A Theory of Brand Positioning: Product-Portfolio View

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Abstract

Beyond real functional differences, brand positioning can have profound effects on the consumers' purchase decisions. From a product portfolio perspective, we provide a microfoundation for why and how brand positioning can deliver credible information that affects consumers' purchase decisions based on the consumer search framework. Consumers form their perceptions of a brand from interactions with all products under the same brand. We conceptualize brand positioning as such aggregate information about characteristics common to products under the same brand name. We operationalize such aggregate information as the mean location of all the products under the brand on a Hotelling-line. When consumers conduct their costly search for product matches, they are guided by how brands are positioned in the market. We show that a niche brand naturally conveys more information than mainstream brands. A mainstream brand has incentives to opportunistically dilute its brand by offering a wide range of products. Even in a monopolistic market, a niche brand may arise as an equilibrium because it serves as a commitment device for no dilution.

Keywords: brand positioning, product portfolio, mainstream and niche positioning, consumer search, hold-up problem

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1 Introduction

She may not like what we've made that following season, but she'll give us the first look because she is a customer of ours, and it's our job to make sure we know what she'll like, and give her some of that.

– *Stuart Weitzman, the founder of Stuart Weitzman*

Brand positioning is one of the most critical and fundamental concepts in marketing and firm strategy. How a firm chooses to position itself in the market significantly affects its competitiveness and performance. While the advantages of honing a firm's unique position in the market may seem obvious, it is easy to take brand positioning for granted. Furthermore, despite the importance and practical relevance of brand positioning, the issue has rarely been the subject of rigorous analysis. What exactly is brand positioning? Why is it so valuable? How does it affect consumers' choices?

Marketing literature defines brand positioning as the way in which consumers perceive the brand (Kotler et al. 2014) and the overall view that consumers have of a brand—a view that is often formed by a unique bundle of associations in the minds of target customers (Avery and Gupta 2014). In other words, brand positioning is the particular place in consumers' mind that a firm seeks to own. Consumers form their perception of the brand from various interactions with the brand's general line of products (that is, several different products under the same brand), which, taken together, identify and refine a brand's distinctiveness.

A consumer who owns Gucci products, for instance, may have in mind an overall image of Gucci as a brand that carries fashionable and sensual handbags and shoes. This perception is formed by the consumer's experiences with Gucci's products, and, critically, this image of the brand will affect the consumer's future apparel purchase decisions. When instead considering Hermes, which is another popular luxury brand, the consumer may have an entirely different brand image given the way the company has positioned itself: timeless elegance and classic luxury.[1] Both Gucci and Hermes keep their brand positions and identities running consistently throughout their products. By doing so, Gucci and Hermes have effectively differentiated themselves from other luxury brands (Yoffie and Kwak, 2001). Consumers who are interested in fashion luxury will seek out Gucci for their next spring collection without foreknowledge of the exact design or style of each individual product. Consumers who prefer classic styles may search Hermes first.

[1]Gucci and Hermes have been quoted in a Harvard Business case, "Gucci Group N.V." (Yoffie and Kwak, 2001) as examples of luxury fashion brands with opposite positioning. According to the case, Gucci even tried to take Hermes' positioning ("classic, timeless luxury") but failed and found a new position of "fashion luxury with sensual appeal".

Brand positioning provides critical information about the characteristics of a firm's products, making it easier for consumers to locate their preferred products and choose where to purchase without blindly searching through several brands to find the right one for them. Suppose that a consumer wants to buy a new jacket for this season. Where does she start her shopping journey? She may go to her favorite brand first, even though she does not know the exact design or style of the jackets offered in this season. This suggests that brand images provide important information that guides consumer search. She could potentially find something she like in other brands' stores that she decided not to visit. The main reason she did not visit those stores is that she has limited time and energy—in other words, search costs. This has also been recognized by marketers, as indicated in our opening quote by Stuart Weitzman. It is not easy to communicate to consumers what they can expect from a brand without a clear positioning. Brand positioning can thus serve as an efficient communication device that invites consumers to search and patronage.

To formalize this idea, we begin by illuminating the fundamental concept of marketing by examining brand positioning from a product portfolio perspective. Our view is that brand positioning is not merely about product positioning. Brand positioning is the overall image of a brand in a consumer's mind. In other words, it is aggregate information about characteristics common to products under the same brand name. To capture this intricate nature of brand positioning as aggregate information that is determined collectively by all of its products, we conceptualize the brand position as the average location of all products under the brand on a Hotelling-line.[2] Consumers are uncertain about the location of each product, but they are aware of the average location of the brand's products—that is, its brand position. Based on this information, consumers can decide whether to search for a specific brand by visiting the store.

We build a micro-model for how and why brand positioning can deliver credible information to consumers and provide a rationale for a firm's specific brand positioning strategy: "niche" or "mainstream" positioning. We first consider a monopolist of two products that are substitutes. The firm chooses the location of its two products on the Hotelling-line on which a continuum of consumers are distributed. A product located closer to a consumer is more likely to satisfy the consumer's need. Consumers do not observe the individual product locations, but they can observe the brand positioning, which is the average location of two products. We assume there are two

[2] Mean location is just *one* operationalization to capture the fact that brand positioning is determined collectively by all of its products under the same brand in a tractable way. Such "aggregate information" represents the average style of each individual product or characteristic common to products under the same brand name. For example, Gucci products tend to be more fashionable while Hermes more classic.

types of consumers in the market: regular consumers who incur a positive search cost and shoppers who can search for a brand without incurring such cost. Under this model setup, the brand is characterized by two parts: the brand positioning, captured by the average location of its two products, and the brand strength, captured by the spread of the two products. After observing the brand positioning, consumers form rational expectations about the spread of two products and decide whether to engages in costly visit to the firm's store. When a consumer visits, she discovers the locations of the two products and can determine how well they match her preferences. She then makes a purchase decision. If neither of the two products matches her needs, she incurs only search costs—visiting a store but leaving empty-handed. Brand positioning, therefore, serves as a crucial device of communication to convey important information to its customers and reduce uncertainty, thereby prompting consumers to initiate the search process.

We first establish that there are two types of brand positioning in equilibrium: *mainstream positioning*, where the brand is positioned close to the center of the Hotelling-line to appeal to the majority of consumers, and *niche positioning*, where the brand is positioned near an end-point to appeal to a small portion of consumers. Then, we show that the presence of heterogeneity in consumer search frictions (i.e., the co-existence of regular consumers who incur a positive search cost and shoppers who do not incur a search cost) creates the critical issue of the *hold-up problem*. Specifically, the brand has an incentive to increase the spread between the two products in order to better serve shoppers who have heterogeneous tastes and visit the store at no cost irrespective of brand positioning or spread. Regular consumers anticipate the brand's opportunistic behavior of spreading out and update their beliefs about the spread; this, in turn, lowers their expected utility and discourages them from paying the search cost and visiting the brand.

We also find that a brand has little incentive to deviate from the expected level of spread when there is sufficient number of regular consumers who are willing to visit the brand, thus supporting a *strong mainstream* brand positioning. Otherwise, a mainstream-positioned brand has a greater incentive to spread its product locations, leading to brand dilution. Therefore, brand positioning information alone may then be insufficient to convince regular consumer to visit the brand. Given this, a brand sometimes needs to credibly communicate its product spread, specifically a smaller spread by committing to a niche positioning, to increase the regular consumers' expected utility. As brand positioning becomes more niche, the spread of individual products becomes smaller as the range of possible spreads shrinks. This reduced spread of a specific brand's position can convey more information about individual products. Hence, we provide a new explanation for consumers' appeal of niche brands—niche brand positioning can serve as the commitment mechanism that allows a brand to communicate credible information about its product spread to consumers.

Finally, we identify the conditions under which brands tend to adopt niche over mainstream positioning strategy after deriving the optimal level of nicheness (or variance). We find that search friction tends toward a proliferation of niche brands. In particular, unless there exist too many or too few shoppers, brands are better off with a niche brand position, one that appeals to a smaller range of consumers but has a higher chance of matching their preferences when search cost is high. In this way, the presence of shoppers can alter the equilibrium dynamics and dramatically change the firm's profit. Our equilibrium analysis sheds light on the optimal conditions and economic tradeoffs under which a firm should position its brand as either "mainstream" or "niche". Therefore, we provide a formal economic structure on the concept of brand positioning.

The article is organized as follows. Section 2 discusses the related literature. Section 3 develops the formal model and points out the main tradeoff behind the model, and Section 4 analyzes it. In Section 5, we show the robustness of our main results to three-product case and provide an example that relaxes exogenous pricing. Finally, Section 6 concludes. All proofs are in the Appendix.

2 Literature Review

Our study contributes to several streams of research about brand positioning in marketing and economics. First, there is a large body of literature studying the concept of brand positioning from the psychological perspective focusing on how consumers perceive, think and feel about brands (Trout and Ries 1986; Davis et al. 2000). Along this line of research, many papers point out that positioning is a process of emphasizing the brand's distinctive and motivating attributes, establishing both the point of unique difference and the point of parity association with the category (Jan Alsem and Kostelijk 2008; Keller et al. 2011). We follow the definition of the traditional approach and provide a formal economic modeling structure to understand the concept of brand positioning.

Also, a vast literature investigates the issue of branding from an economic perspective.[3] Especially in the context of umbrella branding, researchers have examined whether firms can credibly convey vertical information about quality (Sappington and Wernerfelt 1985; Wernerfelt 1988, 1991; Cabral 2000; Zhang 2015; Neeman et al. 2019; Klein et al. 2019; Yu 2021) or horizontal match (Sappington

[3] Also, see Bronnenberg et al. (2019) for an excellent review of the economics of brand and branding. Recent studies have focused on measuring the brand value in a static setting (Goldfarb et al. 2009), and dynamic environment (Borkovsky et al. 2017).

and Wernerfelt 1985; Kuksov 2007; Kuksov et al. 2013) through branding. In our paper, a brand affects which type of consumers will buy its products by choosing the kind of products it will make available in the market. In that sense, the brand decides its horizontal positioning through its product portfolio targeting customers of different preferences.

Several researchers study product positioning with strategic interactions with competition (Hauser and Shugan 1983; Moorthy 1988; Kuksov 2004; Lauga and Ofek 2011). Recent studies have analyzed the dynamic aspects of product positioning, namely repositioning of the product over time (Sweeting 2013; Jeziorski 2014). Villas-Boas (2018) analyzed a monopolist's optimal repositioning strategy under changing consumer preferences. Cong and Zhou (2019) focused on the role of commitment for repositioning under competition. Our paper differs from the existing literature by highlighting the difference between brand positioning and individual product positioning.

Closer to our paper is the research on how product line design or assortment can affect consumer inference and search decision. A stream of related studies (Wernerfelt 1995; Kamenica 2008; Orhun 2009; Villas-Boas 2004) highlights the importance of considering consumer information and communication when designing product lines and shows how the number of products affects consumer inference of the likelihood of fit in different contexts. While the previous literature focuses on the number of products offered in the market, our focus is on the role of the aggregate information about characteristics common to all the products under the same brand, and how this aggregate information, *brand positioning*, can affect consumer search decision.

Also, our paper is related to the literature on strategic disclosure of verifiable information (Crawford and Sobel 1982; Milgrom and Roberts 1986) and recent development in the information design (Kamenica and Gentzkow 2011; Bergemann and Morris 2016, 2019). In this literature, the focus is how to optimally design information structure to foster efficient communication. Unlike papers in this area, we are not modeling the brand positioning as an optimal piece of information that the firm passes to consumers. Conceptually, we study a different paradigm of information environment, where the sender makes multiple decisions (i.e., style or design of individual products), but only the aggregate information (i.e., brand positioning) is observable to the receivers. We identify brand positioning as a natural application of this new information communication paradigm.

Finally, our paper builds on the stream of literature on consumer search, especially ordered search (Armstrong 2017). Armstrong et al. (2009) shows the value of being the first destination or being "prominent" when consumers engage in costly search. Several papers have examined how firms can guide consumer search by using various instruments such as a lower price (Chen and He 2011;

Armstrong and Zhou 2011; Zhou 2011), advertisement (Anderson and Renault 2006; Mayzlin and Shin 2011; Lu and Shin 2018), product design (Kuksov 2004; Bar-Isaac et al. 2012), service (Shin 2007; Janssen and Ke 2020), and targeting (Shin and Yu 2021). Our paper examines the role of brand positioning as another instrument through which the firm can influence consumer search decisions.[4]

3 Model

A multi-product firm serves a unit mass of consumers who are represented by their horizontal preferences for product styles or designs. Consumers' preferences are uniformly distributed on a Hotelling-line in $[-1, 1]$. The firm chooses the design of its product portfolio by locating its products in $[-1/2, 1/2]$ on this line.[5] Each product either satisfies a consumer's need, or it does not. So, the realized utility of the consumer located at x from product i is $u_i(x) = 1$ or 0. If the consumer's ideal design is closer to the product location, then the probability of the product satisfying the consumer's need, $\Pr(u_i(x) = 1)$, is higher. Specifically, the consumer receives utility $u_i(x)$ from buying product $i \in \{1, 2, ..., n\}$ located at x_i, where

$$u_i(x) = \begin{cases} 1, & \text{with probability } \theta - t|x - x_i|, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Thus, even a consumer located exactly at the same location as product i may receive $u_i(x) = 1$ with probability $\theta \le 1$.[6] Conditional on the location of the consumer $x \in [-1/2, 1/2]$, $u_i(x)$'s are independent across different products. Hence, it is possible that a consumer realizes a match with a product located farther away from her location than another product located closer to her. Then, the consumer's utility from the brand is $u(x) = \max_{1 \le i \le n} u_i(x)$. The products are substitutes for one another, and if multiple products satisfy the consumer's need, the consumer is indifferent and therefore buys one at random. This stochastic nature of the matching utility allows for other factors that affect consumer utility beyond the preference matching on the focal dimension of product design.

[4] In particular, Rhodes (2014) is closely related to our study. Rhodes (2014) study a retail setting where consumers decide whether to search for firms carrying multiple products that are not substitutes. It finds that carrying multiple products can provide the firm with the commitment power to maintain low prices, thus encouraging consumer visits. Both his and our papers can be considered as providing different mechanisms on how firms can guide consumer search through the assortment of products. While Rhodes (2014) is more about retailing, our paper is about branding.

[5] We impose a restriction that the support for the firm's product positions is a subset of the consumer market. This assumption eliminates the cases of truncated demand at endpoints and thereby simplifies our equilibrium analysis.

[6] To ensure that $\theta - t|x - x_i| \in [0, 1]$ for any $x \in [-1, 1]$ and $x_i \in [-1/2, 1/2]$, we impose the restriction that $0 < 3t/2 \le \theta \le 1$. The farthest possible distance between a consumer and a product is $3/2$, so we need $\theta - 3t/2 \ge 0$. Also, the consumer can be located precisely at either product, so $\theta \le 1$.

In reality, consumers may perceive brands in multiple dimensions (e.g., Keller (1993, 2003); John et al. (2006); Lehmann et al. (2008)). While we focus on brand positioning decisions in only one dimension, we recognize the multidimensional nature of consumer preferences through the parameter θ, which is allowed to be less than one to capture the possibility of a mismatch between a consumer's ideal product on the focal dimension and the consumer's need.

Moreover, the stochastic matching utility, together with this conditional independence assumption, ensures that the consumer's expected utility depends on the locations of the entire product portfolio as opposed to one particular product closest to her location.[7] In particular, a consumer prefers several distinct products to be located close to her because the total probability of finding a good match with the firm will be higher. Thus, the brand's decision for its product portfolio influences consumers' expected utility and their decisions.[8]

Consumers have a deterministic outside option which gives zero utility. Consumers prefer a matched product to the outside option and prefer the outside option to an unmatched product.[9] In our main analysis, we abstract away the firm's pricing decision. While price image could be an important vertical dimension of brand positioning in reality, we focus on positioning decisions on one horizontal dimension while holding other possible dimensions constant. The current model, therefore, applies to brand positioning situations in which prices are held constant. The consumers' utility from a matched product is normalized to one, and that of an unmatched product zero.

Brand Positioning

Facing a brand with a portfolio of products, consumers find it prohibitively costly to learn about the exact design or style of all the products, which therefore remains largely unobservable. However, consumers have a certain perception of a brand based on various interactions with the brand's several different products. Such a perception is often shaped not by any one particular product, and rather by the designs of the brand's entire product portfolio. In other words, brand positioning is a summary

[7]In a standard deterministic Hotelling model where $u_i(x) = \theta - t|x - x_i|$, the consumer's utility is determined by a product closer to the consumer's location, irrespective of the design of remaining products in the product portfolio.

[8]Our stochastic utility setup where consumers realize a binary utility after incurring a positive search cost is similar to Athey and Ellison (2011). In their model of ordered sponsored search ads, each link i will generate a binary utility from consumer, where the probability of the good outcome q_i is drawn from a common distribution F. The exact outcome of each link is realized only after the user incurs a positive click cost. So, though in expectation a link with a higher q_i is more likely to generate a good outcome, the realized outcome need not always be. Similarly in our model, a product located at x_i provides a good match to the consumer at x with probability $\theta - t|x - x_i|$. A product located close to the consumer is more likely to satisfy the consumer's need, but it sometimes results in a worse outcome.

[9]In a more general model setup, we can have consumers' utility of the outside option as $u_0 \in [0, 1)$. One can show that the general model is equivalent to our current model if we scale the search cost in the general model by $1/(1 - u_0)$.

statistic which provides an aggregate information about the brand's product-line. We operationalize this idea by defining B as the average location of products under the brand. More specifically, a brand that carries n products chooses the location of each of its products x_k for $1 \leq k \leq n$ on the Hoteling line. In our main analysis, we assume the simplest case of two products ($n = 2$).[10] Then, the brand positioning B is as follows:

$$B = \frac{\sum_{k=1}^{n} x_k}{n} = \frac{x_1 + x_2}{2}. \tag{2}$$

Based on the observed brand positioning B, consumers form expectations about the exact designs of the brand's products (x_k's) and accordingly decide whether to visit the store by paying a search cost. Then, we can rewrite $x_1 = B - \Delta$ and $x_2 = B + \Delta$, where $\Delta \equiv (x_2 - x_1)/2$ represents the spread or variance of the brand, defined by the distance between its two products. Without loss of generality, let $\Delta \geq 0$, or equivalently, $x_2 \leq x_1$. Therefore, there are two pieces of information relevant to consumer's decisions: brand positioning B as the average location of products and brand variance Δ as a deviation from B. Consumers consider both the brand positioning B (which is observable) and their expectations about the spread $\widetilde{\Delta}$ (which is unobservable) to make their search decision.

A typical brand carries multiple product lines, so it is not feasible to convey all of its product information due to the limited bandwidth in communication. Instead, the consumers usually have a certain overall image of the brand, which is the brand positioning. For example, consumers have a brand image that Gucci is a fashion luxury through prior experience or advertisements without knowing the exact design or style of individual products for the current season. Also, readers know (through their past experiences) that CNN and Fox generally report news from a more liberal and conservative perspective, respectively. Based on such awareness, they form expectations about the range of news reported by each brand (CNN and Fox), which is the brand variance. Consumers who prefer more liberal articles will go to CNN without exactly knowing the tone of each individual news article believing that they would find news articles tailored to a more liberal audience. The same

[10]Our framework of brand positioning as the average of the brand's product portfolio is a simple yet robust way to capture stylized facts about how consumers' perception of brands is formed by the brand's product portfolio. For instance, consider a Hotelling-line where the two opposite ends represent "formal" and "bold" designs. Our formulation implies that if a brand carries most of its products in classic designs, the brand will be perceived as classic. *Ann Taylor* fits this category. Consumers recognize the brand for its classic and feminine fashion style and visit the store to buy their work wears. *Studio Tomboy* offers a line of products in bold designs and, thus, is perceived as more casual and boyish. Certainly, consumers do not expect to find a formal dress for dinner parties at *Studio Tomboy*'s store. A brand, such as *Banana Republic*, offers a product portfolio about evenly mixed between formal and casual designs. Consumers' perception of the brand is indeed somewhat moderate, lying somewhere in between their perceptions of *Ann Taylor* and *Studio Tomboy*.

thing happens for a more conservative audience, who frequent Fox News channel and webpage.[11]

Based on these stylized patterns, we take it as given that consumers observe the brand positioning B without micro-modeling how B becomes known to consumers. By doing so, we are able to focus on analyzing the role of brand positioning and how it can affect consumer decisions. In reality, the process of consumers becoming aware of the brand positioning can take different forms, such as observations, advertisements, word of mouth, and prior purchase or use of products.

Consumer Type: Regular Consumers vs Shoppers

We assume that there are heterogeneous consumer types to examine the relationship between the role of brand positioning and the extent of search friction in the market. An α fraction of consumers are regular consumers who have a positive search cost $s > 0$. As described above; they can only observe B but not Δ, so they form beliefs $\widetilde{\Delta}$ about Δ. Only after paying a search cost can they see the true Δ and the realized utilities $u_1(x)$ and $u_2(x)$. The remaining $1-\alpha$ fraction of consumers are shoppers with zero search cost who therefore observe everything freely.

We impose some restrictions on the search cost to simplify the equilibrium analysis and focus on the most interesting case. First, if the search cost is too high, no regular consumers will visit the brand even if the brand provides the highest possible benefits by locating its two products at the same location. To avoid this trivial case, we make the following assumption that ensures that regular consumers are willing to visit the brand if she is at the same location as the two products.

Assumption 1 $s < 1 - (1-\theta)^2$.

Moreover, we want to simplify the analysis by requiring "no truncation" of demand for regular consumers. If demand truncation happens, it leads to unnecessary complications of corner cases and non-smooth demand, which makes the analysis rather tedious without adding insight. Particularly, we require the consumer located at 1 (or at -1) to not visit the brand even if both products are co-located as closely as possible at $1/2$ (or at $-1/2$). This translates into the following assumption.

Assumption 2 $s \geq 1 - (1-\theta+t/2)^2$.

All the subsequent equilibrium analysis is performed under Assumptions 1 and 2.

[11] These examples also demonstrate that that a Hotelling-line (as opposed to other demand models without opposite ends, such as a circular city or logit model) provides a natural and appropriate environment to capture the heterogeneous horizontal preferences for consumers such as designs or styles in a spectrum. A circular city or a logit model with symmetric demand distribution is not conducive to an analysis of an important managerial decision on whether to become a mainstream or niche brand.

The game sequence is as follows: First, the brand decides the locations of its two products, x_1 and x_2, to maximize the expected profit. Second, regular consumers observe the brand $B = (x_1 + x_2)/2$ and decide whether to pay the search cost and visit the firm. Upon the visit, they discover the relevant information and decide which product to buy or take the outside option. Shoppers always visit at zero cost and make purchase decisions after observing the exact matching values with both products. Given our assumption of exogenous prices in the main model, the brand's *de facto* objective is to maximize the expected demand.

Breadth of Appeal vs Clarity of Information Communicated

Our model characterizes the image of a brand in two parts: (1) the average location of its two products B, which is observed, and (2) the spread of two products Δ, which is not observed.

If B is close to 0, then the average location is close to the center of the Hotelling-line where a greater mass of consumers (both shoppers and regular consumers) is concentrated. Therefore, we define a brand with B close to 0 as a *mainstream* brand, which can potentially appeal to the majority of consumers. On the other hand, if B is close to either end of $\pm 1/2$, the brand could only appeal to a smaller mass of consumers away from the center of the Hotelling-line. Therefore, we describe a brand with B close to $\pm 1/2$ as a *niche* brand.

The brand's spread Δ (and consumers' expectation $\widetilde{\Delta}$) crucially affects the regular consumers' search decision and, hence, the firm's expected demand. If the spread is small, then both products are located close to B, and consumers located close to the brand have a high chance of having a good match and, thus, may be willing to incur the search cost. So, if the firm can sustain a small enough Δ in equilibrium, it can encourage more visits by neighboring regular consumers. Thus, we interpret a small brand spread in equilibrium as a *strong* brand. On the other hand, if the equilibrium brand spread is large, then regular consumers' probability of a good match with the brand will be smaller. So, it is possible that no regular consumer are willing to search the brand. We specify a brand with a spread too large to serve any regular consumer as a *weak* brand. Though we adopt these terms that are seemingly binary to describe brand positioning (i.e., mainstream vs. niche) and spread (i.e., strong vs. weak), respectively, it must be noted that we analyze the firm optimal decision on the entire interval, i.e., $B \in [-1/2, 1/2]$ and $\Delta \in [0, 1/2 - |B|]$.[12]

As consumers have heterogeneous preferences, the brand has an incentive to keep a somewhat large spread and capture more demand by locating its products distant. Moreover, this would lead

[12]We define a mainstream vs. niche positioning and strong vs. weak brand more carefully in Section 4.

to a brand positioning closer to the center, $B \approx 0$, a mainstream brand positioning. On the other hand, upon observing a brand positioned $B \approx 0$, consumers may be more uncertain about the exact spread of the brand, i.e., the true locations of the brand's *products*. This is because there can be many different combinations that can all generate $B = 0$ as long as $x_1 = -x_2 \in [0, 1/2]$. This potentially reduced scope of a mainstream-positioned brand's ability to communicate information about the brand's product may discourage consumers' search and, therefore, hurt the brand.

By comparison, a niche brand positioning can convey clearer *product* information to consumers than a mainstream brand positioning. Particularly, in an extreme case, by observing $B = 1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = 1/2$; similarly, by observing $B = -1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = -1/2$. More generally, any brand position B can be supported by $x_1 = B - \Delta$ and $x_2 = B + \Delta$ for $\Delta \in [0, \min\{1/2 + B, 1/2 - B\}]$. As B goes from 0 to $\pm 1/2$, the range of possible Δ gets smaller. By claiming a niche brand positioning, the brand can communicate credibly that all of its products are indeed consistent with the overall brand image. Then, consumers who are close to the brand positioning will knowingly visit the brand. Thus, in the presence of consumer search costs, the brand can convince consumers to incur search costs due to the increased information value from positioning.

Choosing a mainstream average location (i.e., the center of the line) can appeal to a broader set of *shoppers*. On the other hand, a tighter expected spread between the two products can appeal to the local *regular consumers*. This tradeoff indicates that the brand may wish to position itself at the center (catering larger set of consumers) with smaller spreads of the products to induce consumer search from regular consumers. However, as we show in our analysis, if the brand's position is close to the center, it can face temptations to opportunistically dilute its brand by locating two products in opposite directions further than expected. The tradeoff between breadth of appeal and clarity of the information communicated is central to brand positioning. We explore this tradeoff in the subsequent equilibrium analysis. We identify conditions (essentially depending on the degree of search frictions in the market) for the brand to choose a niche versus mainstream brand positioning.[13]

[13] Our model setup—specifically, our definition of brand positioning —constrains a niche brand to be a strong brand with a small enough spread. This restriction may reflect reality. Most brands perceived by consumers as niche indeed carry products of similar styles that are consistently positioned in a niche market. For instance, Lefty's specializes in a niche market of left-handed products. If Lefty's offer some products for right-handed users, Lefty's would no longer be the same niche brand for left-handed consumers that it used to be. Other examples include Studio Tomboy and Urban Outfitters, which are perceived as "bold and casual" and "hip and contemporary," respectively. Both brands offer niche designs mostly consistent with their images. Had Studio Tomboy or Urban Outfitters carried several formal dresses, consumers' perception of the brands would be altered and no longer be so niche anymore.

4 Equilibrium Analysis

We start our analysis by characterizing a representative consumer's search decisions and demand and then formulating the equilibrium condition. After that, we analyze two benchmark cases, one with only shoppers and the other with only regular consumers. These cases can be obtained by setting $\alpha = 0$ and $\alpha = 1$, respectively, in the general model. These benchmark models help us isolate the effect of search frictions on the optimal brand positioning, which we subsequently explore.

4.1 Consumer's Search Decision, Demand and Equilibrium Concept

Given the brand's choice of brand positioning B, we calculate the mass of regular consumers who search (or visit the store). Shoppers have zero search cost, so they will always search.

By searching or visiting the brand, a regular consumer at location x pays the search cost s, and discovers the match utility with both products, $u_1(x)$ and $u_2(x)$. Her expected utility is,

$$\begin{aligned} \mathrm{E}[u(x)] = \mathrm{E}\left[\max\{0, u_1(x), u_2(x)\}\right] &= 1 - (1-\theta+t|x-\widetilde{x}_1|)\,(1-\theta+t|x-\widetilde{x}_2|) \\ &= 1 - \left(1-\theta+t|x-B+\widetilde{\Delta}|\right)\left(1-\theta+t|x-B-\widetilde{\Delta}|\right). \end{aligned} \tag{3}$$

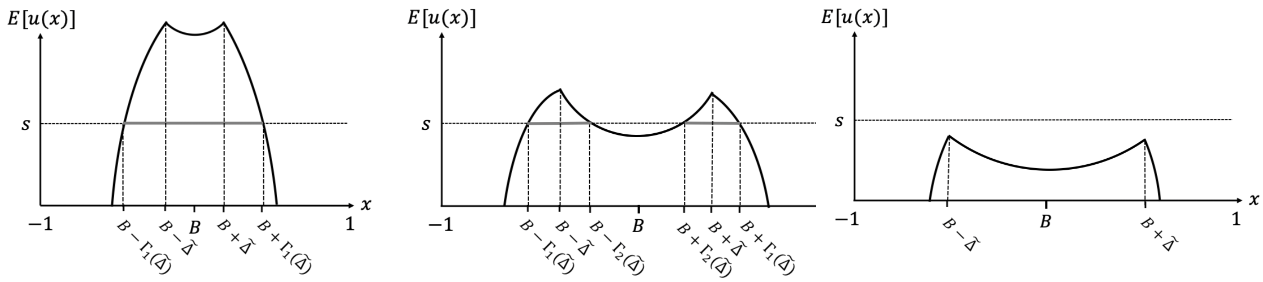
The regular consumer will visit the brand if and only if the benefit of search is greater than the cost of search: $\mathrm{E}[u(x)] \geq s$. So, we can now calculate the total number of visitors (or those who search the brand) among regular consumers. We denote the set of visitors on the Hotelling-line as $\mathcal{V}(B, \widetilde{\Delta})$, where,

$$\mathcal{V}(B,\widetilde{\Delta}) \equiv \begin{cases} \left[B-\Gamma_1(\widetilde{\Delta}), B+\Gamma_1(\widetilde{\Delta})\right] \text{ if } \widetilde{\Delta} \leq \Delta_1, \\ \left[B-\Gamma_1(\widetilde{\Delta}), B-\Gamma_2(\widetilde{\Delta})\right] \cup \left[B+\Gamma_2(\widetilde{\Delta}), B+\Gamma_1(\widetilde{\Delta})\right] \text{ if } \Delta_1 < \widetilde{\Delta} < \Delta_2, \\ \emptyset, \text{ otherwise,} \end{cases} \tag{4}$$

where $\Gamma_1(\widetilde{\Delta}) \equiv \sqrt{\widetilde{\Delta}^2 + (1-s)/t^2} - (1-\theta)/t$, $\Gamma_2(\widetilde{\Delta}) \equiv \sqrt{\left(\widetilde{\Delta} + (1-\theta)/t\right)^2 - (1-s)/t^2}$, $\Delta_1 \equiv \sqrt{1-s}/t - (1-\theta)/t$, and $\Delta_2 \equiv \left[(1-s) - (1-\theta)^2\right]/\left[2t(1-\theta)\right]$.[14]

Equation (4) shows that the total number of regular consumers who visit the brand can be characterized by three different cases, depending on how large the expected spread $\widetilde{\Delta}$ is. First, if $\widetilde{\Delta} \leq \Delta_1$, every consumer in between the two products is willing to visit the brand by incurring the

[14]For any given brand positioning $B \in [-1/2, 1/2]$, we have $\mathcal{V}(B, \widetilde{\Delta}) \in (-1, 1)$ under Assumption 2, and therefore the demand truncation never happens.



(a) A strong brand ($\widetilde{\Delta} \leq \Delta_1$) (b) A medium brand ($\Delta_1 < \widetilde{\Delta} < \Delta_2$) (c) A weak brand ($\widetilde{\Delta} \geq \Delta_2$)

Figure 1: The set of regular consumers who visit the brand.

search cost of s. We have a *strong brand* in this case, as shown by Figure 1-(a), where the gray line represents the set of regular consumers who visit the brand, $\mathcal{V}(B, \widetilde{\Delta}) = \left[B - \Gamma_1(\widetilde{\Delta}),\, B + \Gamma_1(\widetilde{\Delta})\right]$. Second, $\Delta_1 < \widetilde{\Delta} < \Delta_2$, some consumers located in between the two products find it not worthwhile to visit the brand because neither product is expected to provide a good match. We have a *medium brand* in this case, as shown by Figure 1-(b), where regular consumers who visit the brand come from two disjoint intervals that are, roughly speaking, located around $B - \widetilde{\Delta}$ (i.e., $x \in \left[B - \Gamma_1(\widetilde{\Delta}),\, B - \Gamma_2(\widetilde{\Delta})\right]$) and $B + \widetilde{\Delta}$ (i.e., $x \in \left[B + \Gamma_2(\widetilde{\Delta}), B + \Gamma_1(\widetilde{\Delta})\right]$), respectively. Finally, when $\widetilde{\Delta} \geq \Delta_2$, as illustrated by Figure 1-(c), the two products are so distant from each other that no regular consumer will visit the brand. We have a *weak brand* that is unattractive to regular consumers in this case, $\mathcal{V}(B, \widetilde{\Delta}) = \varnothing$. These definitions for a strong, medium, and weak brands are used throughout the paper.

Given the total number of regular consumers who visit, $\mathcal{V}(B, \widetilde{\Delta})$, we can derive the total expected demand from both regular consumers and shoppers. The total expected demand $D(B, \widetilde{\Delta}, \Delta)$ is a function of brand positioning B, consumer's expected spread $\widetilde{\Delta}$ (which affects regular consumers' visit decisions), and the actual spread Δ (which affects all consumers' purchase decisions):

$$
\begin{aligned}
D(B, \widetilde{\Delta}, \Delta) = \frac{\alpha}{2} \int_{x \in \mathcal{V}(B, \widetilde{\Delta})} & [1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)]\, dx \\
& + \frac{1-\alpha}{2} \int_{-1}^{1} [1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)]\, dx. \quad (5)
\end{aligned}
$$

Using $D(B, \widetilde{\Delta}, \Delta)$ above, we formulate the firm's problem next. The firm chooses B and Δ based on the consumer's expected spread $\widetilde{\Delta}$ to maximize the expected demand, where the equilibrium brand positioning, B^* and equilibrium brand spread, Δ^* are given by,

$$
(B^*, \Delta^*) = \underset{(B,\Delta) \in \left\{0 \leq \Delta \leq \frac{1}{2} - |B|\right\}}{\arg\max} D(B, \Delta^*, \Delta). \quad (6)
$$

The right-hand side in Equation (6) involves a constrained optimization problem, where the constraint, $0 \leq \Delta \leq 1/2 - |B|$ is due to limited product space on the Hotelling-line for product positioning. From the perspective of regular consumers who do not observe Δ, we have their expected spread, $\widetilde{\Delta}$ must satisfy $0 \leq \widetilde{\Delta} \leq 1/2 - |B|$. This signifies that the firm can use brand positioning B to *directly* influence consumers' belief $\widetilde{\Delta}$, which is a unique feature to our model. In equilibrium, we require consumers' expectation $\widetilde{\Delta}$ coincides with the firm's optimal choice, Δ^*.

We are interested in the sequential equilibrium of the game (Kreps and Wilson 1982). Compared with perfect Bayesian equilibrium, sequential equilibrium uses trembling-hand perturbations to reach nodes that are off the equilibrium path, and require the players' strategies and beliefs to be sequentially rational and consistent on these nodes. Hence, even if the game reaches off the equilibrium path, the players choose actions optimally from then on, consistent with the equilibrium strategy. More specifically, consumers are aware of the model primitives α, θ and s, and have some expectations about the possible set of equilibria for B, which they observe. If regular consumers observe the firm's choice of B' outside this set, then their beliefs about the unobserved spread $\widetilde{\Delta}$ is consistent with the firm's optimal choice, $\Delta^*(B')$ given the deviated position choice of B'.

To solve for the sequential equilibria of the game, we proceed as the following two steps. First, we take the brand B as given and consider the brand's optimal decision on the spread Δ that is consistent with consumers' anticipation for the spread Δ^*. There are three cases to consider, according to Equation (4). The set of regular consumers who visit the brand, $\mathcal{V}(B, \Delta^*)$, is one interval when $\Delta^* \leq \Delta_1$ (which is the strong brand case), two disjoint intervals when $\Delta_1 < \Delta^* < \Delta_2$ (which is the medium brand case), or an empty-set when $\Delta^* \geq \Delta_2$ (which is the weak brand case). For each of the three cases, we calculate the firm's expepcted demand $D(B, \Delta^*, \Delta)$ based on Equation (5), and determine the equilibrium $\Delta^*(B)$ by taking derivatives of $D(B, \Delta^*, \Delta)$ with respect to Δ and solving the corresponding optimality conditions taking into account of the constraint of $0 \leq \Delta \leq 1/2 - |B|$. Second, we maximize $D(B, \Delta^*(B), \Delta^*(B))$ with respect to B to solve for the equilibrium brand positioning, B^*.

In the next section, we examine two benchmark cases where only one type of consumers exist in the market (shoppers only and regular consumers only). This helps us to identify the underlying forces for the firm's optimal brand positioning decision.

4.2 Benchmarks: Shoppers Only ($\alpha = 0$) and Regular Consumers Only ($\alpha = 1$)

As the first benchmark, we consider the case with only shoppers, which is by setting $\alpha = 0$ in the main model. In this case, all consumers in the market do not incur search costs to visit the brand, and thus, both the brand positioning B and the spread of products Δ are observable to consumers. Therefore, the problem reverts to a simple product positioning problem, and there is no special role of brand positioning beyond the product differentiation. The later comparison between the main model and this benchmark can highlight the unique role of brand positioning. It is easy to solve the complete-information game, the equilibrium of which is presented by the following proposition.

Proposition 1 (Shoppers Only) *If there are only shoppers in the market ($\alpha = 0$), there exists a unique equilibrium with $(B^*, \Delta^*) = (0, \max\{\Delta^*_{\alpha=0}, 0\})$, where $\Delta^*_{\alpha=0} \equiv [1 - (1 - \theta)/t]/2$. Moreover, $\Delta^*_{\alpha=0}$ increases in θ and t: $\partial\Delta^*_{\alpha=0}/\partial\theta \geq 0$ and $\partial\Delta^*_{\alpha=0}/\partial t \geq 0$.*

If there are only shoppers in the market, the firm will position the brand in the center with $B^* = 0$, which is the standard result in product positioning problem. As θ increases, each product provides a good match for nearby consumers with a higher probability, the firm does not see benefits of juxtaposing its products in proximity. Also, as t increases, consumers' preferences become more heterogeneous. Both results in a larger optimal spread, which is quite intuitive.

Next, we study the alternative benchmark case with only regular consumers in the market (that is, $\alpha = 1$). This case is crucially different from the previous benchmark because the regular consumers only observe B and not Δ. Therefore, in equilibrium, the unobservable Δ must be credibly communicated through the firm's choice of B.

Proposition 2 (Regular Consumers Only) *If there are only regular consumers in the market ($\alpha = 1$), there exist a unique set of equilibria such that $\Delta^* = \max\{\Delta^*_{\alpha=1}, 0\}$ and B^* takes any value in $[-(1/2 - \Delta^*), 1/2 - \Delta^*]$, where $\Delta^*_{\alpha=1} \equiv [\sqrt{3(1-s) + 4(1-\theta)^2} - 4(1-\theta)]/(3t) \leq \Delta_1$. Therefore, some regular consumers will visit the brand in equilibrium, who come from one interval on the Hotelling-line. Moreover, $\Delta^*_{\alpha=1}$ decreases in search cost s and increases in θ: $\partial\Delta^*_{\alpha=1}/\partial s \leq 0$ and $\partial\Delta^*_{\alpha=1}/\partial\theta \geq 0$.*

We find that, with only regular consumers, the unobserved product information Δ can be communicated credibly, which is a stark contrast to the main model later. Moreover, in equilibrium, the firm chooses a strong brand with a small spread of $\Delta^* \leq \Delta_1$. This is intuitive because the spread has to be sufficiently small for regular consumers to pay a visit to the store. Based on the observed B

and the expected small spread Δ^*, the regular consumers close to B will visit the store. Therefore, the firm will indeed choose a small spread to serve these regular consumers.

It is immediate to see that a higher search cost s leads to a smaller Δ^* and, thus, a stronger brand.[15] The brand needs to compensate the consumer's high search costs by placing two products closer, which increases their expected utility. Similar to the shoppers-only case, as θ increase the firm wants to keep its products more distant because each product alone is likely to match the needs of consumers close to the product. Also notice that $B^* = 0 \in [-(1/2 - \Delta^*), 1/2 - \Delta^*]$, and therefore, the mainstream brand positioning of $B^* = 0$ is always an equilibrium. Altogether, a *strong* ($\Delta^* \leq \Delta_1$), *mainstream* ($B^* = 0$) brand positioning can be an equilibrium in the case with only regular consumers.

We can now compare the equilibrium spread Δ^* across the two benchmark cases when there are only shoppers ($\alpha = 0$), and only regular consumers ($\alpha = 1$).

Proposition 3 *The equilibrium spread, Δ^* in the case with only regular consumers ($\alpha = 1$) is less than or equal to that in the case with only shoppers ($\alpha = 0$):* $\max\{\Delta^*_{\alpha=1}, 0\} \leq \max\{\Delta^*_{\alpha=0}, 0\}$.

For the regular consumers who must pay a search cost to visit the brand, the brand needs to provide enough benefits by locating two products sufficiently close. In contrast, for shoppers who can visit freely, the brand wants to spread out its products sufficiently to maximize market coverage. If both shoppers and regular consumers co-exist in the market, the brand faces a trade-off in choosing the spread between serving the two types of consumers.

4.3 Equilibrium Brand Positioning

In this section, we analyze the main model with $\alpha \in (0, 1)$. In order to encourage some regular consumers to visit its store, the brand would want to locate their products close to each other by choosing a small spread. However, once regular consumers (who cannot observe the spread of the product directly but only anticipate the spread) visit the brand, the brand is tempted to deviate by increasing the spread to better serve the shoppers. This is the classic *hold-up problem.* Regular consumers are rational and can anticipate the brand's opportunistic behavior. This can be costly for the brand because regular consumers can be discouraged from visiting the brand in the first place. Therefore, we will show that the brand may discipline itself by choosing its brand location B close to

[15]However, the threshold for a strong brand Δ_1 decreases in s at a faster rate. Therefore, as we state in the results in Section 4.3, as s increases, the brand is harder to maintain a strong brand in equilibrium.

either end of the Hotelling-line. In other words, the brand may endogenously choose a niche brand as a commitment device to preserve a strong brand (or the small spread) and thereby serve both the shoppers and the regular consumers.

Importantly, this hold-up problem does not arise under the two benchmark cases. When there are only shoppers, the brand cannot hold them up once they visit the store. When there are only regular consumers, there is no incentive for the brand to deviate by spreading the products further apart. Regular consumers make a visit decision based on their own expectations. Therefore, such a deviation does not increase the number of regular consumers who visit the store. Moreover, such deviation (i.e., increasing the ex-post spread) will reduce the match probability of those who visit the store, further decreasing the demand eventually. Thus, this hold-up problem uniquely arises in the main model where there is heterogeneity in search frictions in the market.

We now solve the sequential equilibrium under the main model. Given any $B \in [-1/2, 1/2]$, there are three possible cases of $\Delta^* \leq \Delta_1$ (a strong brand), $\Delta_1 < \Delta^* < \Delta_2$ (a medium brand) and $\Delta^* \geq \Delta_2$ (a weak brand). For each case, we first relax the constraint $\Delta \leq 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then consider when the constraint $\Delta \leq 1/2 - |B|$ will be binding. In order to highlight the tradeoff in the firm's branding decision, our analysis focuses on the two more interesting cases of a strong brand and a weak brand.[16]

Let's analyze the first case of a strong brand. Given $\Delta^* \leq \Delta_1$, all visitors come from one interval on the Hotelling-line. Based on Equations (4) and (5), we have the first- and second-order optimality conditions imply $\Delta^* = \max\{\Delta^*_\alpha, 0\}$, where, the unconstrained optimal spread Δ^*_α is

$$\Delta^*_\alpha \equiv \frac{\alpha\sqrt{[(1+\alpha)(1-\theta)-(1-\alpha)t]^2+(4-\alpha^2)(1-s)}-2[(1+\alpha)(1-\theta)-(1-\alpha)t]}{(4-\alpha^2)t},$$

which does not depend on B. The following lemma generalizes the binary comparison in Proposition 3 to all $\alpha \in [0, 1]$.

Lemma 1 *The unconstrained optimal spread Δ^*_α decreases with α: $\partial\Delta^*_\alpha/\partial\alpha \leq 0$.*

The constraint is not binding if $\max\{\Delta^*_\alpha, 0\} \leq 1/2 - |B|$; otherwise, we have the constraint

[16]In the Appendix, we analyze all three cases including a medium brand and identify conditions under which a medium brand cannot be an equilibrium. See Lemma A-1 in the Proof of Proposition 5.

binding and $\Delta^* = 1/2 - |B|$. To summarize,

$$\Delta^*(B) = \begin{cases} \max\{\Delta^*_\alpha, 0\}, & \text{if } |B| \le 1/2 - \max\{\Delta^*_\alpha, 0\}, \\ 1/2 - |B|, & \text{otherwise.} \end{cases} \tag{7}$$

This spread can be part of an equilibrium only if it satisfies the assumed condition for a strong brand that $\Delta^*(B) \le \Delta_1$ so that the firm's optimal choice of the spread is consistent with the consumers' expected spread.

It is important to know whether a strong brand can be maintained because a strong brand can serve both shoppers and regular consumers. If only a weak brand can be sustained, the firm is unable to serve the regular consumers completely, thus potentially hurting the firm's profit. The following proposition identifies conditions under which a strong brand can be sustained in equilibrium.

Proposition 4 (Strong brand) *A strong brand can be sustained in equilibrium as follows:*

1. *If $s \le 1 - (1 - \theta + t)^2/4$ or $\alpha \ge \widehat{\alpha}$, then $\Delta^*(B) \le \Delta_1$ for all $B \in [-1/2, 1/2]$. Therefore, any positioning (regardless of whether it is a mainstream or a niche positioning) can sustain a strong brand.*

2. *Otherwise (i.e., if $s > 1 - (1 - \theta + t)^2/4$ and $\alpha < \widehat{\alpha}$), then $\Delta^*(B) \le \Delta_1$ if and only if $|B| \ge 1/2 - \Delta_1$. Therefore, only a niche brand positioning (i.e., $|B| \ge 1/2 - \Delta_1$) can sustain a strong brand.*

This proposition suggests that a strong brand with a small spread $\Delta^* \le \Delta_1$ can always be achieved by a niche positioning, i.e., $|B| \ge 1/2 - \Delta_1$. However, a mainstream positioning (i.e., around the center location of the Hotelling-line such that $|B| < 1/2 - \Delta_1$) can only sustain a strong brand when s is sufficiently small, or α is sufficiently large.

More precisely, when s is relatively small, or α is relatively large, there is enough number of regular consumers who are willing to visit the brand. Then, the firm can maintain a strong brand with a relatively small spread ($\Delta^* \le \Delta_1$) regardless of its brand positioning $B \in [-1/2, 1/2]$. On the other hand, if s is sufficiently large and α is sufficiently small, a brand positioned in the middle is tempted to increase the spread to better serve the large segment of shoppers (which is evident from Lemma 1 that $\partial\Delta^*_\alpha/\partial\alpha \le 0$). Moreover, a high search cost requires an extremely narrow spread to provide sufficient benefit from consumer search. Putting these two effects together, a mainstream brand's unconstrained optimal choice of spread cannot coincide with the consumers' expected spread that can justify their search costs. This can cost the firm significantly because regular consumers will no longer trust it to serve products close to their tastes and decide not to visit. Again, this is the hold-up problem mentioned before.

Altogether, the proposition implies that the firm may sometimes needs to choose a niche positioning to overcome the hold-up problem when it wants to serve regular consumers. The benefit of choosing a niche positioning is that the firm can attract both shoppers and regular consumers. However, by choosing a brand's position far away from the center of the Hotelling-line, the firm is unable to serve the shoppers optimally (as shown in Proposition 1). Given these tradeoffs, the firm will choose the brand positioning B^* optimally to maximize $D(B, \Delta^*(B), \Delta^*(B))$.[17] Note that, in equilibrium, the firm chooses not only whether to be a mainstream or niche, but also how close to the central location a mainstream brand will be, and how close to an end a niche brand will be.

We show that two types of equilibria can arise under different parameter regions: mainstream and niche. First, as the next result shows, a mainstream brand equilibrium is always positioned at the center of the Hotelling-line, $B^* = 0$, but it may sometimes be a strong or weak brand.

Proposition 5 (Mainstream Positioning) *Mainstream positioning can be an equilibrium either as a strong brand or a weak brand.*

1. *If $s \leq 1-(t+1-\theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, then a strong mainstream brand $(B^*, \Delta^*) = (0, \max\{\Delta^*_\alpha, 0\})$ is an equilibrium. The brand serves both regular consumers and shoppers.*[18]

2. *If $s > 1 - t(1-\theta)$ and $\alpha < \overline{\alpha}_{main} (< \widehat{\alpha})$, then a weak mainstream brand $(B^*, \Delta^*) = (0, \Delta^*_{\alpha=0})$ is an equilibrium, and the brand serves only the shoppers.*

The composition of consumers (i.e., the size of α), consumer's search cost s, and match probability of each ideal product θ affect the brand's optimal brand positioning decision critically. As implied by Proposition 4, when s is relatively small, or α is relatively high, there are enough number of regular consumers who are willing to visit the brand, supporting a strong brand for any positioning $B \in [-1/2, 1/2]$. Therefore, even a mainstream positioning can attain a strong brand, and it is then an equilibrium to set $\Delta^* = \max\{\Delta^*_\alpha, 0\}$ that attracts both regular consumers and shoppers to visit.

However, if consumer search cost is sufficiently high such that $s > 1 - t(1-\theta)$, and there exist a sufficiently large number of shoppers ($\alpha < \overline{\alpha}_{main}$), only a weak brand is feasible for a mainstream

[17] In the Appendix (proof to Proposition 5), we follow a similar procedure to analyze the other two cases with $\Delta_1 < \Delta^* < \Delta_2$ and $\Delta^* \geq \Delta_2$.

[18] We can further prove the uniqueness of this equilibrium under $s \leq 1-(t+1-\theta)^2/4$.

positioning, forgoing the regular consumers.[19] When the mainstream brand is weak, the firm may find it more profitable to choose a niche brand positioning to achieve a strong brand. While the brand would appeal to a smaller portion of the entire population, it can attract both regular consumers and shoppers. This benefit of monetizing on the regular consumers is not so significant if α (the size of the regular consumers) is small. Therefore, appealing to a small group of regular consumers by choosing a niche cannot justify forsaking the mainstream brand positioning when the firm can serve the large group of shoppers. This is the main tradeoff between choosing the mainstream vs. niche brand positioning. Also, we find that, given mainstream positioning, it is optimal to position at the center of the Hotelling-line, $B^* = 0$, in order to best serve the shoppers.

Next, we turn to the conditions under which a niche positioning can be the firm's equilibrium choice in the following proposition. The firm chooses a niche brand positioning with $|B^*| = 1/2 - \Delta_1 > 0$ and one product exactly at one endpoint with $|B^*| + \Delta^* = 1/2$.

Proposition 6 (Niche Positioning) *If $s > 1 - t(1-\theta)$ and $\underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche} (< \widehat{\alpha})$, then the unique set of equilibria are strong niche brands: $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$.*[20]

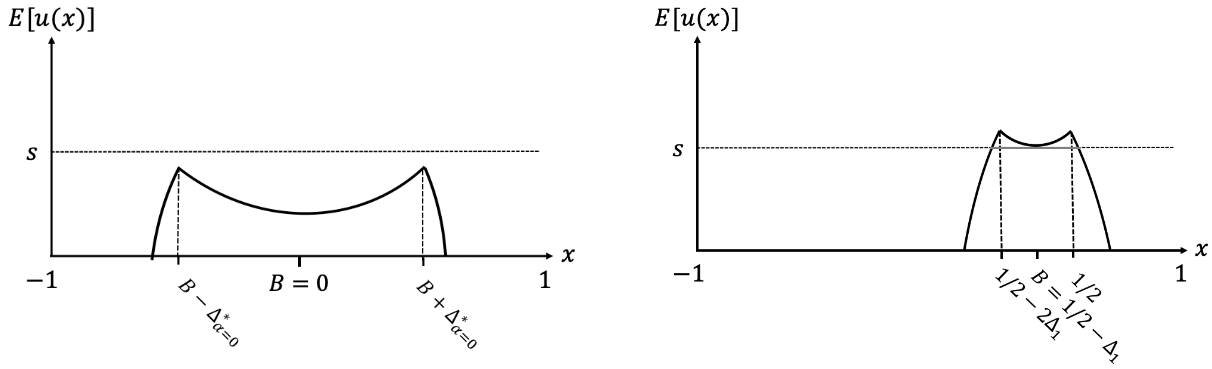
Following the discussions for Proposition 5, the firm finds niche positioning more profitable if the following two conditions are met: (1) a mainstream positioning suffers from a hold-up problem, and (2) the benefit of attracting regular consumers as a niche positioning is large enough.

First, for condition (1), consumer's search cost has to be sufficiently large or θ not too large ($s > 1 - t(1-\theta)$), as shown in Proposition 4 and Proposition 5. In this case, the regular consumers will not visit the brand if it is positioned in the center of the Hotelling-line ($B = 0$), which is illustrated in Figure 2-(a). This is because of the hold-up problem, where the existence of shoppers induces the firm to spread the two products apart. Still, regular consumers demand a smaller spread to justify their search costs. Then, as this proposition suggests, a niche brand positioning can serve as a commitment not to spread its products excessively. However, if there are many regular consumers ($\alpha \geq \overline{\alpha}_{niche}$), even the mainstream can sustain a strong brand (Proposition 5) and serve regular consumers. Therefore, the hold-up problem matters when consumer's search cost has to be sufficiently large ($s > 1 - t(1-\theta)$), and there are not too many regular consumers ($\alpha < \overline{\alpha}_{niche}$).

Next, condition (2) is satisfied if $\alpha > \underline{\alpha}_{niche}$ such that there are enough regular consumers in

[19]The condition $s > 1 - t(1-\theta)$ implies $\Delta^*_{\alpha=0} > \Delta_2$ such that the spread $\Delta^*_{\alpha=0}$ is part of an equilibrium for a mainstream brand where no regular consumers visit the brand.

[20]We have $\underline{\alpha}_{niche} \geq \overline{\alpha}_{main}$. Therefore, the interval of α, $(\underline{\alpha}_{niche}, \overline{\alpha}_{niche})$, for which a niche brand positioning is optimal in Proposition 6 is to the right of the interval $[0, \overline{\alpha}_{main})$ for which a weak mainstream brand is optimal in Part 2 of Proposition 5.



(a) A weak mainstream brand

(b) A strong niche brand

Figure 2: Regular consumers' search decision when facing a niche vs. a mainstream brand under $s > 1 - t(1-\theta)$ and $\underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche}$.

the market. Then, the brand finds it optimal to sacrifice the central location to position its brand toward an endpoint. Unlike mainstream brand positioning which can only serve shoppers (depicted in Figure 2-(a)), the brand can serve some regular consumers, which is illustrated in Figure 2-(b).

Finally, we find that the firm chooses B such that the niche brand is placed as close as possible to the center of the Hotelling-line. This is because the firm can benefit from moving closer to the center position and thereby serve more shoppers while keeping regular consumers' expected benefit greater than search costs. Based on these two forces coming from incentives to serve regular consumers vs. shoppers, each pulling the brand positioning in opposite directions, the firm chooses an optimal location $(|B^*|, \Delta^*) = (1/2 - \Delta_1, \Delta_1)$. This is precisely the equilibrium location B^* where it must be close enough to the endpoints so that regular consumers are willing to visit, while at the same time as close as possible to the center to maximize the demand from the shoppers.

Mainstream vs. Niche Brand Positioning

We have characterized two different types of equilibria—mainstream positioning and niche positioning. Proposition 5 and 6 together demonstrate the opposing forces induced by two segments of consumers and how their interplay fundamentally influences the brand's positioning choice between mainstream and niche positioning. For regular consumers, the brand has an incentive to establish a strong brand positioning by placing its products close to each other. However, the presence of shoppers tempts the brand to spread out its product locations to accommodate the shopper's heterogeneous preferences more efficiently. The more shoppers there are, the stronger this temptation becomes. This will dilute the brand image and discourage regular consumers from visiting the brand.

Therefore, the brand needs a mechanism to overcome the temptation if it aims to serve both shoppers and regular consumers. We showed that niche brand positioning can do this role by helping the firm communicate a strong brand (or small variance) credibly to consumers and prevent brand dilution.

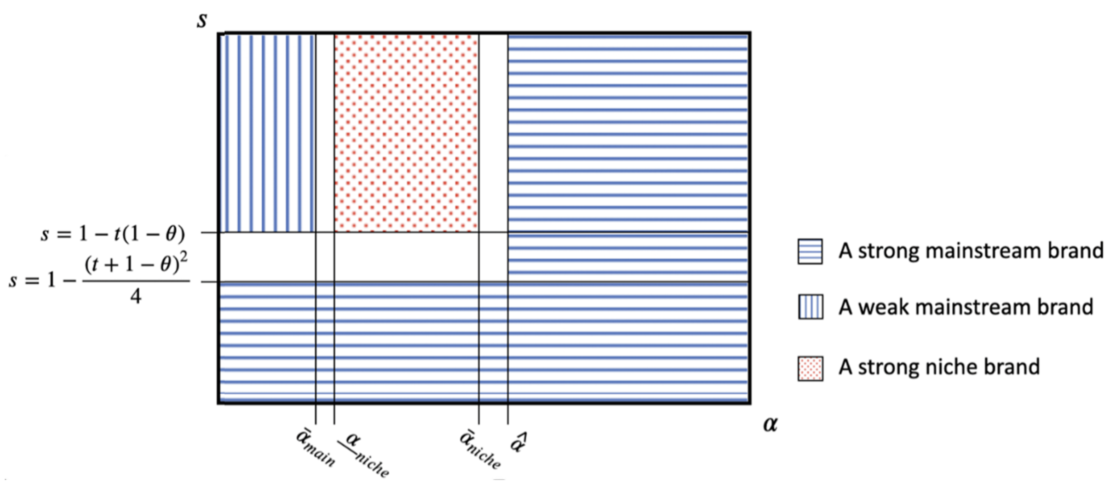


Figure 3: The regions for the existence of different types of equilibrium brand positioning.

A niche brand can be more profitable than a mainstream brand under a specific condition, i.e., if α is in an intermediate interval $(\underline{\alpha}_{niche}, \overline{\alpha}_{niche})$, and the consumer search cost is sufficiently large, i.e. $s > 1 - t(1-\theta)$. This condition is represented by the red dotted area in Figure 3. This implies that even if niche brand can provide more information about the brand's products, under a wide range of parameter space, the brand prefers a mainstream brand to a niche brand. More specifically, whenever the strong mainstream brand (illustrated with horizontal lines in Figure 3) exists, it is an equilibrium. On the other hand, a weak mainstream brand (depicted by vertical lines in Figure 3) can only be an equilibrium if α is sufficiently small. A weak mainstream branding is dominated by a niche branding in an intermediate region and by a strong mainstream brand when α becomes sufficiently large. In summary, the figure shows that when the search cost is sufficiently high and there exist neither too many nor too few shoppers, a monopolistic firm can be better off with a niche brand position, which is a unique equilibrium.

Our findings provide an explanation for why some brands that target niche markets thrive and sometimes outperform other brands that aim to serve the mainstream market. For instance, Lululemon, which is regarded as a niche brand targeting young yoga-lovers, has become the fifth largest market share in the entire athletic apparel industry (source: Statistica 2019/2020). The clarity in Lululemon's niche brand positioning achieved through their product portfolio may have generated an intensive engagement from what may seem like a smaller niche market. Lululemon can assure consumers that they can find different yoga pants in various styles and colors. It is more likely for the shopper to find what she needs by visiting Lululemon than Nike who also carries some yoga pants, but positioned as a mainstream "sport wear" by offering a general line of product portfolio.

Figure 3, as a visual summary of Propositions 5 and 6, focuses on the existence of different types of equilibria. It is a sequential game where a firm first chooses B based on the market situation such as s and α to maximize its profit between mainstream and niche branding. Therefore, three distinct regions in Figure 3 for a strong mainstream, a weak mainstream, and a strong niche brand positioning do not overlap. However, we cannot rule out the co-existence of medium brand positioning.[21] Nevertheless, for some cases, we still prove the uniqueness of the equilibrium (even considering the possibility of medium brand equilibrium); most prominently, the uniqueness of the strong niche brand equilibrium in Proposition 6. For the mainstream brand, we can only show the uniqueness result for the strong mainstream brand equilibrium under $s \leq 1 - (t + 1 - \theta)^2/4$.

Our approach for brand positioning is based on product portfolio view. The results from our analysis help explain some of real-world observations about how mainstream brands can be diluted. For example, fashion brands such as Ellen Tracy, Anne Klein and Dana Buchman documented in an HBS case on Eileen Fisher (Keinan et al., 2012) struggled because brands changed their product offerings to appeal the younger generation in early 2000s. Such attempts alienated the brands' existing customers, and they turned into diluted and weak mainstream brands (Keinan et al., 2012). Moreover, our analysis predicts that a lower search cost of regular consumers will mitigate temptations of dilution of mainstream brands. This prediction resonates with what we have observed in several industries including fashion. In the past, fashion brands mostly depended on off-line channels for sales as well as communications, which entail a high search cost. However, over the past decades, fashion brands moved their sizable businesses and communications to online channels. Also, fashion-related information is no longer reserved only for fashion critics and experts, which means that the regular consumers' search cost has dropped significantly. These are, by and large, consistent with our observations of several successful strong mainstream brands, such as Uniqlo and GAP, who maintain the strong mainstream positioning by offering a product portfolio about evenly mixed between formal and casual designs.

[21] Note that Proposition 5 and 6 together do not cover the entire parameter space in Figure 3; notably, equilibrium in one intermediate region for s and two intermediate intervals of α are not identified. This is because we have identified a sufficient condition to rule out the existence of a medium brand equilibrium where regular consumers visit the brand from two disjoint intervals. Therefore, in these missing regions, a medium brand equilibrium may exist. However, an analysis of a medium brand would add significant technical complexities, and thus, we focus on a strong brand and a weak brand. This decision helps us clearly understand the important tradeoffs in the firm's brand positioning decision and how it affects the role of branding as a communication mechanism.

5 Extensions

5.1 Three-product case

The main model analyzes a simple case where the firm has two products. This section analyzes a case when the firm carries three products and shows that our main results are robust, and thus generalizable beyond the simple two-product cases. In this extension, the firm chooses the styles, or locations, of three products x_1, x_2, and x_3. The shoppers of size $1 - \alpha$ observe individual product locations without incurring any positive cost. Regular consumers of size α only observe the brand positioning $B = \frac{x_1+x_2+x_3}{3}$ after they incur a search cost $s > 0$ to visit the firm's store. After observing the brand positioning B, regular consumers decide whether to visit the brand. Without loss of generality, it is assumed that $x_1 \leq B \leq x_2 \leq x_3$. We can define $x_1 = B - \Delta_1$, $x_2 = B + \Delta_2$, and $x_3 = B + \Delta_3$, which implies $\Delta_1 \geq \Delta_3 \geq \Delta_2 \geq 0$ and $-\Delta_1 + \Delta_2 + \Delta_3 = 0$. If $\Delta_2 = 0$, then we have $\Delta_1 = \Delta_3$ such that the middle product is located precisely at B and two other products are equally distant (in the opposite directions) from B. We refer to this special case with $\Delta_2 = 0$ as a *symmetric* positioning.

Repeating the steps of analysis of the two-product model, we show that the qualitative results are robust to the case with more number of products. We briefly state the results and relegate a more detailed analysis to the Appendix.

Proposition 7 (Two Benchmarks) *Suppose that the firm carries three products. Then, equilibrium of the two benchmark models are characterized as follows:*

1. *Suppose $\alpha = 0$ (shoppers only). There exists a unique equilibrium with $B^* = 0$, $\Delta_2^* = 0$ and $\Delta_1^* = \Delta_3^* = \max\{\Delta_{\alpha=0}^*, 0\}$, where $\Delta_{\alpha=0}^* \equiv \frac{\sqrt{8t^2+16t(1-\theta)+9(1-\theta)^2}-5(1-\theta)}{4t}$.*

2. *Suppose $\alpha = 1$ (regular consumers only). If s is not too small, there exist a set of symmetric equilibria such that $\Delta_2^* = 0$ and $\Delta_1^* = \Delta_3^* = \Delta_{\alpha=1}^*$, and $B^* \in [-(1/2 - \Delta_{\alpha=1}^*), 1/2 - \Delta_{\alpha=1}^*]$.*

Similar to the main model with two products, we have $\Delta_{\alpha=0}^* > \Delta_{\alpha=1}^*$ such that the equilibrium spread is greater when the firm faces shoppers only than when it serves only regular consumers. In Appendix, we show that there exists $\overline{\Delta} > 0$ such that the equilibrium spread, $\Delta_{\alpha=0}^* > \overline{\Delta}$. Thuns, had there been any regular consumer, she would not visit the firm. It implies that in a more general model with both shoppers and regular consumers $\alpha \in [0, 1]$, the firm can face a hold-up problem if there are sufficiently many shoppers in the market. Therefore, the firm can lose the opportunity to serve any regular consumers. This leads to the following result analogous to Proposition 6 for the two-product case.

Proposition 8 (Niche Positioning) *If α is in an intermediate range, then the firm's expected revenue can be greater as a niche positioning brand than as a mainstream positioning brand.*

5.2 Relaxing exogenous pricing: An example

In our main analysis above, we have assumed that the prices of the two products are exogenously given, and the normalized consumer utility of a matched product is one. One might worry that our main analysis and results will not be robust if the firm chooses its product prices endogenously. In this section, we present one example with the firm's choice of prices and a slight modification in our utility setup to demonstrate the robustness of our main analysis without endogenous pricing.

The firm sets price p_i for each product i. Consumers' match utility distribution is modified from Equation (1) as follows:

$$u_i(x) = \begin{cases} \max\{v_i - p_i, 0\}, & \text{with probability } \theta - t|x - x_i|, \\ \max\{-p_i, 0\} = 0, & \text{otherwise.} \end{cases}$$

where v_i is consumer-specific consumption value, which follows a binary distribution with $v_i = v_0$ with probability φ and $v_i = v_0 + 1$ with probability $1 - \varphi$. The term v_i represents the ex-ante heterogeneity among consumers which is observable to consumers at the beginning of the game. The firm sets the prices and the positions of the two products simultaneously, and those prices are not observable to regular consumers before they pay the search cost and visit the store.[22]

We find that, in the modified game in which the firm chooses its product prices, there exists an equilibrium in which the firm sets $p_i = v_0$ for both products and the firm's decisions for brand positioning B and brand spread Δ are equivalent to the equilibrium branding decision under exogenous pricing in Propositions 5 and 6, with an appropriate scaling of parameters α.[23] In this

[22] The assumption that consumers do not observe the prices until they visit the store is conventional in the consumer search literature reviewed in Section 2. This assumption allows us to check for robustness of our results without undermining brand positioning as the main channel of information communication. Alternatively, if prices were observable to consumers prior to visiting a store, then it may be possible that the prices convey some information about individual products. Analyzing such a model in addition to our main model with an implicit communication through brand positioning is beyond the scope of this paper, which we leave for future research.

[23] It is straightforward to argue that the firm will set the equilibrium price for each product as either $v_0 + 1$ or v_0. If the equilibrium price is set at $v_0 + 1$, only the regular consumers with $v_i = v_0 + 1$ will make a purchase if they find a match, in which case they derive zero utility. Consequently, it is not worthwhile for them to pay the search cost and visit the store in the first place. It becomes a trivial case, so we focus on a more interesting case where $p_i = v_0$ when v_0 is sufficiently high and/or φ is sufficiently high

equilibrium, no regular consumers with lower value $v_i = v_0$ will incur the positive search cost. Under the equilibrium price, the regular consumers with $v_i = v_0 + 1$ has the following utility distribution:

$$u_i(x) = \begin{cases} 1, & \text{with probability } \theta - t|x - x_i|, \\ 0, & \text{otherwise.} \end{cases}$$

which is equivalent to the utility distribution in Equation (1) under the exogenous pricing assumption. So, the fraction of effective regular consumers who may visit the firm is $\alpha^{price} = \frac{\alpha(1-\varphi)}{\alpha(1-\varphi)+(1-\alpha)}$. The shoppers' decision is also the same as the model with exogenous pricing. These equivalence properties show that our main analysis of the firm's branding decision under exogenous pricing is robust to this particular example with endogenous pricing.

6 Conclusion

A brand's positioning is one of a firm's most valuable assets: it can have a significant impact on consumers' purchase decisions; it helps firms create market differentiation by providing or articulating critical information about product characteristics; it facilitates consumer search and aids them in determining where to purchase products without having to search through multiple brands. However, the critical issue of brand positioning remains an under-researched topic in economics and marketing. There is a lack of a framework for thinking about the design of brand positioning in a unified and consistent way.

In this paper, we conceptualize the role of brand positioning through a product portfolio and consumer search framework. First, we provide a micro-foundation for how and why brand positioning can deliver credible information to consumers, and we provide a rationale for a firm's specific brand positioning strategy: "niche" or "mainstream." Consumers are uncertain about each product location of the brand but are aware of the average location of the brand's products—in other words, its brand positioning. Based on this information, consumers can make decisions about whether to visit a brand (or firm), while brands can simultaneously determine their individual product locations. We identify the conditions under which brands tend to adopt a niche positioning strategy over a mainstream strategy and find that niche brands prevail when the fraction of regular consumers in the population is within the intermediate range and search cost is sufficiently high. We also find that a mainstream-positioned brand has a greater incentive to spread its individual product locations, which leads to brand dilution, while the niche-positioned brand has less incentive to do so. The niche-positioned brand can thus serve as a commitment tool for a brand not to spread its individual product location and thereby invite more consumers to visit the store. Therefore, our results shed light on the coexistence of different brand positioning and provide firms insights into forming an optimal brand positioning strategy.

The current research proposes one possible mechanism of how brand positioning can affect consumer information, their search decisions, and market outcomes. In doing so, we adopted a simple framework of single-dimensional brand positioning of a monopolist, which cannot fully capture the intricacy of multi-dimensional characteristics of brand positioning in consumers' perceptual space. A natural extension would be to construct a more general framework of brand positioning in multiple dimensions, especially including a price image. Also, we have examined the monopolistic firm's brand positioning choice. We believe our main finding that even a monopolist sometimes finds it optimal to choose a niche positioning to commit not to dilute its own brand can be of a valuable addition to the branding literature. Nevertheless, it would be interesting to investigate, for example, how strategic considerations can change the optimal brand positioning under competition. Extending the current framework to a competitive situation can be a fruitful and important venue for further investigation, which will broaden the implications of our study. Also, we investigated credibility and effects of a particular piece of information, i.e., brand positioning in this paper. Another important issue is, in line with a growing body of research in information design, endogenizing the firm's choice of the piece of information to be communicated. We hope that our modeling framework of the brand position will serve as a basis for advancing further discussions and stimulating more studies on this fundamental concept of marketing for further research.

Appendix

Proof of Proposition 1

Under $\alpha = 0$, by Equation (5), $D(B, \Delta^*, \Delta)$ does not depend on Δ^*. For simplicity we use the shorthand notation of D for $D(B, \Delta^*, \Delta)$ when there is no confusion. We have

$$\begin{aligned}
\left.\frac{\partial D}{\partial B}\right|_{B=B^*} &= -2t(1-\theta+t)B^* = 0, \text{ and } \frac{\partial^2 D}{\partial B^2} = -2t(1-\theta+t) < 0 \Rightarrow B^* = 0. \\
\left.\frac{\partial D}{\partial \Delta}\right|_{\Delta=\Delta^*} &= -4t^2\Delta^*\left[\Delta^* - \tfrac{1}{2}\left(1-\tfrac{1-\theta}{t}\right)\right] = 0, \text{ and} \\
\left.\frac{\partial^2 D}{\partial \Delta^2}\right|_{\Delta=\Delta^*} &= -8t^2\left[\Delta^* - \tfrac{1}{4}\left(1-\tfrac{1-\theta}{t}\right)\right] \leq 0 \Rightarrow \Delta^* = \max\left\{\tfrac{1}{2}\left(1-\tfrac{1-\theta}{t}\right), 0\right\}.
\end{aligned}$$

Notice that we always have $\Delta^* \leq 1/2$. The comparative statics of Δ^* with respect to t and θ is straightforward to obtain. ■

Proof of Proposition 2

Given any $B \in [-1/2, 1/2]$, let's consider three cases below depending on Δ^* according to Equation (4). For each case, we will first relax the constraint $\Delta \leq 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then consider when will the constraint $\Delta \leq 1/2 - |B|$ be binding. For simplicity we use the shorthand notation of D for $D(B, \Delta^*, \Delta)$ when there is no confusion.

First, $\Delta^* \leq \Delta_1$, in which case, all visitors come from one interval on the Hotelling-line. Based on equations (4) and (5), we have the first- and second-order optimality conditions:

$$\begin{aligned}
\left.\frac{\partial D}{\partial \Delta}\right|_{\Delta=\Delta^*} &= 2t^2\Delta^*\left[\sqrt{\tfrac{1-s}{t^2} + (\Delta^*)^2} - 2\Delta^* - \tfrac{2(1-\theta)}{t}\right] = 0, \\
\left.\frac{\partial^2 D}{\partial \Delta^2}\right|_{\Delta=\Delta^*} &= 2t^2\left[\sqrt{\tfrac{1-s}{t^2} + (\Delta^*)^2} - 4\Delta^* - \tfrac{2(1-\theta)}{t}\right] \leq 0.
\end{aligned}$$

This implies that $\Delta^* = \max\left\{[\sqrt{3(1-s) + 4(1-\theta)^2} - 4(1-\theta)]/(3t), 0\right\} = \max\{\Delta^*_{\alpha=1}, 0\}$, which does not depend on B. This implies that,

$$\Delta^*(B) = \begin{cases} \max\{\Delta^*_{\alpha=1}, 0\}, & \text{if } |B| \leq 1/2 - \max\{\Delta^*_{\alpha=1}, 0\}, \\ 1/2 - |B|, & \text{otherwise.} \end{cases} \tag{8}$$

Lastly, We can easily verify that under Assumption 1, $\Delta^*_{\alpha=1} \leq \Delta_1$ and thus $\Delta^*(B) \leq \Delta_1$.

Second, $\Delta_1 < \Delta^* < \Delta_2^*$, in which case, visitors come from two disjoint intervals on the Hotelling-line. Based on equations (4) and (5), we find that,

$$\left.\frac{\partial D}{\partial \Delta}\right|_{\Delta=\Delta^*} = 2t^2 \left[\Delta^* \sqrt{\Delta^{*2} + \tfrac{1-s}{t^2}} - \left(\Delta^* + \tfrac{1-\theta}{t}\right) \left(2\Delta^* - \sqrt{\left(\Delta^* + \tfrac{1-\theta}{t}\right)^2 - \tfrac{1-s}{t^2}}\right)\right].$$

Next, we will show that $\Delta^* < \Delta_2$ implies that $\partial_\Delta D|_{\Delta=\Delta^*} < 0$. In fact, $\Delta^* < \Delta_2$ is equivalent to,

$$\Delta^{*2} + \tfrac{1-s}{t^2} - \left(\Delta^* + \tfrac{1-\theta}{t}\right)^2 > 0. \tag{9}$$

$\partial_\Delta D|_{\Delta=\Delta^*} < 0$ is equivalent to,

$$\Delta^* \left[\sqrt{\Delta^{*2} + \tfrac{1-s}{t^2}} - \left(\Delta^* + \tfrac{1-\theta}{t}\right)\right] < \left(\Delta^* + \tfrac{1-\theta}{t}\right) \left[\Delta^* - \sqrt{\left(\Delta^* + \tfrac{1-\theta}{t}\right)^2 - \tfrac{1-s}{t^2}}\right]. \tag{10}$$

Further notice that $\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t}\right) = \frac{\Delta^{*2} + \frac{1-s}{t^2} - \left(\Delta^* + \frac{1-\theta}{t}\right)^2}{\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} + \left(\Delta^* + \frac{1-\theta}{t}\right)}$, and $\Delta^* - \sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}} = \frac{\Delta^{*2} + \frac{1-s}{t^2} - \left(\Delta^* + \frac{1-\theta}{t}\right)^2}{\Delta^* + \sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}}}$. Based on these two equations, Equation (10) can be equivalently rewritten as,

$$\Delta^* \left[\Delta^* + \sqrt{\left(\Delta^* + \tfrac{1-\theta}{t}\right)^2 - \tfrac{1-s}{t^2}}\right] < \left(\Delta^* + \tfrac{1-\theta}{t}\right) \left[\sqrt{\Delta^{*2} + \tfrac{1-s}{t^2}} + \left(\Delta^* + \tfrac{1-\theta}{t}\right)\right], \tag{11}$$

By multiplying both sides of Equation (10) with $(\Delta^* + (1-\theta)/t)$ and multiplying both sides of Equation (11) with Δ^* and summing them up, we have that $\partial_\Delta D|_{\Delta=\Delta^*} < 0$ is equivalent to, $\sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}} \left[\Delta^{*2} + \left(\Delta^* + \frac{1-\theta}{t}\right)^2\right] < \Delta^* \left[3\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \Delta^{*2}\right]$, which holds because $\sqrt{(\Delta^* + (1-\theta)/t)^2 - (1-s)/t^2} < \Delta^*$ by Equation (9) and $\Delta^{*2} + (\Delta^* + (1-\theta)/t)^2 < 3(\Delta^* + (1-\theta)/t)^2 - \Delta^{*2}$. Therefore, we have proved that under $\Delta_1 < \Delta^* < \Delta_2$, we have $\partial_\Delta D|_{\Delta=\Delta^*} < 0$. This further implies that there does not exist an equilibrium with $\Delta_1 < \Delta^* < \Delta_2$.

Thirdly, $\Delta^* \geq \Delta_2$, which results in zero demand and cannot be in equilibrium.

So far, we have identified an equilibrium candidate for a given B as in Equation (8). Next, we solve for $B^* = \arg\max_B D(B, \Delta^*(B), \Delta^*(B))$.

Consider $\Delta^*(B)$ given by Equation (8). By symmetry, it is without loss of generality to consider $B \geq 0$. We have the following four observations: (1) $\partial_B D(B, \Delta^*, \Delta) = 0$. (2) From Equation (8), it is obvious that $\Delta^{*\prime}(B) \leq 0$ for $B \geq 0$. (3) By the first-order optimality condition, we have that $\partial_\Delta D(B, \Delta^*(B), \Delta^*(B)) \geq 0$, where the equality holds when $|B| + \Delta^*(B) < 1/2$, and the inequality holds when $|B| + \Delta^*(B) = 1/2$. (4) $\partial_{\Delta^*} D(B, \Delta^*(B), \Delta^*(B)) = s\Delta^*(B)/\sqrt{\Delta^*(B)^2 + (1-s)/t^2} > 0$.

These four observations together imply that,

$$\frac{dD(B,\Delta^*(B),\Delta^*(B))}{dB} = \partial_B D(B,\Delta^*(B),\Delta^*(B)) + [\partial_{\Delta^*} D(B,\Delta^*(B),\Delta^*(B)) + \partial_\Delta D(B,\Delta^*(B),\Delta^*(B))]\, \Delta^{*\prime}(B) \leq 0. \quad (12)$$

This implies that the equilibrium brand positioning, B^* takes any value such that $|B^*| \leq 1/2 - \max\{\Delta^*_{\alpha=1}, 0\}$, and correspondingly, the equilibrium spread, $\Delta^* = \max\{\Delta^*_{\alpha=1}, 0\}$. Moreover, later in Proposition 3, we show that $\max\{\Delta^*_{\alpha=1}, 0\} \leq \max\{\Delta^*_{\alpha=0}, 0\} \leq 1/2$. Notice that this equilibrium exists for the entire parameter space under Assumptions 1 and 2. ■

Proof of Proposition 3

We prove that $\max\{\Delta^*_{\alpha=0}, 0\} \geq \max\{\Delta^*_{\alpha=1}, 0\}$. Notice that $\Delta^*_{\alpha=1}$ decreases with s, and $s \geq 1-(1-\theta+t/2)^2$ under Assumption 2. This implies that $\Delta^*_{\alpha=1}$ takes the maximum value of $[\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)]/(3t)$ at $s = 1-(1-\theta+t/2)^2$. Therefore, we only need to prove that

$$\max\{\Delta^*_{\alpha=0}, 0\} \geq \max\left\{\frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t}, 0\right\}.$$

To prove this, we only need to prove that $\frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} > 0 \Rightarrow \Delta^*_{\alpha=0} \geq \frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t}$. Further notice that $\frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} > 0 \Leftrightarrow t > 2(1-\theta)$, and $\Delta^*_{\alpha=0} \geq \frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} \Leftrightarrow$ $6t^2+18(1-\theta)t-3(1-\theta)^2 \geq 0$. $t > 2(1-\theta)$ implies that $6t^2+18(1-\theta)t-3(1-\theta)^2 \geq 57(1-\theta)^2 \geq 0$. This completes the proof. ■

Proof of Lemma 1

First differentiating D with respect to Δ results in $\frac{\partial D}{\partial \Delta} = 2t\cdot\Delta^*_\alpha\cdot\Big(-(1+\alpha)(1-\theta)+t\big(1-2\Delta^*_\alpha - \alpha(1-\sqrt{(\Delta^*_\alpha)^2+\frac{1-s}{t^2}})\big)\Big) = 0$. Differentiating both sides with respect to α, we have $-(1-\theta)-t\cdot\Big(1-\sqrt{(\Delta^*_\alpha)^2+\frac{1-s}{t^2}}\Big) = \frac{\partial\Delta^*_\alpha}{\partial\alpha}\cdot t\cdot\Big(2-\alpha\cdot\frac{\Delta^*_\alpha}{\sqrt{(\Delta^*_\alpha)^2+\frac{1-s}{t^2}}}\Big)$, where $\Big(2-\alpha\cdot\frac{\Delta^*_\alpha}{\sqrt{(\Delta^*_\alpha)^2+\frac{1-s}{t^2}}}\Big) > 0$. The left-hand side is negative if and only if $t^2(\Delta^*_\alpha)^2+1-s < (1-\theta+t)^2$. Given $\Delta^*_\alpha \leq \frac{1}{2}$, we only need to show $\frac{t^2}{4}+1-s < (1-\theta+t)^2$ ($\Leftrightarrow s > 1-\big(1-\theta+\frac{t}{2}\big)\big(1-\theta+\frac{3t}{2}\big)$). This holds by Assumption 2.■

Proof of Proposition 4

Using $\Delta^*_{\alpha=1} \leq \Delta_1$ and $\partial\Delta^*_\alpha/\partial\alpha < 0$, it is straightforward to show that if $\Delta^*_{\alpha=0} > \Delta_1$, then there exists $\widehat{\alpha} \in (0,1)$ such that $\Delta^*_\alpha \leq \Delta_1$ if and only if $\alpha \geq \widehat{\alpha}$. Note that $\Delta^*_{\alpha=0} > \Delta_1$ if and only if $s > 1-(t+1-\theta)^2/4$. Also, if $s \leq 1-(t+1-\theta)^2/4$ such that $\Delta^*_{\alpha=0} \leq \Delta_1$, then $\Delta^*_\alpha \leq \Delta_1$ for all $\alpha \in [0,1]$. Putting all these conditions together, $\Delta^*_\alpha < \Delta_1$ if $s \leq 1-(1-\theta+t)^2/4$ or $\alpha \geq \widehat{\alpha}$.

Otherwise, if $s > 1 - (1 - \theta + t)^2/4$ and $\alpha \leq \widehat{\alpha}$, then $\Delta^*_\alpha \geq \Delta_1$. Then, the results in Part 1 and 2 follow immediately from Equation (7). ■

Proof of Proposition 5

Two propositions will be proven in the following steps. (1) We rule out equilibria involving a medium brand in which the equilibrium spread Δ^* satisfies $\Delta_1 < \Delta^* < \Delta_2$. (2) Equilibria for a mainstream positioning are analyzed for a strong brand ($\Delta^* \leq \Delta_1$) and weak brand ($\Delta^* \geq \Delta_2$) cases. (3) An equilibrium with a niche positioning is identified. (4) We compare the profits and characterize the equilibrium brand positioning decision.

Lemma A-1 *A medium brand cannot be an equilibrium given mainstream positioning if either* $s \leq 1 - (t + 1 - \theta)^2/4$ *or* $s > 1 - t(1 - \theta)$ *and* $\alpha < \overline{\alpha}_{niche} (\leq \widehat{\alpha})$. *A medium brand cannot be an equilibrium given niche positioning if* $\alpha < \underline{\underline{\alpha}}$ *or* $\alpha > \underline{\alpha}$.

Proof. Suppose such an equilibrium existed such that an equilibrium spread Δ^* satisfies $\Delta_1 < \Delta^* < \Delta_2$. Then, the regular consumers who visit the brand come from two disjoint intervals. Similar to the analysis of a strong brand in the main text, we first consider an unconstrained optimization problem. A medium brand implies that $|B|$ cannot be to close to an end, i.e., $|B| < 1/2 - \Delta_1$.

In the proof of Proposition 2, we have proved that $\partial_\Delta D|_{\Delta=\Delta^*,\alpha=1} < 0$. Proof of Proposition 1 shows that if $\Delta^*_{\alpha=0} \leq \Delta_1$, or $s \leq 1 - (t + 1 - \theta)^2/4$, then $\partial_\Delta D|_{\Delta=\Delta^*,\alpha=0} < 0$. Also, if $\Delta^*_{\alpha=0} \geq \Delta_2$, or equivalently, $s > 1 - t(1 - \theta)$, then $\partial_\Delta D|_{\Delta=\Delta^*,\alpha=0} > 0$. Note that $\partial_\Delta D|_{\Delta=\Delta^*} = \alpha \partial_\Delta D|_{\Delta=\Delta^*,\alpha=1} + (1 - \alpha)\partial_\Delta D|_{\Delta=\Delta^*,\alpha=0}$. So, if $s \leq 1 - (t + 1 - \theta)^2/4$, then $\partial_\Delta D|_{\Delta=\Delta^*} < 0$ for all $\alpha \in [0, 1]$, and hence medium brand cannot be sustained given a mainstream positioning. Also, if $s > 1 - t(1 - \theta)$ and α is not too large, i.e., $\alpha < \overline{\alpha}_{niche}$ for some $\overline{\alpha}_{niche} \in (0, 1)$, then $\partial_\Delta D|_{\Delta=\Delta^*} < 0$, which means that medium brand cannot be an equilibrium.

A medium brand may be feasible with a niche positioning where $1/2 - \Delta_2 \leq |B| < 1/2 - \Delta_1$ and $\Delta^*(B) = 1/2 - B$. It remains to characterize the optimal choice of B. For this, we compute $dD(B, \Delta^*(B), \Delta^*(B))/dB$. In particular, we rule out any equilibrium with a medium brand by finding a condition that $\frac{dD(B,\Delta^*(B),\Delta^*(B))}{dB} > 0$, which would imply that the firm wants to position its brand close enough to an end until it reaches $|B| = 1/2 - \Delta_1$. When there is no confusion, $D(B, \Delta^*(B), \Delta^*(B))$ is abbreviated by $D^*(B)$. Equation (12) applies here, where $D^*(B) = \alpha \cdot D^*(B)|_{\alpha=1} + (1 - \alpha) \cdot D^*(B)|_{\alpha=0}$.

First, for $\alpha = 1$ case, $\left.\frac{\partial D}{\partial \Delta^*}\right|_{\Delta=\Delta^*,\alpha=1} = \frac{s\left[\Delta^*\sqrt{\left(\Delta^*+\frac{1-\theta}{t}\right)^2-\frac{1-s}{t^2}}-\left(\Delta^*+\frac{1-\theta}{t}\right)\sqrt{\Delta^{*2}+\frac{1-s}{t^2}}\right]}{\sqrt{\left(\Delta^*+\frac{1-\theta}{t}\right)^2-\frac{1-s}{t^2}}\sqrt{\Delta^{*2}+\frac{1-s}{t^2}}} < 0$. It is equivalent to $\Delta^*\sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t}\right)\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} < 0 \Leftrightarrow \Delta^{*2}\left[\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}\right] - \left(\Delta^* + \frac{1-\theta}{t}\right)^2\left(\Delta^{*2} + \frac{1-s}{t^2}\right) < 0 \Leftrightarrow -\frac{1-s}{t^2}\left[\left(\Delta^* + \frac{1-\theta}{t}\right)^2 + \Delta^{*2}\right] < 0$. This proves that $\partial_{\Delta^*} D^*(B)|_{\Delta=\Delta^*,\alpha=1} < 0$. Moreover, it is easy to show that $\partial_B D|_{\alpha=1} = 0$ and $\Delta^{*\prime}(B) = -1$. This implies that $\frac{dD^*(B)}{dB}|_{\alpha=1} = \partial_B D^*(B)|_{\alpha=1} - (\partial_\Delta D^*(B)|_{\Delta=\Delta^*,\alpha=1} + \partial_{\Delta^*} D^*(B)|_{\Delta=\Delta^*,\alpha=1}) > 0$.

Second, for $\alpha = 0$ case, $\left.\frac{dD^*(B)}{dB}\right|_{\alpha=0} = t\left[4t\Delta^*(B)^2 + 4(1-\theta)\Delta^*(B) - t - (1-\theta)\right] < 0 \Leftrightarrow \Delta^*(B) < \frac{\sqrt{t^2+t(1-\theta)+(1-\theta)^2}-(1-\theta)}{2t}$. Notice that $\Delta^*(B) < \Delta_2$, so to show $dD^*(B)/dB|_{\alpha=0} < 0$, we only need to show that $\Delta_2 \leq [\sqrt{t^2 + t(1-\theta) + (1-\theta)^2} - (1-\theta)]/(2t)$, or, $s \geq 1 - (1-\theta)\sqrt{t^2 + t(1-\theta) + (1-\theta)^2}$, which holds if $s > 1 - t(1-\theta)$. This shows that $dD^*(B)/dB|_{\alpha=0} < 0$.

Putting together the two benchmark cases of $\alpha = 1$ and $\alpha = 0$, there exist $\underline{\alpha}$ and $\underline{\underline{\alpha}}$ such that $dD(B, \Delta^*(B), \Delta^*(B))/dB \leq 0$ for $\alpha < \underline{\alpha}$ and $dD(B, \Delta^*(B), \Delta^*(B))/dB \geq 0$ for $\alpha > \underline{\underline{\alpha}}$. In summary, the medium brand cannot be an equilibrium for both mainstream and niche positioning under the following conditions: (1) $\alpha \leq \underline{\underline{\alpha}}$, or $\alpha \geq \underline{\alpha}$, and (2) either $s \leq 1 - (t + 1 - \theta)^2/4$ or $s > 1 - t(1-\theta)$ and $\alpha \leq \overline{\alpha}_{niche}$. ■

Lemma A-2 *Given a mainstream positioning (i.e., $|B| < 1/2 - \Delta_1$), a weak brand ($\Delta^* \geq \Delta_2$) can be an equilibrium only if $s > 1 - t(1-\theta)$.*

Proof. No regular consumers will visit the firm, and accordingly the firm's optimal branding decision is equivalent to the firm's decision in the first benchmark with only the shoppers. Therefore, the firm sets $(B^*, \Delta^*) = (0, \Delta^*_{\alpha=0})$. Then, regular consumers' decision not to visit the firm is optimal if only if $\Delta^*_{\alpha=0} \geq \Delta_2$, which holds if $s > 1 - t(1-\theta)$. This is a necessary condition for a weak mainstream brand to be an equilibrium as the firm's potential deviation to a niche positioning needs to be checked (which is done in the end of Proof of Proposition 5.) ■

Lemma A-3 *The optimal niche positioning is $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$.*

Proof. In Lemma A-1, we ruled out a medium brand case for a niche positioning. The only remaining possibility for an equilibrium with a niche positioning is a strong brand where $\Delta^* \leq \Delta_1$. For $|B| \geq 1/2 - \Delta_1$, we have $\Delta^*(B) = 1/2 - |B|$ given by Equation (7). In this case, by the proofs of Propositions 1 and 2, we know both the shoppers' demand and the regular consumers' demand decreases with $|B|$, and therefore the total demand, $D(B, \Delta^*(B), \Delta^*(B))$ achieves the maximum at $|B^*| = 1/2 - \Delta_1$. ■

We have thus far identified optimal positioning for each of three types of positioning: a strong mainstream, a weak mainstream and strong niche positioning. It remains to check for profitable deviations from each of these cases. In particular, from a mainstream positioning, the firm can deviate to the optimal niche positioning, and vice versa. The demand under each of the three identified candidates for equilibrium are respectively denoted by $D_{main}^{strong}, D_{main}^{weak} = (1-\alpha)\cdot D^*(0, \Delta^*_{\alpha=0})|_{\alpha=0}, D_{niche}^{strong} = \alpha \cdot D^*(1/2 - \Delta_1, \Delta_1)|_{\alpha=1} + (1-\alpha)\cdot D^*(1/2 - \Delta_1, \Delta_1)|_{\alpha=0}$. There are the following three cases:

1. A strong mainstream brand: When it exists as the optimal and consistent positioning decision, there is no profitable deviation to a niche positioning. Therefore, if either $s < 1-(t+1-\theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, then a strong brand $(B, \Delta^*) = (0, \Delta^*_\alpha)$ is an equilibrium.

2. A weak mainstream brand: The firm only serves the shoppers. If the firm deviates to a niche positioning $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$, regular consumers update beliefs accordingly and some of them visit. Therefore, the firm serves both shoppers and regular consumers. This is not a profitable deviation, i.e., $D_{main}^{weak} > D_{niche}^{strong}$, if and only if α is sufficiently small, $\alpha < \overline{\alpha}$. So, if $s > 1 - t(1-\theta)$ and $\alpha < \overline{\alpha}_{main} := \min\{\overline{\alpha}, \underline{\underline{\alpha}}\}$, then a weak mainstream brand $(B, \Delta^*) = (0, \Delta^*_{\alpha=0})$ is an equilibrium.

3. A strong niche brand: Whenever a strong mainstream brand positioning exists, it is profitable to deviate to a mainstream positioning. If a weak mainstream brand exists, niche positioning is more profitable if $s > 1 - t(1-\theta)$ and $\underline{\alpha}_{niche} := \max\{\underline{\alpha}, \overline{\alpha}\} < \alpha < \overline{\alpha}_{niche}$.

This completes the proof. ■

Proof of Proposition 6

The proof follows immediately from the proof of Proposition 5 case 3 when $s > 1 - t(1-\theta)$ and $\underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche}$. Finally, for this result, the interval $\alpha \in (\underline{\alpha}_{niche}, \overline{\alpha}_{niche})$ must be nonempty. We are unable to show these properties for the general parameter space. Instead, we present a numeric analysis to show that these intervals are nonempty under some parameter setting. Take $s = 0.9, \theta = 0.8, t = 0.7$. Then, $\widehat{\alpha} = \overline{\alpha}_{niche} = 0.475$, $\underline{\alpha}_{niche} = 0.271$, and $\overline{\alpha}_{main} = 0.047$. So, our main result holds for all parameter regions $\alpha \in (0.271, 0.475)$. ■

Proof of Proposition 7

Case 1. $\alpha = 0$. A shopper located at $x \in [-1, 1]$ expects to receive utility $\mathbb{E}[u(x)] = 1 - (1-\theta+t|x-B+\Delta_1|)\,(1-\theta+t|x-B-\Delta_1+\Delta_3|)\,(1-\theta+t|x-B-\Delta_3|)$. The firm's expected revenue is $D_{\alpha=0}(B,\Delta_1,\Delta_3) := \int_{-1}^{1} \mathbb{E}[u(x)]\, dx$, which the firm maximizes by setting B, Δ_1, Δ_2 and Δ_3. The first-order condition should satisfy: $\partial D(B,\Delta_1,\Delta_3)/\partial B = 0$, $\partial D(B,\Delta_1,\Delta_3)/\partial \Delta_1 = 0$, and $\partial D(B,\Delta_1,\Delta_3)/\partial \Delta_3 = 0$.

$$\frac{\partial D(B,\Delta_1,\Delta_3)}{\partial B} = -t\big(B^3t^2 + B\big(t^2(3-\Delta_1^2+\Delta_1\Delta_3-\Delta_3^2)+6t(1-\theta)+3(1-\theta)^2\big)+t^2\Delta_1\Delta_3(\Delta_3-\Delta_1)\big)$$
$$\frac{\partial D(B,\Delta_1,\Delta_3)}{\partial \Delta_1} = -\frac{t(\Delta_3-2\Delta_1)}{2}\big(-\Delta_1\big(6t^2\Delta_3+4t(1-\theta)\big)+t^2(1+B^2+2B\Delta_3+4\Delta_3^2)+t(2-\Delta_3)(1-\theta)-2(2-\theta)^2\big)$$
$$\frac{\partial D(B,\Delta_1,\Delta_3)}{\partial \Delta_3} = -\frac{t(\Delta_1-2\Delta_3)}{2}\big(-\Delta_3\big(6t^2\Delta_1+4t(1-\theta)\big)+t^2(1+B^2-2B\Delta_1+4\Delta_1^2)+t(2-\Delta_1)(1-\theta)-2(2-\theta)^2\big)$$

First, note that $\frac{\partial D(B,\Delta_1,\Delta_3)}{\partial B} = 0$ if and only if $B = 0$ and $\Delta_1\Delta_3(\Delta_3-\Delta_1) = 0$. This is because the expression inside the parenthesis is non-negative, and it can be zero only if the terms multiplied by B vanish, and so does the term $t^2\Delta_1\Delta_3(\Delta_3-\Delta_1)$. Given that $B = 0$, we have $\frac{\partial D(B,\Delta_1,\Delta_3)}{\partial \Delta_1} = \frac{\partial D(B,\Delta_3,\Delta_1)}{\partial \Delta_1}$, i.e., there is symmetry between Δ_1 and Δ_3. This implies that there the optimal levels of Δ_1 and Δ_3 will coincide, i.e., $\Delta_1^* = \Delta_3^*$. Plugging this property back into the second or third equation above results in $\frac{\partial D(B,\Delta^*,\Delta^*)}{\partial \Delta_1}|_{\Delta_1=\Delta} = t\Delta^* \cdot g_{\alpha=0}(\Delta^*) = 0$, where

$$g_{\alpha=0}(\Delta) := t^2 - 2\Delta^2 \cdot t^2 - 5\Delta \cdot t(1-\theta) + 2t(1-\theta) - 2(1-\theta)^2, \tag{13}$$

for which $\Delta^* = \max\{0, \Delta^*_{\alpha=0}\}$, where $\Delta^*_{\alpha=0} = \frac{\sqrt{8t^2+16t(1-\theta)+9(1-\theta)^2}-5(1-\theta)}{4t}$.

Case 2: $\alpha = 1$. Assume that $\alpha = 1$ and $\widetilde{\Delta}_1 = \widetilde{\Delta}_3 = \widetilde{\Delta}$. a regular consumer located at x expects to receive utility $\mathbb{E}[u(x);\widetilde{\Delta}] = 1-\big(1-\theta+t|x-B+\widetilde{\Delta}_1|\big)\big(1-\theta+t|x-B-\widetilde{\Delta}_1+\widetilde{\Delta}_3|\big)\big(1-\theta+t|x-B-\widetilde{\Delta}_3|\big)$. The consumer will decide to visit the firm if and only if $\mathbb{E}[u(x);\widetilde{\Delta}] > s$. The set of regular consumers who visit the firm can be expressed as an interval $[\underline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3), \overline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3)]$. The two endpoints of the interval are solutions to the equation $\mathbb{E}[u(x);\widetilde{\Delta}] = s$. If the anticipated spread is too large, i.e., $\widetilde{\Delta} > \overline{\Delta} := -\frac{1-\theta}{t} + \frac{1}{t}\sqrt{\frac{1-s}{1-\theta}}$, then no regular consumer will engage in search. Assuming symmetry, i.e., $\widetilde{\Delta}_2 = 0$ (hence, $\widetilde{\Delta}_1 = \widetilde{\Delta}_3 = \widetilde{\Delta}$), we can obtain $\underline{x} = B - f(\widetilde{\Delta})$ and $\overline{x} = B + f(\widetilde{\Delta})$, where $0 \le f(\widetilde{\Delta}) \le 1 - |B|$ and

$$f(\widetilde{\Delta}) := -\frac{t}{1-\theta} + \frac{\sqrt[3]{2}\cdot t\cdot \widetilde{\Delta}^2}{\sqrt[3]{27(1-s)-\sqrt{27^2(1-s)^2-108\cdot t^6\cdot\widetilde{\Delta}^6}}} + \frac{\sqrt[3]{27(1-s)-\sqrt{27^2(1-s)^2-108\cdot t^6\cdot\widetilde{\Delta}^6}}}{3\sqrt[3]{2}\cdot t}.^{24} \tag{14}$$

For simplicity, we focus on a symmetric case for from here on. The total consumer demand $D_{\alpha=1}(B,\Delta_1,\Delta_3;\widetilde{\Delta}_1,\widetilde{\Delta}_3) := \frac{1}{2}\int_{\underline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3)}^{\overline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3)} [1-(1-\theta+t|x-B+\Delta_1|)(1-\theta+t|x-B-\Delta_1+\Delta_3|)(1-\theta+t|x-B-\Delta_3|)]\, dx$.

In equilibrium denoted by $(B^*,\Delta_1^*,\Delta_3^*)$, the first-order conditions must hold: $\frac{\partial D(B,\Delta_1,\Delta_3;\Delta_1^*,\Delta_3^*)}{\partial \Delta_1}|_{\Delta_1=\Delta_1^*} =$

[24] This requires $s < 1 - \frac{1}{2}2\sqrt{3}\cdot t^3\cdot\widetilde{\Delta}^3 9 (\le 1 - \frac{2\sqrt{3}t^3}{72})$ to ensure that the function is real-valued. Further requires an assumption such that $\mathcal{V}(B,\widetilde{\Delta},\widetilde{\Delta})$ is one interval. Also, it is that $f(\cdot)$ is an increasing function whenever it is real-valued.

0 and $\frac{\partial D(B,\Delta_1,\Delta_3;\Delta_1^*,\Delta_3^*)}{\partial \Delta_3}|_{\Delta_3=\Delta_3^*} = 0$. Given symmetry $\Delta_1^* = \Delta_3^* = \Delta^*$, the conditions become $\frac{\partial D(B,\Delta_1,\Delta_3;\Delta^*)}{\partial \Delta_1}|_{\Delta_1=\Delta^*} = 0$ and $\frac{\partial D(B,\Delta_1,\Delta_3;\Delta^*)}{\partial \Delta_3}|_{\Delta_3=\Delta^*} = 0$. Moreover, the optimality condition for B is $\frac{\partial D(B^*,\Delta^*,\Delta^*;\Delta^*)}{\partial B} = 0$. Using the definition of $D_{\alpha=1}(B,\Delta_1,\Delta_3;\widetilde{\Delta}_1,\widetilde{\Delta}_3)$ above, we can compute $\frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial B}$, $\frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial \Delta_1}$ and $\frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial \Delta_3}$.

First, by the symmetry, $\frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial B} = 0$ given that $|B| + \widetilde{\Delta} < 1/2$.

Second, $\frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial \Delta_1}|_{\Delta_1=\Delta_3=\widetilde{\Delta}} = \frac{\partial D(B,\Delta_1,\Delta_3;\widetilde{\Delta})}{\partial \Delta_3}|_{\Delta_1=\Delta_3=\widetilde{\Delta}} = \frac{\widetilde{\Delta}\cdot t}{2} \cdot g_{\alpha=1}(\widetilde{\Delta})$, where $g_{\alpha=1}(\Delta) := -2t^2\Delta^2 - 5t(1-\theta)\Delta - 2(1-\theta)^2 + 2t(1-\theta)f(\Delta) + t^2 f(\Delta)^2$. ($f(\cdot)$ is defined in Equation (14)). This vanishes if and only if $\widetilde{\Delta} = 0$ or $-2t^2\widetilde{\Delta}^2 - 5t(1-\theta)\widetilde{\Delta} - 2(1-\theta)^2 + 2t(1-\theta)f(\widetilde{\Delta}) + t^2 \cdot f(\widetilde{\Delta})^2 = 0$. When evaluated at $\widetilde{\Delta} = 0$, $-3(1-\theta)^2 < 0$. Next, the same expression evaluated at $\widetilde{\Delta} = 1/2$ is $\frac{1}{48}\big(- 8(6(1-\theta) + t)(3(1-\theta) + 2t) + \frac{3\cdot\sqrt[3]{4}t^4}{\left(\sqrt[3]{-27(1-s)+\sqrt{27^2(1-s)^2-27t^6/16}}\right)^2} + 8/3\sqrt[3]{2}\big(\sqrt[3]{-27(1-s) + \sqrt{27^2(1-s)^2 - 27t^6/16}}\big)^2\big) \geq \frac{1}{48}\big(-8(6(1-\theta)+t)(3(1-\theta)+2t)+\frac{t^4}{3\sqrt[3]{(1-s)^2/4}}\big)$,[25] which is non-negative if and only if $s > 1 - \frac{t^6}{144\sqrt{6}}\sqrt{\frac{1}{(6(1-\theta)+t)^3(3(1-\theta)+2t)^3}}$. Then, under this condition, there exists a solution to the first order equation $\widetilde{\Delta} > 0$. ■

Corollary 1 *1. If the expected spread is sufficiently large, i.e.,* $\Delta^* > \overline{\Delta} := -\frac{1-\theta}{t} + \frac{1}{t}\sqrt{\frac{1-s}{1-\theta}}$*, then no regular consumer will visit the firm. 2. If s is sufficiently large, then the optimal spread when there are only shoppers satisfies* $\Delta^*_{\alpha=0} > \overline{\Delta}$*. 3. The equilibrium spread,* Δ^* *in the case with only regular consumers* ($\alpha = 1$) *is less than or equal to that in the case with only shoppers* ($\alpha = 0$)*:* $\max\{\Delta^*_{\alpha=1}, 0\} \leq \max\{\Delta^*_{\alpha=0}, 0\}$.

Proof. 1. Given symmetry, a consumer located exactly at $x = B$ has the largest expected utility from the firm among all consumers. So, if this consumer finds it optimal not to visit, then no regular consumers will visit. This is the case if $1-(1-\theta+t(B-B+\Delta^*))(1-\theta)(1-\theta+t(B+\Delta^*-B)) < s$, which implies that $\Delta^* > \overline{\Delta} := -\frac{1-\theta}{t} + \frac{1}{t}\sqrt{\frac{1-s}{1-\theta}}$.

2. Finally, $\Delta^*_{\alpha=0} > \overline{\Delta}$ if and only if $\sqrt{8t^2 + 16t(1-\theta) + 9(1-\theta)^2} - (1-\theta) > 4\sqrt{\frac{1-s}{1-\theta}}$. This inequality clearly holds as the consumer search cost s becomes sufficiently large, i.e., $s > 1 - \frac{1-\theta}{16}\big(\sqrt{8t^2 + 16t(1-\theta) + 9(1-\theta)^2} - (1-\theta)\big)^2$.

3. From Proposition 7, $\Delta_{\alpha=0}$ makes the inside the parentheses of Equation (13) vanish, and $\Delta_{\alpha=1}$ the same for $g_{\alpha=1}(\Delta)$ Computing the difference between the expressions inside the parentheses results in $g_{\alpha=0}(\Delta)-2g_{\alpha=1}(\Delta) = t(1-f(\Delta))\big(2(1-\theta)+t(1-f(\Delta))\big) \geq 0$. Therefore, $g_{\alpha=1}(\Delta^*_{\alpha=1}) = 0$, and $g_{\alpha=0}(\Delta^*_{\alpha=1}) \geq 0$. Since $g_{\alpha=0}(\Delta)$ is decreasing in Δ, this shows that $\Delta^*_{\alpha=0} \geq \Delta^*_{\alpha=1}$. ■

[25] The last inequality is due to $\sqrt[3]{-27(1-s)} \leq \sqrt[3]{-27(1-s) + \sqrt{27^2(1-s)^2 - 27t^6/16}} \leq 0$, and therefore squaring the terms result in $\sqrt[3]{-27(1-s)}^2 \geq \sqrt[3]{-27(1-s) + \sqrt{27^2(1-s)^2 - 27t^6/16}}^2 \geq 0$

Proof of Proposition 8

We now consider a more general case where a market consists of $\alpha \in (0,1)$ fraction of regular consumers and $1-\alpha$ of shoppers. The expected revenue of the firm is $D_\alpha = \alpha \cdot D_{\alpha=1} + (1-\alpha)\cdot D_{\alpha=0}$. The expected demand $D_\alpha(B, \Delta_1, \Delta_3; \widetilde{\Delta}_1, \widetilde{\Delta}_3) =$

$$\begin{aligned}&\tfrac{\alpha}{2}\int_{\underline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3)}^{\overline{x}(B,\widetilde{\Delta}_1,\widetilde{\Delta}_3)} [1-(1-\theta+t|x-B+\Delta_1|)(1-\theta+t|x-B-\Delta_1+\Delta_3|)(1-\theta+t|x-B-\Delta_3|)]\,dx \\ &\quad+\tfrac{1-\alpha}{2}\int_{-1}^{1} [1-(1-\theta+t|x-B+\Delta_1|)(1-\theta+t|x-B-\Delta_1+\Delta_3|)(1-\theta+t|x-B-\Delta_3|)]\,dx\end{aligned} \tag{15}$$

The firm will choose B, Δ_1 and Δ_3 to maximize this expected demand.

If α is sufficiently small so that the firm cannot serve regular consumers, then as a mainstream positioning brand, the maximum expected demand is $D_\alpha(0, \Delta^*_{\alpha=0}) = (1-\alpha)\cdot D_{\alpha=0}(0, \Delta^*_{\alpha=0})$.

On the other hand, if the firm chooses a niche positioning $B^* = 1-\overline{\Delta}$, where $\overline{\Delta} = -\frac{1-\theta}{t} + \frac{1}{t}\sqrt{\frac{1-s}{1-\theta}}$, the expected demand is $D_\alpha(\frac{1}{2}-\overline{\Delta}, \overline{\Delta}) = \alpha\cdot D_{\alpha=1}(\frac{1}{2}-\overline{\Delta}, \overline{\Delta}) + (1-\alpha)\cdot D_{\alpha=0}(\frac{1}{2}-\overline{\Delta}, \overline{\Delta})$. More precisely, $D_{\alpha=1}(B^*, \overline{\Delta}) = \frac{1}{2}\int_{\underline{x}(\frac{1}{2}-\overline{\Delta},\overline{\Delta})}^{\overline{x}(\frac{1}{2}-\overline{\Delta},\overline{\Delta})} \left[1-(1-\theta+t|x-B^*+\overline{\Delta}|)(1-\theta+t|x-B^*|)(1-\theta+t|x-B^*-\overline{\Delta}|)\right] dx$.

Comparing the two expected demands, $D_\alpha(1/2-\overline{\Delta}, \overline{\Delta}) > D_\alpha(0, \Delta^*_{\alpha=0})$ if α is sufficiently large. Therefore, α can neither be too large nor too small.[26]

It remains to show an existence of parameters for which the expected demand is greater as a niche-positioned brand. The equilibrium spread Δ^*_α under $B=0$ should satisfy $\alpha\cdot g_{\alpha=1}(\Delta) + (1-\alpha)\cdot g_{\alpha=0}(\Delta) = 0$, where the left-hand side of the equation is decreasing in Δ. We will find a lower bound of the left-hand side to obtain a lower bound of Δ^*_α and compute conditions under which this lower bound is greater than $\overline{\Delta}$.

Note that $f(\cdot) \geq 0$, and therefore $g_{\alpha=1}(\Delta) \geq \underline{g}_{\alpha=1}(\Delta) := -2t^2\Delta^2 - 5t(1-\theta)\Delta - 2(1-\theta)^2$. Then, the solution to the equation $\alpha\cdot\underline{g}_{\alpha=1}(\Delta) + (1-\alpha)\cdot g_{\alpha=0}(\Delta) = 0$ can be computed and denoted by $\underline{\Delta}^*_\alpha = -\frac{15(2-\alpha)(1-\theta)t-\sqrt{3}\sqrt{t^2(12(8t^2-16t(-1+\theta)+9(-1+\theta)^2)-4\alpha(40t^2-80t(-1+\theta)+31(-1+\theta)^2)+\alpha^2(64t^2-128t(-1+\theta)+43(-1+\theta)^2))}}{8(3-2\alpha)t}$.

And, $\underline{\Delta}^*_\alpha > \overline{\Delta}$ if and only if α is sufficiently small and s large enough.

Lastly, we find a numeric example which satisfies sufficient conditions for the result stated this proposition to hold: $\theta = 0.8, t = 0.5, s = 0.97$ and $0.375 \leq \alpha \leq 0.388$ ■

[26] Note that here we do not show that $(B^*, \Delta*) = (1/2-\overline{\Delta}, \overline{\Delta})$ is the optimal niche brand positioning decision, which is analytically cumbersome for the three-product case. However, as we compare profits under the optimal mainstream positioning to those under a possibly suboptimal niche positioning, we make a conservative statement in the proposition.

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