PL")**.** *If product valuation is moderate ($\underline{v}^{PL} < v < \overline{v}^{PL}$), then in the absence of a sponsored ad slot and presence of a private label, the platform sets a price for the private label even higher than the price of a third-party seller, implying $p_l^{PL} \geq p_i^{PL}$.*

The intuition behind Lemma 3 is that the platform is less motivated to compete on price with the third-party sellers because the platform earns commission on the sales of third-party sellers. To see this, consider the extreme case when the platform does not earn any commission ($\phi = 0$). Then, the platform would compete on price just like any third-party seller, and we have $p_l^{PL} = p_i^{PL} = \frac{\alpha(N-4)(N-3)(2v-1)+6(N-3)v+6}{6(7\alpha-5)+2N(\alpha(2N-13)+6)}$. But if $\phi > 0$, the platform takes a share of third-party sellers' revenue. Then, the platform charges a strictly higher price for the private label (that is, $p_l^{PL} > p_i^{PL}$) with the intention of softening price competition in the market. Moreover, the difference between the two prices increases with the commission rate $\phi$.

*Players' Payoffs:* The platform's total revenue $\Pi_p^{PL}$ and each seller's net profit $\pi_i^{PL}$ are as follows:

$$\begin{cases} \Pi_p^{PL} = & \frac{1}{3}\left(p_l^{PL}\ D_l^{win} + \phi\ 2\ p_i^{PL}\ d_i^{lose1}\right) + \frac{2}{3}\left(p_l^{PL}\ D_l^{lose} + \phi\ p_i^{PL}\ d_i^{win} + \phi\ p_i^{PL}\ d_i^{lose2}\right), \\ \pi_i^{PL} = & (1-\phi)\ p_i^{PL}\left(\frac{1}{3}\ d_i^{lose1} + \frac{1}{3}\ d_i^{lose2} + \frac{1}{3}\ d_i^{win}\right). \end{cases}$$

To follow the expression for the platform's total revenue $\Pi_p^{PL}$, notice that the private label obtains the first organic slot with a probability of $\frac{1}{3}$ and the corresponding total revenue of the platform is $p_l^{PL}\ D_l^{win} + \phi\ 2\ p_i^{PL}\ d_i^{lose1}$, where $D_l^{win}$ is the private label's demand in the first organic slot and $d_i^{lose1}$ is the third-party seller's demand in a non-top slot. But one of the other third-party sellers $i$ could obtain the first organic slot with probability $\frac{2}{3}$, and the corresponding total revenue of the platform becomes $p_l^{PL}\ D_l^{lose} + \phi\ p_i^{PL}\ d_i^{win} + \phi\ p_i^{PL}\ d_i^{lose2}$, where the private label's demand in a non-top slot is $D_l^{lose}$, seller $i$'s demand in the top slot is $d_i^{win}$ and seller $j$'s demand in a non-top slot is $d_i^{lose2}$. Next, to follow the expression for a third-party seller's revenue $\pi_i^{PL}$, note that seller $i$'s demand is: $d_i^{lose1}$ when the private label brand is in the first organic slot, $d_i^{lose2}$ when seller $j$ is in the first organic slot, and $d_i^{win}$ when seller $i$ is in the first organic slot. The consumer surplus $CS^{PL}$ and total surplus $TS^{PL}$ are presented in Appendix A2.3.

before $T = 2$) or at the same time (i.e., $T = 2$). Under these alternative timings, the same calculations and results would apply.

Compared to the benchmark case (Case "BM"), we notice that the prices of all the three products in the marketplace are higher (given that the product valuation is a subset of $1 < v < 2$). That is, $p_l^{PL} > p_i^{PL} > p_i^{BM}$. We obtain this result because adding a product to the market reduces the fraction of consumers in the monopoly segment and increases the fraction of consumers in the competitive segment. As discussed earlier, when consumers' product valuation is moderate, consumers in the monopoly segment are more price sensitive than those in the competitive segment. Thus, introducing a private label reduces the price competition among sellers and raises the price of all products.

### 4.4 In the presence of sponsored advertising and the private label brand (Case "APL")

Now we consider the main case where the platform features a sponsored ad slot and introduces a private label. As noted earlier, we need to consider two possible subcases. The platform can choose not to bid in the ad auction and thereby concede the ad slot to the third-party sellers (Case "Concede"). Alternatively, the platform can favor its private label by placing it in the ad slot (Case "Contest"). Table 1 illustrates the rank-order of the positions of the three firms corresponding to these two cases. Recall that only the top position is prominent and the product featured in the ad slot benefits from its prominence (whereas those in the first and second positions among the organic search results do not benefit).

| Case "Concede" | Case "Contest" |
|---|---|
| $z : p_z^{Concede}$ | $l : p_l^{Contest}$ |
| $k : p_k^{Concede}$ | $i : p_i^{Contest}$ |
| $l : p_l^{Concede}$ | $j : p_i^{Contest}$ |

Table 1: Platform arrangement in Case "Concede" and Case "Contest."

#### 4.4.1 Platform conceding the ad slot (Subcase "Concede")

In this case, to solve for the equilibrium prices and bids of the two third-party sellers, denoted by $(p_z^*, b_z^*, p_k^*, b_k^*)$, we follow the same logic as in Case "Ad". The equilibrium private label price $p_l^*$ can be solved as in Case "PL" using rational expectation.

*Pricing decision:* We know from Lemma 2 that the third-party seller in the ad slot $z$ tends to price lower than the other third-party seller $k$. We also know from Lemma 3 that the private label is priced higher than the price of a third-party seller. Appendix A2.4 shows that the equilibrium prices are $p_z^* = p_z^{Concede}, p_k^* = p_k^{Concede}, p_l^* = p_l^{Concede}$. Upon comparing the three prices, we have the following finding:

**Lemma 4** (Equilibrium Price under Case "Concede")**.** *If product valuation is moderate (* $\underline{v}^{Concede} < v < \overline{v}^{Concede}$*) and if the platform concedes the ad slot to a third-party seller, seller z, who wins the ad auction, sets the lowest price followed by seller k in the organic slot and then private label l, implying* $p_z^{Concede} < p_k^{Concede} < p_l^{Concede}$.

Next we provide a brief outline of the solution for equilibrium prices. Assume $p_z^* < p_k^* < p_l^*$. When expecting the private label to be price $p_l^*$, third-party sellers $z$ and $k$ who deviate to price $p_z$ and $p_k$, respectively, face a net profit function of $\pi_z(p_z, p_k^*, p_l^*)$ and $\pi_k(p_k, p_z^*, p_l^*)$. The solution process follows the same logic as in Case "Ad". Appendix A2.4 presents the results. From $\frac{\partial(\pi_z(p_z,p_k^*,p_l^*))}{p_z}\Big|_{p_z=p_z^*} = 0$ and $\frac{\partial(\pi_k(p_k,p_z^*,p_l^*))}{p_k}\Big|_{p_k=p_k^*} = 0$, we obtain the third-party sellers' prices $p_z^*$ and $p_k^*$ as a function of the equilibrium private label price $p_l^*$.

Next, given the third-party seller's equilibrium prices $p_z^*$ and $p_k^*$, let $\Pi_P(p_l, p_k^*, p_z^*)$ denote the platform's revenue upon deviating to set the price for the private label at $p_l$. Notice that $p_l$ affects the private label's demand, $D_l^{lose}(p_l, p_z^*, p_k^*)$. It also affects the third-party seller's demand $D_z^{win}(p_z^*, p_k^*, p_l)$ and $D_k^{lose}(p_k^*, p_z^*, p_l)$, thus influencing the platform's commission revenue. We can write down the platform's total revenue as

$$\Pi_P(p_l, p_k^*, p_z^*) = p_l \ D_l^{lose}(p_l, p_z^*, p_k^*) + \phi \left( p_z D_z^{win}(p_z^*, p_k^*, p_l) + \phi D_k^{lose}(p_k^*, p_z^*, p_l) \right) + b_k^*,$$

where $D_l^{lose}(p_l, p_z^*, p_k^*)$, $D_z^{win}(p_z^*, p_k^*, p_l)$ and $D_k^{lose}(p_k^*, p_z^*, p_l)$ represent the demand of $l$, $z$ and $k$, respectively, and are presented in Appendix A2.4. Noting $\frac{\partial \Pi_P(p_l,p_k^*,p_z^*)}{p_l}\Big|_{p_l=p_l^*} = 0$, we obtain the equilibrium prices $p_z^* = p_z^{Concede}, p_k^* = p_k^{Concede}, p_l^* = p_l^{Concede}$, which are also presented in Appendix A2.4.

We can verify that the solution satisfies the condition $v - 1 < p_z^{Concede} < p_k^{Concede} < p_l^{Concede} < v - \frac{1}{2}$ and that it is indeed an equilibrium when $\underline{v}^{Concede} < v < \overline{v}^{Concede}$.

*Advertising decision:* Using the equilibrium prices, we derive the envy-free equilibrium bids. Note that the two third-party sellers are competing for the ad slot in this subcase. If seller $z$ wins the ad auction, its sales revenue is $(1-\phi) \ p_z^{Concede} \ D_z^{win}(p_z^{Concede}, p_k^{Concede}, p_l^{Concede})$; if it loses the ad auction, its sales revenue is $(1-\phi)\ p_k^{Concede}\ D_z^{lose}(p_z^{Concede}, p_k^{Concede}, p_l^{Concede})$. Therefore, seller $z$'s envy-free equilibrium bid is given by:

$$b_z^* = b_z^{Concede} = (1-\phi)\ p_z^{Concede}\ \Big(D_z^{win}(p_z^{Concede}, p_k^{Concede}, p_l^{Concede}) - D_z^{lose}(p_z^{Concede}, p_k^{Concede}, p_l^{Concede})\Big).$$

Turning to seller $k$, if it wins, its demand is $D_k^{win}(p_k^{Concede}, p_z^{Concede}, p_l^{Concede})$. Seller $k$'s equilibrium bid is

$$b_k^* = b_k^{Concede} = (1-\phi)\ p_k^{Concede}\ \Big(D_k^{win}(p_k^{Concede}, p_z^{Concede}, p_l^{Concede}) - D_k^{lose}(p_k^{Concede}, p_z^{Concede}, p_l^{Concede})\Big).$$

We find that $b_z^{Concede} > b_k^{Concede} > 0$ under $\underline{v}^{Concede} < v < \overline{v}^{Concede}$. This means that seller $z$ indeed overbids seller $k$ and wins the ad auction in equilibrium, confirming that our assumption indeed holds.

*Players' Payoffs:* Following the same logic as in Case "Ad", the winning seller $z$ and the losing seller $k$ expect the following net profits:

$$\begin{cases} \pi_z^{Concede} = (1-\phi)\ p_z^{Concede}\ D_z^{win} - b_k^{Concede}, \\ \pi_k^{Concede} = (1-\phi)\ p_k^{Concede}\ D_k^{lose}. \end{cases}$$

Upon conceding the ad slot, the platform earns private label sales revenue, commission from third-party sales, and advertising revenue. Hence, we have:

$$\Pi_p^{Concede} = \underbrace{p_l^{Concede}\ D_l}_{\text{Private label sales}} + \underbrace{\phi\ p_z^{Concede}\ D_z^{win} + \phi\ p_k^{Concede}\ D_k^{lose}}_{\text{Commission revenue}} + \underbrace{b_k^{Concede}}_{\text{Ad revenue}}.$$

The derivations of consumer surplus $CS^{Concede}$ and the total surplus $TS^{Concede}$ are presented in Appendix A2.4.

### 4.4.2 Platform contesting for the ad slot (Subcase "Contest")

*Pricing decision:* Now we turn attention to the subcase where the platform contests for the ad slot and assumes the top position. Denote the equilibrium price of the private label in the ad slot as $p_l^*$ and the price of a third-party seller $i$ in the organic slot as $p_i^*$. Again, the ranking of the two third-party sellers in the organic listing does not affect the outcome because only the ad slot is prominent. We find that

$p_l^* > p_i^*$ cannot be observed in equilibrium, and it suffices to focus on $p_l^* < p_i^*$ (see Appendix A2.5).

Assume $p_l^* < p_i^*$. At $T = 2$, if the third-party seller $i$ sets its price at $p_i$, then its sales revenue is $(1-\phi)p_i D_i(p_i, p_i^*, p_l^*))$, where $D_i(p_i, p_i^*, p_l^*)$ is the demand function (see Appendix A2.5). When anticipating the other third-party seller prices to be $p_i^*$ and rationally expecting the private label prices to be $p_l^*$, seller $i$ should have no incentive to deviate from $p_i^*$. This requires $\frac{\partial((1-\phi)p_i D_i(p_i,p_i^*,p_l^*))}{p_i}\Big|_{p_i=p_i^*} = 0$.

Let $D_l(p_l, p_i^*)$ denote the expected demand of the private label at price $p_l$ in $T = 3$ (see Appendix A2.5). Then, given that the two third-party sellers price at $p_i^*$, the platform's total revenue is given by: $\Pi_p(p_l, p_i^*) = p_l D_l(p_l, p_i^*) + 2\ \phi\ p_i^* * \Big(q_{ao}^2(p_l, p_i^*) + q_{oo}(p_i^*, p_i^*) + (N-3)q_m(p_i^*)\Big)$. In this expression, the first term is the platform's private label sales revenue, and the second term is the platform's commission revenue. Note that if the platform contests for the ad slot and assumes the top position, it forgoes potential advertising revenue from third-party sellers. In order for $p_l^*$ to be observed in equilibrium, we should have that $\frac{\partial \Pi_p(p_l,p_i^*)}{\partial p_l}\Big|_{p_l=p_l^*} = 0$. Based on this condition, we find that $p_l^* = p_l^{Contest}$ and $p_i^* = p_i^{Contest}$ (the expressions for $p_l^{Contest}$ and $p_i^{Contest}$ are presented in Appendix A2.5). The solution satisfies the condition $v-1 < p_l^{Contest} < p_i^{Contest} < v - \frac{1}{2}$, and hence it is an equilibrium when $\underline{v}^{Contest} < v < \overline{v}^{Contest}$. On comparing the two prices, we have the following result.

**Lemma 5** (Equilibrium Price under subcase "Contest")**.** *If product valuation is moderate ($\underline{v}^{Contest} < v < \overline{v}^{Contest}$) and if the platform contests for the ad slot, the private label is priced lower than the price of a third-party seller's product in the organic slot, implying $p_l^{Contest} < p_i^{Contest}$.*

To follow the intuition, note that two countervailing forces are at play. On the one hand, the platform has less incentive to compete on price and might like to charge for the private label a price higher than a third-party seller's price. On the other hand, the platform might like to charge a lower price for the private label to take advantage of its prominent position and increase sales. Overall, the latter force dominates when product valuation is moderate ($\underline{v}^{Contest} < v < \overline{v}^{Contest}$). Therefore, the platform prices its private label lower than the third-party sellers.

*Players' Payoffs:* Each third-party seller's profit is given by:

$$\pi_i^{Contest} = (1-\phi)\ p_i^{Contest}\ D_i.$$

The platform's total revenue includes private label sales and commission revenue, and is given by:

$$\Pi_p^{Contest} = \underbrace{p_l^{Contest}\ D_l}_{\text{Private label sales}} + \underbrace{2\ \phi\ p_i^{Contest}\ D_i}_{\text{Commission revenue}}.$$

We obtain the consumer surplus $CS^{Contest}$ and total surplus $TS^{Contest}$ following the same logic as in the benchmark case (see Appendix A2.5).

# 5 Platform's Optimal Strategy

In this section, we derive the platform's optimal strategy by comparing the total revenue of the platform $\Pi_p$ under the four different cases discussed in Section 4 (i.e., cases "BM", "Ad", "PL" and "APL"). To begin, we assume that the commission rate $\phi$ is exogenous to the model (see Section 5.1). This assumption reflects the reality that in the short run the commission rate is fixed and typically conforms to the prevailing rate for a given product category. On average, Amazon charges a commission of 15% with some variation depending on the product category. In the long run, however, the platform may optimize its commission rate to maximize the total revenue from third-party sellers, the private label, and sponsored advertising. To capture this situation, we later endogenize the commission rate $\phi$ (see Section 5.2).

## 5.1 Exogenous commission rate

Recall that the sponsored ad slot allows the platform to extract more surplus from third-party sellers by charging them for a prominent placement. Furthermore, the private label enables the platform to compete with third-party sellers for a share of the market and also expand the market by providing consumers more varieties. Consequently, the platform earns higher revenue in the presence of both the ad slot and the private label (i.e., Case "APL") than in the absence of one or both of them (i.e., cases "BM", "Ad", and "PL"). Hence, we focus on Case "APL" and scrutinize the subcases "Concede" and "Contest" to determine the optimal strategy for the platform.

***Platform's Strategy.*** On the question of whether the platform should concede or contest the ad slot, one may be inclined to think that the platform's best strategy is to contest for the ad slot and make its private label prominent, thus evincing self-preference. This is because the platform earns a higher margin from selling the private label than a third-party product. As such, the incremental value of placing the private label in the ad slot may be higher than that for a third-party product. Thus, one could argue that although conceding allows the platform to monetize the prominent slot, the ad revenue alone is unlikely to match the higher private label sales the platform could garner by contesting for the ad slot.

However, we find that the above intuition does not always hold. Broadly, when deciding on whether to concede or to contest, the platform faces a trade-off between a) conceding the ad slot to a third-party seller and increasing the total surplus and b) contesting for the ad slot, placing the private label in the top position, and selling more of the high-margin profit label. More specifically, the upside of conceding the ad slot to a third-party seller is that it increases the total surplus, implying $TS^{Concede} > TS^{Contest}$. The surplus is higher because when the platform concedes, the prominent third-party seller sets a price lower than the likely price of the private label in the ad slot, suggesting $p_z^{Concede} < p_l^{Contest}$. The platform charges a higher price for the private label because it collects commissions from third-party sellers and hence is not motivated to engage in intense price competition with them. The lower price of the prominent third-party seller expands the sales to the monopoly segment and raises the total surplus. Thus, by conceding the ad slot to a third-party seller instead of contesting it, the platform increases the total pie that can be shared among the third-party sellers, consumers, and the platform. This benefits the platform, all else being equal.

To help investigate the platform's optimal strategy, define $\alpha^*(\phi)$ as the threshold at which the platform is indifferent between conceding and contesting, implying $\Pi_P^{Concede} = \Pi_P^{Contest}$. Regarding the question of when the platform should concede or contest the ad slot, one may think that the platform should contest for the ad slot when ad effectiveness ($\alpha$) is large. This is because when sponsored advertising is more effective in raising demand, the platform should have a more significant incentive to promote its private label, which delivers a higher margin than a third-party product. However, on examining the equilibrium behavior of the platform, we obtain the following result.

**Proposition 1.** *a) It is not always optimal for the platform to give prominence to the private label by placing it in the ad slot. b) When product valuation is moderate and the commission is exogenously fixed, it is optimal for the platform to concede the ad slot to a third-party seller if $\alpha > \alpha^*(\phi)$, but contest for the ad slot and evince self-preference if $\alpha < \alpha^*(\phi)$. c) Furthermore, the threshold $\alpha^*(\phi)$ weakly decreases with $\phi$.*

Counter to our intuition, Proposition 1 shows that the platform optimally chooses to concede (rather than contest) the ad slot when $\alpha > \alpha^*(\phi)$. In this parameter space, conceding increases the total pie significantly because more consumers are influenced by ads (due to larger $\alpha$). Consequently, the platform strives to increase the total surplus rather than increasing private label sales, and thus concedes the ad slot to third-party sellers. However, if $\alpha < \alpha^*(\phi)$, it is optimal for the platform to contest for the ad slot, as it permits the platform to sell more of the high-margin private label.

We note that the cut-off value $\alpha^*(\phi)$ weakly decreases with $\phi$. In other words, as the commission rate $\phi$ increases, the platform chooses to concede in a larger parameter space. Intuitively, when the platform takes a large percentage of the third-party sales, it benefits more from conceding the ad slot and improving third-party sales.

The above discussion shows that a strategic platform does not always self-preference its private label in sponsored search advertising. This is because if the platform does so, it foregoes the opportunity to monetize the prominent ad slot and earns lower commissions from third-party sales. Taking into consideration the third-party sellers' strategic advertising decisions and pricing decisions, the platform chooses the strategy that maximizes the total revenue from all three sources: private label, commission from third-party sales, and advertising.

***Welfare implications.*** A frequent criticism leveled at e-commerce platforms is that they leverage their private labels to compete with the third-party sellers and furthermore give the private labels a favorable position.[7,8] The counterargument to this is that the private label provides more product varieties and is offered at a lower price, implying that consumers are better off because of the private label.[9,10] To examine whether the private label hurts the third-party sellers and consumers, we consider not only the price but also the placement of the private label (i.e., whether or not the private label is in a prominent ad slot). We have the following result.

**Proposition 2.** *If the platform gives prominence to the private label by placing it in the ad slot (instead of conceding the ad slot to third-party sellers), the platform hurts consumer welfare and total welfare. The impact on third-party sellers is asymmetric: while it hurts the profits of the third-party seller who wins under conceding, it improves the profits of the third-party seller who loses under conceding.*

Counter to the complaints of third party sellers, Proposition 2 shows that self-preferencing by the

[7]https://themarkup.org/amazons-advantage/2021/10/14/amazon-puts-its-own-brands-first-above-better-rated-products.

[8]https://www.yalelawjournal.org/note/amazons-antitrust-paradox.

[9]https://cei.org/blog/amazons-private labels-dont-threaten-competition/.

[10]https://www.nytimes.com/2018/06/23/business/amazon-the-brand-buster.html.

platform can be beneficial to some third-party sellers. Furthermore, in contrast to the platitudes of e-commerce platforms, self-preferencing by the platform can hurt consumers. The proposition highlights that in evaluating the effect of private label brands on consumers and third-party sellers, one needs to take into account both *prices* and *positions*.

We find that a lower-priced private label does not necessarily benefit consumers. It actually hurts consumers when the private label takes the ad slot (i.e., $CS^{Contest} < CS^{Concede}$). Notice that the platform charges a lower price for the private label (i.e., $p_l^{Contest} < p_i^{Contest}$) to take advantage of its prominent position and increase sales. This may raise the question of why consumer surplus is hurt if the private label takes the prominent position. To see this, note that if the platform allows third-party sellers an equal chance to compete for the ad position, the winning third-party seller will set an even lower price than the price of the private label in the ad slot (i.e., $p_z^{Contest} < p_l^{Contest}$), thus benefiting consumers even more.

Our analysis also shows that although the platform benefits from hosting the sponsored ad, consumer surplus can be the highest in the presence of the private label and absence of sponsored advertising (Case "PL"). To understand why sponsored advertising hurts consumer surplus, note that although the sponsored ad pushes the seller who wins the slot to set a lower price, it forces sellers who are not in the ad slot to charge higher prices. On average, therefore consumers may pay a higher price in the presence of an ad slot.

Another counter-intuitive finding is that if the platform takes the ad slot for its private label, its top position does not necessarily hurt third-party sellers. In fact, when the platform takes the ad slot, it has an asymmetric effect on third-party sellers: it benefits one third-party seller but hurts the other third-party seller. Note that in the counterfactual situation where the platform concedes the ad slot, third-party seller $z$ wins the auction while third-party seller $k$ loses the auction. In comparison to this counterfactual, when the platform takes the ad slot, seller $z$ earns lower profits while seller $k$ earns higher profits, implying $\pi_z^{Concede} > \pi_i^{Contest} > \pi_k^{Concede}$. The reason why the third-party seller $k$ earns higher profits is because it now competes with the higher-priced private label in the ad slot and loses fewer consumers to the private label. But since seller $z$ loses its prominent placement, if the platform contests, the seller earns lower profits. By considering both the price and position of the private label, our analysis offers insight into how self-preferencing in sponsored advertising affects the players in an ecommerce marketplace.

***Value of sponsored advertising and the private label brand for the platform.*** Clearly, the ad slot and the private label are two important strategic instruments at the disposal of the platform to improve its profits. It is an open question whether these two instruments function as complements or substitutes in improving the platform's profits. Furthermore, one may think that if the two instruments function as complements, it may be profitable for the platform to use the ad slot for the private label rather than concede it to the third-party sellers.

To probe these issues, we define sponsored advertising and the private label as complements in improving platform's profits if the incremental value of adding an ad slot is higher in the presence of a private label than in the absence of a private label (i.e., $\max[\Pi_p^{Concede}, \Pi_p^{Contest}] - \Pi_p^{PL} > \Pi_p^{Ad} - \Pi_p^{BK}$). That is, when the value of adding a private label is higher in the presence of an ad slot than in the absence of an ad slot (i.e., $\max[\Pi_p^{Concede}, \Pi_p^{Contest}] - \Pi_p^{Ad} > \Pi_p^{PL} - \Pi_p^{BK}$). On the other hand, we define the two instruments as substitutes if the incremental value of adding an ad slot is more in the absence of a private label ($\Pi_p^{Ad} - \Pi_p^{BK}$) than in the presence of a private label ($\max[\Pi_p^{Concede}.\Pi_p^{Contest}] - \Pi_p^{PL}$). This is equivalent to stating that the incremental value of adding a private label in the absence of an ad slot ($\Pi_p^{PL} - \Pi_p^{BK}$) is higher than in its presence ( $\max[\Pi_p^{Concede}, \Pi_p^{Contest}] - \Pi_p^{Ad}$).

Upon analyzing the role of the private label and ad slot in improving the platform's profits, we have the following finding.

**Proposition 3.** *When $\phi < \phi^*(\alpha)$, sponsored advertising and the private label function as substitutes. When $\phi > \phi^*(\alpha)$, sponsored advertising and the private label play a complementary role in improving the platform's revenue. Furthermore, $\phi^*(\alpha)$ first decreases then increases with $\alpha$.*

Proposition 3 and Figure 2 help us appreciate how sponsored advertising and the private label interact with each other depending on the value of $\alpha$ and $\phi$ and influence the platform's strategy. Note that that when $\phi > \phi^*(\alpha)$, the two instruments are complementary, implying $\Pi_p^{Ad} - \Pi_p^{BK} < \max[\Pi_p^{Concede}, \Pi_p^{Contest}] - \Pi_p^{PL}$. However, when $\phi < \phi^*(\alpha)$, the two instruments become substitutes and we have $\Pi_p^{Ad} - \Pi_p^{BK} > \max[\Pi_p^{Concede}, \Pi_p^{Contest}] - \Pi_p^{PL}$.

Some may naïvely expect sponsored advertising and private label as two independent sources of revenue for the platform. Some could view sponsored advertising and the private label as substitutes. This is because the private label competes with third-party sellers for a share of the market and, in turn, reduces the value of the ad slot for a third-party seller. However, our analysis shows that this intuition

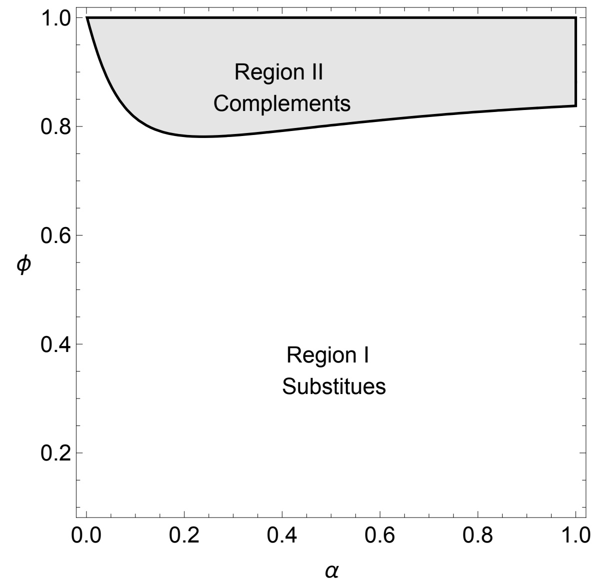


Figure 2: The value of sponsored advertising and the private label for the platform ($v = 1.5, N = 20$).

holds in Region I, not in Region II. In Region II, where $\phi$ is large and $\alpha$ is moderate, the ad slot and the private label function as complements. While the ad slot helps the platform extract some advertising revenue from third-party sellers, it increases price competition in the market and hurts the platform's commission revenue. In this context, the presence of a private label can soften the price competition induced by the ad slot. We find that when $\phi(\alpha)$ is large, the platform is not interested in cutting the price of the private label and increasing competition in the market because as it hurts the commissions from third-party sellers. Thus, in such situations the two instruments function as complements (rather than substitutes).

Next, turn attention to the effect of $\alpha$. When $\alpha$ is small, a winning third-party seller is not interested in cutting the price significantly. When $\alpha$ is large, even the private label in the ad slot is forced to cut prices deeply to gain more sales. In both cases, the existence of the private label has little effect on softening the price competition accentuated by the ad slot. Therefore, when $\alpha$ is small or large, the private label and the ad slot remain substitutes (as in Region I). Only when $\alpha$ is moderate, do they play a complementary role in improving the platform's revenue (as in Region II).

This analysis clarifies that private label and sponsored advertising are not two independent sources of revenue for the platform. It is possible for the platform to leverage one of them to gain more from the other. In particular, the tendency of sponsored advertising to intensify price competition can be counterbalanced by the ability of the private label to soften the competition among the third-party sellers. In such instances, the two instruments function as complements. But they function as substitutes otherwise.

## 5.2 Endogenous commission rate

In the preceding analysis, we assumed that the commission rate is exogenously fixed and identified the conditions when the private label and sponsored advertising function as substitutes and complements in improving the platform's profits. Now we proceed to examine the strategic implications of the platform choosing the commission rate. Assume that each third-party seller's outside option is $\pi_0 \geq 0$, implying that a third-party seller participates in the marketplace only if its expected profits from selling on the platform are not less than the outside option. The seller's outside option can be viewed as the opportunity cost of selling on the platform. This parsimonious set up permits us to focus on the platform's decisions regarding the private label and the ad slot and abstract away from competition between marketplaces.

Clearly, when the commission rate increases, the third-party sellers' profits decrease whereas the platform's total profits increase. Therefore, in equilibrium, the platform sets the commission rate at the highest possible level that ensures both third-party sellers participate in the platform. In subcase "Concede", the commission rate should be just enough to ensure that the losing seller $k$ participates (and the winning seller $z$ earns a positive surplus). Under subcase "Contest", as the two third-party sellers expect the same profits, the commission rate is just enough to ensure both sellers participate. The equilibrium commission rates under subcases "Concede" and "Contest" respectively, are as follows:

$$\phi^{Concede} = 1 - \frac{\pi_0}{\pi_k^{Concede}(\phi = 0)}, \; \phi^{Contest} = 1 - \frac{\pi_0}{\pi_i^{Contest}(\phi = 0)}. \tag{2}$$

In the above expression, $\pi_0$ is the third-party seller's outside option profits. $\pi_k^{Concede}(\phi = 0)$ is the losing seller $k$'s profits if the commission rate is zero under subcase "Concede". $\pi_i^{Contest}(\phi = 0)$ is each third-party seller's profits if the commission rate is zero under subcase "Contest". We further find that $\pi_k^{Concede}(\phi = 0) = \pi_k^{Contest}(\phi = 0)$, implying the optimal commission rate is the same in the two subcases. Intuitively, when $\phi = 0$, the platform does not earn any profits from third-party sales, and hence would compete on price just as if it is another third-party seller. Therefore, contesting and conceding yield the same profits for the losing third-party seller(s).

Given that the endogenous commission rate $\phi^*$ is a function of the seller's outside option $\pi_0$, we can derive the threshold $\alpha^*(\pi_0)$ that makes the platform indifferent between conceding the ad slot and competing for it.

***Platform's strategy.*** We know from Proposition 1 that the platform will concede the ad slot if $\alpha > \alpha^*(\phi)$. Moreover, the platform concedes the ad slot when the exogenous commission rate is high. Upon examining the platform's optimal strategy when the commission rate is endogenous, we have the following result.

**Proposition 4.** *If product valuation is moderate and if the platform chooses the commission rate, the platform's optimal strategy is to concede the ad slot to a third-party seller if $\alpha > \alpha^*(\pi_0)$ but to contest for the ad slot if $\alpha < \alpha^*(\pi_0)$. Furthermore, the cut-off value $\alpha^*(\pi_0)$ weakly increases with $\pi_0$.*

Consistent with Proposition 1, the platform concedes the ad slot when $\alpha$ is above a threshold. But now threshold $\alpha^*$ is a function of $\pi_0$ rather than $\phi$. Recall that when the commission rate is exogenous, the platform concedes the ad slot to a third-party seller when $\phi$ is large. But now the platform chooses to concede when $\pi_0$ is small. This observation is counter-intuitive because one may expect that the platform would choose to concede to please third-party sellers when they have an attractive outside option. The intuition for the result is as follows. The optimal commission rate $\phi$ decreases with the seller's outside option $\pi_0$ (see Equation (2)). Thus, as the seller's outside option increases, the maximum commission that the platform can charge the third-party sellers, and yet ensure they participate in the marketplace decreases. Consequently, the platform obtains a small fraction of third-party sales as commission. This reduced incentive for conceding the ad slot motivates the platform to contest for the ad slot and increase private label sales.

***Welfare Implications.*** Broadly, the welfare implications are similar to those observed when the commission rate is exogenous (See Proposition 2). We have the following result.

**Proposition 5.** *When the platform chooses the commission rate, if the platform gives prominence to the private label by placing it in the ad slot (instead of conceding the ad slot to third-party sellers), the platform hurts consumer welfare and total welfare. The impact on third-party sellers is asymmetric: while it hurts the profits of the third-party seller who wins under conceding, it does not affect the profits of the third-party seller who loses under conceding.*

When the platform optimally chooses to concede instead of contesting for the ad slot, it improves consumer surplus and total surplus. But if the platform decides to contest rather than concede the ad slot, it hurts consumers and reduces total surplus. Both the third-party sellers are assured of their outside option profits ($\pi_0$) if the platform contests for the ad slot. But if the platform concedes the ad slot, the losing seller $k$ is assured of the outside option profits, whereas the winning seller $z$ earns a positive surplus beyond the outside option profits. Therefore, contesting hurts the winning seller $z$ but does not affect the losing seller $k$.

Next, we proceed to examine the incremental value of the ad slot and the private label for the platform when the commission rate is endogenously decided.

***Value of sponsored advertising and the private label brand for the platform.*** Recall that when the commission rate is exogenous, sponsored advertising and the private label can be either substitutes or complements depending on the parameter space (see Proposition 3). When the commission rate is endogenous we obtain a different result.

**Corollary 1.** *When the platform chooses the commission rate, sponsored advertising and the private label play a complementary role in improving the platform's revenue.*

In principle, the platform can strategically use both the ad slot and the private label brand to lower price, offer more varieties to consumers, and thereby improve total welfare. If the exogenous commission rate is low, sponsored ads and private labels can be substitutes in improving the platform's profits. However, if the platform chooses the commission rate, it can optimize $\phi$ to get the most out of sponsored advertising and the private label by making them complementary.

## 6 Model Extension

In the preceding analysis, we focused on products of moderate valuation. Recall that the price sensitivity of consumers in the monopoly segment is more than that of those in the competitive segment. We identified the conditions when the platform will choose to concede the ad slot to third-party sellers (see Proposition 1). We also discussed when the ad slot and private label could work as complements to improve the platform's profits (Propositions 3 and 4). Now we turn our attention toward products of high valuation ($v \geq p^* + 1$) and examine whether the earlier results will hold when valuation is high. The demand for high-valuation products is qualitatively different from the pattern we observed for moderate-valuation products in the previous section. Now consumers in the monopoly segment are less price-sensitive than those in the competitive segment. Moreover the platform's optimal strategy is also different.

***Pricing equilibrium.*** To ensure a pure strategy pricing equilibrium, we focus on the parameter space where the prominence of the ad slot $\alpha$ is not too large and the commission rate $\phi$ is significant. Specifically,

$$\alpha < \min\{\frac{-2N+v+2}{N^2-Nv-3N+2v+2}, \frac{2N-2v+1}{N-v-1}\},$$
$$\phi > \max\{\frac{-12\alpha-\alpha N^2+7\alpha N+2\alpha Nv-6N-6\alpha v+6v+12}{12\alpha-4\alpha N+2\alpha Nv-6\alpha v+6v-12}, \frac{2\alpha-\alpha N^2+\alpha N+2\alpha Nv-2N-4\alpha v+2v}{8\alpha-4\alpha N+2\alpha Nv-4\alpha v+2v-4},$$
$$N+\frac{1}{2} < v < N+2, \tag{3}$$

Following the same logic as in §4 we derive the demand and the equilibrium prices for each of the four cases. Below, we summarize the price corresponding to each of the cases, relegating the details of the analysis to Appendix OA2.

- Case "BM." We find that $p_i^{BM} = v-1$, implying that both third-party sellers charge the monopoly price. Consequently, all the consumers in a third-party seller's monopoly segment purchase from it whereas those in the competitive segment purchase from the seller closer to them.

- Case "Ad." In this case, the equilibrium price is $p_z^{Ad} = p_k^{Ad} = v-1$. Thus, both the winning and the losing third-party sellers charge the monopoly price $v-1$. As the sellers' pricing strategies are the same as in Case "BM", the presence of the ad slot does not change a third-party seller's price in the absence of a private label (although the ad slot is still valuable to sellers as it increases their demand). This finding is qualitatively different from the result when product valuation is moderate. When valuation is moderate, the winning third-party seller charges a lower price compared to the losing seller. The reason for the difference in the results is that when product valuation is high (i.e., $v^* \geq p+1$), consumers in the monopoly segment are less price-sensitive than those in the competitive segment. Specifically, the demand of third-party sellers $z$ and $k$ are as follows:

$$\begin{cases} D_z = \underbrace{\frac{1}{N}\frac{1}{N-1}(1+p_k-p_z)}_{\text{Competitive segment: } Q_{ao}^1(p_z,p_k)} + \underbrace{(N-2)\Big[\frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\Big(\alpha+(1-\alpha)\frac{1}{N-1}\Big)\Big]}_{\text{Monopoly segment: } \overline{Q_m^2}}, \\ D_k = \underbrace{\frac{1}{N}\Big(\alpha+(1-\alpha)\frac{1}{N-1}\Big)+\frac{1}{N}\frac{1}{N-1}(p_z-p_k)}_{\text{Competitive segment: } q_{ao}^1(p_z,p_k)} + \underbrace{(N-2)(1-\alpha)\ 2\frac{1}{N}\frac{1}{N-1}}_{\text{Monopoly segment: } \overline{q_m}}. \end{cases}$$

It is easy to see that when product valuation is high ($v^* > p + 1$), all consumers in the monopoly segment buy a product and they are price-insensitive. Furthermore, the third-party seller $z$ has a larger monopoly segment compared to that of seller $k$, implying $\overline{Q_m^2} > \overline{q_m}$. As they compete for a share of the same pool of consumers in the competitive segment who are price sensitive (with a size of $\frac{1}{N}\frac{1}{N-1}$), the third-party seller in the ad slot sells to relatively more price-insensitive consumers and is less motivated to cut price. In equilibrium, both the third-party sellers set the price at $v-1$ to capture all consumers in their respective monopoly segments.

- <u>Case "PL."</u> The private label is priced higher than the price of a third-party seller, and we have $p_i^{PL} = v-2, p_l^{PL} = v-1$. This is because the platform has less incentive to compete on price compared to the third-party sellers. The private label brand charges the monopoly price of $v-1$ and only sells to consumers in its monopoly segment whereas a third-party seller charges a price as low as $v-2$ so that it can sell to consumers in both its monopoly segment and the competitive segment (for whom the other preferred seller is the other third-party seller). In contrast to the benchmark Case "BM", we have an additional product in the market (i.e., the private label) which makes the market more competitive if product valuation is high. This impels the third-party sellers to reduce their prices from $p_i^{BM} = v-1$ to $p_i^{PL} = v-2$.

- <u>Case "APL."</u> Recall that in this case, the platform can potentially leverage both the private label and ad slot to improve profits. Specifically, the platform could concede the ad slot or contest for the ad slot and make the private label prominent.

  - *Subcase "Platforms concede."* In this subcase, all the sellers choose the same prices as in Case "PL", implying $p_z^{Concede} = p_k^{Concede} = v-2, p_l^{Concede} = v-1$. The existence of an ad slot does not change a third-party seller's price even in the presence of a private label brand.

  - *Subcase "Platforms contest."* Here we observe prices as in subcase "Concede" and subcase "PL": $p_i^{Contest} = v-2, p_l^{Contest} = v-1$. Thus, whether the platform concedes the ad slot or contests for the ad slot, or whether there is an ad slot does not affect third-party sellers' prices. We see two forces impinging on the private label's price, and both of them act in the same way: Being a private label forces the price to be higher than a third-party seller's price, and occupying the top position also impels the price to be higher. Given that the private label price is already as high as the monopoly price $v-1$, whether the platform concedes or contests for the ad slot does not change its price.

In sum, in the absence of a private label, the two third-party sellers charge the monopoly price of $v-1$, and capture all the demand from the monopoly segment. In the presence of a private label brand, the private label charges the monopoly price $v-1$, but the third-party sellers lower their prices to $v-2$ so that they can compete for the consumers in the competitive segment (whose other preferred brand is the private label). Furthermore, the presence of an ad slot and the auction outcome have no bearing on sellers' prices.

**Platform's strategy**

Recall that when product valuation is moderate, the platform may sometimes find it profitable to concede the ad slot to soften the price competition between third-party sellers and increase the total surplus (see Proposition 1). Focusing on high valuation products (see Condition 3) and then comparing the platform's total revenue under the different cases, we obtain the following result.

**Proposition 6.** *If product valuation is high, the platform's optimal strategy is to contest for the ad slot and place the private label in the top position. This holds true irrespective of whether the commission rate $\phi$ is exogenously given or endogenously determined.*

Proposition 6 suggests that though it can be optimal for the platform to concede the ad slot to third-party sellers when product valuation is moderate, it is sub-optimal to do so when product valuation is high. If product valuation is moderate, the platform can concede the ad slot and induce the prominent third-party seller to cut prices, expand the sales to consumers in the monopoly segment, and increase total surplus. However, this opportunity vanishes when product valuation is high because the consumers in the monopoly segment are price-insensitive and purchase a product anyway.

On examining the welfare implications, we have the following result.

**Corollary 2.** *If product valuation is high and commission rate is exogenous, the platform contests for the ad slot and places the private label in the top position in equilibrium. Such self-preferencing hurts consumers (by $\frac{\alpha(N-3)(2N-1)}{2(N-1)N}$), benefits sellers (by $\frac{\alpha(v-2)(1-\phi)}{(N-1)N}$) and improves total surplus (by $\frac{\alpha(N-3)}{2(N-1)N}$) compared to conceding the ad slot to third-party sellers.*

If the platform concedes the ad slot, it lowers total surplus by $TS^{Concede} - TS^{Contest} = -\frac{\alpha(N-3)}{2(N-1)N}$. We observe this because conceding the ad slot worsens consumer match. Specifically, when the platform concedes, the lower-priced third-party seller $z$ induces consumers whose first-preferred variety is the private label to consume their second-preferred variety $z$ which is farther away from them. Finally, when product valuation is high, it is beneficial for third-party sellers if the platform contests for the ad slot. This because if the platform concedes the ad slot, the third-party sellers compete so intensely in the ad auction that they end up in a prisoner's dilemma. Thus if the platform contests for the ad slot and places the private label in the top position, it helps third-party sellers to avoid a prisoner's dilemma.

We obtain a qualitatively similar result if the commission rate is endogenous except that sellers get their outside option profits irrespective of whether they contest or concede. In particular, when the commission rate is endogenously decided, if the platform contests, it is better off, consumers are worse off, and the total surplus increases. However, third-party sellers obtain the same profits as when the platform concedes the ad slot.

## 7 Conclusions

Private label sales are growing in significance in e-commerce marketplaces. Third-party sellers are complaining that self preferencing the private label in sponsored advertising by the platform hurts third-party sales. Platforms argue that the private label is priced lower than comparable products of third-party sellers and that the private label provides consumers more variety, implying consumers are benefiting from the private label. Given the multiple sides of an e-commerce marketplace and the variety of strategic players operating in the market, it is difficult to make causal inferences regarding such issues by examining the data from one third-party seller or consumer. It is even more difficult to provide guidelines to managers or policy makers. In this paper, we take a step toward theoretically examining how sponsored advertising and private label influence the behavior of the platform, third-party sellers, and consumers. Our analysis provides useful insights on several questions of significance to managers and regulators.

- *Does the platform always benefit from self-preferencing the private label in sponsored advertising?* The answer is no. Because the platform earns a higher margin from private label sales than the sales commission from third-party sellers, one may naturally think it would be beneficial for the platform to place the private label in the prominent slot. This line of reasoning is not always valid because it does not consider the fact that conceding the ad slot to third-party sellers can increase the total surplus and increase the total pie. Our analysis shows that it is indeed optimal for the platform to concede the ad slot to third-party sellers if ad effectiveness is above a threshold ($\alpha > \alpha^*(\phi)$, see Proposition 1; $\alpha > \alpha^*(\pi_0)$, see Proposition 4)

- *Does self-preferencing the private label in sponsored advertising always hurt the third-party sellers?* One may conjecture that placing the private label in the prominent ad slot will hurt the profits of third-party sellers. This is because of the apprehension that the gain of the private label could be at the expense of third-party sellers. We find that self-preferencing the private label in sponsored advertising helps one third-party seller but hurts the other. To follow the rationale, note that in the counterfactual situation where the platform concedes the ad slot, the winning third-party seller cuts prices deeply and captures significant demand from the losing third-party seller. This losing third-party seller stands to benefit if the platform contests and takes the prominent ad slot. This is because when the platform contests for the ad slot, the private label is less motivated to cut price, increase price competition in the marketplace, and lose commissions on third-party sales. Consequently, the price of the private label in the ad slot is higher, and the third-party seller loses fewer consumers to the private label. This in turn improves the third-party seller's profits (see Proposition 2)

- *Is self-preferencing the private label in sponsored advertising beneficial to consumers?* Because the private label is priced lower and because it increases the variety of products available in the market, we may believe that self-preferencing the private label will be beneficial to consumers. On the contrary, we find that such conduct on the part of the platform reduces consumer surplus. To see this, note that if a third-party seller wins the sponsored slot, its price will be even lower than the price of the private label in the sponsored ad slot, thus raising consumer surplus (see Propositions 2 and 5)

- *Is it profitable for the platform to contest for the ad slot when sponsored advertising significantly adds to the prominence of the private label?* The answer is no. When the platform contests for the ad slot and places the private label in the top position, it increases the sales of the private label. But the platform loses potential advertising revenue from third-party sellers and commissions from third-party sales. Our analysis shows that if sponsored advertising makes the private label highly prominent, the platform chooses to concede rather than contest. We observe this because the loss of commission and advertising revenue is larger than the gain in private label sales if the platform contests for the ad slot instead of conceding it (see Proposition 1).

- *Are the private label and sponsored advertising two independent sources of revenue for the platform?* They are not two independent sources of revenue. Since the product in the ad slot is priced lower to take advantage of its prominence, sponsored advertising increases price competition in the market. By placing the private label in the ad slot, the platform can strategically soften price competition in the market. Note that because the platform earns commissions from third-party sales, it is less inclined than any of the third-party sellers to increase price competition in the market. Consequently, when the commission rate is high, ad effectiveness is moderate and product valuation is moderate, we see a complementary relationship between these two sources of profits of the platform. Otherwise, the two strategic instruments function as substitutes in improving the platform's profits (see Proposition 3 and Corollary 2) .

- *Is the platform's strategy contingent on product valuation?* The answer is yes. It is optimal for the platform to concede the ad slot only when product valuation is moderate and the sellers can improve market coverage by reducing price. If product valuation is high, there is no opportunity to improve coverage by reducing price. Moreover, if the platform concedes the ad slot to a third-party seller, then some consumers may buy the lower priced product featured in the ad slot despite it not being their first-preferred product, thus reducing total surplus. Hence, the platform is better off contesting for the ad slot if product valuation is high (see Proposition 6).

**Limitations and directions for further research.** Our analysis takes an initial step in examining how the platform can self-preference its private label in sponsored advertising. There are other avenues by which the platform can give a preferential treatment to its private label. For example, Amazon can leverage data gathered on third-party sales to inform its product launch decisions (e.g., Jiang et al. 2011). It is conceivable that there is uncertainty about the size of new product markets, and third-party sellers are willing to enter these markets even if the expected profits are low. The platform can use third-party sales data to inform its beliefs about the market and accordingly time its entry decision (e.g., Long et al. 2021). The platform can also potentially manipulate the organic search results to favor its private label (e.g., Zennyo 2021). These issues await further scrutiny. Next, in developing our model, we assumed that the products are horizontally differentiated. It would be useful to examine the implications of self-preferencing the private label in a vertically differentiated market (e.g., Amaldoss and Shin 2015). Moreover, in this initial analysis, we examined a monopoly platform. Future research can seek to develop a tractable formulation of competing platforms, and examine the impact of self-preferencing the private label in a competitive market (e.g., Etro 2021a). Finally, it would be useful to confront the predictions of our model with data.

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# Appendix

## A1 Proof for the Demand Model in Lemma 1

**Demand of seller $z$ in the top slot**

- The consumer's first-preferred product and second-preferred product are both available. This entails two scenarios:

  – Seller $z$ is the consumer's first-preferred product and seller $k$ in one of the non-top slots is the consumer's second-preferred brand with a probability of $\frac{1}{N}\frac{1}{N-1}$, where $k \in \{1...n\}, k \neq z$. Such consumers are uniformly distributed on $l_z \sim [0, \frac{1}{2}]$. If $p_z > p_k$, consumers buy from seller $z$ when $l_z \leq \frac{1+p_k-p_z}{2}$; if $p_z < p_k$, all consumers on $l_z$ buy from seller $z$. The demand of seller $z$ is

$$\sum_{k\in\{1...n\},k\neq z} \frac{1}{N}\frac{1}{N-1} \begin{cases} 0, & \text{if } p_z - p_k > 1, \\ \frac{\frac{1+p_k-p_z}{2}}{\frac{1}{2}} = 1 + p_k - p_z, & \text{if } 0 < p_z - p_k \leq 1, \\ 1, & \text{if } p_z - p_k \leq 0. \end{cases}$$

  – The consumer's first-preferred product $k$ is available in one of the non-top slots and seller $z$ is the consumer's second-preferred product. This happens with a probability of $\frac{1}{N}\Big(\alpha + (1-\alpha)\frac{1}{N-1}\Big)$. Such consumers are uniformly distributed on $l_k \sim [0, \frac{1}{2}]$ and they choose product $z$ if $l_k + p_k > 1 - l_k + p_z$, i.e., $l_k > \frac{1+p_z-p_k}{2}$. Seller $z$'s demand is

$$\sum_{k\in\{1...n\},k\neq z} \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) \begin{cases} 0, & \text{if } p_z - p_k > 0, \\ \frac{\frac{1}{2}-\frac{1+p_z-p_k}{2}}{\frac{1}{2}} = p_k - p_z, & \text{if } -1 < p_z - p_k \leq 0, \\ 1, & \text{if } p_z - p_k \leq -1. \end{cases}$$

  Combining the above two scenarios, seller $z$'s demand when both the consumer's first-preferred and second-preferred products are available is

$$\begin{cases} 0, & \text{if } p_z - p_k > 1, \\ \sum_{k\in\{1\ldots n\},k\neq z} Q^1_{ao}(p_z,p_k), \text{ where } Q^1_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1}\left(1+p_k-p_z\right), & \text{if } 0 < p_z - p_k \leq 1, \\ \sum_{k\in\{1\ldots n\},k\neq z} Q^2_{ao}(p_z,p_k), \text{ where } Q^2_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)(p_k-p_z), & \text{if } -1 < p_z - p_k \leq 0, \\ \sum_{k\in\{1\ldots n\},k\neq z} \overline{Q^2_{ao}}, \text{ where } \overline{Q^2_{ao}} \equiv \left(\frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\right), & \text{if } p_z - p_k \leq -1. \end{cases}$$

- Only one of the consumer's two preferred products is available. This again breaks down into two scenarios.

  - Seller $z$ is the first-preferred brand (with a probability of $\frac{1}{N}$) and the consumer's second-preferred brand is not available (with a probability of $\frac{N-n}{N-1}$). Such consumers are uniformly distributed on $l_z \sim [0, \frac{1}{2}]$ and they choose product $z$ if $l_z + p_z < v$, i.e., $l_z < v - p_z$. Hence seller $z$'s demand is

$$\frac{1}{N}\frac{N-n}{N-1}\begin{cases} \frac{v-p_z}{\frac{1}{2}} = 2(v-p_z), & \text{if } v - p_z \leq \frac{1}{2}, \\ 1, & \text{if } v - p_z > \frac{1}{2}. \end{cases}$$

  - The consumer's first-preferred brand is not available (with a probability of $\frac{N-n}{N}$) and seller $z$ is the second-preferred brand (with a probability of $\alpha + (1-\alpha)\frac{1}{N-1}$). Denote the consumer's first-preferred brand as $j$. Such consumers are uniformly distributed on $l_j \sim [0, \frac{1}{2}]$ and they choose product $z$ if $1 - l_j + p_z < v$, i.e., $l_j > 1 + p_z - v$. Hence seller $z$'s demand is

$$\frac{N-n}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\begin{cases} 0, & \text{if } v - p_z \leq \frac{1}{2}, \\ \frac{\frac{1}{2}-(1+p_z-v)}{\frac{1}{2}} = 2(v-p_z) - 1, & \text{if } \frac{1}{2} < v - p_z \leq 1, \\ 1, & \text{if } v - p_z > 1. \end{cases}$$

  Put together, seller $z$'s demand when only one of the consumer's preferred brands is available comes as

$$\begin{cases} (N-n)Q_m^1(p_z), \text{ where } Q_m^1(p_z) \equiv \frac{2}{N}\frac{1}{N-1}(v-p_z), & \text{if } v-p_z \leq \frac{1}{2}, \\ (N-n)Q_m^2(p_z), \text{ where } Q_m^2(p_z) \equiv \frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\big(2(v-p_z)-1\big), & \text{if } \frac{1}{2} < v-p_z \leq 1, \\ (N-n)\overline{Q_m^2}, \text{ where } \overline{Q_m^2} \equiv \frac{1}{N}\frac{1}{N-1} + \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right), & \text{if } v-p_z > 1. \end{cases}$$

Finally, the number of clicks on seller $z$ in the prominent slot (denoted by $c$) is equal to the probability that seller $z$ is in the consumer's consideration set.

$$c = \sum_{k\in\{1\ldots n\},k\neq z} \frac{1}{N}\frac{1}{N-1} + \sum_{k\in\{1\ldots n\},k\neq z} \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) + \frac{1}{N}\frac{N-n}{N-1} + \frac{N-n}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right). \tag{A1}$$

**Demand of seller $k$ in a non-top slot**

- The consumer's first-preferred product and second-preferred product are both available. There are three possibilities here.
  - Seller $z$ is the consumer's first-preferred product and seller $k$ is the consumer's second-preferred product. This is the mirror of Case 1 above. Such consumers are uniformly distributed on $l_z \sim [0, \frac{1}{2}]$. If $p_z > p_k$, consumers buy from $k$ if and only if $l_z > \frac{1+p_k-p_z}{2}$; if $p_z < p_k$, no one buys from $k$. Seller $k$'s demand is

$$\frac{1}{N}\frac{1}{N-1}\begin{cases} 1, & \text{if } p_z - p_k > 1, \\ \frac{\frac{1}{2}-\frac{1+p_k-p_z}{2}}{\frac{1}{2}} = p_z - p_k, & \text{if } 0 < p_z - p_k \leq 1, \\ 0, & \text{if } p_z - p_k \leq 0. \end{cases}$$

  - The consumer's first-preferred product is $k$ and her second-preferred product is $z$. This is the mirror of Case 2 above. Such consumers are uniformly distributed on $l_k \sim [0, \frac{1}{2}]$ and they choose product $k$ if $l_k \leq \frac{1+p_z-p_k}{2}$. Seller $k$'s demand is

$$\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\begin{cases}1, & \text{if } p_z-p_k>0,\\ \frac{\frac{1+p_z-p_k}{2}}{\frac{1}{2}}=1+p_z-p_k, & \text{if } -1<p_z-p_k\leq 0,\\ 0, & \text{if } p_z-p_k\leq -1.\end{cases}$$

Combining the above two cases, seller $k$'s demand when both the consumer's preferred products are available and one of them is seller $k$ is

$$\begin{cases}\overline{q^1_{ao}} \equiv \frac{1}{N}\frac{1}{N-1}+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right), & \text{if } p_z-p_k>1,\\ q^1_{ao}(p_z,p_k) \equiv \frac{1}{N}\frac{1}{N-1}(p_z-p_k)+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right), & \text{if } 0<p_z-p_k\leq 1,\\ q^2_{ao}(p_z,p_k) \equiv \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)(1+p_z-p_k), & \text{if } -1<p_z-p_k\leq 0,\\ 0, & \text{if } p_z-p_k\leq -1.\end{cases}$$

- Both seller $k$ and $j$ in the non-top slots are in the consumer's consideration set with a probability $(1-\alpha)\frac{2}{N(N-1)}$. Such consumers are uniformly distributed on $l_k \sim [0,1]$, and they choose $l_k$ if $l_k \leq \frac{1+p_j-p_k}{2}$. Seller $k$'s demand in this scenario is $\sum_{j\in\{1\ldots n\},j\neq z,j\neq k}(1-\alpha)2\frac{1}{N}\frac{1}{N-1}\ \frac{1+p_j-p_k}{2}$, which is equal to

$$\sum_{j\in\{1\ldots n\},j\neq z,j\neq k}\begin{cases}\overline{q_{oo}},\ \text{where } \overline{q_{oo}} \equiv (1-\alpha)\frac{1}{N}\frac{2}{N-1}, & \text{if } p_j-p_k>1,\\ q_{oo}(p_j,p_k),\ \text{where } q_{oo}(p_j,p_k)\equiv(1-\alpha)\frac{1}{N}\frac{1}{N-1}(1+p_j-p_k), & \text{if } -1<p_j-p_k\leq 1,\\ 0, & \text{if } p_j-p_k\leq -1.\end{cases}$$

- Only one of the consumer's preferred products is available.
  - Seller $k$ is in the consumer's consideration set but the consumer's other preferred brand is not available (with a probability of $(1-\alpha)2\frac{1}{N}\frac{N-n}{N-1}$). Such consumers are uniformly distributed on $l_k \sim [0,1]$ and the cut-off consumer's location is $p_k+l_k=v$, i.e., $l_k=v-p_k$. Seller $k$'s

demand is

$$\begin{cases} (N-n)q_m(p_k), \text{ where } q_m(p_k) \equiv (1-\alpha)\ 2\frac{1}{N}\frac{1}{N-1}(v-p_k), & \text{if } v-p_k \leq 1, \\ (N-n)\overline{q_m}, \text{ where } \overline{q_m} \equiv (1-\alpha)\ 2\frac{1}{N}\frac{N-1}{N-1}, & \text{if } v-p_k > 1. \end{cases}$$

# A2 Appendix for the Case with a Moderate Product Valuation

In this appendix, we solve the equilibrium prices for a moderate product valuation. To simply notations, in this appendix, we use $p^*$ interchangeably when analyzing each case (whereas, in the main text, we represent the equilibrium solution with a superscript representing each case). In each case, we first solve the equilibrium price assuming that $v-1 < p^* < v-\frac{1}{2}$. We then find the parameter space where the solution satisfies $v-1 < p^* < v-\frac{1}{2}$. Finally, we prove that given other sellers' prices at the equilibrium level, a seller has no incentive to deviate to $p < v-1$ or $p > v-\frac{1}{2}$.

## A2.1 Solution for Case "BM"

Assume that the other third-party seller prices at the equilibrium price level $v-1 < p^* < v-\frac{1}{2}$. First, if seller $i$ deviates to charge a price $p_i > p^*$, based on Lemma 1, its demand is

$$\begin{aligned} D_i(p_i, p^*) &= \frac{1}{2}\Big(Q^1_{ao}(p_i, p^*) + (N-2)Q^2_m(p_i)\Big) + \frac{1}{2}\Big(q^2_{ao}(p^*, p_i) + (N-2)q_m(p_i)\Big), \\ &= \frac{-\alpha(N-2)(2Np_i - 2Nv + N - 5p_i - p^* + 6v - 3) + (6-4N)p_i + 4(N-2)v + 2p^* + 2}{2(N-1)N}. \end{aligned}$$

Following a similar logic, if seller $i$ deviates to charge a lower price $p_i < p^*$, its demand is

$$\begin{aligned} D_i(p_i, p^*) &= \frac{1}{2}\Big(Q^2_{ao}(p_i, p^*) + (N-2)Q^2_m(p_i)\Big) + \frac{1}{2}\Big(q^1_{ao}(p^*, p_i) + (N-2)q_m(p_i)\Big), \\ &= \frac{-\alpha(N-2)(2Np_i - 2Nv + N - 5p_i - p^* + 6v - 3) + (6-4N)p_i + 4(N-2)v + 2p^* + 2}{2(N-1)N}. \end{aligned}$$

We can observe that seller $i$'s demand function when deviating to price higher is the same as that when deviating to price lower. Intuitively, a seller's demand is a continuous function of its price when averaging across all the possibilities of obtaining a top slot and a non-top slot, so there is no discrete jump in a seller's demand as a function of the price. We summarize this insight, which we will refer to in following analyses, in the following result.

**Result 1.** *Keeping the prices of other sellers fixed, a seller's demand function is continuous in its price when the platform ranks sellers randomly.*

For any $p_i$, seller $i$'s sales revenue is $(1-\phi)p_i D_i(p_i, p^*)$, where $\phi$ is the commission rate. To make sure that seller $i$ has no incentive to deviate from $p^*$, we need

$$\frac{\partial((1-\phi)p_i D_i(p_i, p^*))}{\partial p_i}\bigg|_{p_i=p^*} = 0.$$

This leads to

$$p_i^* = p_i^{BM} \equiv \frac{\alpha(N-3)(N-2)(2v-1)+4(N-2)v+2}{22\alpha+N(\alpha(4N-19)+8)-14}.$$

The above result satisfies the condition $v-1 < p_i^* < v-\frac{1}{2}$ when $\underline{v_1}^{BM} < v < \overline{v}^{BM}$, where $\underline{v_1}^{BM} = \frac{10\alpha+2\alpha N^2-9\alpha N+8N-10}{20\alpha+4\alpha N^2-18\alpha N+8N-12}$, $\overline{v}^{BM} = \frac{16\alpha+3\alpha N^2-14\alpha N+8N-12}{10\alpha+2\alpha N^2-9\alpha N+4N-6}$. Notice that $\overline{v}^{BM} < 2$. Therefore, the parameter space specified in Condition 1 is a subset of $1 < v < 2$.

As a final step, we need to find the condition under which seller $i$ has no incentive to deviate to $p_i < v-1$ or $p_i > v-\frac{1}{2}$ given the other sellers price at $p_i^*$. Online Appendix OA1.1 shows that $p_i^*$ is an equilibrium when $\underline{v_1}^{BM} < v < \overline{v}^{BM}$ and $v > \underline{v_2}^{BM}$. Since $\underline{v_2}^{BM} > \underline{v_1}^{BM}$, this simplifies into $\underline{v}^{BM} < v < \overline{v}^{BM}$, where $\underline{v}^{BM} = \underline{v_2}^{BM}$.

### A2.2 Solution for Case "Ad"

Consider the possibility where $p_z^* < p_k^*$. We will show that this is the only possible equilibrium under the parameter space we consider. First, consider seller $z$'s pricing decision. Expecting that seller $k$ prices at $p_k^*$ and loses the ad auction by bidding $b_k^*$, when seller $z$ deviates to price at $p_z$, its demand is $D_z^{win}(p_z, p_k^*) = Q_{ao}^2(p_z, p_k^*) + (N-2)Q_m^2(p_z)$. Seller $z$'s net profit accounting for the advertising cost is

$$\pi_z(p_z, p_k^*) = (1-\phi)p_z D_z^{win}(p_z, p_k^*) - b_k^*.$$

Note that seller $z$'s advertising cost is equal to seller $k$'s bid $b_k^*$ and does not depend on seller $z$'s price. Furthermore, based on envy-free equilibrium, seller $z$'s bid is equal to

$$b_z(p_z, p_k^*) = (1-\phi)p_z\Big(D_z^{win}(p_z, p_k^*) - D_z^{lose}(p_z, p_k^*)\Big), \tag{A2}$$

where $D_z^{lose}(p_z, p_k^*) = q_{ao}^1(p_k^*, p_z) + (N-2)q_m(p_z)$ represents seller $z$'s demand upon losing the ad auction given that seller $k$ prices at $p_k^*$.

Similarly, expecting that seller $z$ prices at $p_z^*$ and wins the ad auction by bidding $b_z^*$, when seller $k$ deviates to price at $p_k$, its demand is $D_k^{lose}(p_k, p_z^*) = q_{ao}^2(p_z^*, p_k) + (N-2)q_m(p_k)$. Its net profit is

$$\pi_k(p_k, p_z^*) = (1-\phi)p_k D_k^{lose}(p_k, p_z^*).$$

According from the envy-free equilibrium, seller $k$'s bid is equal to

$$b_k(p_k, p_z^*) = (1-\phi)p_k\Big(D_k^{win}(p_k, p_z^*) - D_k^{lose}(p_k, p_z^*)\Big). \tag{A3}$$

In the above expression, where $D_k^{win}(p_k, p_z^*) = Q_{ao}^1(p_k, p_z^*) + (N-2)Q_m^2(p_k)$ represents seller $k$'s demand upon winning the ad auction given that seller $z$ prices at $p_z^*$.

From $\frac{\partial \pi_z(p_z, p_k^*)}{p_z}\big|_{p_z = p_z^*} = 0$ and $\frac{\partial \pi_k(p_k, p_z^*)}{p_k}\big|_{p_k = p_k^*} = 0$, we obtain the solution for $p_z^*$ and $p_k^*$ as follows

$$\begin{cases} p_z^* = p_z^{Ad} \equiv \frac{\alpha^2(N-2)^2(2N-3)(2v-1)-2\alpha(N-2)(N(N(4v-2)-15v+7)+13v-6)-(4N-5)(2(N-2)v+1)}{(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)(\alpha(N-2)+1)}, \\ p_k^* = p_k^{Ad} \equiv \frac{\alpha(N-2)(3N-4)(2v-1)-(4N-5)(2(N-2)v+1)}{22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35}. \end{cases}$$

We can verify that the above solution satisfies $v - 1 < p_z^* < v - \frac{1}{2}$ and $v - 1 < p_k^* < v - \frac{1}{2}$ when $v$ is at a moderate level given by $\underline{v}^{Ad} < v < \overline{v}^{Ad}$, where $\underline{v}^{Ad} = \frac{(9-4N)N-5}{(2N-3)(-2\alpha+(\alpha-4)N+5)} + \frac{1}{2}$ and $\overline{v}^{Ad} = \frac{2(\alpha(N-2)(3N-4)+4N-5)}{(4N-5)(\alpha(N-2)+1)}$. Furthermore, $p_z^* < p_k^*$ is satisfied under $\underline{v}^{Ad} < v < \overline{v}^{Ad}$. Online Appendix OA1.2 further shows that under $\underline{v}^{Ad} < v < \overline{v}^{Ad}$, neither seller has an incentive to deviate out of the range of $v - 1 < p < v - \frac{1}{2}$ given that the other seller prices at the equilibrium price level.

Substituting $p_z^*$ and $p_k^*$ into the bid functions, we obtain the equilibrium bids of the two third-party sellers as

$$\begin{cases} b_z^* = b_z^{Ad} \equiv (1-\phi)\alpha(N-2)\frac{\left(4\alpha^2(N-2)^2(N-1)(2v-1)-\alpha(N-2)(4N(N(4v-2)-13v+8)+38v-25)-(4N-5)(4N(v-1)-6v+5)\right)\left(\alpha^2(N-2)^2(2N-3)(2v-1)-2\alpha(N-2)(N(N(4v-2)-15v+7)+13v-6)-(4N-5)(2(N-2)v+1)\right)}{N(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)^2(\alpha(N-2)+1)^2}, \\ b_k^* = b_k^{Ad} \equiv (1-\phi)\alpha(N-2)\frac{(\alpha(N-2)(3N-4)(2v-1)-(4N-5)(2(N-2)v+1))\left(2\alpha^2(N-2)^2(N-1)(2v-1)-\alpha(N-2)(2N(8(N-3)v-8N+23)+34v-31)-(4N-5)(4N(v-1)-6v+5)\right)}{(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)^2(\alpha(N-2)N+N)}. \end{cases}$$

We can verify that $b_z^{Ad} > b_k^{Ad}$, so that seller $z$ overbids seller $z$ in equilibrium (as we have assumed).

Finally, the net profits of the two third-party sellers are

$$\begin{cases} \pi_z^* = \pi_z^{Ad} \equiv (1-\phi)\frac{C^{Ad}}{(N-1)N(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)^2(\alpha(N-2)+1)}, \\ \pi_k^* = \pi_k^{Ad} \equiv (1-\phi)\frac{(2\alpha+(2-\alpha)N-3)((4N-5)(2(N-2)v+1)-\alpha(N-2)(3N-4)(2v-1))^2}{(N-1)N(22\alpha+N(-27\alpha+8(\alpha-2)N+48)-35)^2}, \end{cases}$$

where $C^{Ad} \equiv \alpha^2(N-2)^2(-4\left(N\left(N\left(64N^2-436N+1079\right)-1158\right)+457\right)v^2$ $+2N(N(4N(8N-93)+1189)-1488)v+N(N(8N(4(N-5)N+49)-647)+644)+3(434v-89))+$ $\alpha^4(N-2)^4\left(2(N-4)^2N-19\right)(1-2v)^2-\alpha^3(N-2)^3(2v-1)(N(N(2N(8N-46v-25)+410v+1)-$ $602v+111)+292v-82)+\alpha(N-2)(4N-5)(2(N-2)v+1)(N(4N((4N-27)v+6)+210v-67)-$ $126v+47)+(5-4N)^2(2N-3)(2(N-2)v+1)^2$.

**Ruling out other possible equilibria in Case “Ad”**

Consider the other possibility where $p_z^* > p_k^*$. Seller $z$'s demand upon deviating to price at $p_z$ is

$$D_z^{win}(p_z, p_k^*) = Q_{ao}^1(p_z, p_k^*) + (N-2)Q_m^2(p_z).$$

When seller $k$ deviates to price at $p_k$, its demand is

$$D_k^{lose}(p_z^*, p_k) = q_{ao}^1(p_z^*, p_k) + (N-2)q_m(p_k).$$

From $\frac{\partial\left((1-\phi)p_z D_z^{win}(p_z,p_k^*)-b_k^*\right)}{p_z}\Big|_{p_z=p_z^*} = 0$ and $\frac{\partial\left((1-\phi)p_k D_k^{lose}(p_z^*,p_k)\right)}{p_k}\Big|_{p_k=p_k^*} = 0$, we obtain the solution for $p_z^*$ and $p_k^*$ as

$$\begin{cases} p_z^* = -\frac{-4\alpha^2(N-2)^3(2v-1)+\alpha(N-2)(2N(N(4v-2)-18v+7)+38v-15)+(4N-5)(2(N-2)v+1)}{-16N^2+16\alpha^2(N-2)^3-8\alpha(N-3)(2N-3)(N-2)+48N-35}, \\ p_k^* = -\frac{-4\alpha^2(N-2)^3(2v-1)+\alpha(N-2)(N((8N-38)v+7)+40v-12)+(4N-5)(2(N-2)v+1)}{-16N^2+16\alpha^2(N-2)^3-8\alpha(N-3)(2N-3)(N-2)+48N-35}. \end{cases}$$

The above solution violates $p_z^* > p_k^*$ conditional on $\underline{v}^{Ad} < v < \overline{v}_{Ad}$ and it cannot be in equilibrium.

**Consumer surplus and total surplus in Case "Ad"**

$$
\begin{aligned}
TS^{Ad} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-2)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_{1+p_z^{Ad}-v}^{\frac{1}{2}}(v-(1-x))2dx \\
&+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\Big[\int_0^{\frac{1-p_k^{Ad}+p_z^{Ad}}{2}}(v-x)2dx+\int_{\frac{1-p_k^{Ad}+p_z^{Ad}}{2}}^{\frac{1}{2}}(v-(1-x))2dx\Big] \\
&+(N-2)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_k^{Ad}}(v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $z$ (with a probability of $\frac{1}{N}$) who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and whose second-preferred variety is $z$. Consumers in this segment purchase $z$ if and only if the distance with their most-preferred seller $x \geq 1 + p_z^{Ad} - v$. The third term represents consumers whose first-preferred variety is $k$ and second-preferred variety is $z$. Since $p_k^{Ad} > p_z^{Ad}$, consumers in this segment purchase $k$ if the distance with seller $k$ is $x < \frac{1-p_k^{Ad}+p_z^{Ad}}{2}$, generating a total surplus of $v - x$. They purchase $z$ if $x > \frac{1-p_k^{Ad}+p_z^{Ad}}{2}$, generating a total surplus of $v-(1-x)$. The fourth term represents consumers whose only available variety is $k$. Such consumers buy from seller $k$ if and only if their distance with seller $k$, $x$, is smaller than $v - p_k^{Ad}$, where $x \sim U[0,1]$.

The consumer surplus in the benchmark case comes as

$$
CS^{Ad} = TS^{Ad} - \Pi_p^{Ad} - \pi_z^{Ad} - \pi_k^{Ad}.
$$

### A2.3 Solution for Case "PL"

Assume that $p_l^* > p_i^*$ (we will show later that this is the only equilibrium, i.e., $p_l^* < p_i^*$ cannot be in equilibrium). To derive the equilibrium price, we need to make sure that if both third-party sellers set prices at $T = 2$ anticipating that $p_l = p_l^*$ at $T = 3$, then the platform has no incentive to deviate to price its private label at $p_l \neq p_l^*$.

First consider a third-party seller $i$'s pricing decision at $T = 2$, expecting the other third-party seller chooses the equilibrium price $p_i^*$, and anticipating that the platform chooses $p_l = p_l^*$ at $T = 3$. To derive a third-party seller $i$'s demand function upon deviation, it suffices to consider case where seller $i$ prices at $p_i > p_i^*$ based on Result 1. [11] Depending on the organic ranking, there are three possible outcomes

[11] Specifically, when seller $i$ deviates to price lower, we can obtain seller $i$'s demand by replacing $q_{ao}^2$ with $q_{ao}^1$ in $d_i^{lose2}$,

for seller $i$. The demand function in each outcome follows Lemma 1 by applying $n = 3$ (see Table A1).

| Case 1 | Case 2 | Case 3 |
| --- | --- | --- |
| $l : p_l^*$ | $j : p_i^*$ | $i : p_i$ |
| $i : p_i$ | $i : p_i$ | $j : p_i^*$ |
| $j : p_i^*$ | $l : p_l^*$ | $l : p_l^*$ |

Table A1: Demand when $i$ deviates to price at $p_i$ in Case "PL".

- Case 1: The private label brand obtains the top slot (with a probability of $\frac{1}{3}$). In this case, seller $i$'s demand when the private label lies in the ad slot is

$$d_i^{lose1}(p_i, p_i^*, p_l^*) = q_{ao}^1(p_l^*, p_i) + q_{oo}(p_i^*, p_i) + (N-3)q_m(p_i).$$

- Case 2: The other third-party seller $j$ obtains the top slot (with a probability of $\frac{1}{3}$). Seller $i$'s demand when the other third-party seller obtains the ad slot is

$$d_i^{lose2}(p_i, p_i^*, p_l^*) = q_{ao}^2(p_i^*, p_i) + q_{oo}(p_l^*, p_i) + (N-3)q_m(p_i).$$

- Case 3: Seller $i$ obtains the top slot (with a probability of $\frac{1}{3}$). The corresponding demand is

$$d_i^{win}(p_i, p_i^*, p_l^*) = Q_{ao}^1(p_i, p_i^*) + Q_{ao}^2(p_i, p_l^*) + (N-3)Q_m^2(p_i).$$

Average across the above three possibilities, seller $i$'s demand when deviating to price at $p_i$ comes as

$$D_i(p_i, p_i^*, p_l^*) = \frac{1}{3}d_i^{lose1}(p_i, p_i^*, p_l^*) + \frac{1}{3}d_i^{lose2}(p_i, p_i^*, p_l^*) + \frac{1}{3}d_i^{win}(p_i, p_i^*, p_l^*).$$

By $\frac{\partial(1-\phi)p_i D_i(p_i,p_i^*,p_l^*)}{p_i}\Big|_{p_i=p_i^*} = 0$, we have

$$p_i^* = \frac{\alpha(N-3)(2(N-4)v - N + p_l^* + 4) + 3(2(N-3)v + p_l^* + 2)}{39\alpha + N(\alpha(4N-25)+12) - 27}.$$

and replace $Q_{ao}^1$ with $Q_{ao}^2$ in $d_i^{win}$. The resulting expected demand $D_i = \frac{1}{3}d_i^{lose1} + \frac{1}{3}d_i^{lose2} + \frac{1}{3}d_i^{win}$ is the same as $D_i$ when deviating to price higher.

Now consider the platform's profit when deviating to price the private label at $p_l \neq p_l^*$ at $T = 3$, given that both third-party sellers price at the above $p_i^*$ (which is a function of $p_l^*$). Since the platform earns revenue from both private label brand and third-party sellers, we need to consider the demand of all three sellers. There are two possible demand outcomes based on Table A2.

| Case 1 | Case 2 |
|---|---|
| $l : p_l$ | $i : p_i^*$ |
| $i \ : \ p_i^*$ | $l : p_l$ |
| $j \ : \ p_i^*$ | $j : p_i^*$ |

Table A2: Demand when $l$ deviates to price at $p_l$ in Case "PL".

- Case 1: The private label brand obtains the top slot (with a probability of $\frac{1}{3}$). In this case, we have the demand of the third party as $d_i^{lose1}(p_i^*, p_i^*, p_l) = q_{ao}^1(p_l, p_i^*) + q_{oo}(p_i^*, p_i^*) + (N-3)q_m(p_i^*)$. The demand of the private label brand is

$$D_l^{win}(p_l, p_i^*, p_i^*) = 2Q_{ao}^1(p_l, p_i^*) + (N-3)Q_m^2(p_l).$$

- Case 2: The private label brand obtains one of the non-top slots (with a probability of $\frac{2}{3}$). In this case, we have the demand of a third-party seller in the top slot as $d_i^{win}(p_i^*, p_i^*, p_l) = Q_{ao}^1(p_i^*, p_i^*) + Q_{ao}^2(p_i^*, p_l) + (N-3)Q_m^2(p_i^*)$, and the demand of a third-party seller in a non-top slot as $d_i^{lose2}(p_i^*, p_i^*, p_l) = q_{ao}^1(p_i^*, p_i^*) + q_{oo}(p_l, p_i^*) + (N-3)q_m(p_i^*)$. The demand of the private label brand comes as

$$D_l^{lose}(p_l, p_i^*, p_i^*) = q_{ao}^2(p_i^*, p_l) + q_{oo}(p_i^*, p_l) + (N-3)q_m(p_l).$$

Recall that when the platform chooses the sales price for the private label brand, sellers already submit their prices. The platform's profit is

$$\begin{aligned}\Pi_p(p_l, p_i^*, p_i^*) =& \frac{1}{3}\Big(p_l \ D_l^{win}(p_l, p_i^*, p_i^*) + \phi \ 2 \ p_i^* \ d_i^{lose1}(p_i^*, p_i^*, p_l)\Big) \\ &+ \frac{2}{3}\Big(p_l D_l^{lose}(p_l, p_i^*, p_i^*) + \phi \ p_i^* \ d_i^{win}(p_i^*, p_i^*, p_l) + \phi \ p_i^* \ d_i^{lose2}(p_i^*, p_i^*, p_l)\Big).\end{aligned}$$

By $\frac{\partial \Pi_p(p_l, p_i^*, p_i^*)}{p_l}\Big|_{p_l = p_l^*} = 0$, we have

$$p_l^* = \frac{2p_i^*(\phi+1)(\alpha(N-3)+3) + \alpha(N-4)(N-3)(2v-1) + 6(N-3)v + 6}{4\alpha(N-3)^2 + 12(N-2)}.$$

Replacing $p_i^*$ as a function of $p_l^*$ that we derived earlier, we obtain the equilibrium price

$$\begin{cases} p_l^* = p_l^{PL} \equiv \frac{(33\alpha+2\alpha(N-3)\phi+N(\alpha(4N-23)+12)+6\phi-21)(\alpha(N-4)(N-3)(2v-1)+6(N-3)v+6)}{2(9(77\alpha^2+(\alpha-1)^2(-\phi)-104\alpha+35)+8\alpha^2N^4+2(24-49\alpha)\alpha N^3+N^2(72-\alpha(\alpha(\phi-449)+396))+6N(\alpha(\alpha(\phi-152)-\phi+178)-51))}, \\ p_i^* = p_i^{PL} \equiv \frac{(33\alpha+N(\alpha(4N-23)+12)-21)(\alpha(N-4)(N-3)(2v-1)+6(N-3)v+6)}{2(9(77\alpha^2+(\alpha-1)^2(-\phi)-104\alpha+35)+8\alpha^2N^4+2(24-49\alpha)\alpha N^3+N^2(72-\alpha(\alpha(\phi-449)+396))+6N(\alpha(\alpha(\phi-152)-\phi+178)-51))}. \end{cases}$$

We can verify that the above solution satisfies $v-1 < p_l^* < v - \frac{1}{2}$ and $v-1 < p_i^* < v - \frac{1}{2}$ when $v$ is at a moderate level given by $\underline{v}^{PL} < v < \overline{v}^{PL}$, where $\underline{v_1}^{PL}$ and $\overline{v}^{PL}$ are given in Appendix A4. Furthermore, $p_l^* > p_i^*$ is satisfied when $\underline{v_1}^{PL} < v < \overline{v}^{PL}$.

To establish the equilibrium, we also need to find the condition under which given $p_l^*$, it is sub-optimal for seller $i$ to price at $p_i < v-1$ or $p_i > v - \frac{1}{2}$ and given $p_i^*$, it is sub-optimal for the platform to price the private label at $p_l < v-1$ or $p_l > v - \frac{1}{2}$. Online Appendix A2.3 proves that $p_l^*$ and $p_i^*$ is in equilibrium when $\max\{\underline{v_1}^{PL}, \underline{v_2}^{PL}, \underline{v_3}^{PL}\} < v < \overline{v}^{PL}$. Since $\underline{v_3}^{PL}$ is larger than $\underline{v_1}^{PL}$ and $\underline{v_2}^{PL}$, the above condition simplifies into $\underline{v}^{PL} < v < \overline{v}^{PL}$, where $\underline{v}^{PL} = \underline{v_3}^{PL}$.

**Ruling out other possible equilibria in Case "PL"**

Finally, we can prove that the other scenario where the private label is priced at $p_l^* < p_i^*$ cannot hold in equilibrium. Following the intuition of Result 1, once we average across the possible rankings, the private label brand's demand function is the same upon pricing lower or higher than $p_i^*$. [12] Therefore, we will reach the same equilibrium price solution, contradicting with the premise that $p_l^* < p_i^*$.

**Consumer surplus and total surplus in Case "PL"**

We first derive the total surplus given a third-party seller or a private label brand is in the top slot.

- Consider the case where one of the third-party sellers (denoted by $z$) is in the top slot, with a probability of $\frac{2}{3}$. Denote the other third-party seller in the non-top slot as $k$. In this case

[12] Specifically, under the premise that $p_l^* < p_i^*$, the demand function can be obtained in a similar fashion. First, if seller $i$ deviates, the demand function follows that in Table A1 by replacing $q_{ao}^1$ with $q_{ao}^2$ in $d_i^{lose1}$, and replacing $Q_{ao}^2$ with $Q_{ao}^1$ in $d_i^{win}$. If the platform deviates on the private label pricing, the demand function follows Table A2 by replacing $Q_{ao}^1$ with $Q_{ao}^2$ in $D_l^{win}$, replacing $q_{ao}^1$ with $q_{ao}^2$ in $d_i^{lose1}$, replacing $q_{ao}^2$ with $q_{ao}^1$ in $D_i^T$, and replacing $Q_{ao}^2(p_i^*, p_l)$ with $Q_{ao}^1(p_i^*, p_l)$ in $d_i^{lose2}$. We resolve the expected demand and find that the same demand function as in the case where $p_l^* > p_i^*$.

$p_z = p_k = p_i^{PL}$.

$$
\begin{aligned}
TS^{i\ on\ top} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-3)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_{1+p_i^{PL}-v}^{\frac{1}{2}}\big(v-(1-x)\big)2dx \\
&+ \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1}{2}}(v-x)2dx + \int_{\frac{1}{2}}^{\frac{1}{2}}(v-(1-x))2dx\right] \\
&+ \frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1-p_l^{PL}+p_i^{PL}}{2}}(v-x)2dx + \int_{\frac{1-p_l^{PL}+p_i^{PL}}{2}}^{\frac{1}{2}}(v-(1-x))2dx\right] \\
&+ (1-\alpha)\frac{2}{N(N-1)}\left[\int_0^{\frac{1-p_k^{PL}+p_l^{PL}}{2}}(v-x)dx + \int_{\frac{1-p_k^{PL}+p_l^{PL}}{2}}^{1}(v-(1-x))dx\right] \\
&+ (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_k^{PL}}(v-x)dx, \\
&+ (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_l^{PL}}(v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $z$ who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and their second-preferred variety is $z$. They purchase $z$ if and only if the distance with their most-preferred seller $x \geq 1 + p_i^{PL} - v$. The third term represents consumers on the spoke of $k$ and whose second-preferred variety is $z$, with a distance from $k$ following $x \sim U[0, \frac{1}{2}]$. Since $p_k = p_z = p_i^{PL}$, consumers in this segment purchase $k$ if and only if $x < \frac{1}{2}$, generating a total surplus of $v - x$; otherwise, they purchase $z$, generating a total surplus of $v - (1 - x)$. The four term represents consumers on the spoke of $l$ and whose second-preferred variety is $z$, with a distance from $l$ following $x \sim U[0, \frac{1}{2}]$. Since $p_l^{PL} > p_z = p_i^{PL}$, consumers in this segment purchase $l$ if and only if $x < \frac{1-p_l^{PL}+p_i^{PL}}{2}$, generating a total surplus of $v - x$; otherwise, they purchase $z$, generating a total surplus of $v - (1 - x)$. The fifth term represents consumers whose two preferred brands are $k$ and $l$ in the organic slots, with a distance from seller $k$ following $x \sim U[0, 1]$. Consumers in this segment purchase $k$ if $x < \frac{1-p_k^{PL}+p_l^{PL}}{2}$ and they purchase seller $z$ otherwise. The last two terms represent consumers whose only available variety is $k$ and $l$, respectively. Consumers in $k$'s monopoly segment buy from seller $k$ if and only if their distance with seller $k$, $x < v - p_k^{PL}$, where $x \sim U[0, 1]$. A similar logic applies to consumers in seller $l$'s monopoly segment.

- Consider the other where the private label brand is in the top slot, with a probability of $\frac{1}{3}$.

$$
\begin{aligned}
TS^{l\ on\ top} =& 2 * \frac{1}{N(N-1)} \left[ \int_0^{\frac{1-p_l^{PL}+p_i^{PL}}{2}} (v-x)2dx + \int_{\frac{1-p_l^{PL}+p_i^{PL}}{2}}^{\frac{1}{2}} (v-(1-x))2dx \right] \\
&+ (N-3)\frac{1}{N(N-1)} \int_0^{\frac{1}{2}} \big(v-(1-x)\big)2 \\
&+ (N-3)\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) \int_{1+p_l^{PL}-v}^{\frac{1}{2}} \big(v-(1-x)\big)2dx \\
&+ 2 * \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) \left[ \int_0^{\frac{1}{2}} (v-x)2dx \right] \\
&+ (1-\alpha)\frac{2}{N(N-1)} \left[ \int_0^{\frac{1}{2}} (v-x)dx + \int_{\frac{1}{2}}^{1} (v-(1-x))dx \right] \\
&+ 2 * (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1} \int_0^{v-p_i^{PL}} (v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $l$ and whose other-preferred seller is one of the two third-party seller. The second term represents consumers on the spoke of seller $l$ and whose other-preferred seller is not available. The third term represents consumers whose most preferred variety is not available and their second-preferred variety is $l$. The fourth term represents consumers on the spoke of one of the sellers in the organic slots, and whose second-preferred variety is $l$. Consumers in this segment receive their most-preferred variety. The fifth term represents consumers whose preferred brands are the two third-party sellers with the same price. The last term represents consumers in the monopoly segment of one of the third-party sellers.

Combined, the expected total surplus in this scenario is

$$TS^{PL} = \frac{2}{3}TS^{i\ on\ top} + \frac{1}{3}TS^{l\ on\ top}.$$

The consumer surplus is

$$CS^{PL} = TS^{PL} - \Pi_p^{PL} - 2\pi_i^{PL}.$$

### A2.4 Solution for Case "Concede"

As we have shown in Lemma 3, the private label brand tends to price higher than a third-party seller. Therefore, we assume $p_l^* > p_k^*$ (we will verify this is satisfied under the equilibrium shortly). Given this, it suffices to discuss three potential outcomes: (1) $p_l^* > p_k^* > p_z^*$, (2) $p_l^* > p_z^* > p_k^*$, (3) $p_z^* > p_l^* > p_k^*$. We first solve the equilibrium price assuming $p_l^* > p_k^* > p_z^*$ and then find the condition when the solution satisfies the assumption.

For the winning seller $z$, when it makes pricing decisions, it expects seller $k$ to price at $p_k^*$ at $T = 2$ and it rationally expects the private label to be priced at $p_l^*$ at $T = 3$. Upon deviating to price at $p_z$, it expects a net profit of

$$\pi_z(p_z, p_k^*, p_l^*) = (1 - \phi)p_z D_z^{win}(p_z, p_k^*, p_l^*) - b_k^*,$$

where $D_z^{win}(p_z, p_k^*, p_l^*) = Q_{ao}^2(p_z, p_k^*) + Q_{ao}^2(p_z, p_l^*) + (N - 3)Q_m^2(p_z)$ is seller $z$'s demand upon winning, expecting that seller $k$ prices at $p_k^*$ and seller $l$ will price at $p_l^*$. From $\frac{\partial \pi_z(p_z, p_k^*, p_l^*)}{p_z}\Big|_{p_z = p_z^*} = 0$, we have

$$p_z^* = \frac{\alpha(N-2)(2(N-3)v - N + p_k^* + p_l^* + 3) + 2Nv + p_k^* + p_l^* - 6v + 2}{4\left(\alpha(N-2)^2 + N - 2\right)}. \tag{A4}$$

Similarly, for the losing seller $k$, by deviating to price at $p_k$, it expects a demand of $D_k^{lose}(p_k, p_z^*, p_l^*) = q_{ao}^2(p_z^*, p_k) + q_{oo}(p_l^*, p_k) + (N - 3)q_m(p_k)$. Seller $k$'s net profit upon such deviation is

$$\pi_k(p_k, p_z^*, p_l^*) = (1 - \phi)p_k D_k^{lose}(p_k, p_z^*, p_l^*).$$

Based on $\frac{\partial \pi_k(p_k, p_z^*, p_l^*)}{p_k}\Big|_{p_k = p_k^*} = 0$, we have

$$p_k^* = \frac{-p_z^*(\alpha(N-2) + 1) + \alpha(N-3)(2v-1) - 2Nv + (\alpha - 1)p_l^* + 6v - 2}{-6\alpha + 2(\alpha - 2)N + 8}. \tag{A5}$$

Now we analyze the platform's optimal pricing strategy for its private label at $T = 3$, given that the two third-party sellers make the price and bid decisions with the expectation that $p_l = p_l^*$. Upon deviating to price the private label at $p_l$, the platform's revenue $\Pi_P(p_l, p_k^*, p_z^*)$ is more involved. $p_l$ affects the private label's demand, $D_l^{lose}(p_l, p_z^*, p_k^*)$. It also affects the third-party seller's demand $D_z^{win}(p_z^*, p_k^*, p_l)$ and $D_k^{lose}(p_k^*, p_z^*, p_l)$, thus influencing the platform's commission revenue. We can write down the platform's total revenue as

$$\Pi_P(p_l, p_k^*, p_z^*) = p_l D_l^{lose}(p_l, p_z^*, p_k^*) + \phi\left(p_z D_z^{win}(p_z^*, p_k^*, p_l) + D_k^{lose}(p_k^*, p_z^*, p_l)\right) + b_k^*$$

where $D_l^{lose}(p_l, p_z^*, p_k^*) = q_{ao}^2(p_z^*, p_l) + q_{oo}(p_k^*, p_l) + (N-3)q_m(p_l)$ is the private label brand's demand in one of the organic slots. From $\frac{\partial \Pi_P(p_l, p_k^*, p_z^*)}{p_l}\Big|_{p_l = p_l^*} = 0$, we have

$$p_l^* = \frac{-p_z^*(\phi+1)(\alpha(N-2)+1) + \alpha(N-3)(2v-1) - 2Nv + (\alpha-1)p_k^*(\phi+1) + 6v - 2}{-6\alpha + 2(\alpha-2)N + 8}. \tag{A6}$$

Combined equation (A4) (A5) and (A6), the equilibrium price assuming assuming $p_l^* > p_k^* > p_z^*$ is

$$\begin{cases} p_z^* = p_z^{Concede} \equiv \frac{-(\alpha-1)\alpha(N-1)^2\phi + \alpha^3(5-2N)^2(N-3)(N-2)(2v-1) - 2\alpha^2(2N-5)(N(4(N-7)N(2v-1)+129v-63)-99v+48) + \alpha(4N-7)(N(N(N(8v-4)-62v+29)+162v-71)-144v+64) + 2(7-4N)^2((N-3)v+1)}{2(\alpha(N-2)+1)(\alpha(8(\alpha-2)\phi-130\alpha+357)+8(\alpha-2)^2N^3+(\alpha-2)N^2(\alpha(\phi-62)+96)+N(\alpha(-6(\alpha-2)\phi+157\alpha-491)-4\phi+378)+7(\phi-35))}, \\ p_k^* = p_k^{Concede} \equiv \frac{(-5\alpha+2(\alpha-2)N+7)(3\alpha(N-3)(N-2)(2v-1)-2(4N-7)((N-3)v+1)) - \alpha(N-1)^2\phi}{2(\alpha(8(\alpha-2)\phi-130\alpha+357)+8(\alpha-2)^2N^3+(\alpha-2)N^2(\alpha(\phi-62)+96)+N(\alpha(-6(\alpha-2)\phi+157\alpha-491)-4\phi+378)+7(\phi-35))}, \\ p_l^* = p_l^{Concede} \equiv \frac{\phi\big(\alpha\big(-4N^3+31N^2+8(N-4)(N-3)(N-2)v-88N+85\big)-2\alpha^2(N-4)(N-3)(N-2)(2v-1)+4(4N-7)((N-3)v+1)\big)+(-5\alpha+2(\alpha-2)N+7)(3\alpha(N-3)(N-2)(2v-1)-2(4N-7)((N-3)v+1))}{2(\alpha(8(\alpha-2)\phi-130\alpha+357)+8(\alpha-2)^2N^3+(\alpha-2)N^2(\alpha(\phi-62)+96)+N(\alpha(-6(\alpha-2)\phi+157\alpha-491)-4\phi+378)+7(\phi-35))}. \end{cases}$$

The above solution satisfies $v - 1 < p_l^*, p_k^*, p_z^* < v - \frac{1}{2}$ when $\underline{v}^{Concede} < v < \overline{v}^{Concede}$ (defined in Appendix A4). Furthermore, $p_l^* > p_k^* > p_z^*$ is met given $\underline{v}^{Concede} < v < \overline{v}^{Concede}$. To further establish the above solution as an equilibrium, Online Appendix OA1.4 shows that a seller has no incentive to deviate to $p < v - 1$ or $p \geq v - \frac{1}{2}$ under $\underline{v}^{Concede} < v < \overline{v}^{Concede}$.

Finally, based on envy-free equilibrium, the equilibrium bid of seller $z$ is

$$b_z^* = (1-\phi)p_z^*\Big(D_z^{win}(p_z^*, p_k^*, p_l^*) - D_z^{lose}(p_z^*, p_k^*, p_l^*)\Big),$$

where $D_z^{lose}(p_z^*, p_k^*, p_l^*) = q_{ao}^1(p_k^*, p_z^*) + q_{oo}(p_l^*, p_z^*) + (N-3)q_m(p_z^*)$ represents seller $z$'s demand if losing the auction.

The equilibrium bid of seller $k$ is

$$b_k^* = (1-\phi)p_k^*\Big(D_k^{win}(p_k^*, p_z^*, p_l^*) - D_k^{lose}(p_k^*, p_z^*, p_l^*)\Big),$$

where $D_k^{win}(p_k^*, p_z^*, p_l^*) = Q_{ao}^1(p_k^*, p_z^*) + Q_{ao}^2(p_k^*, p_l^*) + (N-3)Q_m^2(p_k^*)$ is seller $k$'s demand if winning the ad auction.

**Ruling out other possible equilibria in Case "Concede"**

Finally, we prove that the other two possible outcomes cannot be in equilibrium. Consider the second possible outcome $p_l^* > p_z^* > p_k^*$. The demand function can be obtained by replacing $Q_{ao}^2$ with $Q_{ao}^1$ in $D_z^{win}$, and replacing $q_{ao}^2$ with $q_{ao}^1$ in $D_k^{lose}$. The resulting solution is given in Online Appendix OA3. The solution does not satisfy $p_z^* > p_k^*$ when $1 < v < 2$, thus cannot be in equilibrium under Condition 1 (which is a subset of $1 < v < 2$ as proved in Appendix A2.1).

To solve the third possible outcome with $p_z^* > p_l^* > p_k^*$, in the demand function, besides replacing $Q_{ao}^2$ with $Q_{ao}^1$ in $D_z^{win}$ and replacing $q_{ao}^2$ with $q_{ao}^2$ in $D_k^{lose}$, we also need to replace $Q_{ao}^2$ to $Q_{ao}^1$ in $D_z^{win}$ and replace $q_{ao}^2$ with $q_{ao}^1$ in $D_l^{lose}$. However, the resulting solution (see Online Online Appendix OA3 for details) contradicts with the premise that $p_z^* \geq p_l^*$ when $1 < v < 2$ thus cannot be in equilibrium under Condition 1.

**Consumer surplus and total surplus in Case "Concede"**

$$
\begin{aligned}
TS^{Concede} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-3)\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\int_{1+p_z^{Concede}-v}^{\frac{1}{2}}(v-(1-x))2dx \\
&+ \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1-p_k^{Concede}+p_z^{Concede}}{2}}(v-x)2dx + \int_{\frac{1-p_k^{Concede}+p_z^{Concede}}{2}}^{\frac{1}{2}}(v-(1-x))2dx\right] \\
&+ \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1-p_l^{Concede}+p_z^{Concede}}{2}}(v-x)2dx + \int_{\frac{1-p_l^{Concede}+p_z^{Concede}}{2}}^{\frac{1}{2}}(v-(1-x))2dx\right] \\
&+ (1-\alpha)\frac{2}{N(N-1)}\left[\int_0^{\frac{1-p_k^{Concede}+p_l^{Concede}}{2}}(v-x)dx + \int_{\frac{1-p_k^{Concede}+p_l^{Concede}}{2}}^{1}(v-(1-x))dx\right] \\
&+ (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_k^{Concede}}(v-x)dx, \\
&+ (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_l^{Concede}}(v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $z$who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and their second-preferred variety is $z$. Consumers in this segment purchase $z$ if and only if the distance with their most-preferred seller $x \geq 1 + p_z^{Concede} - v$. The third term represents consumers on the spoke of $k$ and whose second-preferred variety is $z$, with a distance from $k$ following $x \sim U[0, \frac{1}{2}]$. Since $p_k^{Concede} > p_z^{Concede}$, consumers in this segment purchase $k$ if and only if $x < \frac{1-p_k^{Concede}+p_z^{Concede}}{2}$, generating a total surplus of $v-x$; otherwise, they purchase $z$, generating a total surplus of $v-(1-x)$. The fourth term follows the same logic except for that the consumers are on the spoke of $l$ rather than $k$. The fifth term represents consumers whose two preferred brands are $k$ and $l$ in the organic slots, with a distance from seller $k$ following $x \sim U[0,1]$. Consumers in this segment purchase $k$ if $x < \frac{1-p_k^{Concede}+p_l^{Concede}}{2}$ and they purchase seller $z$ otherwise. The last two terms represent consumers whose only available variety is $k$ and $l$, respectively. Consumers in $k$'s monopoly segment buy from seller $k$ if and only if their distance with seller $k$, $x < v - p_k^{Concede}$, where $x \sim U[0,1]$. A similar logic applies to consumers in seller $l$'s monopoly segment.

The consumer surplus in the benchmark case comes as

$$CS^{Concede} = TS^{Concede} - \Pi_p^{Concede} - \pi_z^{Concede} - \pi_k^{Concede}.$$

### A2.5 Solution for Case "Contest"

Assume $p_l^* < p_i^*$. For the third-party seller $i$, when it makes pricing decisions, it expects the other third-party seller to price at $p_i^*$ at $T=2$ and it rationally expects the private label to be priced at $p_l^*$ at $T=3$. Its sales revenue is $(1-\phi)p_i D_i(p_i, p_i^*, p_l^*)$, where $D_i(p_i, p_i^*, p_l^*)$ is the demand function given by

$$D_i(p_i, p_i^*, p_l^*) = q_{ao}^2(p_l^*, p_i) + q_{oo}(p_i^*, p_i) + (N-3)q_m(p_i).$$

Now consider the private label brand's pricing at $T=3$. Given the two third-party sellers pricing at $p_l^*$, when pricing at $p_l$, it expects a demand of

$$D_l(p_l, p_i^*) = 2Q_{ao}^2(p_l, p_i^*) + (N-3)Q_m^2(p_l).$$

The platform's total revenue is $\Pi_p(p_l, p_i^*) = p_l D_l(p_l, p_i^*) + 2\,\phi\, p_i^* * \Big(q_{ao}^2(p_l, p_i^*) + q_{oo}(p_i^*, p_i^*) + (N-3)q_m(p_i^*)\Big)$.

Based on $\frac{\partial((1-\phi)p_i D_i(p_i, p_i^*, p_l^*))}{p_i}\Big|_{p_i=p_i^*} = 0$ and $\frac{\partial \Pi_p(p_l, p_i^*)}{\partial p_l}\Big|_{p_l=p_l^*} = 0$, we obtain $p_l^*$ and $p_i^*$ as below

$$\begin{cases} p_l * = p_l^{Contest} \equiv \frac{C^{Contest}}{2(\alpha(N-2)+1)(26\alpha+\alpha(N-2)\phi+N(-21\alpha+4(\alpha-2)N+34)+\phi-35)}, \\ p_i^* = p_i^{Contest} \equiv \frac{3\alpha(N-3)(N-2)(2v-1)-2(4N-7)((N-3)v+1)}{2(26\alpha+\alpha(N-2)\phi+N(-21\alpha+4(\alpha-2)N+34)+\phi-35)}, \end{cases}$$

where $C^{Contest} \equiv 2(\alpha-2)\alpha N^3(2v-1) + N^2(\alpha(-2\alpha\phi+15\alpha-29)+v(2\alpha(2(\alpha-1)\phi-15\alpha+29)-8)) + N(\alpha(10\alpha\phi-37\alpha-6\phi+67)+v(2\alpha(-10\alpha\phi+37\alpha+12\phi-70)-4\phi+38)-8)+2(\alpha(-6\alpha\phi+15\alpha+7\phi-27)+3(2\alpha-1)v(2(\alpha-1)\phi-5\alpha+7)-2\phi+7)$. The above solution satisfies $v-1 < p_l^* < p_i^* < v-\frac{1}{2}$ when $\underline{v}^{Contest} < v < \overline{v}^{Contest}$ (see Appendix A4 for the expression).

Next, we derive the condition under which a seller will not deviate to $p < v-1$ or $p > v-\frac{1}{2}$. Online Appendix OA1.5 proves that $p_i^*$ and $p_l^*$ is an equilibrium when $N \geq 5, \underline{v}^{Bais} < v < \overline{v}^{Contest}$.

**Ruling out other possible equilibria in Case "Contest"**

In this subsection, we prove that the other case where $p_l^* > p_i^*$ cannot hold. The solution can be obtained similarly by replacing $q_{ao}^2$ with $q_{ao}^1$ in $D_i$ and replacing $Q_{ao}^2$ with $Q_{ao}^1$ in $D_l$. The solution for the equilibrium prices under the updated demand function is given in Online Appendix OA3. However, the solution violates the condition of $v-1 < p_i^* < p_l^* < v-\frac{1}{2}$ given $\underline{v}^{Bais} < v < \overline{v}^{Contest}$ therefore cannot be in equilibrium under Condition 1.

**Consumer surplus and total surplus in Case "Contest"**

$$
\begin{aligned}
TS^{Contest} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-3)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_{1+p_l^{Contest}-v}^{\frac{1}{2}}\big(v-(1-x)\big)2dx \\
&+ 2*\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1-p_i^{Contest}+p_l^{Contest}}{2}}(v-x)2dx + \int_{\frac{1-p_i^{Contest}+p_l^{Contest}}{2}}^{\frac{1}{2}}(v-(1-x))2dx\right] \\
&+ (1-\alpha)\frac{2}{N(N-1)}\left[\int_0^{\frac{1}{2}}(v-x)dx + \int_{\frac{1}{2}}^{1}(v-(1-x))dx\right] \\
&+ 2*(N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{v-p_i^{Contest}}(v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $l$ (with a probability of $\frac{1}{N}$) who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and their second-preferred variety is $l$. Consumers in this segment purchase $l$ if and only if the distance with their most-preferred seller $x \geq 1+p_l^{Contest}-v$. The third term represents consumers on the spoke of one of the sellers in the organic slots $i$, and whose second-preferred variety is $l$, with a distance from $i$ following $x \sim U[0,\frac{1}{2}]$. Since $p_i^{Contest} > p_l^{Contest}$, consumers in this segment purchase $i$ if and only if $x < \frac{1-p_i^{Contest}+p_l^{Contest}}{2}$, generating a total surplus of $v-x$; otherwise, they purchase $l$, generating a total surplus of $v-(1-x)$. The fourth term represents consumers whose two preferred brands are $i$ and $j$ in the organic slots, with a distance from seller $i$ following $x \sim U[0,1]$. Given that seller $i$ and $j$ have the same price, consumers in this segment purchase $i$ if $x < \frac{1}{2}$ and they purchase seller $j$ if $x > \frac{1}{2}$. The last term represents consumers in the monopoly segment of one of the third-party sellers. Consumers in $i$'s monopoly segment buy from seller $i$ if and only if their distance with seller $i$ is $x < v - p_i^{Contest}$, where $x \sim U[0,1]$.

The consumer surplus in the benchmark case comes as

$$CS^{Contest} = TS^{Contest} - \Pi_p^{Contest} - 2\pi_i^{Contest}.$$

## A3 Comparison of Players' Payoffs

This appendix compares each player's payoff (including consumer surplus, total surplus, and third-party seller' net profits) as well as the platform's three revenue streams under each scenario. We first keep the commission rate exogenous and then move to the endogenous commission rate case.

### A3.1 Comparison with an exogenous commission rate

- The private label's price satisfies $p_l^{Contest} < p_l^{PL} < p_l^{Concede}$. In terms of the third-party seller's price, in the absence of a private label, we have $p_z^{Ad} < p_i^{BM} < p_k^{Ad}$; in the presence of a private label, we have $p_z^{Concede} < p_i^{PL} < p_k^{Concede} < p_i^{Contest}$.

- The total surplus satisfies $\max\{TS^{PL}, TS^{Concede}\} > \max\{TS^{BM}, TS^{Ad}, TS^{Contest}\}$. The total surplus can be the highest either under Case "PL" or Case "Concede". The trade-off between the two cases is that Case "Concede" induces the winning seller to price lower, leading to a market expansion effect, but it also induces consumers to choose their less-preferred seller, sacrificing consumer fit value. Second, conceding the ad slot generates a higher total surplus relative to contesting, i.e., $TS^{Concede} > TS^{Contest}$, because the private label brand has less incentive to reduce prices. Third, $\min\{TS^{PL}, TS^{Concede}, TS^{Contest}\} > \max\{TS^{Ad}, TS^{BM}\}$. This suggests that the total surplus is higher when there is a private label than without it because it provides more varieties to consumers.

- For consumer surplus, we have $\max\{CS^{PL}, CS^{Concede}\} > \max\{CS^{BM}, CS^{Ad}, CS^{Contest}\}$. In other words, consumer surplus can be the highest under Case "PL" or Case "Concede", following the same reason as the discussion on total surplus. Finally, we have $CS^{Contest} < CS^{Concede}$, implying that contesting the ad slot hurts consumer surplus.

- Regarding the third-party sellers, if there is an ad slot, the winning seller is better off than the losing seller. However, it is possible for both sellers to be worse off when there is a sponsored ad compared with in the absence of an ad slot.

  To see this, without a private label brand (i.e., Case "BM" and "Ad"), we have $\min\{\pi_i^{BM}, \pi_z^{Ad}\} > \pi_k^{Ad}$. Here, $\pi_i^{BM}$ can be larger or smaller than $\pi_z^{Ad}$. In other words, it is possible (but not always) for both sellers to be worse off when there is a sponsored slot as they fall into a prisoner's dilemma situation. $\pi_z^{Ad} > \pi_k^{Ad}$ implies that the losing seller is hurt more than the winning seller.

  When there is a private label (i.e., Case "PL", "Concede" and "Contest"), we have $\min\{\pi_i^{PL}, \pi_z^{Concede}\} > \pi_i^{Contest} > \pi_k^{Concede}$. Relative to conceding, the platform contesting the ad slot hurts the winning seller $z$ but benefits the losing seller $k$. The reason is that a winning private label $l$ prices higher than a winning third-party seller $z$ and steals less demand from the losing third-party seller $k$. Note that having an ad slot may (but not always) benefit the winning third-party seller relative to not having the ad slot.

Now consider the platform's three revenue streams. Generally speaking, the platform's ad revenue is higher without a private label (than with a private label) because the third-party sellers benefit more from winning the ad slot. The platform's commission revenue is higher without a private label because third-party sellers have a higher market share and more sales. Finally, the platform's private label revenue is the highest under Case "Contest", followed by Case "PL", followed by Case "Concede".

### A3.2 Comparison with an endogenous commission rate

The comparisons of the total surplus and the consumer surplus across different platform arrangements stay qualitatively the same as in the exogenous commission rate case. This is because endogenizing the commission rate largely re-splits the combined surplus between third-party sellers and the platform. Specifically, both the total surplus and consumer surplus are the highest under Case "PL" or Case "Concede". Furthermore, Case "Concede" still dominates Case "Contest" in the total surplus and consumer surplus, because the two cases lead to the same commission rate.

The net profits of third-party sellers are more involved. In Case "BM", "PL", and "Contest" (where the two third-party sellers have the same net profit given a fixed commission rate), with an optimally chosen commission rate, both third-party sellers' net profits are equal to the outside option (i.e., $\pi_i^{BM} = \pi_i^{PL} = \pi_i^{Contest} = u_0$). In Case "Ad" and "Concede" (where the two third-party sellers have different net profits given a fixed commission rate), the losing third-party seller $k$ still gets its outside option (i.e., $\pi_k^{Ad} = \pi_k^{Concede} = u_0$), whereas the winning third-party seller earns a positive surplus (i.e., $\pi_z^{Ad} > u_0$ and$\pi_z^{Concede} > u_0$). Following this, although Case "Concede" and "Contest" have the same commission rate, the former leaves the winning seller with a positive surplus, while the latter extracts all surplus from both sellers.

The platform's revenue streams under different scenarios are qualitatively the same as in the exogenous commission rate case, with a couple of differences. For example, the commission revenue is lower under Case "BM" than Case "Ad" with an exogenous commission rate, but it is higher with endogenous commission rate. This is because sellers get more profit under case "BM", which allows the platform to charge a higher commission.

## A4 Variable Definitions

$$\underline{v_1}^{PL} = \frac{9(\alpha-1)(\alpha(7\phi+33)-3(\phi+7))+4\alpha^2N^4+\alpha N^3(36-\alpha(2\phi+47))+N^2(\alpha(19\alpha-6)\phi+3\alpha(69\alpha-89)+72)-3N(\alpha(5\alpha(4\phi+27)-16\phi-213)+78)}{18(\alpha(\alpha(7\phi+33)-12\phi-43)+5\phi+14)+8\alpha^2N^4-2\alpha N^3(\alpha(2\phi+47)-24)+N^2(\alpha^2(38\phi+414)-12\alpha(2\phi+31)+72)-6N(\alpha(5\alpha(4\phi+27)-24\phi-157)+6\phi+45)}.$$

$$\overline{v}^{PL} = \frac{18\big(55\alpha^2+(\alpha-1)^2(-\phi)-79\alpha+28\big)+12\alpha^2N^4+(84-145\alpha)\alpha N^3+N^2(\alpha(-2\alpha(\phi-328)-663)+144)+3N(\alpha(4(\alpha-1)\phi-439\alpha+569)-180)}{2(9(33\alpha^2+(\alpha-1)^2(-\phi)-43\alpha+14)+4\alpha^2N^4+(24-47\alpha)\alpha N^3+N^2(36-\alpha(\alpha(\phi-207)+186))+3N(\alpha(2(\alpha-1)\phi-135\alpha+157)-45))}.$$

$$\underline{v_2}^{PL} = \frac{108\big(-11\alpha^3+62\alpha^2+(\alpha-1)^3\phi-68\alpha+21\big)+4(\alpha-12)\alpha^2N^5+\alpha N^4(\alpha(\alpha(\phi-63)+660)-288)+N^3\big(\alpha\big(\alpha^2(395-13\phi)+12\alpha(\phi-300)+2700\big)-432\big)+3N^2\big(\alpha\big(-411\alpha^2+3(\alpha-1)(7\alpha-5)\phi+3236\alpha-3081\big)+756\big)+9N\big(213\alpha^3-1432\alpha^2-3(\alpha-1)^2(5\alpha-2)\phi+1517\alpha-438\big)}{2(54(-22\alpha^3+91\alpha^2+2(\alpha-1)^3\phi-93\alpha+28)+4(\alpha-6)\alpha^2N^5+\alpha N^4(\alpha(\alpha(\phi-63)+354)-144)+N^3(\alpha(\alpha^2(395-13\phi)+12\alpha(\phi-173)+1440)-216)+3N^2(\alpha(-411\alpha^2+3(\alpha-1)(7\alpha-5)\phi+2012\alpha-1767)+414)+9N(213\alpha^3-964\alpha^2-3(\alpha-1)^2(5\alpha-2)\phi+945\alpha-264))}.$$

$$\underline{v_3}^{PL} = \frac{-108(\alpha(\alpha(11\alpha(\phi+1)-23\phi-62)+15\phi+68)-3(\phi+7))+4\alpha^2N^5(\alpha\phi+\alpha-12)-3\alpha N^4\big(21\alpha^2(\phi+1)-4\alpha(2\phi+55)+96\big)+N^3\big(395\alpha^3(\phi+1)-24\alpha^2(13\phi+150)+36\alpha(\phi+75)-432\big)-3N^2\big(\alpha\big(411\alpha^2(\phi+1)-4\alpha(125\phi+809)+141\phi+3081\big)-756\big)+9N\big(\alpha\big(213\alpha^2(\phi+1)-8\alpha(44\phi+179)+165\phi+1517\big)-6(3\phi+73)\big)}{2(-54(\alpha(22\alpha^2(\phi+1)-13\alpha(4\phi+7)+40\phi+93)-2(5\phi+14))+4\alpha^2N^5(\alpha\phi+\alpha-6)-3\alpha N^4(21\alpha^2(\phi+1)-2\alpha(6\phi+59)+48)+N^3(395\alpha^3(\phi+1)-12\alpha^2(36\phi+173)+36\alpha(3\phi+40)-216)-3N^2(411\alpha^3(\phi+1)-4\alpha^2(161\phi+503)+3\alpha(99\phi+589)-18(2\phi+23))+9N(\alpha(\alpha(213\alpha(\phi+1)-424\phi-964)+269\phi+945)-6(9\phi+44)))}.$$

$$\underline{v}^{Concede} = \max\{\frac{\alpha(8\alpha(\phi-5)-17\phi+161)+2(\alpha-8)(\alpha-2)N^3+N^2(\alpha((\alpha-3)\phi-17\alpha+123)-160)+N(2\alpha(\alpha(23-3\phi)+7\phi-123)-4\phi+266)+7(\phi-21)}{2\phi(\alpha^2(N-4)(N-2)-2\alpha(N-4)(N-2)-4N+7)+2(N-2)(-5\alpha+2(\alpha-2)N+7)(-4\alpha+(\alpha-4)N+7)}, \frac{\alpha(\alpha(\alpha(370-32\phi)+79\phi-1208)-59\phi+1246)+12(\alpha-2)^2\alpha N^4+2(\alpha-2)N^3(\alpha(\alpha(\phi-58)+100)-16)+N^2(\alpha(\alpha(\alpha(413-16\phi)+33\phi-1454)-11\phi+1477)-352)+N(\alpha(\alpha(\alpha(40\phi-643)-90\phi+2170)+52\phi-2231)-8\phi+644)+14(\phi-28)}{2(\alpha(N-2)+1)(\alpha(8(\alpha-2)\phi-55\alpha+147)+4(\alpha-2)^2N^3+(\alpha-2)N^2(\alpha(\phi-30)+44)+N(\alpha(-6(\alpha-2)\phi+72\alpha-217)-4\phi+161)+7(\phi-14))}\}$$

$$\overline{v}^{Concede} = \min\{\frac{-40\alpha^2(\phi+1)+23\alpha(3\phi+7)+2(\alpha-2)N^3(\alpha\phi+\alpha-8)+N^2(\alpha(-17\alpha(\phi+1)+29\phi+123)-160)+2N(\alpha(23\alpha(\phi+1)-38\phi-123)+6\phi+133)-21(\phi+7)}{2(2N-5)\phi(\alpha^2(N-4)(N-2)-2\alpha(N-4)(N-2)-4N+7)+2(N-2)(-5\alpha+2(\alpha-2)N+7)(-4\alpha+(\alpha-4)N+7)}, \frac{\phi\big(2\alpha^2(N-4)(N-2)^2+\alpha(N((27-4N)N-64)+53)+8N-14\big)+(N-2)(-5\alpha+2(\alpha-2)N+7)(-17\alpha+(5\alpha-16)N+28)}{2(2N-5)\phi(\alpha^2(N-4)(N-2)-2\alpha(N-4)(N-2)-4N+7)+2(N-2)(-5\alpha+2(\alpha-2)N+7)(-4\alpha+(\alpha-4)N+7)}\}.$$

$$\underline{v_1}^{Contest} = \max\{\tfrac{2\alpha(-\alpha(4\phi+11)+5\phi+21)+2(\alpha-2)\alpha N^3-N^2(\alpha(\alpha(\phi+14)-25)+8)+N(\alpha(\alpha(6\phi+31)-4\phi-57)+26)-3(\phi+7)}{2(\alpha(N-2)+1)(\alpha(4\phi+11)+2(\alpha-2)N^2+N(-\alpha(\phi+10)+2\phi+15)-5\phi-14)}, \tfrac{-2\left(\alpha^2(2\phi+37)-3\alpha(\phi+23)+\phi+28\right)+6(\alpha-2)\alpha N^3+((79-43\alpha)\alpha-16)N^2+N(\alpha(2(\alpha-1)\phi+99\alpha-181)+60)}{2(\alpha(N-2)+1)(\alpha(4\phi+11)+2(\alpha-2)N^2+N(-\alpha(\phi+10)+2\phi+15)-5\phi-14)}\}.$$

$$\overline{v}^{Contest} = \min\{\tfrac{1}{2} - \frac{(N-1)(4N-7)}{2\phi(\alpha(N-2)+1)+2(N-2)(-4\alpha+(\alpha-4)N+7)}, \frac{2\phi(\alpha(N-2)+1)+(N-2)(-17\alpha+(5\alpha-16)N+28)}{2\phi(\alpha(N-2)+1)+2(N-2)(-4\alpha+(\alpha-4)N+7)}, \frac{\alpha(N-1)(6\alpha+N(-5\alpha+(\alpha+4)N-14)+8)-2\phi(\alpha(N-3)+2)(\alpha(N-2)+1)}{2(N-3)(\alpha(N-2)+1)(\alpha(N-2\phi-1)+2\phi)}\}.$$

# Online Appendix

## OA1 Global Optimality of $p_i^*$ in the Main Analysis

In this appendix, we prove that the local equilibrium we derived in the main analysis is also a global equilibrium under Condition 1.

### OA1.1 Optimality of $p_i^{BM}$ in Case "BM"

As a final step, we need to find the condition under which seller $i$ has no incentive to deviate to $p_i < v-1$ or $p_i > v - \frac{1}{2}$ given the other sellers price at $p^*$.

- It's easy to see that seller $i$ has no incentive to deviate to $p_i < v-1$. This is because seller $i$'s profit function upon such an deviation, $D_i(p_i)' = \frac{1}{2}\left(Q^2_{ao}(p_i,p^*)+(N-2)\overline{Q^2_m}\right)+\frac{1}{2}\left(q^1_{ao}(p^*,p_i)+(N-2)\overline{q_m}\right)$, is strictly less than its demand function $D_i(p_i)$ when $v-1 < p_i < v-\frac{1}{2}$ (because $Q^2_m(p_i)$ is capped at $\overline{Q^2_m}$, and $q_m(p_i)$ is capped at $\overline{q_m}$). Since seller $i$ has no incentive to deviate to $p_i < p_i^*$ under the higher demand function $D_i(p_i)$, it neither has an incentive to do so under the smaller demand function $D_i(p_i)'$.

- Consider the seller $i$'s profit by deviating to $p_i > v - \frac{1}{2}$. In this case seller $i$'s demand is $\frac{1}{2}\left(Q^1_{ao}(p_i,p^*) + (N-2)Q^1_m\right) + \frac{1}{2}\left(q^2_{ao}(p^*,p_i) + (N-2)q_m\right)$. Seller $i$'s profit is maximized at $p_i' = \frac{\alpha^2(N-2)^2(3N-8)(2v-1)-2\alpha(N-2)(N((8N-42)v+7)+48v-14)-8(2N-3)(2(N-2)v+1)}{2(-2\alpha+(\alpha-4)N+6)(22\alpha+N(\alpha(4N-19)+8)-14)}$. A sufficient condition to ensure that seller $i$ will not deviate to $p_i > v-\frac{1}{2}$ is $p_i' < v - \frac{1}{2}$. This requires $v > \underline{v_2}^{BM}$, where $\underline{v_2}^{BM} \equiv \frac{-12\alpha^2+104\alpha+\alpha^2N^3-16\alpha N^3-7\alpha^2N^2+94\alpha N^2-32N^2+16\alpha^2N-176\alpha N+88N-60}{-24\alpha^2+128\alpha+2\alpha^2N^3-16\alpha N^3-14\alpha^2N^2+100\alpha N^2-32N^2+32\alpha^2N-200\alpha N+96N-72}$.

To summarize, $p_i^*$ is an equilibrium when $\underline{v_1}^{BM} < v < \overline{v}^{BM}$ and $v > \underline{v_2}^{BM}$. Since $\underline{v_2}^{BM} > \underline{v_1}^{BM}$, this simplifies into $\underline{v}^{BM} < v < \overline{v}^{BM}$, where $\underline{v}^{BM} = \underline{v_2}^{BM}$.

### OA1.2 Optimality of $p_z^{Ad}$ and $p_k^{Ad}$ in Case "Ad"

Now we show that under $\underline{v}^{Ad} < v < \overline{v}^{Ad}$, neither seller has an incentive to deviate out of the range of $v-1 < p < v - \frac{1}{2}$ given that the other seller prices at the equilibrium price level.

- For the winning seller $z$, since seller $z$ pays for seller $k$'s bid, which does not depend on seller $z$'s price, below, we only compare seller $z$'s sales revenue upon deviation.

– If $p_z < v-1$, the upper bound of seller $z$'s demand is $D_z(p_z) = Q^2_{ao}(p_z, p^*_k) + (N-2)\overline{Q^2_m} = \frac{\alpha(N-2)(N+p^*_k-p_z-2)+2N+p^*_k-p_z-3}{(N-1)N}$. From $\frac{\partial(1-\phi)\left(p_z D_z(p_z)\right)}{p_z}\Big|_{p_z=p'_z} = 0$ we have $p'_z = \frac{1}{2}\left(\frac{N-1}{\alpha(N-2)+1} + N + p^*_k - 2\right)$. Since $p'_z > v-1$ when $1 < v < 2$, seller $z$ has no incentive to price lower than $v-1$.

– If $p_z > v - \frac{1}{2}$, seller $z$'s demand is $D_z(p_z) = Q^1_{ao}(p_z, p^*_k) + (N-2)Q^1_m(p_z) = \frac{(3-2N)p_z+2(N-2)v+p^*_k+1}{(N-1)N}$. $\frac{\partial(1-\phi)\left(p_z D_z(p_z)\right)}{p_z}\Big|_{p_z=p'_z} = 0$ leads to $p'_z = \frac{2(N-2)v+p^*_k+1}{4N-6}$. To ensure that it's unprofitable for seller $z$ to deviate, a sufficient condition is $p'_z < v - \frac{1}{2}$, which requires $v > \frac{-52\alpha+16\alpha N^3-32N^3-73\alpha N^2+128N^2+108\alpha N-170N+75}{-60\alpha+16\alpha N^3-32N^3-76\alpha N^2+136N^2+118\alpha N-192N+90}$. Since the above amount is smaller than $\underline{v}^{Ad}$, it's unprofitable for seller $z$ to deviate to this range.

- Now consider the losing seller $k$'s pricing strategy.

  – If $p_k < v-1$, the upper bound of seller $k$'s demand comes as $D_k(p_k) = q^1_{ao}(p^*_z, p_k) + (N-2)\overline{q_m} = \frac{-2\alpha+\alpha N-2N+p_k-p^*_z+3}{N-N^2}$. From $\frac{\partial(1-\phi)\left(p_k D_k(p_k)\right)}{p_k}\Big|_{p_k=p'_k} = 0$ we have $p'_k = \frac{1}{2}(2\alpha-\alpha N+2N+p^*_z-3)$. As $p'_k > v-1$ under $1 < v < 2$, seller $k$ has no incentive to deviate to price lower than $v-1$.

  – If $p_k \geq v - \frac{1}{2}$, we have seller $k$'s demand as $D_k(p_k) = q^2_{ao}(p^*_z, p_k) + (N-2)q_m(p_k) = \frac{\alpha(N-2)(p_k+p^*_z-2v+1)-2Np_k+2Nv+3p_k+p^*_z-4v+1}{(N-1)N}$. $\frac{\partial(1-\phi)\left(p_k D_k(p_k)\right)}{p_k}\Big|_{p_k=p'_k} = 0$ results in $p'_k = -\frac{\alpha(N-2)(p^*_z-2v+1)+2Nv+p^*_z-4v+1}{-4\alpha+2(\alpha-2)N+6}$. A sufficient condition for seller $k$ to not deviate is $p'_k < v - \frac{1}{2}$, which translates into $v > \underline{v}^{Ad}$, with $\underline{v}^{Ad}$ as given before.

### OA1.3 Optimality of $p^{PL}_l$ and $p^{PL}_i$ in Case "PL"

To establish the equilibrium, we also need to find the condition under which given $p^*_l$, it is sub-optimal for seller $i$ to price at $p_i < v-1$ or $p_i > v - \frac{1}{2}$ and given $p^*_i$, it is sub-optimal for the platform to price the private label at $p_l < v-1$ or $p_l > v - \frac{1}{2}$.

- Consider seller $i$ first.

  – If $p_i < v-1$, we derive the upper bound of seller $i$'s demand upon such deviation (i.e., without considering the cap on its demand when $p_i < p^*_i - 1$ or $p_i < p^*_l - 1$) and show that seller $i$ has no incentive to deviate even if the deviation results in the upper bound of its profit. Following the same notations as the case with $v-1 < p_i < v - \frac{1}{2}$, the upper bound of seller $i$'s demand when the private label gets the top slot is $d^{lose1}_i = q^1_{ao}(p^*_l, p_i) + q_{oo}(p^*_i, p_i) + (N-3)\overline{q_m}$. Its demand's upper bound when the other third-party sellers obtains the top slot is $d_i^{lose2} = q_{ao}^1(p_i^*, p_i) + q_{oo}(p_l^*, p_i) + (N-3)\overline{q_m}$. If seller $i$ obtains the top slot, the upper bound of its demand is $d_i^{win} = Q_{ao}^2(p_i, p_i^*) + Q_{ao}^2(p_i, p_l^*) + (N-3)\overline{Q_m^2}$. On average, seller $i$'s demand when deviating to price at $p_i < v-1$ is no more than $D_i = \frac{1}{3}d_i^{lose1} + \frac{1}{3}d_i^{lose2} + \frac{1}{3}d_i^{win} = \frac{\alpha(N-3)(N-2p_i+p_i^*+p_l^*-4)+3(2N-2p_i+p_i^*+p_l^*-4)}{3(N-1)N}$. We find that $(1-\phi)p_iD_i$ is maximized at $p_i' = \frac{1}{4}\left(\frac{3N}{\alpha(N-3)+3} + N + p_i^* + p_l^* - 4\right)$. Furthermore, $p_i' > v-1$, suggesting that seller $i$ has no incentive to deviate to $p_i < v-1$ even under the upper bound of its demand.

– If $p_i > v - \frac{1}{2}$, seller $i$'s demand is $D_i = \frac{1}{3}d_i^{lose1} + \frac{1}{3}d_i^{lose2} + \frac{1}{3}d_i^{win}$, where $d_i^{lose1} = q_{ao}^2(p_l^*, p_i) + q_{oo}(p_i^*, p_i) + (N-3)q_m(p_i)$, $d_i^{lose2} = q_{ao}^2(p_i^*, p_i) + q_{oo}(p_l^*, p_i) + (N-3)q_m(p_i)$, $d_i^{win} = Q_{ao}^1(p_i, p_i^*) + Q_{ao}^1(p_i, p_l^*) + (N-3)Q_m^1(p_i)$. We find that seller $i$ maximizes its profit at $p_i' = -\frac{\alpha(N-3)(p_i^*+p_l^*-4v+2)+3(2(N-3)v+}{4(-3\alpha+(\alpha-3)N+6)}$
To ensure that seller $i$ has no incentive to deviate, we need $v > \underline{v_2}^{PL}$, where $\underline{v_2}^{PL}$ is defined in Appendix A4.

- Now consider the price of the private label.

  – If $p_l^* < v-1$, the platform's profit function is $\Pi_p = \frac{1}{3}\left(p_l D_l^{win} + \phi * 2 * p_i^* d_i^{lose1}\right) + \frac{2}{3}\left(p_l D_l^{lose} + \phi * p_i^* d_i^{win} + \phi * p_i^* d_i^{lose2}\right)$, where

$$
\begin{aligned}
D_l^{win} &= 2Q_{ao}^2(p_l, p_i^*) + (N-3)\overline{Q_m^2}, \\
d_i^{lose1} &= q_{ao}^2(p_l, p_i^*) + q_{oo}(p_i^*, p_i^*) + (N-3)q_m(p_i^*), \\
D_l^{lose} &= q_{ao}^1(p_i^*, p_l) + q_{oo}(p_i^*, p_l) + (N-3)\overline{q_m}, \\
d_i^{win} &= Q_{ao}^1(p_i^*, p_i^*) + Q_{ao}^1(p_i^*, p_l) + (N-3)Q_m^2(p_i^*), \\
d_i^{lose2} &= q_{ao}^1(p_i^*, p_i^*) + q_{oo}(p_l, p_i^*) + (N-3)q_m(p_i^*).
\end{aligned}
$$

  We find that the level of $p_l'$ that maximizes $\Pi_p$ is larger than $v-1$. Consequently, the platform has no incentive to reduce the private label price below $v-1$.

  – If $p_l^* > v - \frac{1}{2}$, the demand is the same as in the case with $v - 1 < p_l^* < v - \frac{1}{2}$ except for $Q_m^2(p_l)$ in $D_l^{win}$ is replaced by $Q_m^1(p_l)$. The platform maximizes its profit at $p_l' = -\frac{\alpha(N-3)+p_i^*(\phi+1)(\alpha(N-3)+3)-(2\alpha-3)(N-3)v+3}{2(-3\alpha+(\alpha-3)N+6)}$. A sufficient condition to ensure that the platform does not deviate is $p_l' < v - \frac{1}{2}$. This requires $v > \underline{v_3}^{PL}$ (given in Appendix A4).

Combined, $p_l^*$ and $p_i^*$ is in equilibrium when $\max\{\underline{v_1}^{PL}, \underline{v_2}^{PL}, \underline{v_3}^{PL}\} < v < \overline{v}^{PL}$. Since $\underline{v_3}^{PL}$ is larger than $\underline{v_1}^{PL}$ and $\underline{v_2}^{PL}$, the above condition simplifies into $\underline{v}^{PL} < v < \overline{v}^{PL}$, where $\underline{v}^{PL} = \underline{v_3}^{PL}$.

### OA1.4 Optimality of $p_z^{Concede}, p_k^{Concede}$ and $p_l^{Concede}$ in Case "Concede"

To further establish the above solution as an equilibrium, we show that a seller has no incentive to deviate to $p < v - 1$ or $p \geq v - \frac{1}{2}$ under $\underline{v}^{Concede} < v < \overline{v}^{Concede}$.

- Consider seller $z$'s profit when deviating. Again, since seller $z$ pays for seller $k$'s bid, which does not depend on seller $z$'s price, below, we only compare seller $z$'s sales revenue upon deviation.
  - If $p_z < v - 1$, the upper bound of seller $z$'s demand in this case is $D_z(p_z) = Q_{ao}^2(p_z, p_k^*) + Q_{ao}^2(p_z, p_l^*) + (N-3)\overline{Q_m^2} = \frac{\alpha(N-2)(N+p_k^*+p_l^*-2p_z-3)+2N+p_k^*+p_l^*-2p_z-4}{(N-1)N}$. Seller $z$'s sales revenue $(1-\phi)p_z D_z(p_z)$ is maximized at $p_z' = \frac{1}{4}\left(\frac{N-1}{\alpha(N-2)+1} + N + p_k^* + p_l^* - 3\right)$. Furthermore, we find that $p_z' > v - 1$ when $N \geq 5, 1 < v < 2$. Therefore, deviating to $p_z < v - 1$ is sub-optimal for seller $z$.
  - When $p_z > v - \frac{1}{2}$, seller $z$'s demand comes as $(p_z, p_k^*) + Q_{ao}^1(p_z, p_l^*) + (N-3)Q_m^1(p_z) = \frac{-2Np_z+2(N-3)v+p_k^*+p_l^*+4p_z+2}{(N-1)N}$. Seller $z$'s revenue is maximized at $p_z' = \frac{2(N-3)v+p_k^*+p_l^*+2}{4(N-2)}$. To ensure that seller $z$ has no incentive to deviate to $p_z > v - \frac{1}{2}$, a sufficient condition is $p_z' < v - \frac{1}{2}$ which requires $v > \frac{\alpha(5\alpha(4\phi-35)-37\phi+455)+16(\alpha-2)^2N^3+(\alpha-2)N^2(3\alpha(\phi-38)+160)+N(\alpha(-17\alpha(\phi-15)+31\phi-743)-8\phi+532)+14(\phi-21)}{4(\alpha(8(\alpha-2)\phi-55\alpha+147)+4(\alpha-2)^2N^3+(\alpha-2)N^2(\alpha(\phi-30)+44)+N(\alpha(-6(\alpha-2)\phi+72\alpha-217)-4\phi+161)+7(\phi-14))}$. Since the above amount is smaller than $\underline{v}^{Concede}$, seller $z$ will not deviate to $p_z > v - \frac{1}{2}$ under the condition that $\underline{v}^{Concede} < v < \overline{v}^{Concede}$.

- Consider seller $k$'s profit when deviating.
  - If $p_k < v - 1$, the upper bound of seller $i$'s demand in this case is $D_k(p_k) = q_{ao}^1(p_z^*, p_k) + q_{oo}(p_l^*, p_k) + (N-3)\overline{q_m} = \frac{-\alpha(N-p_k+p_l^*-3)+2N-2p_k+p_l^*+p_z^*-4}{(N-1)N}$. Seller $k$'s profit $(1-\phi)p_k D_k(p_k)$ is maximized at $p_k' = \frac{1}{2}\left(N + \frac{p_l^*-p_z^*-2}{\alpha-2} + p_l^* - 3\right)$. We find that $p_k' > v - 1$ when $1 < v < 2$. Therefore, deviating to $p_k < v - 1$ is sub-optimal for seller $k$.
  - If deviating to $p_k > v - \frac{1}{2}$, seller $k$'s demand is $D_k(p_k) = q_{ao}^2(p_l^*, p_k) + q_{oo}(p_i^*, p_k) + (N-3)q_m(p_k)$, $= \frac{\alpha(N(p_k+p_z^*-2v+1)-3p_k-p_l^*-2p_z^*+6v-3)-2Np_k+2Nv+4p_k+p_l^*+p_z^*-6v+2}{(N-1)N}$. Seller $k$'s sales revenue is maximized at $p_k' = \frac{-p_z^*(\alpha(N-2)+1)+\alpha(N-3)(2v-1)-2Nv+(\alpha-1)p_l^*+6v-2}{-6\alpha+2(\alpha-2)N+8}$. A sufficient condition to avoid seller $k$ from deviating is $p_k' < v - \frac{1}{2}$ which requires $v > \frac{-2\alpha\phi+8\alpha+\alpha N^2-8N^2+\alpha N\phi-6\alpha N+26N+\phi-21}{-4\alpha\phi+16\alpha+2\alpha N^2-8N^2+2\alpha N\phi-12\alpha N+30N+2\phi-28}$.

Since the above amount is smaller than $\underline{v}^{Concede}$, seller $k$ does not benefit by choosing $p_k > v-\frac{1}{2}$ under the condition that $\underline{v}^{Concede} < v < \overline{v}^{Concede}$.

- Consider the platform's profit upon deviation.
  - If $p_l < v-1$, the private label's demand is $D_l(p_l) = q^1_{ao}(p^*_z, p_l) + q_{oo}(p^*_k, p_l) + (N-3)\overline{q_m}$. Seller $k$'s demand is $D_k(p^*_k) = q^2_{ao}(p^*_z, p^*_k) + q_{oo}(p_l, p^*_k) + (N-3)q_m(q^*_k)$. Seller $z$'s demand is $D_z(p^*_z) = Q^2_{ao}(p^*_z, p^*_k) + Q^1_{ao}(p^*_z, p_l) + (N-3)Q^2_m(q^*_z)$. The platform's profit, $p_l D_l(p_l) + \phi p^*_z D_z(p^*_z) + \phi p^*_k D_k(p^*_k)$, is maximized at $p'_l = \frac{-3\alpha+(\alpha-2)N+(\alpha-1)p^*_k(\phi+1)-p^*_z(\phi+1)+4}{2(\alpha-2)}$. Since $p'_l > v-1$ when $1 < v < 2$, the platform does not benefit from choosing $p_l < v-1$.
  - Consider the case with $p_l > v-\frac{1}{2}$. In this case, all sellers' demand is the same as in the main case with $v-1 < p_l < v-\frac{1}{2}$. Consequently, the platform's optimal price is $p^*_l$ and it does not benefit from deviating to $p_l > v-\frac{1}{2}$.

### OA1.5 Optimality of $p_l^{Contest}$ and $p_i^{Contest}$ in Case "Contest"

Next, we derive the condition under which a seller will not deviate to $p < v-1$ or $p > v-\frac{1}{2}$.

- Consider seller $i$'s profit when deviating.
  - First, it's easy to see that seller $i$ has no incentive to deviate to $p_i < v-1$, as it's demand function is weakly smaller than its demand function with $v-1 < p_i < v-\frac{1}{2}$ (because $q_o$ will be capped at $\overline{q_o}$).
  - If $p_i > v-\frac{1}{2}$, seller $i$'s demand is $D_i(p_i) = q^2_{ao}(p^*_l, p_i) + q_{oo}(p^*_i, p_i) + (N-3)q_m(p_i) = \frac{\alpha(N(p_i+p^*_l-2v+1)-3p_i-p^*_i-2p^*_l+6v-3)-2Np_i+2Nv+4p_i+p^*_i+p^*_l-6v+2}{(N-1)N}$. Seller $i$'s profit is maximized at $p'_i = \frac{-p^*_l(\alpha(N-2)+1)+\alpha(N-3)(2v-1)-2Nv+(\alpha-1)p^*_i+6v-2}{-6\alpha+2(\alpha-2)N+8}$. To ensure that seller $i$ has no incentive to deviate to $p_i > v-\frac{1}{2}$, a sufficient condition is $p'_i < v-\frac{1}{2}$ which requires $v > \frac{-2\alpha\phi+8\alpha+\alpha N^2-8N^2+\alpha N\phi-6\alpha N+26N+\phi-21}{-4\alpha\phi+16\alpha+2\alpha N^2-8N^2+2\alpha N\phi-12\alpha N+30N+2\phi-28}$. Since the above amount is smaller than $\underline{v}^{Contest}$, seller $i$ will not deviate to $p_i > v-\frac{1}{2}$ under the condition that $\underline{v}^{Contest} < v < \overline{v}^{Contest}$.
- Next, consider the platform's profit upon deviation.
  - If $p_l < v-1$, the private label's demand is $D_l(p_l) = 2Q^2_{ao}(p_l, p^*_i) + (N-3)\overline{Q^2_m}$. Seller $i$'s demand is $D_i(p^*_i) = q^2_{ao}(p_l, p^*_i) + q_{oo}(p^*_i, p^*_i) + q_m(q^*_i)$. The platform's profit, $p_l D_l(p_l) + 2\phi p^*_i D_i(p^*_i)$, is

maximized at $p_l' = \frac{6\alpha+\alpha N^2+N(\alpha(2p_i^*(\phi+1)-5)+2)-2(2\alpha-1)p_i^*(\phi+1)-4}{4\alpha(N-2)+4}$. A sufficient condition for the platform not to deviate is $p_l' > v-1$, which is met under $N \geq 5, 1 < v < 2$.

– It's easy to see that the platform has no incentive to deviate to $p_l > v - \frac{1}{2}$, as it leads to a lower demand function than the case with $v - 1 < p_l < v - \frac{1}{2}$. As the platform will not price $p_l$ higher than $p_l^*$ even under the larger demand function, it will not benefit from doing so with the lower demand function.

In summary, $p_i^*$ and $p_l^*$ is an equilibrium when $N \geq 5, \underline{v}^{Bais} < v < \overline{v}^{Contest}$.

## OA2 Online Appendix for the Case with a High Product Valuation

In this appendix, we solve the equilibrium for the high product valuation case. We show that in the benchmark case, $p_i^* = v - 1$ is an equilibrium when $v \geq 6, N < v < 2N-2, 0 < \alpha < \frac{-4N+2v+4}{N^2-Nv-4N+2v+4}$. In Case "PL", the equilibrium, $p_i^* = v-2, p_l^* = v-1$, holds when $N + \frac{1}{2} < v < N+2$ and $\phi > \frac{-12\alpha-\alpha N^2+7\alpha N+2\alpha Nv-6N-6\alpha v+6v+12}{12\alpha-4\alpha N+2\alpha Nv-6\alpha v+6v-12}$. In Case "Ad", the equilibrium is $p_z^* = p_k^* = v-1$ when $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$. In Case "Concede", $p_l^* = v-1, p_z^* = p_k^* = v-2$ is an equilibrium under the condition that $N + \frac{1}{2} < v < 2\alpha - \alpha N + 2N - 1$. In Case "Contest", $p_i^* = v-2, p_l^* = v-1$ is an equilibrium in the parameter space of $0 < \alpha < \min\{\frac{2N-2v+1}{N-v-1}, \frac{2N-8}{3N-9}\}$, $\frac{2\alpha-\alpha N^2+\alpha N+2\alpha Nv-2N-4\alpha v+2v}{8\alpha-4\alpha N+2\alpha Nv-4\alpha v+2v-4} < \phi < 1$, $v \leq \frac{1}{2}\left(6\alpha+\alpha N^2-5\alpha N+2N-4\right)$. Furthermore, as $\frac{-4N+2v+4}{N^2-Nv-4N+2v+4} > \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$, the joint set of the above conditions is equivalent to Condition 3.

### OA2.1 Solution for Case "BM"

Assume that the other third-party seller prices at the equilibrium price level $p^* = v-1$. We need to find the condition which, given that the other seller prices at $p^* = v-1$, seller $i$ has no incentive to deviate to any $p_i \neq p^* = v-1$. Below we show that the condition required is $v \geq 6, N < v < 2N-2, 0 < \alpha < \frac{-4N+2v+4}{N^2-Nv-4N+2v+4}$.

- First, it is sub-optimal for seller $i$ to charge $p_i < v-2$, because it leads to the same demand as charging $p_i = v-2$ but lower revenue.

- If seller $i$ deviates to charge a lower price $v-2 < p_i < v-1$, based on Lemma 1, its demand is

$$D_i(p_i) = \frac{1}{2}\Big(Q^2_{ao}(p_i, p^*) + (N-2)\overline{Q^2_m}\Big) + \frac{1}{2}\Big(q^1_{ao}(p^*, p_i) + (N-2)\overline{q_m}\Big),$$
$$= \frac{\alpha(N-2)(N-p_i+v-4) + 2(2N-p_i+v-4)}{2(N-1)N}.$$

To ensure that seller $i$ has no incentive to deviate to price lower than $v-1$, we need the value of $p_i$ which maximizes $(1-\phi)p_iD_i(p_i)$ to be larger than $p^* = v-1$. That is,

$$\frac{N}{\alpha(N-2)+2} + \frac{1}{2}(N+v-4) \geq v-1.$$

A sufficient condition for the above inequality to hold is $v \geq 6, v < N < 2N-2, 0 < \alpha < \frac{-4N+2v+4}{N^2-Nv-4N+2v+4}$.

- Next, consider the deviation with $v-1 < p_i < v-\frac{1}{2}$. In this case, seller $i$'s demand is,

$$D_i(p_i) = \frac{1}{2}\Big(Q^1_{ao}(p_i, p^*) + (N-2)Q^2_m(p_i)\Big) + \frac{1}{2}\Big(q^2_{ao}(p^*, p_i) + (N-2)q_m(p_i)\Big),$$
$$= -\frac{\alpha(N-2)(2Np_i - 2Nv + N - 5p_i + 5v - 2) + 2(2N-3)(p_i - v)}{2(N-1)N}.$$

To stop seller $i$ from deviating to any $v-1 < p_i < v-\frac{1}{2}$, we need the value of $p_i$ which maximizes $(1-\phi)p_iD_i(p_i)$, $\frac{1}{2}\left(v - \frac{\alpha(N-2)^2}{10\alpha+N(\alpha(2N-9)+4)-6}\right)$, to be smaller than $v-1$. A sufficient condition to ensure this is $v > \frac{16\alpha+3\alpha N^2-14\alpha N+8N-12}{10\alpha+2\alpha N^2-9\alpha N+4N-6}$. Since the RHS is smaller than $N$, it is satisfied under $\frac{v+2}{2} < N < v$.

- Finally, if $p_i$ deviates to price at $p_i > v-\frac{1}{2}$, its demand is

$$D_i(p_i) = \frac{1}{2}\Big(Q^1_{ao}(p_i, p^*) + (N-2)Q^1_m(p_i)\Big) + \frac{1}{2}\Big(q^2_{ao}(p^*, p_i) + (N-2)q_m(p_i)\Big),$$
$$= \frac{(-2\alpha + (\alpha-4)N + 6)(p_i - v)}{2(N-1)N}.$$

Since the value of $p_i$ which maximizes $(1-\phi)p_iD_i(p_i)$, i.e., $\frac{v}{2}$, is smaller than $p^* = v - \frac{1}{2}$ for any $v \geq 1$, seller $i$ has no incentive to deviate to $p_i > v-\frac{1}{2}$.

Overall, $p_i^* = v-1$ is an equilibrium when $v \geq 6, N < v < 2N-2, 0 < \alpha < \frac{-4N+2v+4}{N^2-Nv-4N+2v+4}$. Note that we can further show that any interior solution with $p^* < v-1$ cannot be in equilibrium in this parameter space.

The total surplus in the benchmark case is as follows.

$$
\begin{aligned}
TS^{BM} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-2)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_0^{\frac{1}{2}}(v-(1-x))2dx \\
&+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_0^{\frac{1}{2}}(v-x)2dx + (N-2)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^{1}(v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $z$ who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and they all buy their second-preferred variety $z$. The third term represents consumers whose first-preferred variety is $k$ and their second-preferred variety is $z$. Consumers in this segment all purchase from $k$ given $p_k = p_z = v-1$. The fourth term represents consumers who purchase their only available variety $k$.

### OA2.2 Solution for Case "PL"

In this case, the equilibrium is $p_i^* = v-2, p_l^* = v-1$ which holds when $N+\frac{1}{2} < v < N+2$ and $\phi > \frac{-12\alpha-\alpha N^2+7\alpha N+2\alpha Nv-6N-6\alpha v+6v+12}{12\alpha-4\alpha N+2\alpha Nv-6\alpha v+6v-12}$.

- Consider seller $i$ first. We show that given the other third-party seller prices at $v-2$ and the private label prices at $v-1$, seller $i$ has no incentive to deviate to the following price levels.
  - First, seller $i$ does not improve its profit by deviating to $p_i < v-2$. If deviating to price at $p_i < v-2$, following the same notation as in Appendix A2.3, we have seller $i$'s demand as follows,

$$
\begin{cases}
d_i^{lose1} = \overline{q_{ao}^1}(v-1,p_i) + q_{oo}(v-2,p_i) + (N-3)\overline{q_m}, \\
d_i^{lose2} = q_{ao}^1(v-2,p_i) + \overline{q_{oo}}(v-1,p_i) + (N-3)\overline{q_m}, \\
d_i^{win} = Q_{ao}^2(p_i,v-2) + \overline{Q_{ao}^2}(p_i,v-1) + (N-3)\overline{Q_m^2}.
\end{cases}
$$

    Average across the above three possibilities, seller $i$'s demand when deviating to price at $p_i$ is,

$$
D_i = \frac{\alpha(N-3)(N-p_i+v-5)+3(2N-p_i+v-5)}{3(N-1)N}.
$$

We need the level of $p_i$ that maximize $(1-\phi)p_iD_i$ to be larger than $v-2$, that is,

$$\frac{1}{2}\left(\frac{3N}{\alpha(N-3)+3}+N+v-5\right)>v-2.$$

The above condition is satisfied for any $v<N+2$.

– Second, seller $i$ has no incentive to deviate to $v-2<p_i<v-1$. If deviating to price at $v-2<p_i<v-1$, we have

$$\begin{cases} d_i^{lose1}=q_{ao}^1(v-1,p_i)+q_{oo}(v-2,p_i)+(N-3)\overline{q_m}, \\ d_i^{lose2}=q_{ao}^2(v-2,p_i)+q_{oo}(v-1,p_i)+(N-3)\overline{q_m}, \\ d_i^{win}=Q_{ao}^1(p_i,v-2)+Q_{ao}^2(p_i,v-1)+(N-3)\overline{Q_m^2}. \end{cases}$$

Average across the above three possibilities, seller $i$'s demand when deviating to price at $p_i$ is,

$$D_i=\frac{1}{3}d_i^{lose1}+\frac{1}{3}d_i^{lose2}+\frac{1}{3}d_i^{win}=\frac{\alpha(N-3)(N-2p_i+2v-7)+6N-6p_i+6v-21}{3(N-1)N}.$$

The optimal $p_i$ in this case, $\frac{1}{4}\left(\frac{3N}{\alpha(N-3)+3}+N+2v-7\right)$, is smaller than $v-2$ when $v>N+\frac{1}{2}$.

– Next, we need to show that seller $i$ will not deviate to $v-1\leq p_i\leq v-\frac{1}{2}$. If deviating, we have

$$\begin{cases} d_i^{lose1}=q_{ao}^2(v-1,p_i)+(N-3)q_m(p_i), \\ d_i^{lose2}=q_{oo}(v-1,p_i)+(N-3)q_m(p_i), \\ d_i^{win}=Q_{ao}^1(p_i,v-1)+(N-3)Q_m^2(p_i). \end{cases}$$

Average across the above three possibilities, seller $i$'s demand when deviating to price at $p_i$ is,

$$D_i=-\frac{\alpha(N-3)(2Np_i-2Nv+N-7p_i+7v-2)+3(2N-5)(p_i-v)}{3(N-1)N}.$$

We need the level of $p_i$ that maximize $(1-\phi)p_iD_i$, $\frac{\alpha(N-3)(N(2v-1)-7v+2)+3(2N-5)v}{6(7\alpha-5)+2N(\alpha(2N-13)+6)}<v-1$, to be smaller than $v-1$. This is ensured when $v>N$.

– Finally, seller $i$ will not benefit from $p_i>v-\frac{1}{2}$. The expected demand in this case is $D_i=\frac{(-3\alpha+(\alpha-2)N+5)(p_i-v)}{(N-1)N}$. The optimal $p_i$ $\frac{v}{2}$, is smaller than $v-\frac{1}{2}$ for any $v\geq 1$.

– Put together with $N-1 < v \leq N$, a sufficient condition to ensure that seller $i$ has no incentive to deviate from $N + \frac{1}{2} < v < N + 2$.

- Next, consider the platform. We show that given the two third-party sellers price at $v-2$, the private label has no incentive to deviate from $v-1$. This entails the following scenarios.

  – When $p_l < v-2$, the platform's average profit is the same as in the case below (from Result 1) so we discuss it together with the case below.

  – When $v-2 < p_l < v-1$, following the same notation as in Appendix A2.3, demand function is

$$\begin{aligned}
D_l^{win} &= 2Q_{ao}^1(p_l, v-2) + (N-3)\overline{Q_m^2},\\
d_i^{lose1} &= q_{ao}^1(p_l, v-2) + q_{oo}(v-2, v-2) + (N-3)\overline{q_m},\\
D_l^{lose} &= q_{ao}^2(v-2, p_l) + q_{oo}(v-2, p_l) + (N-3)\overline{q_m},\\
d_i^{win} &= Q_{ao}^1(v-2, v-2) + Q_{ao}^2(v-2, p_l) + (N-3)\overline{Q_m^2},\\
d_i^{lose2} &= q_{ao}^1(v-2, v-2) + q_{oo}(p_l, v-2) + (N-3)\overline{q_m}.
\end{aligned}$$

Consequently, the platform's profit is,

$$\begin{aligned}
\Pi_p =& \frac{1}{3}\Big(p_l D_l^{win} + \phi * 2 * (v-2) d_i^{lose1}\Big) + \frac{2}{3}\Big(p_l D_l^{lose} + \phi * (v-2) * d_i^{win} + \phi * (v-2) d_i^{lose2}\Big),\\
=& \frac{1}{3(N-1)N}\Big(2(v-2)\phi(\alpha(N-3)(N+p_l-v-2)+\\
& 3(2N+p_l-v-2)) + \alpha(N-3)p_l(N-2p_l+2v-8) + 6p_l(N-p_l+v-4)\Big).
\end{aligned}$$

To make sure that the platform has no incentive to price its private label at $p \leq v-1$, we need the level of $p_l$ that maximizes $\Pi_p$ to be larger than $v-1$, that is,

$$\frac{\alpha N^2 + N(\alpha(2(v-2)\phi + 2v - 11) + 6) - 6(\alpha-1)((v-2)\phi + v - 4)}{4\alpha(N-3) + 12} > v - 1.$$

This is equivalent to

$$\phi > \frac{-12\alpha - \alpha N^2 + 7\alpha N + 2\alpha N v - 6N - 6\alpha v + 6v + 12}{12\alpha - 4\alpha N + 2\alpha N v - 6\alpha v + 6v - 12}.$$

– When $v - 1 < p_l < v - \frac{1}{2}$, the demand function is

$$\begin{aligned} D_l^{win} &= (N-3)Q_m^2(p_l), \\ d_i^{lose1} &= \overline{q_{ao}^1} + q_{oo}(v-2, v-2) + (N-3)\overline{q_m}, \\ D_l^{lose} &= (N-3)q_m(p_l), \\ d_i^{win} &= Q_{ao}^1(v-2, v-2) + \overline{Q_{ao}^2} + (N-3)\overline{Q_m^2}, \\ d_i^{lose2} &= q_{ao}^1(v-2, v-2) + \overline{q_{oo}} + (N-3)\overline{q_m}. \end{aligned}$$

The platform's profit is $\Pi_p = \frac{2(v-2)\phi\left(\alpha(N-3)^2+6N-9\right)-(N-3)p_l(\alpha(N-2)+2\alpha(N-4)(p_l-v)+6p_l-6v)}{3(N-1)N}$. The price $p_l$ that maximizes the above profit, $\frac{v}{2} - \frac{\alpha(N-2)}{4(\alpha(N-4)+3)}$, is smaller than $v-1$. As a result, the platform does not benefit from pricing higher than $v-1$.

– When $p_l > v - \frac{1}{2}$, the demand function can be obtained by replacing $Q_m^2(p_l)$ with $Q_m^1(p_l)$ in $D_l^{win}$ in the above case. The platform's profit $\frac{2(2\alpha-3)(N-3)p_l(p_l-v)+2(v-2)\phi\left(\alpha(N-3)^2+6N-9\right)}{3(N-1)N}$ is maximized at $\frac{v}{2} < v - \frac{1}{2}$. Therefore, the platform does not benefit from deviating to price at $p_l > v - \frac{1}{2}$.

– Combined, the platform has no incentive to price its private label at $p_l \neq v-1$ under the condition that $\phi > \frac{-12\alpha-\alpha N^2+7\alpha N+2\alpha Nv-6N-6\alpha v+6v+12}{12\alpha-4\alpha N+2\alpha Nv-6\alpha v+6v-12}$.

Finally, we can show that the following interior solutions cannot be in equilibrium.

- There is no equilibrium satisfying $p_l^* < p_i^* \leq v-1$. Assuming $p_l^* < p_i^* < v-1$, the solution is $p_l^* = -\frac{(2\phi+5)(\alpha(N-4)(N-3)+6(N-2))}{2(\phi-5)(\alpha(N-3)+3)}$ and $-\frac{5\alpha(N-4)(N-3)+30(N-2)}{2(\phi-5)(\alpha(N-3)+3)}$. However, the parameter space where $0 < p_l^* < p_i^* < v-1$ is empty.

- There is no interior equilibrium with $p_l^* > p_i^*$ and $p_l^* - p_i^* < 1$. Under the assumption that $0 < p_l^* - p_i^* < 1$, the solution is $-\frac{(2\phi+5)(\alpha(N-4)(N-3)+6(N-2))}{2(\phi-5)(\alpha(N-3)+3)}$ and $-\frac{5\alpha(N-4)(N-3)+30(N-2)}{2(\phi-5)(\alpha(N-3)+3)}$. However, under any given $\phi$ and $\alpha$, we can find a profitable global price deviation for the third-party seller or the private label.

The total surplus in this case is

$$TS^{PL} = \frac{2}{3}TS^{Concede} + \frac{1}{3}TS^{Contest},$$

where $TS^{Concede}$ and $TS^{Contest}$ are derived later in Case "Concede" and Case "Contest".

### OA2.3 Solution for Case "Ad"

Below we prove that the equilibrium is $p_z^* = p_k^* = v - 1$ under the condition $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$.

- For the winning seller $z$, given that seller $k$ prices at $p_k^* = v-1$, we show that seller $z$ has no incentive to deviate to any $p_z \neq v-1$. Since seller $z$ pays for seller $k$'s bid, which does not depend on seller $z$'s price, we only compare seller $z$'s sales revenue below.
  - If $p_z < v-2$, seller $z$'s profit when pricing lower than $v-2$ is smaller than its profit when pricing at $v-2$. Therefore, $p_z < v-2$ is sub-optimal for seller $z$.
  - If $v-2 \leq p_z < v-1$, seller $z$'s demand is

    $$D_z(p_z) = Q_{ao}^2(p_z, v-1) + (N-2)\overline{Q_m^2} = \frac{\alpha(N-2)(N-p_z+v-3)+2N-p_z+v-4}{(N-1)N}.$$

    From $\frac{\partial(1-\phi)\left(p_z D_z(p_z)\right)}{p_z}\Big|_{p_z=p_z'} = 0$ we have $p_z' = \frac{1}{2}\left(\frac{N-1}{\alpha(N-2)+1} + N + v - 3\right)$. We need $p_z' > v-1$ so that seller $z$ has no incentive to deviate to price lower than $v-1$. A sufficient condition to ensure this is $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$ (under the condition that $v \geq N + \frac{1}{2}$).
  - If $v-1 \leq p_z < v - \frac{1}{2}$, seller $z$'s demand is

    $$D_z(p_z) = Q_{ao}^1(p_z, v-1) + (N-2)Q_m^2(p_z) = -\frac{\alpha(N-2)^2(2p_z-2v+1)+(2N-3)(p_z-v)}{(N-1)N}.$$

    From $\frac{\partial(1-\phi)\left(p_z D_z(p_z)\right)}{p_z}\Big|_{p_z=p_z'} = 0$ we have $p_z' = \frac{1}{2}\left(v - \frac{\alpha(N-2)^2}{2\alpha(N-2)^2+2N-3}\right)$. Since $p_z' < v-1$ when $v > N$, seller $z$ has no incentive to deviate to price higher at $v-1 \leq p_z < v-\frac{1}{2}$ from $v-1$.
  - If $p_z \geq v - \frac{1}{2}$, seller $z$'s demand is

    $$D_z(p_z) = Q_{ao}^1(p_z, v-1) + (N-2)Q_m^1(p_z) = -\frac{(2N-3)(p_z-v)}{(N-1)N}.$$

$\frac{\partial(1-\phi)\left(p_z D_z(p_z)\right)}{p_z}\Big|_{p_z=p_z'} = 0$ leads to $p_z' = \frac{v}{2}$. As $p_z' < v - \frac{1}{2}$ when $v \geq 1$, seller $z$ does not benefit from deviating to price higher than $v - \frac{1}{2}$.

  - Combined, seller $z$'s optimal price given that seller $k$ prices at $v-1$ is $p_z^* = v - 1$ when $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$ (under the condition that $v \geq N + \frac{1}{2}$).

- Now consider the losing seller $k$'s pricing strategy given that $p_z^* = v - 1$.
  - $p_k < v - 2$ is sub-optimal for seller $z$ as it leads to same demand but lower price thus lower revenue compared with $p_k = v - 2$.
  - If $v - 2 \leq p_k < v - 1$, seller $k$'s demand is

$$D_k(p_k) = q_{ao}^1(v-1, p_k) + (N-2)\overline{q_m} = \frac{-2\alpha + \alpha N - 2N + p_k - v + 4}{N - N^2}.$$

    Based on $\frac{\partial(1-\phi)\left(p_k D_k(p_k)\right)}{p_k}\Big|_{p_k=p_k'} = 0$, we find that the optimal price is $p_k' = \alpha - \frac{\alpha N}{2} + N + \frac{v}{2} - 2$. To ensure $p_k' > v - 1$ so that seller $k$ has no incentive to deviate to price lower than $v - 1$, a sufficient condition is $\alpha < \frac{2N-v-2}{N-2}$ (under the condition that $v > N$).

  - If $p_k \geq v - 1$, seller $k$'s demand is

$$D_k(p_k) = q_{ao}^2(v-1, p_k) + (N-2)q_m(p_k) = \frac{(-2\alpha + (\alpha-2)N + 3)(p_k - v)}{(N-1)N}.$$

    From $\frac{\partial(1-\phi)\left(p_k D_k(p_k)\right)}{p_k}\Big|_{p_k=p_k'} = 0$ we have $p_k' = \frac{v}{2} < v - 1$. Consequently, seller $k$ will not price higher than $\frac{v}{2}$.

  - Combined, seller $k$'s optimal price is $p_k^* = v - 1$ given that $p_z^* = v - 1$ when $\alpha < \frac{2N-v-2}{N-2}$ (under the condition that $v > N$).

Overall, $p_z^* = p_k^* = v - 1$ is an equilibrium when $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$, $\alpha < \frac{2N-v-2}{N-2}$ and $v > N + \frac{1}{2}$. Since $\frac{-2N+v+2}{N^2-Nv-3N+2v+2} < \frac{2N-v-2}{N-2}$ when $N < v < 2N - 2$, the condition simplifies into $\alpha < \frac{-2N+v+2}{N^2-Nv-3N+2v+2}$.

Furthermore,we can show that the following interior solutions cannot be in equilibrium.

- $p_k^* < p_z^* < v - 1$. The solution follows this assumption is $p_z^* = \frac{1}{3}\left(\frac{4(N-1)}{\alpha(N-2)+1} + 2N - 5\right)$ and $p_k^* = \frac{8\alpha + N(\alpha(N-6)+6) - 9}{3\alpha(N-2)+3}$. However, the solution contradicts $p_k^* < p_z^*$.

- $p_z^* < p_k^* < v-1$. The solution follows this assumption is $p_z^* = \frac{1}{3}\alpha(N-2)(2N-5)+2N-3$ and $p_k^* = \frac{1}{3}\alpha(N-4)(N-2)+2N-3$. However, the solution does not satisfy the premise that $0 < p_z^* < p_k^* < v-1$.

Total surplus and consumer surplus in Case "Ad" are the same as in Case "BM" because sellers price the same.

### OA2.4 Solution for Case "Concede"

Below we show that $p_l^* = v-1, p_z^* = p_k^* = v-2$ is an equilibrium under the condition that $N+\frac{1}{2} < v < 2\alpha - \alpha N + 2N - 1$.

- Consider seller $z$'s profit upon deviating to $p_z \neq v-2$.
  - If $p_z < v-2$, seller $z$'s demand comes as

    $$D_z(p_z) = Q_{ao}^2(p_z, v-2) + \overline{Q_{ao}^2} + (N-3)\overline{Q_m^2} = \frac{\alpha(N-2)(N-p_z+v-4)+2N-p_z+v-5}{(N-1)N}.$$

    Seller $z$ maximizes its revenue at $p_z' = \frac{1}{2}\left(\frac{N-1}{\alpha(N-2)+1} + N + v - 4\right)$. The above amount is larger than $v-2$ under $v \leq \frac{\alpha N^2 - 2\alpha N + 2N - 1}{-2\alpha + \alpha N + 1}$.
  - If $v-2 < p_z < v-1$, seller $z$'s demand is equal to,

    $$D_z(p_z) = Q_{ao}^1(p_z, v-2) + Q_{ao}^2(p_z, v-1) + (N-3)\overline{Q_m^2} = \frac{\alpha(N-2)(N-p_z+v-4)+2N-2p_z+2v-7}{(N-1)N}.$$

    The optimal price for seller $z$ is $p_z' = \frac{1}{2}\left(\frac{1}{\alpha(N-2)+2} + N + v - 4\right)$. To avoid seller $z$ from deviating, we need $p_z' < v-2$. This is equivalent to $\frac{N^2+1}{N} < v < \frac{1}{2}(2N+1)$ and $\frac{-2N+2v-1}{N^2-Nv-2N+2v} < \alpha < 1$, or $v > N + \frac{1}{2}$.
  - If $p_z > v-1$, an upper bound on seller $z$'s demand is,

    $$\begin{aligned} D_z(p_z) &= Q_{ao}^1(p_z, v-1) + (N-3)Q_m^2(p_z), \\ &= -\frac{\alpha(N-3)(N-2)(2p_z-2v+1)+(2N-5)(p_z-v)}{(N-1)N}. \end{aligned}$$

    The optimal price for seller $z$ associated with the above upper-bound demand function is

$p_z' = \frac{1}{4}\left(\frac{2N-5}{2\alpha(N-3)(N-2)+2N-5} + 2v - 1\right)$. The above amount is smaller than $v-1$ under $v > N-1$. Therefore, seller $z$ will not deviate to price at $v - 1 < p_z < v - \frac{1}{2}$.

  - Put together, seller $z$ will not deviate when $v \leq \frac{\alpha N^2 - 2\alpha N + 2N - 1}{-2\alpha + \alpha N + 1}$, and $\frac{N^2+1}{N} < v < \frac{1}{2}(2N + 1)$ and $\frac{-2N+2v-1}{N^2-Nv-2N+2v} < \alpha < 1$ or $v > N + \frac{1}{2}$. Since $\frac{\alpha N^2-2\alpha N+2N-1}{-2\alpha+\alpha N+1} \leq N + \frac{1}{2}$, a sufficient condition is that $v \geq N + \frac{1}{2}$.

- Consider seller $k$'s profit upon deviating to $p_k \neq v - 2$.

  - When $p_k < v - 2$, the upper bound of seller $k$'s demand is

$$D_k(p_k) = q_{ao}^1(v-2, p_k) + \overline{q_{oo}} + (N-3)\overline{q_m} = \frac{-2\alpha + \alpha N - 2N + p_k - v + 5}{N - N^2}.$$

    Seller $k$ maximizes its sales revenue $(1-\phi)p_k D_k(p_k)$ at $p_k' = \frac{1}{2}(2\alpha - (\alpha-2)N + v - 5)$. To ensure $p_k' > v - 2$, a sufficient condition is $v < 2\alpha - \alpha N + 2N - 1$.

  - If $v - 2 < p_k < v - 1$, seller $k$'s demand is

$$\begin{aligned} D_k(p_k) &= q_{ao}^2(v-2, p_k) + q_{oo}(v-1, p_k) + (N-3)\overline{q_m} \\ &= \frac{\alpha(N(-p_k + v - 3) + 3p_k - 3v + 8) + 2N - 2p_k + 2v - 7}{(N-1)N}. \end{aligned}$$

    Furthermore, $(1-\phi)p_k D_k(p_k)$ is maximized at $p_k' = \frac{\alpha(N(v-3)-3v+8)+2N+2v-7}{2\alpha(N-3)+4}$. Seller $k$ will not deviate to $p_k > v - 2$ when $p_k' < v - 2$, which is met when $v > N + \frac{1}{2}$.

  - If $v - 1 < p_k < v - \frac{1}{2}$, seller $k$'s demand is

$$D_k(p_k) = q_{oo}(v-1, p_k) + (N-3)q_m(p_k) = \frac{(\alpha-1)(2N-5)(p_k - v)}{(N-1)N}.$$

    We have $p_k' = \frac{v}{2} < v - 1$, and seller $k$ has no incentive to deviate to price higher than $\frac{v}{2}$.

  - Combined, seller $k$'s optimal price is $p_k^* = v - 1$ given that $p_z^* = v - 2$ and $p_l^* = v - 1$ under the condition that $N + \frac{1}{2} < v < 2\alpha - \alpha N + 2N - 1$.

Furthermore,we can show that the following interior solutions cannot be in equilibrium.

- $v - 1 \geq p_l^* > p_z^* > p_k^*$. The solution is $p_z^* = \frac{\alpha\left(84\alpha^2-\alpha(\phi+308)+\phi+320\right)-4(\alpha-2)\alpha^2N^3+\alpha N^2\left(34\alpha^2-\alpha(\phi+91)+\phi+45\right)+N(\alpha(\alpha(-94\alpha}{2(\alpha^3(N-3)(N-2)(\phi-3)+\alpha^2(N(-2N(\phi-3)+13\phi-43)-20\phi+68)-\alpha(N-3)(}$
  $p_k^* = \frac{\alpha(2\alpha(42\alpha+\phi-148)-\phi+310)-\alpha^2N^3(4\alpha+\phi-9)+\alpha N^2(\alpha(34\alpha+4\phi-91)-\phi+45)+N(\alpha(\alpha(-94\alpha-5\phi+288)+2\phi-235)+50)-100}{2(\alpha^3(N-3)(N-2)(\phi-3)+\alpha^2(N(-2N(\phi-3)+13\phi-43)-20\phi+68)-\alpha(N-3)(6\phi-25)-5(\phi-5))}$

and $p_l^* = \frac{\alpha(4\alpha(3\alpha(3\phi+5)-32\phi-58)+3(43\phi+90))-2(\alpha-2)\alpha^2N^3(\phi+1)+\alpha N^2(2\alpha(2\alpha(4\phi+5)-21\phi-29)+19\phi+35)+N\left(-\alpha\left(62\alpha^2+2(3\alpha-5)(7\alpha\right.\right.}{2(\alpha^3(N-3)(N-2)(\phi-3)+\alpha^2(N(-2N(\phi-3)+13\phi-43)-20\phi+68)-\alpha(N-3)(6\phi-25)-5(\phi-5}$

The parameter space which satisfies $v-1 > p_l^* > p_z^* > p_k^*$ is empty.

- $v-1 \geq p_z^* > p_l^* > p_k^*$. The solution is $p_z^* = \frac{(5-3\alpha)\left(\alpha^2(N-3)(N-2)-\alpha(N-4)(3N-7)-10(N-2)\right)+(\alpha-1)\alpha\phi(6\alpha+N(-5\alpha+(\alpha-1)N\right.}{2(\alpha(2\alpha(\phi-3)-6\phi+25)+5(\phi-5))}$ $p_k^* = \frac{\alpha\phi(6\alpha+N(-5\alpha+(\alpha-1)N+6)-7)+(3\alpha-5)(\alpha(N-6)(N-3)+10(N-2))}{2(\alpha(2\alpha(\phi-3)-6\phi+25)+5(\phi-5))}$ and $p_l^* = \frac{\phi\left(-73\alpha+\alpha^2(N-3)(3N-10)+\alpha(44-5N)N-20N+40\right.}{2(\alpha(2\alpha(\phi-3)-6\phi+25)-}$

  The parameter space which satisfies $v-1 > p_z^* > p_l^* > p_k^*$ is almost empty.

The total surplus in this case comes as,

$$\begin{aligned} TS^{Concede} =& \frac{1}{N}\int_0^{\frac{1}{2}}(v-x)2dx + (N-3)\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\int_0^{\frac{1}{2}}\big(v-(1-x)\big)2dx \\ &+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1}{2}}(v-x)2dx\right] \\ &+\frac{1}{N}\left(\alpha+(1-\alpha)\frac{1}{N-1}\right)\left[\int_0^{\frac{1}{2}}(v-(1-x))2dx\right] \\ &+(1-\alpha)\frac{2}{N(N-1)}\int_0^1(v-x)dx \\ &+2*(N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1}\int_0^1(v-x)dx. \end{aligned}$$

The first term represents consumers on the spoke of seller $z$ who receive their most preferred variety. The second term represents consumers whose most preferred variety is not available and their second-preferred variety is $z$. Consumers in this segment all purchase $z$. The third term represents consumers on the spoke of $k$ and whose second-preferred variety is $z$. Consumers in this segment purchase $k$. The fourth term represents consumers on the spoke of $l$ and whose second-preferred variety is $z$. Consumers in this segment purchase $z$. The fifth term represents consumers whose two preferred brands are $k$ and $l$ in the organic slots. Consumers in this segment purchase $k$. The last term represents consumers whose only available variety is $k$ or $l$.

## OA2.5 Solution for Case "Contest"

Below we show that $p_i^* = v-2, p_l^* = v-1$ is an equilibrium with condition of $0 < \alpha < \frac{2N-2v+1}{N-v-1}$, $\frac{2\alpha-\alpha N^2+\alpha N+2\alpha Nv-2N-4\alpha v+2v}{8\alpha-4\alpha N+2\alpha Nv-4\alpha v+2v-4} < \phi < 1$, $v \leq \frac{1}{2}\left(6\alpha+\alpha N^2-5\alpha N+2N-4\right)$.

- Consider seller $i$'s profit when deviating. It's easy to see that seller $i$ has no incentive to price lower than $v-2$ as it reduces the sales price but leading to the same demand as charging $v-2$.

Therefore, it's sufficient to consider when $p_i \geq v-2$.

- If $v-2 < p_i < v-1$, seller $i$'s demand when pricing at $p_i$ is given by,

$$D_i(p_i) = q^1_{ao}(v-1,p_i) + q_{oo}(v-2,p_i) + (N-3)\overline{q_m} = \frac{-\alpha(N-p_i+v-5)+2N-2p_i+2v-7}{(N-1)N}.$$

  Seller $i$'s profit $(1-\phi)p_iD_i(p_i)$ is maximized at $p_i' = \frac{1}{2}\left(-\frac{3}{\alpha-2}+N+v-5\right)$. We need the above amount smaller than $v-2$ to avoid seller $i$ from charging a higher price than $v-2$. A sufficient condition is $0 < \alpha < \frac{2N-2v+1}{N-v-1}$.

- When $p_i > v-1$, seller $i$'s demand is

$$D_i(p_i) = q^2_{ao}(v-1,p_i) + (N-3)q_m(p_i) = \frac{(-4\alpha+(\alpha-2)N+5)(p_i-v)}{(N-1)N}.$$

  Seller $i$'s revenue is maximized at $p_i' = \frac{2}{v} < v-1$. Therefore, seller $i$ will not deviate to $p_i \geq v-1$.

• Next, consider the platform's profit when pricing its private label at $p_l \neq v-1$.

- If $p_l < v-2$, the upper bound of the private label's demand is $D_l(p_l) = 2Q^2_{ao}(p_l,p_i^*) + (N-3)\overline{Q^2_m}$. Seller $i$'s demand is $D_i(p_i^*) = q^2_{ao}(p_l,p_i^*) + q_{oo}(p_i^*,p_i^*) + \overline{q_m}$. The platform's profit, $p_lD_l(p_l)+2\phi p_i^*D_i(p_i^*)$, is maximized at $p_l' = \frac{14\alpha+\alpha N^2-9\alpha N+2(v-2)\phi(\alpha(N-2)+1)+2\alpha Nv+2N-4\alpha v+2v-8}{4\alpha(N-2)+4}$. A sufficient condition to stop the platform from deviating is $p_l' > v-2$, which is guaranteed when $\frac{2\alpha-\alpha N^2+\alpha N+2\alpha Nv-2N-4\alpha v+2v}{8\alpha-4\alpha N+2\alpha Nv-4\alpha v+2v-4} < \phi < 1$.

- If $v-2 < p_l < v-1$, the private label's demand is $D_l(p_l) = 2Q^1_{ao}(p_l,p_i^*) + (N-3)\overline{Q^2_m}$. Seller $i$'s demand is $D_i(p_i^*) = q^1_{ao}(p_l,p_i^*) + q_{oo}(p_i^*,p_i^*) + \overline{q_m}$. The platform's profit, $p_lD_l(p_l) + 2\phi p_i^*D_i(p_i^*)$, is maximized at $p_l' = \frac{1}{4}(N(\alpha(N-5)+2)+2(3\alpha+(v-2)\phi+v-4))$. To ensure the platform not to deviate, we need $p_l' > v-1$. A sufficient condition for this is $v \leq \frac{1}{2}\left(6\alpha+\alpha N^2-5\alpha N+2N-4\right)$.

- Consider the case with $v-1 < p_l < v-\frac{1}{2}$. In this case, we have $D_l(p_l) = (N-3)Q^2_m(p_l)$ and $D_i(p_i^*) = \overline{q^1_{ao}} + q_{oo}(p_i^*,p_i^*) + \overline{q_m}$. The platform's profit is maximized at $p_l' = \frac{1}{4}\left(\frac{1}{\alpha(N-2)+1}+2v-1\right)$, which is again smaller than $v-1$ under $v > N$.

- Finally, the platform's profit when deviating to $p_l > v - \frac{1}{2}$ is weakly less than its profit at $p_l = v - \frac{1}{2}$ and thus is sub-optimal.

Furthermore, we can show that the following interior solutions are not in equilibrium.

- $p_l^* < p_i^* < v - 1$. The solution following is $p_l^* = \frac{42\alpha^2-60\alpha-2\alpha^2N^3+17\alpha^2N^2-11\alpha N^2-47\alpha^2N+2\phi(-3\alpha+(\alpha-2)N+4)(\alpha(N-2)+1)+51\alpha N-10N+20}{2(\alpha(N-2)+1)(\alpha(N(\phi-3)-2\phi+8)+\phi-5)}$, $p_i^* = -\frac{\alpha(N-6)(N-3)+10(N-2)}{2(\alpha(N(\phi-3)-2\phi+8)+\phi-5)}$, which contradicts with the assumption $p_l^* < p_i^* < v - 1$.
- $p_i^* < p_l^* < v-1$. The solution following is $p_l^* = \frac{2\alpha(3\alpha-3\phi-14)+(\alpha-3)\alpha N^2+N\left(-5\alpha^2+2(\alpha-2)\phi+19\alpha-10\right)+8\phi+20}{2(2\alpha+\phi-5)}$, $p_i^* = -\frac{\alpha(N-6)(N-3)+10(N-2)}{2(2\alpha+\phi-5)}$, which contradicts with the assumption $p_i^* < p_l^* < v - 1$ when $\alpha$ or $\phi$ is not negligible.

The total surplus in this case is given by,

$$
\begin{aligned}
TS^{Contest} =& 2 * \frac{1}{N(N-1)} \int_0^{\frac{1}{2}} (v-(1-x))2dx + (N-3)\frac{1}{N(N-1)} \int_0^{\frac{1}{2}} (v-x)2dx \\
&+ (N-3)\frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) \int_0^{\frac{1}{2}} \big(v-(1-x)\big)2dx \\
&+ 2 * \frac{1}{N}\left(\alpha + (1-\alpha)\frac{1}{N-1}\right) \left[\int_0^{\frac{1}{2}} (v-x)2dx\right] \\
&+ (1-\alpha)\frac{2}{N(N-1)} \left[\int_0^{\frac{1}{2}} (v-x)dx + \int_{\frac{1}{2}}^1 (v-(1-x))dx\right] \\
&+ 2 * (N-3)(1-\alpha)\frac{2}{N}\frac{1}{N-1} \int_0^1 (v-x)dx.
\end{aligned}
$$

The first term represents consumers on the spoke of seller $l$ in the top slot and whose second-preferred product is one of the third-party sellers. Consumers in this segment buy from the third-party seller. The second term represents consumers on the spoke of seller $l$ and whose second-preferred product is not available. Consumers in this segment buy from the private label brand. The third term represents consumers whose most preferred variety is not available and their second-preferred variety is $l$. Consumers in this segment purchase $l$. The fourth term represents consumers on the spoke of one of the third-party sellers in the organic slots $i$, and whose second-preferred variety is $l$. Consumers in this segment purchase $i$. The fifth term represents consumers whose two preferred brands are the two third-party sellers and they purchase their most-preferred variety. The last term represents consumers in the monopoly segment of one of the third-party sellers.

### OA2.6 Comparison with an exogenous commission rate

### OA2.7 Comparison with an endogenous commission rate

## OA3 Rule out other possible equilibrium

In appendix A2.4, assuming $p_l^* > p_z^* > p_k^*$, the solution is

$$p_z^* = \frac{C_1}{C_2}, p_k^* = \frac{C_3}{C_4}, p_l^* = \frac{C_5}{C_6}.$$

In the above expression, $C_1 \equiv -16(\alpha-1)^2 N^3(\alpha(N-2)(2v-1)+2v) + 8(\alpha-1)N^2(\alpha^2(N-2)(N(2v-1)+27v-14)$
$+\alpha(N(14-25v)+80v-32)-26v+4)+N(\alpha(\alpha(-478\alpha+N(143\alpha+(48\alpha-44)N-378)+1084)$
$-(\alpha-1)^2(N-2)\phi)+\alpha(223N-710)+2v(\alpha\left(-2\alpha^2(N-2)(22N+115)+\alpha(4N(10N+87)-1153)-198N+906\right)$
$-217)+112)+\alpha((\alpha-1)\phi(\alpha(N-2)N+1)+\alpha(300\alpha+N(-3\alpha(23N+4)+53N+181)-754))-133\alpha(N-4)+6v(\alpha(2N+5)-7)(\alpha(10\alpha(N-2)-8N+25)-7)-98.$

$C_2 \equiv 2(-32(\alpha-1)^2N^3(\alpha(N-2)+1)+16(\alpha-1)N^2(\alpha^2(N-2)(N+13)+\alpha(38-12N)-12)$
$+2N(\alpha^3(-(N-2))((N-3)\phi+41N+217)+\alpha^2(N((N-8)\phi+37N+332)+13\phi-1071)$
$+3\alpha((N-3)\phi-63N+274)+2\phi-189)+\alpha(2\alpha(15\alpha(\phi-19)-29\phi+731)+\alpha(5\alpha-4)N^2(\phi+21)$
$+N(5\alpha(-5\alpha(\phi-3)+7\phi-73)-12\phi+245)+36\phi-1113)-7(\phi-35)).$

$C_3 \equiv 32(\alpha-1)^2N^3v(\alpha(N-2)+1)-4(\alpha-1)N^2(2\alpha^2(N-2)(2N(v+1)+27v-6)+\alpha(N(11-52v)+162v-26)$
$-52v+8)+N\left(\alpha(\alpha(-47N^2+\alpha(N-2)(N(8N+37)-182)+447N-774)+(\alpha-1)(N-2)\phi(\alpha(N-2)+1)\right)$
$+\alpha(510-161N)+2v\left(\alpha\left(2\alpha^2(N-2)(22N+115)-3\alpha(2N-5)(7N+78)+217N-925\right)+217\right)-112)+$
$\alpha(\alpha(-3\alpha(N-2)(N(7N-4)-50)+20(N-18)N+712)-\phi(\alpha(N-2)+1)(\alpha(N-2)N+1))+7\alpha(21N-68)-6v(\alpha(2N+5)-7)(\alpha(10\alpha(N-2)-9N+26)-7)+98.$

$C_4 \equiv 2(32(\alpha-1)^2N^3(\alpha(N-2)+1)-16(\alpha-1)N^2(\alpha^2(N-2)(N+13)+\alpha(38-12N)-12)$
$+2N(\alpha^3(N-2)((N-3)\phi+41N+217)-\alpha^2(N((N-8)\phi+37N+332)+13\phi-1071)-3\alpha((N-3)\phi$
$-63N+274)-2\phi+189)-5\alpha^3(N-2)(N(\phi+21)-3\phi+57)+\alpha^2(N(4N(\phi+21)-35\phi+365)+58\phi-1462)+\alpha(12N\phi-245N-36\phi+1113)+7(\phi-35)).$

$C_5 \equiv 32(\alpha-1)^2N^3v(\alpha(N-2)+1)+4(\alpha-1)N^2(\alpha(N-2)+1)(\alpha((N-2)\phi-3N+10)-8)$
$-2v\left(\alpha^2(N-2)((N-3)\phi+N+29)+\alpha(3N(\phi-9)-7\phi+83)+2(\phi-13)\right))+N(-2\alpha^3(N-2)$

$(13N\phi-27N-28\phi+92)+\alpha^2(N(24N(\phi-2)-141\phi+451)+190\phi-778)+\alpha(39N\phi-169N-98\phi+526)+2v(2\alpha^3(N-2)(11(N-3)\phi+11N+137)-2\alpha^2(N(10N(\phi+1)-83\phi+235)+8(17\phi-81))+\alpha(-60(N-3)\phi+236N-963)-38\phi+217)+16(\phi-7))+6\alpha v(4\alpha(5\alpha(7-3\phi)+29\phi-81)-2\alpha(5\alpha-4)N^2(\phi+1)+2(5\alpha-4)N(5\alpha(\phi-1)-3\phi+7)-72\phi+231)+10\alpha^3(N-2)(N(4\phi-6)-9\phi+21)+\alpha^2(N(-33N\phi+47N+216\phi-464)-308\phi+812)+\alpha(165-62N)\phi+7\alpha(23N-72)+42v(2\phi-7)-28\phi+98,$

$C_6\equiv 2(32(\alpha-1)^2N^3(\alpha(N-2)+1)-16(\alpha-1)N^2(\alpha^2(N-2)(N+13)+\alpha(38-12N)-12)+2N(\alpha^3(N-2)((N-3)\phi+41N+217)-\alpha^2(N((N-8)\phi+37N+332)+13\phi-1071)-3\alpha((N-3)\phi-63N+274)-2\phi+189)-5\alpha^3(N-2)(N(\phi+21)-3\phi+57)+\alpha^2(N(4N(\phi+21)-35\phi+365)+58\phi-1462)+\alpha(12N\phi-245N-36\phi+1113)+7(\phi-35)).$

In appendix A2.4, assuming $p_z^*>p_l^*>p_k^*$, the solution is

$$p_z^*=\frac{C_7}{C_8},p_k^*=\frac{C_9}{C_{10}},p_l^*=\frac{C_{11}}{C_{12}}.$$

In the above expression, $C_7\equiv 4(\alpha-1)(26(\alpha-1)\alpha N^2(2v-1)+N(\alpha(\alpha(-\phi)+83\alpha+\phi-87)+2v(\alpha((\alpha-1)\phi-83\alpha+93)-20)+16)+2(\alpha((\alpha-1)\phi-31\alpha+30)+2v(\alpha(\alpha(-\phi)+31\alpha+\phi-33)+8)-6))+2((5\alpha-4(\alpha-1)N-3)\left(\alpha^2(N-2)(9N-10)(2v-1)-\alpha(N-1)(9N(2v-1)-50v+20)-16Nv+4N+14v-3\right)-(\alpha-1)\alpha\phi(N(\alpha(N-2)(2v-1)-2Nv+N+4v-1)-1))+(N-1)\left((\alpha-1)\alpha\phi(\alpha(N-2)(2v-1)-2Nv+N+4v-1)+(-5\alpha+4(\alpha-1)N+3)^2(\alpha(N-2)(2v-1)+2v)\right)-16(\alpha-1)^2(2v-1)(3\alpha(N-2)+2).$

$C_8\equiv 2\alpha\phi\left(2\alpha(2\alpha-5)+2(\alpha-1)^2N^2-2(3\alpha-4)(\alpha-1)N+5\right)+8(\alpha-1)(\alpha(-4\alpha(\phi-31)+5\phi-121)+52(\alpha-1)\alpha N^2+2N(\alpha((\alpha-1)\phi-83\alpha+89)-16)-\phi+26)+2(\alpha(2\alpha(200\alpha+\phi-355)-5\phi+407)-144(\alpha-1)^2\alpha N^3-4(\alpha-1)N^2(\alpha((\alpha-1)\phi-157\alpha+159)-28)+4N(\alpha(\alpha(2\alpha(\phi-110)-5\phi+423)+4\phi-253)-\phi+46)+3(\phi-25))+4(N-1)(\alpha(N-2)+1)(-5\alpha+4(\alpha-1)N+3)^2-96(\alpha-1)^2(2\alpha(N-2)+1).$

$C_9\equiv 2(-55\alpha-8(\alpha-1)\alpha N^3(-5\alpha+10(\alpha-1)v+2)+2N^2(-3\alpha(\alpha(28\alpha-41)+17)+2(\alpha-1)(\alpha(84\alpha-89)+16)v+8)+N(\alpha(\alpha(224\alpha-\phi-319)+\phi+141)+2v(52-\alpha(\alpha(224\alpha-\phi-451)+\phi+281))-24)+2\alpha\left(\alpha(-48\alpha+\phi+64)+v\left(96\alpha^2-2\alpha(\phi+92)+2\phi+111\right)-\phi\right)-42v+9)+8(\alpha-1)(2\alpha^2(N-2)(8N-9)(2v-1)$

$+\alpha(N(4N(3-8v)+110v-43)-74v+29)+N(8-20v)+16v-6)+(N-1)(\alpha\phi(-2\alpha+N(\alpha-2\alpha v+$
$2v-1)+4(\alpha-1)v+1)$
$+(-5\alpha+4(\alpha-1)N+3)(\alpha(-4\alpha(N-2)(N-1)-3N+2)+2v(\alpha(N-2)+1)(4\alpha(N-1)-4N+3)))-$
$32(\alpha-1)^2(2v-1)(2\alpha(N-2)+1).$

$C_{10} \equiv 2\alpha\phi\left(2\alpha(2\alpha-5)+2(\alpha-1)^2N^2-2(3\alpha-4)(\alpha-1)N+5\right)+8(\alpha-1)(\alpha(-4\alpha(\phi-31)+5\phi-121)$
$+52(\alpha-1)\alpha N^2+2N(\alpha((\alpha-1)\phi-83\alpha+89)-16)-\phi+26)+2(\alpha(2\alpha(200\alpha+\phi-355)-5\phi+407)$
$-144(\alpha-1)^2\alpha N^3-4(\alpha-1)N^2(\alpha((\alpha-1)\phi-157\alpha+159)-28)+4N(\alpha(\alpha(2\alpha(\phi-110)-5\phi+423)+$
$4\phi-253)-\phi+46)$
$+3(\phi-25))+4(N-1)(\alpha(N-2)+1)(-5\alpha+4(\alpha-1)N+3)^2-96(\alpha-1)^2(2\alpha(N-2)+1).$

$C_{11} \equiv 4(\alpha-1)(8\alpha(\phi-\alpha(\phi-9))-58\alpha+4\alpha N^2(-2\alpha\phi+8\alpha+4(\alpha-1)v(\phi-4)+\phi-6)$
$+2N(\alpha(10\alpha(\phi-5)-8\phi+43)+2v(\alpha(-10\alpha(\phi-5)+14\phi-55)-3\phi+10)+2(\phi-4))+2v(2\alpha(4\alpha(\phi-$
$9)-8\phi+37)+5\phi-16)-3\phi+12)+2(\alpha(4\alpha(-2\alpha(\phi+12)+\phi+32)+\phi-55)$
$+4(\alpha-1)\alpha N^3(\alpha(-\phi)+10\alpha+2(\alpha-1)v(\phi-10)-4)+2N^2(\alpha(\alpha(4\alpha(\phi-21)-7\phi+123)+3(\phi-17))-$
$2(\alpha-1)v(\alpha(4\alpha(\phi-21)-9\phi+89)+2(\phi-8))+8)$
$+N(\alpha(\alpha(4\alpha(\phi+56)+5\phi-319)-6\phi+141)-2v(\alpha(\alpha(4\alpha(\phi+56)+13\phi-451)-22\phi+281)+7\phi-52)-24)$
$+2v(\alpha(8\alpha^2(\phi+12)-4\alpha(\phi+46)-5\phi+111)+3(\phi-7))+9)+(N-1)(\alpha\phi(-2\alpha-4\alpha^2(N-2)(N-1)(2v-1)$
$+\alpha N((8N-30)v+3)+N(-4N+10v+7)+20\alpha v-8v-3)+(-5\alpha+4(\alpha-1)N+3)(\alpha(-4\alpha(N-2)(N-$
$1)-3N+2)$
$+2v(-4\alpha+4(\alpha-1)N+3)(\alpha(N-2)+1)))+16(\alpha-1)^2(2v-1)(\phi-2)(2\alpha(N-2)+1).$

$C_12 \equiv -2\alpha\phi\left(2\alpha(2\alpha-5)+2(\alpha-1)^2N^2-2(3\alpha-4)(\alpha-1)N+5\right)$
$+8(\alpha-1)\left(\alpha(4\alpha(\phi-31)-5\phi+121)-52(\alpha-1)\alpha N^2+2N(\alpha(\alpha(-\phi)+83\alpha+\phi-89)+16)+\phi-26\right)$
$+2(\alpha(-2\alpha(200\alpha+\phi-355)+5\phi-407)+144(\alpha-1)^2\alpha N^3+4(\alpha-1)N^2(\alpha((\alpha-1)\phi-157\alpha+159)-28)$
$+4N(\alpha(\alpha(-2\alpha\phi+220\alpha+5\phi-423)-4\phi+253)+\phi-46)-3\phi+75)-4(N-1)(\alpha(N-2)+1)(-5\alpha+$
$4(\alpha-1)N+3)^2+96(\alpha-1)^2(2\alpha(N-2)+1).$

In appendix A2.5, assuming $p_l^* > p_i^*$, the solution is

$$\begin{cases} p_i^* &= -\frac{-4\alpha^2(3-N)^2(N-2)(2v-1)+\alpha(3-N)(4(2N-5)(2v-1)+N((30-8N)v-3)-20v+2)+(7-4N)(2(3-N)v-2)}{2(70\alpha-4\alpha^2(N-2)(7N-8)+4\alpha N(7N-25)+20N+\phi-17)+4(N-1)(-5\alpha+4(\alpha-1)N+3)(\alpha(N-2)+1)+24(\alpha-1)(2\alpha(N-2)+1)}, \\ p_l^* &= \frac{2\left(\alpha(16\alpha+\phi-16)+7\alpha N^2(\alpha-2(\alpha-1)v-1)+N(-\alpha(22\alpha+\phi-23)+2v(\alpha(22\alpha+\phi-26)-\phi+6)-4)+2v(-\alpha(16\alpha+\phi-19)+\phi-5)+3\right)+4(\alpha-1)(2v-1)(3\alpha(N-2)-\phi+2)+(N-1)(-5\alpha+4(\alpha-1)N+3)(\alpha(N-2)(2v-1)+2v)}{2(70\alpha-4\alpha^2(N-2)(7N-8)+4\alpha N(7N-25)+20N+\phi-17)+4(N-1)(-5\alpha+4(\alpha-1)N+3)(\alpha(N-2)+1)+24(\alpha-1)(2\alpha(N-2)+1)}. \end{cases}$$

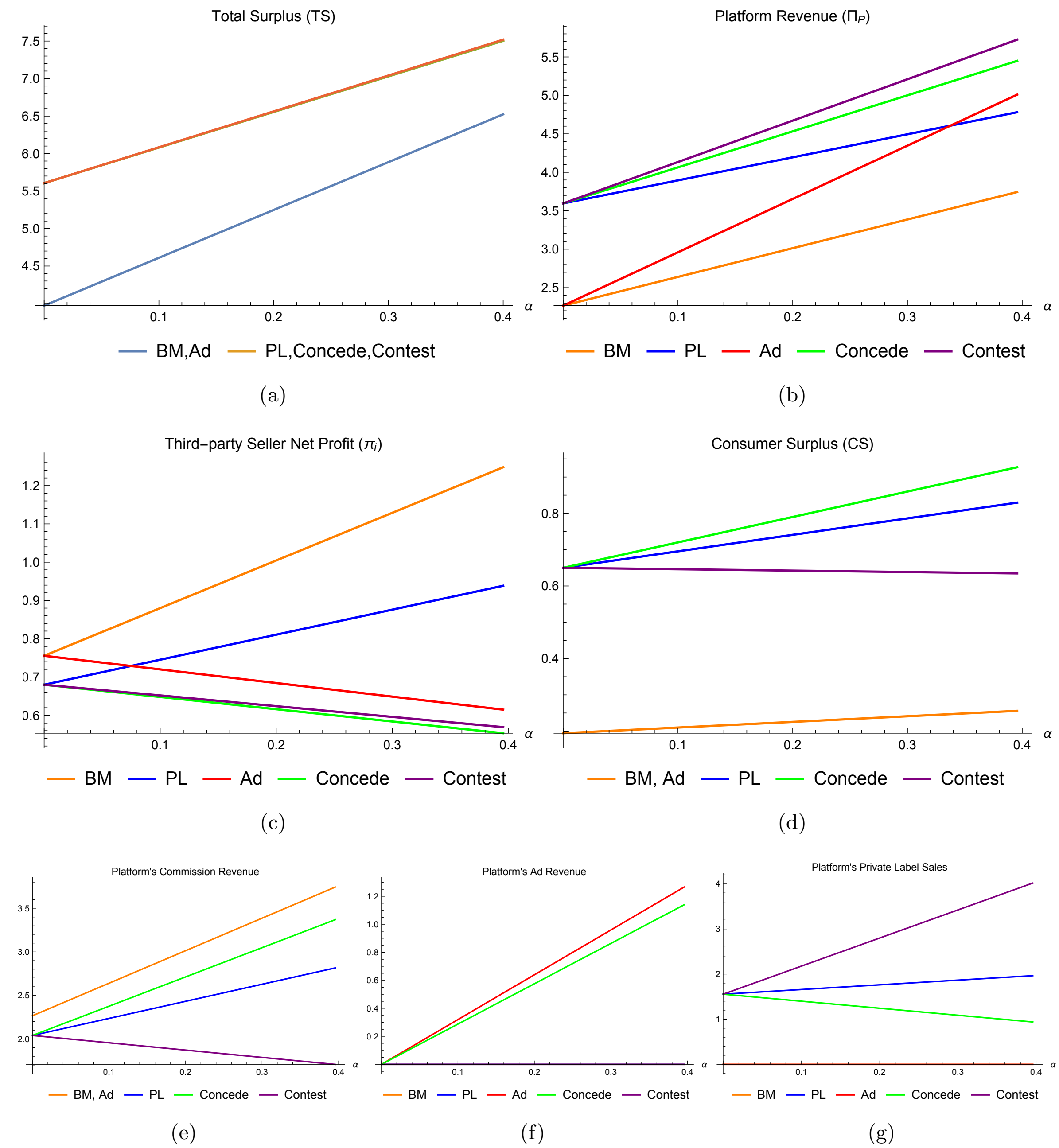


Figure OA1: Comparison of all arrangements under an exogenous commission rate with a high product valuation ($v = 11, N = 10, \phi = 0.6$).

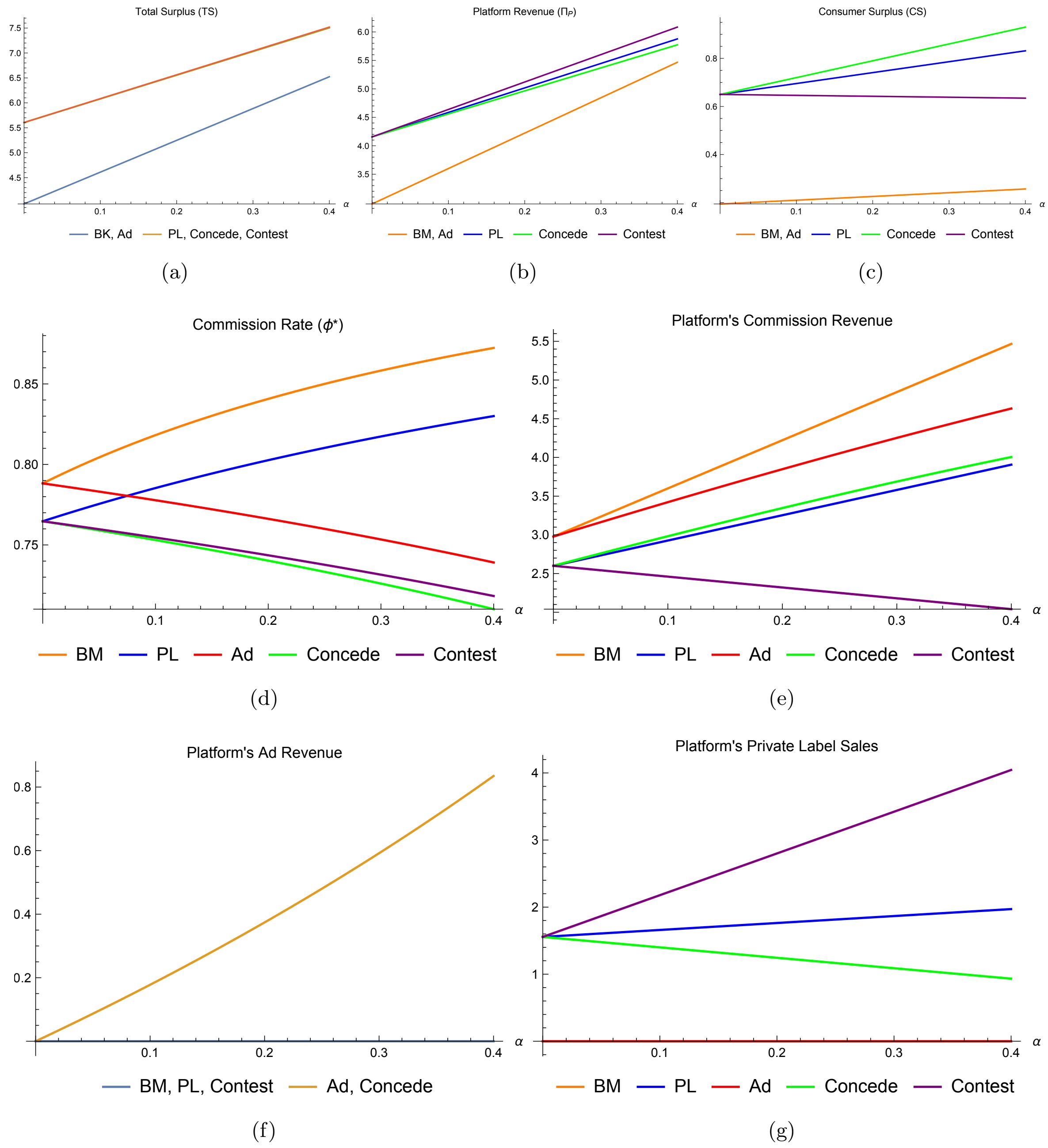


Figure OA2: Comparison of all arrangements under an endogenous commission rate with a high product valuation ($\pi_0 = 0.4, v = 11, N = 10$).