Endogenous Inequality in Decentralized Two-Sided Markets

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Abstract

This paper examines whether and under what conditions one should expect asymmetric equilibrium in markets where agents on both sides look for an optimal alternative on the other side through a costly sequential search. It shows that when the potential match payoff distribution has thick tails, a stable equilibrium may necessarily involve asymmetric search strategies and, consequently, the expected payoff inequality between the two sides even when the two sides are ex-ante symmetric (equal). That is, in some markets, an unambiguous equilibrium outcome from the interaction between two ex-ante equal populations is that one of them will end up being disadvantaged relative to the other. A consequence is that a one-time intervention to correct inequality in a repeated game may not achieve a lasting equality outcome. Furthermore, a search cost reduction may eliminate equilibrium inequality in some markets but induce it in others. Even worse, lower search costs may reduce welfare.

1 Introduction

Any gainful interaction between two (or more) people or economic agents may raise the questions of equity and equality: are the gains from the interaction split equitably? Inequality-averse preference seems to be ingrained in the human mind (see, e.g., Tricomi et al., 2010). Furthermore, inequality attracts extra attention when the gains are not equitably shared between two agents as a function of the agent belonging to some identifiable group, and especially when there is no a priori clear argument for why one group should have an advantage over another (or whether it is fair for it to have one).

Oftentimes, current inequality is attributed to carry-over effects from an initial distortion in equity. For example, some groups have historically occupied a position of power. This may have occurred relatively randomly (e.g., which nation won a war, was lucky to come across a favorable natural environment or discover a useful technology) or explained by factors that should be no longer relevant in the modern world (e.g., physical strength). The consideration that follows is often about how long would it take for the inequity to disappear or to be corrected, as once corrected, it is assumed that a lasting (self-sustaining) equitable equilibrium will ensue.

Whenever an inequality exists between groups that do not currently have substantively different underlying abilities, a theoretically interesting and practically relevant question is whether it is driven by such historical factors, or could the inequality be expected in equilibrium even if the initial conditions (i.e., initial allocations, abilities, and rules) would be equal (symmetric). This question is practically important due to its consequences for establishing equality. If inequalities are due to what I called the historical factors, then resetting the system could lead to the perpetually equal outcomes from then on. However, if they are driven by the market forces, resetting the system once will not achieve a lasting equality. Rather, the system may either revert back to the previous unequal state or the inequalities may flip (given the symmetric fundamentals, existence of an equilibrium with asymmetry towards one group implies the existence of the symmetric equilibrium with reverse inequalities).

Many economic activities are decided by members of one group independently searching for their counterparts from another group, members of which in turn search for their best picks out of the first. The most straightforward example of the above environment is, perhaps, the marriage market, where traditionally, men are looking for women and vice versa. Firms looking for workers and workers looking for jobs is another example. In both of these settings, one can observe the following three market characteristics: i) once two potential agents get in touch with each other, there is an evaluation cost for each agent to be able to decide whether to agree to the match; ii) once a match is made, both agents are (at least temporarily) out of the market (in the case of firms, each open position may be considered an economic agent), and iii) forming a match requires both parties to agree.

An example from a business market could be a product category where making a product requires input from two industries. For example, suppose building a deck or a bookcase requires a carpenter and a painter. Each carpenter and each painter may prefer a particular way of working on the projects or particular projects. When forming a team, would carpenters have their picks of preferable-to-them painters or the other way around?

For an example in the context traditionally considered as in the field of marketing, consider workers, let's say plumbers, looking for jobs and potential customers, let's say homeowners needing to quickly repair a plumbing leak.[1] In this market, one arrangement is that each individual homeowner looks for a plumber, while individual plumbers, having listed their phone numbers on a platform (e.g., yellow pages, or homeadvisor.com) wait for offers.In this case, search cost of a homeowner is the cost of communicating with the plumber to describe the problem and obtain the price quote (which may also require a home visit). The search/evaluation cost on the plumber's side likewise includes the time needed to evaluate the project and give the price quote and also the opportunity cost of waiting for a request from the next homeowner in need. After a deal is done, hopefully, the homeowner does not have value for another plumber for the day, and the plumber is now busy for some time and, therefore, cannot entertain other emergency offers. An additional complication in this example that may not be present in the marriage market and not necessarily present in the usual labor market is that the prices, and therefore, the payoffs from an exogenous match are endogenous and depend on the expectations of search and

[1] Or, assuming that everybody can be helped by the expertise of an economist and a psychologist, one may also rephrase this example in terms of each economist looking for a psychologist for help, and each psychologist looking for an economist for help.

acceptance behavior of the other side of the market.

Note that the above market arrangement is essentially similar to the classic "pre-industrial" marketplace, i.e., a market where individual buyers were looking through individual rows of individual sellers, each selling a limited amount of merchandise. I refer to such markets as decentralized and/or two-sided: both sides (e.g., buyers and sellers) are searching. This is in difference to a market arrangement in which members of at least one of the sides is organized, e.g., workers may be organized into a firm, which, as a result, has substantial market power. The recent development of e-commerce platforms (e.g., home-advisor.com for home repair and upgrade services, amazon.com third-party marketplace and ebay.com for product markets) increased the ubiquity of such decentralized two-sided markets.

The intuition for the possibility of inequity between sides in symmetric two-sided markets is that when decision makers and search decisions are present on both sides of the (market) exchange, a symmetric matching outcome may be unstable. That is, starting from symmetric fundamentals, expectations, and strategies, a small deviation in search strategy by members of one of the sides of the market may lead to the optimal response of the members of the other side in the opposite direction (i.e., if the members of one population accept offers more readily, it makes it optimal for the members of the other population to become more picky), which in turn may facilitate even stronger search distortions of the first side of the market, and this may lead the market outcomes away from equality and towards an asymmetric equilibrium (if it exists). The implication is then that the equilibrium outcome may necessarily and predictably involve substantial inequities between populations.

This paper considers whether and under what conditions one should expect an equitable outcome given equal initial conditions in the context of a market with the above-stated three characteristics. Namely, it considers a market consisting of two large populations of agents which look for pair-wise matches (or deals) with agents of the other population, and assumes the following three market characteristics. First, each agent faces a search or evaluation cost per agent it considers to make a deal with and evaluations are done sequentially. An agent needs to incur this seach/evaluation cost to determine her value of the deal. Second, each agent has a capacity constraint in the sense that once a deal is made, another deal is not possible or has no value, i.e., agents that made the deal leave the market at least for some time. Third, a deal is made when both agents evaluating each other decide to accept it, i.e., each agents has the ability to prevent a deal, but not to force another agent into a deal.

In a centralized market, i.e., where at least one side of the market is organized, a question of fairness and equitable split of surplus is often raised with the natural presumption that the more organized side (e.g., firms) is likely to have an advantage over the decentralized side (e.g., consumers or employees). A traditional solution could then be either regulation (e.g., consumer protection laws) or organizing the other side (e.g., labor unions), although decentralization is also technically possible (e.g., breaking down monopolies into smaller competing firms). Alternatively, in a decentralized economy with a large number of agents on each side making independent decisions, it is not clear whether payoff asymmetry would naturally arise, and if it does, whether it can be corrected by market mechanisms, such as changing the search costs, which would be akin to changing the competitiveness level (given that enforcing regulations on "small" agents is especially difficult).

The main result of this paper is that in two-sided markets with costly search, the unique (up to renaming of sides) equilibrium prediction could be that inequality will arise, i.e., that members of one side will end up receiving higher expected equilibrium payoffs (while being less willing to accept a match with a random draw from the other side). Furthermore, the paper derives a condition under which the equilibrium may involve inequality, which is interpreted as the distribution of payoffs from a random match having thick tails. Under this condition, whether the equilibrium will involve inequality may non-monotonically depend on the search cost, so that decreasing search cost from one level to another may either induce or eliminate asymmetry. In addition, it is possible that decreasing search cost will reduce social welfare.

2 Related Literature

Marketing literature on inequality in marketing and its management is rather sparse, but perhaps of emerging interest (e.g., Fu et al., 2021, examine the market implications of machine learning algorithm restrictions designed to promote equality among consumers, and Zhang et al., 2021, empirically examine the success of an Airbnb IA algorithm to reduce racial economic inequality in the lodging rental market).

There is a number of models in Economics examining whether inequalities starting from unequal initial endowments would persist. For example, Freeman (1996) builds a model where wage inequalities resulting to the human capital (education) inequalities persist across generations due to the expensiveness of human capital investment and under the assumption that agents are unable to borrow money to fund it. Likewise, Matsuyama (2000) shows how an initial inequity in wealth may be sustained through time due to the credit markets where profitable investments are only possible for rich enough households while the poorer ones can only be creditors, and under certain conditions, the expected return on investments is higher than the interest rate in the credit market. These models rely on the initial inequity to explain inequity in the later periods.

Matsuyama (2004) shows how wealth inequality across nations may increase through time due to global financial markets (borrowing and lending). As in the model of the current paper, Matsuyama (2004) shows that with symmetric initial conditions, the symmetric steady state is not stable and over time, the market would develop inequalities. In difference to the models in the above papers, the current model does not rely on the role of financial markets, and obtains the result in a search framework of a static equilibrium (i.e., without reliance of gradual divergence through time periods). Furthermore, in Matsuyama (2004), socially beneficial solution is simply to prohibit global financial market, whereas in the context of this paper, prohibiting trade would be clearly suboptimal (although, obviously, would force equality with zero gains for all).

Starting from Stigler (1961), there is an extensive literature on markets determined by search and matching, in the context of job search (e.g., McCall 1970, Diamond 1982, Salop 1986) and in the context of the marriage market (e.g., Burdett and Coles, 1997). In terms of technical model setup, the fundamental structure of the search model in this paper is similar to the one in Kuksov (2007), although Kuksov (2007) focusses on the agents' uncertainty even after incurring search cost and does not have asymmetric equilibria due to the specific assumptions on the distribution of payoffs.

Perhaps, closest to the results in this paper are the results in Mailath et al. (2000). Looking for the possibility of unequal wages between fundamentally symmetric populations of workers, Mailath et al. (1990) obtain that asymmetric outcome may arise in matching between the two populations of workers on one side and a group of employers on the other side. In their model, the asymmetry arises due to the interaction between worker incentive to invest in their education and the employer strategy whether to incur an expense of searching for skilled labor within one or the other group. In their framework, multiple equilibria may exist because if employers do not check within certain population, it is optimal for the applicants in that population not to invest as much in education, which in turn reinforces the employers' incentive not to search in that population. While the asymmetric equilibrium with inequality between the expected payoffs of the two groups of workers may exist, the symmetric equilibrium always exist (and is stable). The difference in the current paper is that by putting two symmetric populations on the different sides of the market (i.e., they search for each other as opposed to both of them searching for a match with a third group), we obtain that an asymmetric equilibrium may be unique (up to renaming populations). Given the difference in the model assumptions, Mailath et al. (2000) model may be more suitable to be applied to racial inequity in wages of workers, whereas the current model may be more suitable for understanding inequity between the supply and the demand sides of a market, collaborations between different professions (e.g., jobs that require joint effort by two types of skilled workers) or academic disciplines, or gender equality in cross-gender marriages.

3 The Model

Agents from each of two infinite populations I and J are interested in pairwise matching with agents from the other population. For a possibility of a match, an agent must go to a meeting with an agent from the other population. Random meetings are set up at a cost s to each agent in the meeting, with agents of both populations incurring this cost.If agents i and j end up in a match, their payoffs are V_{ij} and V_{ji}, respectively, where the potential payoffs V_{ij} and V_{ji} are *i.i.d.* across all (i, j) pairs. Denote the cumulative distribution function of V_{ij} by $F(\cdot)$ and its density function by $f(\cdot)$.[2] The origin of the payoffs in a match may be coming from a distribution of characteristics and preferences of agents, and

[2] "Potential" refers to that these payoffs only realize if a match occurs, which is an endogenous event defined below.

the agents of different populations care about different things. To simplify the model, monetary transfers between agents to "pay" for the match are not allowed.

At a meeting, agents observe own payoff. It will be clear below that whether an agent observes the other agent's payoff is not consequential, but to fully define the model, let's assume she does not. Then, each of the agents at the meeting makes her decision whether to accept or reject the other agent. It is not consequential whether these decisions are done simultaneous or sequential, but to fully define the model, let's assume the decisions are made simultaneously by the two agents at a meeting. If both accept, the meeting ends with a match, if either does not accept, the meeting ends with no match. In case of a match, agents receive the payoffs defined above and exit the matching process forever. If there is no match, both agents are free to set up further meetings.

Unless explicitly stated otherwise, in the analysis and discussions below, I assume that the expected payoff of a match is high enough relative to the search cost and the never-matched outcome, so that unmatched agents from each population are always interested in paying s and engaging in a meeting. There are two equivalent ways to operationalize this "high enough" assumption. One is by postulating zero payoff of the outside option (withdrawing from the search process without a match) and assuming that the mean of $F(\cdot)$ is high enough, and the other is to assume that the payoff $U_{\text{no\_match}}$ of the outside option is sufficiently negative.[3] I adopt the latter, as it simplifies the notation.

Technically, if all agents from one of the populations are unwilling to pay the search cost, i.e., decide not to engage in search, no meetings can be set up. It is then (weakly) optimal for the agents not to try to be willing to meet. In other words, there is a trivial equilibrium with no search and no matches made (this argument can be applied to any market or interaction if the decision to participate is considered, no matter whether it is costly or not). We will concentrate on the non-trivial equilibria where agents search (which exist as far as the search cost is not too high relative to the difference between the mean of $F(\cdot)$ and $U_{\text{no\_match}}$). Furthermore, we will use stable Bayesian Nash equilibrium as a solution concept.[4]

[3]To be clear, this payoff is at the end of the game, and not per meeting.

[4]Technically, the model is a Bayesian game (game with uncertainty) since agents learn the realization of the exogenous value V_{ij} at a meeting. It is also a game with many stages, but backward induction (e.g., perfect Bayesian equilibrium) is not useful or necessary to narrow down the equilibria in this game because

4 Model Analysis

As in any setup with sequential search and incomplete information, and where agents' possibilities and information sets do not change with the number of searches, the decision to accept follows a reservation rule strategy (Stigler, 1961). That is, the optimal strategy for an agent $k \in K$, where $K \in \{I, J\}$, is to set up meetings until the expected payoff from the match in the meeting is at or above the reservation value R_K. The subscript on R is that of population as opposed to the individual agent because, as we will see below, the optimal reservation value depends on the behavior of all agents in the other population and is unique. In other words, equilibrium reservation values of all agents in within the same population are the same. As will be clear from the equilibrium equations, the equilibrium reservation value is also the value of the game to the agent $k \in K$, i.e., in equilibrium, the expected payoff net of search costs for agent $k \in K$ is R_K.

4.1 Equilibrium Equations

Let us consider the decision problem of agent $i \in I$ keeping in mind that an agent $j \in J$ faces a similar problem, and hence all statements about $i \in I$ also hold for $j \in J$ if indices i and I are swapped with indices j and J, respectively.

Since decision of an agent $i \in I$ to accept or not only matters if the other agent j at the meeting accepts, agent i decides on acceptance conditional on agent j accepting, i.e., each agent makes the accept/reject decisions as if the other agents accepts her. The equilibrium condition on G_I is that the expected benefit of searching until a better alternative is found is equal to the expected cost of searching till then:

$$\mathrm{E}(x \mid x > R_I) = \frac{s}{\mathrm{Prob}(j' \in J \text{ accepts}) \cdot \mathrm{Prob}(x > R_I)}, \tag{1}$$

where x is a draw from distribution $F(\cdot)$ and represents agent i's payoff of the match, and j' stands for a generic agent in J. The above equation can be rewritten as

$$s = (1 - F(R_J)) \int_{x > R_I} (x - R_I) dF(x), \tag{2}$$

players do not respond to each other's actions.

since $\text{Prob}(x > R_I)\text{E}(x \mid x > R_I)$ equals to the integral in the above equation and $\text{Prob}(j' \in J \text{ accepts}) = 1 - F(R_J)$.

Another way to understand the above equation is that it is the standard condition on optimal reservation rule R_I when agent i has the ability to return to the previous meeting and reconsider her decision on acceptance.[5] Since nothing changes between meetings (neither the distribution of agents nor belief of agent i about the chances in the following meetings changes), agent i will never want to reconsider. Therefore, the condition still holds when the reconsideration is not possible, as it was assumed in this model.

Note also that the solution of Equation (2) for R_I is unique, since $\int_{x>R_I}(x - R_I)dF(x)$ strictly decreases in R_I and the other terms do not depend on R_I. This observation implies that all agents in J should have the same reservation value and, therefore, justifies the use of the population subscript on R.

Given reservation rule R_J of population J, the probability that agent $j \in J$ accepts agent $i \in I$ is $1 - F(R_J)$. Therefore, the equilibrium reservation values R_I and R_J for agents in I and J satisfy the following system of equations:

$$\begin{cases} s = (1 - F(R_J)) \int_{x>R_I} (x - R_I)\, dF(x), \\ s = (1 - F(R_I)) \int_{x>R_J} (x - R_J)\, dF(x), \end{cases} \tag{3}$$

where the first and the second equations are, correspondingly, the reaction functions of an agent $i \in I$ and an agent $j \in J$ to the reservation value of the other population. Note that the solution of the first and the second equations for R_I and R_J, respectively, as functions of R_J and R_I, respectively, define the reaction functions $R_I = R_I(R_J)$ and $R_J = R_J(R_I)$ of the optimal strategy of the agents in one population given their beliefs about the strategy of the agents in the other population. Clearly, these reaction functions are downward sloping.

In a symmetric equilibrium, the reservation value $R = R_I = R_J$ of all agents satisfies

$$s = (1 - F(R)) \int_{x>R} (x - R)\, dF(x), \tag{4}$$

which is unique because the right hand side is strictly decreasing in R.

[5]The right hand side is just the probability of the next search presenting a better option times the expected value of the next option conditional on it being better. The next option is better if both of the following conditions are met: i) the next agent accepts, and ii) the next V_{ij} is above R_I. These conditions are independent since V_{ji} is independent of V_{ij}, which implies multiplication of their probabilities.

4.2 Asymmetric Equilibria and Equilibrium Stability

Let us now consider the possibility of asymmetric equilibria for a given distribution $F(\cdot)$. Since the equilibria is defined by reservation values R_I and R_J which satisfy the system of equations (3), we are looking for asymmetric solutions to that system for some s.

Proposition 1. *An asymmetric equilibrium exists for some s if and only if the function*

$$H(R) = \frac{\int_{x>R}(x-R)\,dF(x)}{1-F(R)} \tag{5}$$

is non-decreasing for some R.

Proof. See Appendix. □

Note that the function $H(\cdot)$ is the hazard function of distribution $F(\cdot)$, and if $H(R)$ is increasing for R above some value R_0, then $F(\cdot)$ has thick (right) tail, i.e., weighed relatively heavier on the high draws.

The above proposition shows that inequity may arise in equilibrium of a two-sided matching game between ex-ante identical populations. Once again, the intuition for the possibility of multiple equilibria is that, as can be seen from Equations (3), if agents of one population do not accept easily, the agents of the other population respond by accepting more frequently thereby reducing the effective search costs by the agents of the first population and making optimal their unwillingness to match easily. This result is somewhat similar to Mailath et al. (2000) result that two populations ("red" and "green") of workers may be differently searched for by employers, which results in the difference of how much the workers of the different populations invest in acquiring skills, and the resulting difference in skills, in turn, make it optimal for the employers to discriminate. However, the above proposition shows that when two symmetric populations are interested in matching with each other, the asymmetric outcome may result even without endogenous investments in making agents more valuable in a match, and as we argue below, is stronger in the sense that it asymmetric outcome may be a clear prediction as opposed to just one of the plausible outcomes.

To better understand the condition for the existence of asymmetric equilibria, note that $H(R)$ is decreasing for high R for a normal distribution, but it is increasing for a distribution with the density function of the form $exp(-a(x)x)$ if $a(x)$ is decreasing in x.

In the case of multiple equilibria (i.e., when an asymmetric equilibrium exists), a natural question to ask is which equilibrium is likely to hold. Since populations are ex-ante identical, one cannot argue for one asymmetric equilibrium vs. its mirror image (the corresponding equilibrium with population names swapped). However, an interesting question is whether the symmetric equilibrium is to be expected, or one of the asymmetric ones.

For this purpose, let us look more closely at the reaction curve $R_I(R_J)$ defined by Equation (2), keeping in mind that everything that applies to $R_I(R_J)$ applies to $R_J(R_I)$ with all indices I and J swapped in all claims. As we have already noted, it is always downward slopping, and strictly so at such R_J that $f(R_J) > 0$. Furthermore, as $R_J \to -\infty$, we have that $R_I(R_J)$ asymptotes to a finite maximal value $\overline{R}$ which is the solution of the following equation corresponding to the optimal reservation rule when the other population always accepts:

$$s = \int_{x>\overline{R}} (x - \overline{R})\, dF(x). \tag{6}$$

On the other hand, when $R_J \to +\infty$, we have $R_I(R_J) \to -\infty$. Therefore, if the intersection of the reaction curves is unique, the equilibrium (which is symmetric) is globally stable. On the other hand, if there are multiple equilibria, then the general position outcome will be that the first intersection corresponds to a locally stable equilibrium, followed by a locally unstable, then a locally stable, etc. In particular, we have the following proposition:

Proposition 2. *If asymmetric equilibrium is unique, it is almost always the unique stable equilibrium up to renaming of the populations. If there are no asymmetric equilibria, the symmetric equilibrium is stable.*

The term "almost always" in the above proposition is meant in the sense of "general position," i.e., if it does not hold, an arbitrarily small change in the distribution $F(\cdot)$ is enough to result in unique stable asymmetric equilibrium: this is because the only way the asymmetric equilibrium will not be stable and/or the symmetric one will be stable is if the reaction curves touch but do not intersect at the asymmetric equilibrium reservation values (which one can view as an unlikely knife-edge condition).

In the following section we consider several special cases of $F(\cdot)$ and s to illustrate a number of possibilities of what may happen in the (stable) equilibrium.

5 Examples

First, as an example of a distribution with thick tails, consider a match payoff distribution with the probability density function

$$f(x) = \begin{cases} \frac{1}{2}\mathbf{e}^{-\sqrt{x}} & \text{if } \ x > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The CDF and hazard rate of this distribution on $[0, \infty]$ are, respectively,

$$F(x) = \left(1 - (\sqrt{x} + 1)\mathbf{e}^{-\sqrt{x}}\right) \quad \text{and} \quad H(x) = 2 \cdot \frac{x + 3\sqrt{x} + 3}{\sqrt{x} + 1}. \tag{8}$$

Figure 1 illustrates the reservation rule reaction curves when $s = 1$ (see Appendix for the specification and derivation of the reaction functions). In this and all other figures, the green (gray when printed in black and white) curve is $R_I(R_J)$, the red (black when printed in black and white) curve is $R_J(R_I)$. Intersection of these curves is a stable equilibrium if the green curve intersects the red from below to above and is not stable otherwise.

In this example, if $s = 1$, the symmetric equilibrium is not stable, while in a stable equilibrium, agents in one population always accept and have the reservation value of -6.776, while agents in the other population have the reservation value of 17.600 and their probability of accepting at each meeting is 7.827% (see Appendix for details).[6] The expected per-agent expenditure on search is $1/7.827\% = 12.776$ (the expected per-agent search cost expenditure is always the same for the two populations and is always inversely proportional to the product of acceptance probabilities of the agents on the two sides).

As shown in the Appendix, if $s < 6$, a similar asymmetry to the above is the unique (stable equilibrium) prediction in this example. On the other hand, when $s \geq 6$, the unique (and stable) equilibrium is symmetric with agents in both populations always accepting, and the expected net payoff is $6-s$ per agent. Thus, this example illustrates i) the possibility of asymmetric outcome being a unique prediction, and ii) that a decrease in search costs across populations may change the equilibrium outcome from symmetric to asymmetric, i.e., may create inequality. Furthermore, in the range of s with asymmetric equilibria (i.e.,

[6] Note that the negative reservation value has no behavioral difference from zero reservation value since the potential match payoffs in this example are always non-negative, but still, it has the interpretation of the expected payoff of that population's agents at the end of the game. Thus, to ensure search, the payoff from not engaging in the search-till-match must be below -6.776.

when $s < 6$), a decrease in search costs makes agents in one of the populations strictly worse off, although in this example, the social welfare (average payoff across populations) always increases when the search costs decrease.

It is also possible (for some value distributions), that a decrease in search costs would result in the unique asymmetric equilibrium changing to the unique symmetric one, so that a reduction in search costs could remove the inequality. To see this, consider, for example, the distribution obtained from the one in the above example by changing all payoff values above 100 to $124\,{}^{2}\!/_{11}$ (which is the expected value of the payoff conditional on it being above 100). Then, for $s = 1$, the equilibrium strategies will be exactly as stated at the beginning of this section, i.e., asymmetric. However, for any s, the probability of acceptance by each agent is at least as high as the probability of $124\,{}^{2}\!/_{11}$ being drawn as the potential match payoff. Therefore, for small enough s, the equilibrium strategies on each side will become: accept if and only if the observed potential payoff equals $124\,{}^{2}\!/_{11}$, i.e., the equilibrium is symmetric.

As another example, consider V_{ij} with the probability density function

$$f(x) = \frac{\sqrt{2}}{\pi(x^4 + 1)}. \tag{9}$$

It has a convex hazard rate decreasing until the critical value $r_0 = 0.656$ and increasing after it (see Appendix for the expressions on CDF and the hazard rate). As in the previous example, the equilibrium is symmetric for large s and asymmetric for small s (with the change at around $s = 0.026028$). Figures 2 and 3 illustrate the former and the latter cases. Furthermore, as Figure 3 illustrates, for some s, the reaction functions are not concave. This leads to the social welfare possibly lower in the (not stable) symmetric equilibrium than in the asymmetric equilibrium, which is true in this example for small enough s (approximately, for $s < 0.01$).

On the other hand, the concavity of response functions in the neighborhood of s when the stable equilibrium changes from symmetric to asymmetric leads to the following curious result. Comparing the (unique-up-to-renaming-populations stable asymmetric) equilibrium when $s = 0.017$ with the (unique stable symmetric) equilibrium when $s = 0.026$, we obtain that a decrease in search cost can result in a lower social welfare (see Appendix for details).[7]

[7]This is in addition to social inequality possibly considered as a negative by itself in the minds of fairly

To see the intuition for this result, observe that when s is at the value where the (stable) equilibrium switches from the symmetric to asymmetric, the reaction functions are tangential to each other (since the unique intersection corresponding to the symmetric equilibrium splits into three intersections). Therefore, a small change in s, leading to a small change in the reaction functions, results in a relatively large shift of the asymmetric equilibrium along the reaction curves away from the 45^o line. If the reaction curves are concave, this shift makes a negative effect on the social welfare, and it over-powers the positive effect of the lower search costs shifting the reaction curves up.

Summarizing the possibilities shown in the above examples, we obtain the following proposition:

Proposition 3. *A reduction in search costs may result in any of the following.*

1. *Inequality between populations may be created.*
2. *Inequality between populations may be eliminated.*
3. *One of the populations may become worse off.*
4. *Social welfare may decrease.*

Finally, as a curious example consider the exponential distribution of payoffs in a match between random agents (i.e., $f(x) = \exp(-x)$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$), which is easy to solve analytically. In this case, the reaction function is $R_I = -\ln(s) - R_J$, i.e., any $R_J \in [0, -\ln(s)]$ and the above defined R_I constitutes an equilibrium. This means that the social welfare of the game is $-\ln(s)/2$ per agent and the probability of match is s (making the total expected expenditure on search equal to 1 per agent) regardless of the equilibrium. Multiplicity of equilibria in this case makes it impossible to make predictions for how a change in s would affect the equilibrium outcome.

6 Robustness to Monetary Transfers between Agents

If the inequalities are caused by agents of one population being disadvantaged by too high search inclination of the other population (i.e., being accepted with too small a probability), one may wonder if allowing individual agents to offer money for acceptance to the agents

minded agents (if we are to step outside of the current model).

they are meeting and like could change the equilibrium dynamics and restore symmetry. In the model, this is generally a technically complex consideration because an agent's optimal offer strategy would be a function of her potential match payoff observed at a meeting. With the continuous distribution of the potential payoffs, the optimal offer is then a function, which means solving for equilibrium would require solving for a function (i.e., for an infinite number of parameters). However, in some cases, it is possible to establish the direction of the change in the expected equilibrium payoffs due to the ability of agents to make offers without explicit solutions. We do this below to show the robustness of the equilibrium inequity result.

Formally, let us extend the model by assuming that having observed own potential payoff from the match V_{ij}, agent i is able to make an offer to transfer value $t_{ij} > 0$ to agent j conditional on agent j accepting. To keep the game symmetric, agents of both populations are able make these offers, and they are made simultaneously at the meetings before the simultaneous decision on whether to accept.

Intuitively, one could view the ability to make offers as a positive for the expected net equilibrium payoff of the game to the agents with that ability, since each agent may but does not have to make a strictly positive offer: only whenever the agent sees high enough value, she would choose to encourage acceptance by offering the monetary incentive. However, there is a negative indirect effect: expectations of such payments by the agents of this population can make agents of the other population less willing to accept unless they receive a sufficiently high offer. The net effect is not immediately clear.

Let us examine the equilibrium effects of this change in the model to the example of $f(x) = exp(-\sqrt{x})/2$ with $s = 1$ considered in the previous section (although the same argument would apply to any case where the equilibrium in the model without transfers is asymmetric with one of the population always accepting). As shown in the previous section, in this case absent transfers, agents from one of the populations, lets say population I, always accept. Therefore, agents of populations J have no need to offer any transfers. However, agents of population I have strictly negative reservation value, and therefore strictly benefit from increasing the acceptance rate of agents in J. It is therefore possibly individually rational for agents in I to offer monetary incentive to the agents they meet. The exact optimal offer depends on V_{ij}, and is strictly positive at least for some (high enough) V_{ij}'s. Therefore, in equilibrium, agents in I will at least sometimes make offers to agents in J.

Now, observe that the search problem of agents in J becomes to search for the best $V_{ji} + t_{ij}$ instead of the best V_{ji}. Since V_{ij} and, therefore, t_{ij} is independent of V_{ji}, the distribution of $V_{ji} + t_{ij}$, relative to that of V_{ji} has a higher variance. This means agent $j \in J$ has a strictly higher incentive to search, i.e., the equilibrium probability of agent j accepting actually declines! In turn, this makes agent in I even more willing to always accept. Thus, agents in I, in equilibrium, are in expectation worse off both due to the actual monetary transfers (actual money out of their wallets) and due to the lower rates of being accepted, while agents in J are better off due to the offers (their change in acceptance behavior is of their choosing, so they cannot be worse off due to it). Therefore, we have the following proposition.

Proposition 4. *Allowing agents to make monetary offers to the agents they are meeting, may increase the expected equilibrium payoff inequality.*

While the above proposition is established for a special case where one of the populations accepts with probability 1, one can speculate that the same intuition would apply to any situation where the equilibrium without monetary transfer possibility is asymmetric: the disadvantaged population always accepts with a higher probability (since the reservation value is directly linked with the acceptance probability), and therefore always is more interested in increasing the acceptance rate of the other population. Thus, although when each population acceptance is not guaranteed, agents from each population would sometimes offer (strictly positive) transfers, on average, the disadvantaged population would be the one making higher offers, which probably would result in its offers having a higher variance as well, and thus, it would be encouraging more search of the other population. This result may be viewed also as parallel to the result in Matsuyama (2004) that cross-country financial markets create inequality because the ability of agents to lend makes one of the populations (countries) a net lender and, eventually, worse off (individual agents choose to lend to the other country instead of investing in their own country).

7 Discussion

If equality is a goal, how could a policy maker ensure it? In some models, e.g., the credit market or investment driven economies in Freeman (1996) or Matsuyama (2000, 2004), regulating financial markets or facilitating education loans could remove the budget constraints that lead to the persistency of inequality or its increase over time. In Mailath et al. (2000), the symmetric and asymmetric equilibria are equally likely, and therefore, if market expectations are (approximately) set to correspond to the symmetric one, equality will ensue.

On the other hand, the model of this paper predicts asymmetry (under some parameteric/distribution conditions) even if the initial expectations are next to symmetric. One could speculate that encouraging search (i.e., asymmetric reduction in search cost) of the disadvantaged population or taxing search (i.e., increasing search costs) of the advantaged population could nudge the system to balance. However, this is not the case: there will be either a small effect, or the asymmetry will flip between population and the new outcome will be even more asymmetric (in the opposite direction). Enforcing search rules that are deemed fair may not be practical as the individual payoffs are not likely to be observed by the policymaker. The average search duration cannot be enforced on individual agents since only the random draw of the number of searches is observed (and not the search strategy). The only way to ensure equality through dictating selectiveness is to require accepting all (i.e., no more than one search). This may be very inefficient if the distribution of payoffs has high variance. As we argued above, allowing monetary transfers may just exacerbate inequality.

The condition for the stability of the symmetric equilibrium (thin tails in a certain range) provides one idea how equilibrium forces could be used to encourage equality. Namely, one could try to design markets/platforms or impose incentives to change the distribution of payoffs to have thin tails (i.e., reduce the probability of the high positive draws). In a context with many markets, bundling some markets could be an example of such market design (since, by central limit theorem, bundling will change distributions towards normal, which has thin tails). Alternatively, if the platform designer observes a variable correlated to the payoffs, taxing (or requiring transfers between parties contingent on) this variable could make the distribution of the residual payoffs have thinner tails. That is, an imperfect (but fair) instrument can be used to change the system into one that leads to a symmetric equilibrium.

If local stability of equilibria identifies a unique asymmetric equilibrium up to renaming populations, one expects asymmetry, but it is impossible (within the symmetric model) to predict which way it will go. Therefore, stepping outside of the model, agents of each population may be interested to do something to coordinate on the equilibrium they prefer (to the detriment of the other population). For example, cultural norms may be an instrument of such coordination. Note that in any equilibrium, each agent chooses the strategy that is *uniquely* optimal, i.e., each agent strictly prefers not to deviate. However, each agent would like to convince members of the other population that her own population is as choosy as she can possibly convince the other population. One way of doing this is by convincing members of own population to be as choosy as possible, so that the other population would discover that this population is choosy. One may think of agents engaging in creation of social norms as to discourage high acceptance rates in own population. With multiple equilibria, some "cultural norms" of the sort are self-enforcing (so that not only they are optimal to follow if everybody else follows them, but also small deviations in the beliefs or behavior would revert back to the established norm).

Appendix

Proof of Proposition 1

An asymmetric solution of the system (3) exists for some s if the following equation has an asymmetric solution:

$$(1 - F(R_J)) \int_{x>R_I} (x - R_I)\, dF(x) = (1 - F(R_I)) \int_{x>R_J} (x - R_J)\, dF(x). \tag{10}$$

Dividing both sides by $(1 - F(R_J))(1 - F(R_I))$, we have

$$\frac{\int_{x>R_I} (x - R_I)\, dF(x)}{1 - F(R_I)} = \frac{\int_{x>R_J} (x - R_J)\, dF(x)}{1 - F(R_J)}. \tag{11}$$

In other words, we look at arguments of the function

$$H(R) = \frac{\int_{x>R} (x - R)\, dF(x)}{1 - F(R)} \tag{12}$$

that correspond to the same function value, which exist if and only if the function $H(R)$ is not strictly monotonic. Since $H(R) \to \infty$ as $R \to -\infty$ and therefore, $H(R)$ is decreasing for some R, $H(R)$ is not strictly monotonic if and only if it is non-decreasing for some R.

Solution Details for $\mathbf{f(x)}$ Defined in Equation (7)

Given that Equation (7), Equation (2) defining the reaction function $R_I(R_J)$, i.e., the optimal reservation value R_I of agents $i \in I$ as a function of the reservation value R_J of agents in J, becomes

$$s = 2\left(1 - \sqrt{R_J}\right)\left(R_I + \sqrt{R_I} + 3\right) \mathbf{e}^{-\sqrt{R_J} - \sqrt{R_I}} \tag{13}$$

for $R_J \in [0, R_1]$, where R_1 is a value of R_J at which R_I becomes 0. These boundaries ($R_I > 0$ and $R_J > 0$) are to make sure that the $f(x) = \frac{1}{2}\mathbf{e}^{-\sqrt{(\mathbf{x})}}$ equation would apply to $x > R_k$ for $k = I, J$. From Equation (13), we obtain that R_1 is the solution of $s = 6(1 - \sqrt{R_1})\mathbf{e}^{-\sqrt{R_1}}$, which exists (and is positive) if and only if $s < 6$. We proceed for now under this condition. Since $R_J < 0$ and $R_J = 0$ imply the same acceptance strategy by agents in J (given that $V_{ji} < 0$ is impossible), $R_I(R_J) = R_I(0)$ for $R_J < 0$. Finally, for $R_J > R_1$, Equation (2) reduces to

$$s = \left(\sqrt{R_J} - 1\right)(6 - R_I)\mathbf{e}^{-\sqrt{R_J}}, \tag{14}$$

which implies

$$R_I = 6 - \frac{s\mathbf{e}^{-\sqrt{R_J}}}{\sqrt{R_J} - 1}, \quad \text{for} \quad R_J > R_1. \tag{15}$$

The equations on $R_J(R_I)$ are obtained by swapping the indices I and J in the equations above. Figure 1 is then obtained by plotting these functions for $s = 1$. Given the directions of the intersections, it then follows that the asymmetric equilibria corresponding to the intersections with $R_I < 0$ or $R_J < 0$ are stable and the symmetric equilibrium with $R_k > 0$ for $k = I, J$ is not. These intersection directions are also straightforward to confirm by implicit differentiation the reaction function equations for $s < 6$.

Coming back to the case $s \geq 6$, note that $\mathrm{E}(V_{ij}) = 6$, i.e., the expected value of a match with a random agent (gross of the search cost) is 6. That is, net of the search cost, the worst payoff (i.e., 0) does not justify searching more. Therefore, for $s > 6$, all agents strategy should be to always accept regardless of the other agents strategies. Therefore, the only equilibrium, obtained by elimination of strictly dominated strategies (and therefore, stable) is for all agents in both populations to always accept (and the expected equilibrium payoffs are $6 - s$ for each agent). This completes the proof of the claim in Section 5 that the outcome (of a stable equilibrium) is asymmetric if and only if $s < 6$.

Finally, to prove that, within the range of $s < 6$, a decrease in s results in one of the populations becoming worse off, observe that in this case ($s < 6$), in a stable equilibrium, the expected payoff (which is also the reservation value) of agents in one of the populations is positive and that of agents in the other population is negative. Without loss of generality, let population J have the negative expected payoff, i.e., always accepts. This implies that when s decreases, R_I has to increase (since the acceptance probability of agents in J remains at 1 and the payoff distribution has no mass points). This implies that the probability of acceptance of agents in I decreases (since $f(R_I) > 0$ for $R_I > 0$). In turn, this implies that R_J has to decrease, i.e., population J is worse off. Note that the above argument implicitly assumed that as s decreased, the population with the negative expected payoff remained the same. However, this is not needed to establish the claim: if the expected payoff of one of the population changes from positive to negative, clearly, that population became worse off. This completes the proof of the claim in Section 5 that a decrease of s within the range of $s < 6$ makes the (stable) equilibrium payoff of one of the populations lower.

Solution Details for $\mathbf{f(x)}$ Defined in Equation (9)

For this distribution of the potential match payoffs, the CDF is

$$F(x) = \frac{1}{2} + \frac{1}{4\pi} \ln \frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1} + \frac{\arctan(\sqrt{2}x + 1) + \arctan(\sqrt{2}x - 1)}{2\pi}, \tag{16}$$

and the hazard rate is

$$H(x) = \frac{\pi - 2\arctan(x^2)}{2\sqrt{2}\pi(1 - F(x))} - x. \tag{17}$$

Unlike in the previous example, since the probability density function in this case is always positive and continuous, there are no multiple cases (i.e., equations are not piecewise), however, due to the complexity of the equilibrium equations, the analysis is through solution evaluations for particular values of the search cost.

In particular, when $s = 0.0261$, the unique and stable equilibrium is symmetric with expected payoffs of 0.6553 per agent (analysis of the difference between the reaction functions shows that the intersection is unique for $s > 0.026028$ and triple for the smaller search costs). When $s = 0.0167$, the symmetric equilibrium is not stable, but two asymmetric equilibria (which are mirror images of each other) are. The expected payoffs are -0.5257 per agent in one of the populations, and 1.803 per agent in the other population, with the average per-agent expected payoff of 0.6387. Therefore, in this example, a search costs decrease of 0.0094 resulted in the average per-agent expected payoff (social welfare) decrease of 0.0166.

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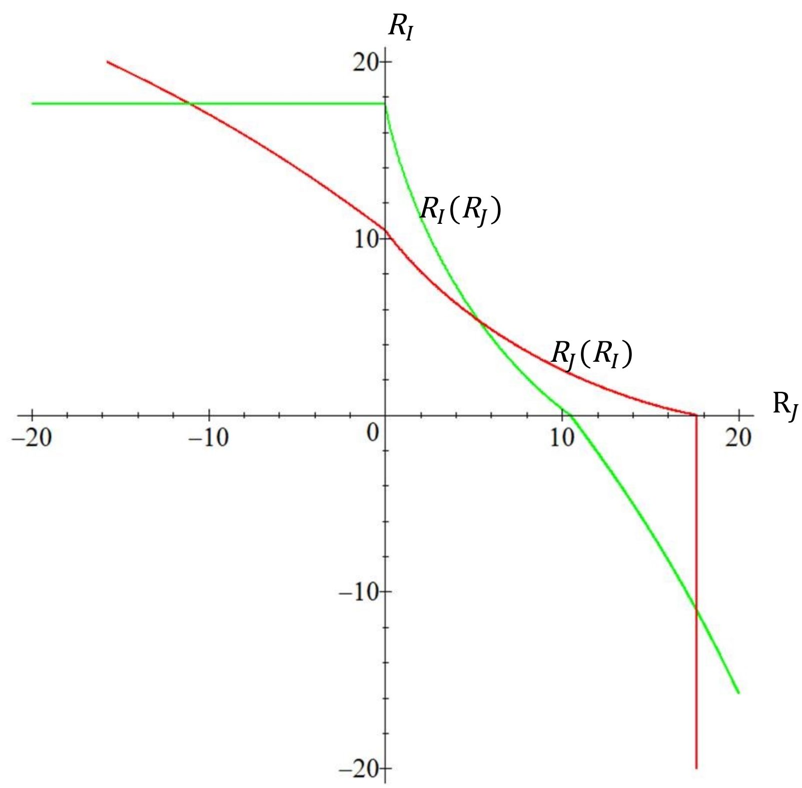

Figure 1: Reaction functions when $f(x) = \mathbf{e}^{-\sqrt{x}}/2$ on $x \geq 0$, and $s = 1$.

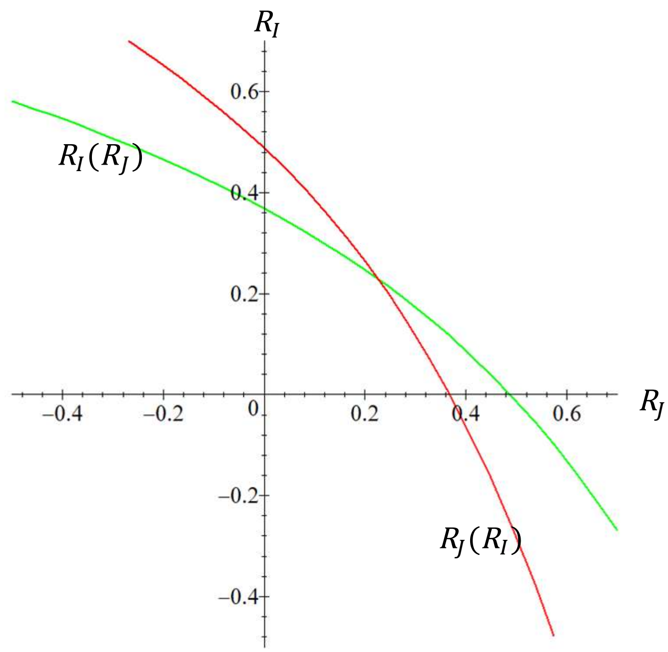

Figure 2: Reaction functions when $f(x) = 1/(x^4 + 1)$ and $s = 0.1$.

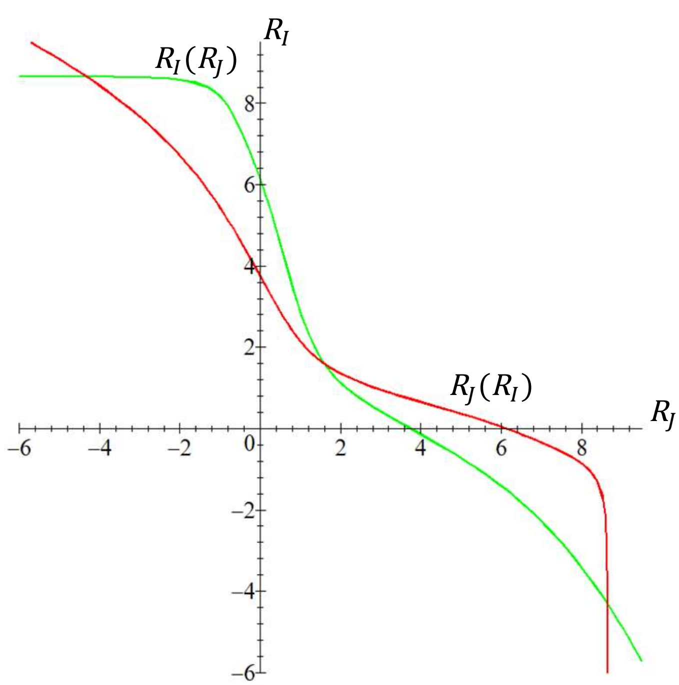

Figure 3: Reaction functions when $f(x) = 1/(x^4 + 1)$ and $s = 0.001$.