Student Loans and Income Share Agreements for Financing Education*

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June 2, 2022

Abstract

Income Share Agreements (ISAs) are seen as an alternative to financing education using student loans, which subject students to default and increase their cost of borrowing (or financial difficulty) in the event of an unfavorable job market outcome. This paper examines a university's decision of offering ISA financing and the impact it has on a student's effort. We find that ISAs induce student moral hazard and result in lower student effort compared to student loans. Yet, the student surplus is higher in the case of ISA financing than student-loan financing because the ISA contract requires payment of only a fraction of after-graduation income. The university prefers student's ISA financing in a market where the student faces large loan-default-related financial difficulties; otherwise, the university prefers student-loan financing. An implication of the university's endogenous contracting decision is that a reduction in the student's financial difficulty can actually reduce expected student surplus. We also find that under certain conditions requiring the university to offer ISA financing can help improve social welfare.

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1 Introduction

College education is an important avenue for income growth. According to the U.S. Bureau of Labor Statistics (BLS) survey, in 2020, workers with a bachelor's degree had a median weekly income of $1,305, which was much higher than $781 for workers with a high school diploma.[1] However, college education in the US is an expensive endeavor. In addition to about $40K lost in yearly income, a student's average 4-year college expenditure ranges from about $100K for in-state public institutions to more than $200K for nonprofit private institutions. Given the large cost of a college degree, most students (more than two third) rely on student loans to pay for tuition and other expenses. The significance of student loans to the US consumers and economy is evident by the observation that in 2021 about 45 million borrowers held a total of $1.7 trillion (which is second only to mortgage loans) in student loans.[2] Financing education using student loans is not without challenges. Students face a default risk: If a student is not successful on the job market, she may fail to make the required repayment and default. In such situations, the student may need to refinance her loan or borrow at higher rates to fulfill her commitments, thus increasing the student's financial difficulty.

Although student loans are the most common approach to financing education, Income Share Agreement (ISA) contracts are being seen as an innovative alternative to financing education through student loans. An ISA contract requires the student to pay a fixed percentage of her after-graduation income in exchange for education funds. For example, students who receive funds from the Purdue University's Back a Boiler — ISA Fund "complete the agreement by paying back a set percentage of their post-education salary over a set number of years."[3] A major advantage of an ISA contract is that unlike in the case of student loans the student is not expected to make a payment if she is not successful on the job market. Therefore, the student faces no default risk.

A number of universities and coding academies (more than 60 as of 2019) have started offering ISA contracts to their students. Robert Morris University's Colonial Success Fund

[1] https://www.bls.gov/careeroutlook/2021/data-on-display/education-pays.htm

[2] https://www.forbes.com/sites/zackfriedman/2021/02/20/student-loan-debt-statistics-in-2021-a-record-17-trillion/?sh=7da785961431

[3] https://www.purdue.edu/backaboiler/overview/index.html

started offering ISA contracts to students in 2020. The contract requires the students to start paying a fixed percentage of their monthly salary six months after graduation. A student is expected to pay only in those months in which the gross monthly income exceeds the annual equivalent of $25,000.[4] The University of Utah's Invest in U is a similar pilot ISA program, which is designed to fill students' funding gaps. The university suggests the program is an ideal tool to help students finish their education in the current uncertain economic environment.[5] The Lewis Income Share Agreement (LISA) program at Clarkson University offers up to $10,000 per year in ISA funding and students pay back a fixed percentage of their salary (if it is higher than $20,000 per year) after graduation.[6] Several coding academies (e.g., Kenzie Academy, Holberton School, Pursuit, Lambda School, etc.) offer similar ISA options to their students.

Although ISAs are gaining popularity, there is no clear understanding of their impact on students and institutions that offer them. How do ISAs incentivize students' effort in comparison to student loans? Are students actually better off with ISAs? Why are only some universities offering ISA? In this paper, we aim to answer these questions. We develop a game theoretic framework with a single university and a single student under which we consider different models to establish an understanding of forces that drive the university's and the students' incentives under the student-loan financing and ISA financing options. We assume the student has no savings and finances the entire cost of her tuition. In the first model, we assume the university sets an upfront tuition fee which the student pays using a student loan. The possibility of an unfavorable job-market outcome subjects the student to default risk. In the event the student is not successful on the job market, she borrows from other sources at a higher rate to meet her repayment schedule. We refer to this higher rate as the financial-difficulty level of the student. In the second model, the university offers an ISA contract which requires the student to pay a proportion of her after-graduation income to the university. Equivalently, the university sets the stake it has in the student's future income. A student's likelihood of success on the job market depends on the effort level

[4]https://www.rmu.edu/about/news/rmu-make-college-more-affordable-through-income-share-agreements

[5]https://isa.utah.edu/

[6]https://www.clarkson.edu/isa

endogenously chosen by the student given the terms of the accepted contract. We compare equilibrium strategies and payoffs between the two models to generate important insights.

A comparison of the students' effort choices between the student-loan financing and ISA financing options reveals ISAs induce student moral hazard and result in lower student effort compared to student loans. The intuition is the following. In the case of student-loan financing, the student is the full claimant of her after-graduation income. But in the case of ISA financing a fraction of the student's income goes to the university. Therefore, the student becomes less motivated to study when financing education using an ISA as compared to student loan. This result is similar in spirit to the well-known result in the context of distribution channels where a retailer provides lower customer service in a decentralized channel compared to a centralized channel (see Tirole, 1988).

We find, however, that ISA financing is desired from the student's perspective. In the loan-financing case, the university extracts all the surplus from the student by setting a tuition fee that leaves the student indifferent between attending and not attending the university. In the ISA financing case, the university does not require any upfront payment and receives only a fixed percentage of the student's after-graduation income. Because the university does not extract all the student surplus in the ISA financing case, the student prefers it over student-loan financing. Interestingly, the university may also prefer ISA financing, particularly when the student's financial-difficulty level is sufficiently large. When the student anticipates facing large financial difficulties in the case of student-loan default, she is less willing to borrow money to pay university tuition. The university responds by cutting tuition fee which drives the university's profits lower. But, because the ISA contract eliminates the possibility of default, its appeal to the student (and therefore the university's stake in the student's future income) does not change with an increase in the student's financial difficulty. Therefore, ISA financing produces higher university profit than student-loan financing in a market where the student faces large financial difficulty upon default. An implication of the university's endogenous contracting decision is that a reduction in the student's financial difficulty can backfire and reduce expected student surplus.

ISA financing is desirable in a broader range of financial difficulty from the welfare perspective than it is from the university's perspective. The reason is that student surplus is always higher in the case of ISA financing whereas the university profit is higher in the ISA financing case only when the student's financial difficulty is sufficiently large. Therefore, in an intermediate range of student's financial difficulty, ISA financing is optimal from the welfare perspective but not from the university's perspective. In such situations, a social planner may incentivize the university to offer ISA financing to the student through a subsidy that is conditional on the university offering ISA financing to the student. Alternatively, the planner may mandate the university to offer ISA financing to the student.

Following this analysis, we allow the university to set a hybrid-financing contract that may include both upfront fee and ISA features. Consistent with the real-world observation that ISAs offered by universities do not typically cover the entire cost of tuition, we find that the university prefers the student's ISA-loan hybrid financing over ISA financing or loan financing. The reason is that the hybrid financing allows the university to both create value for the student by reducing the disutility associated with default (using the ISA feature) and extract this value away from the student (using the up-front price feature). Because the hybrid-financing contract extracts all the surplus away from the student, the ISA-only contract is more desired from the student's perspective. Although hybrid financing produces a higher welfare than loan financing (because university's profit is higher with hybrid financing), ISA financing is ideal from the welfare perspective when student's financial difficulty is sufficiently high (because it fully protects the student from default).

Finally, we demonstrate the robustness of our main results and generate additional insights by considering four extensions of the hybrid-financing model. First, we consider students are heterogeneous in their financial difficulty and find that in the equilibrium the university offers a tuition-stake menu of contracts. The high-financial-difficulty students self select into the low-tuition-high-stake contract whereas the low-financial-difficulty students opt for the high-tuition-low-stake contract. An important implication is that an increase in the income inequality (measured by the difference in students' financial difficulty) can lead the university to rely primarily on high-income (low-financial-difficulty) students (by raising the tuition fee) for its profit. Second, we assume the university cares not only about its profit but also about student surplus. As expected, when the university cares sufficiently about students, it moves from hybrid financing to ISA financing. Third, we examine the university's incentive to invest in education quality and find that the university is more likely to invest in education quality when both student's financial difficulty and cost of enhancing quality are low. Fourth, we consider competing universities and find qualitatively similar results with the exception that students receive positive surplus (due to competition for students between universities).

Our results have important implications for policy makers. To see this, first note the following. We show that upfront tuition-based payment for university education, which we assume is financed fully through a student loan, has the advantage that it makes the student the full residual claimant of her efforts which leads to greater effort exertion and better expected outcomes post education. However, the downside is that if there is a loan default then the student suffers high disutility; fearing this the student might not take up university education if the disutility from default is high, and this hurts students with lower financial standing more. On the other hand, an ISA has the upside that it shields the student from default (as there is no loan involved), which enables the student to pursue education when otherwise she might not have if she had to take out a loan. However, the downside of an ISA is that the student has less motivation to work hard because she is not the full residual claimant of the outcomes of her efforts, which leads to worse post education outcomes.

Good policy can aim to achieve the upside of tuition-based payment, i.e., make the student the residual claimant of her efforts, while reducing its downside, i.e., shield the student from the disutility of a default. This can be done in multiple ways. For instance, the student could be offered a subsidized loan such that the loan amount is lesser which would lead to less disutility even if there is a default. Indeed, 529 programs that allow parents to save for their children's education on a subsidized basis serve a purpose similar to subsidized loans. Another possibility could be capping up-front tuition payments. Alternatively, the student could be offered assistance in the case of a default, i.e., default protection.

In other words, an important implication from our research is that, outcomes can be improved if tuition payments are determined in a manner that does not distort effort exertion by students, while ensuring that they pursue education. Therefore, in comparison to ISAs (or any scheme that prices education based on a share of post-education performance), total welfare will increase more from programs that reduce the liabilities that students carry due to their student loans for tuition payments, which can be achieved through subsidized loans and default assistance. Moreover, if there are price caps on upfront student tuition (i.e., the university cannot extract the full expected surplus from education), then students can also benefit from tuition-based payment with loans.

1.1 Related Literature

The idea of using ISAs to finance professional education was first proposed in Friedman and Kuznets (1945). Nerlove (1975) points out that participants' behavioral responses (e.g., moral hazard) to ISAs may challenge the viability of risk-pooling ISAs. We show ISAs may not only be viable but also be more attractive than student-loan financing for both universities and students even in the presence of moral hazard. Grout (1983) shows ISAs are appealing to students when job market is highly uncertain and students are risk averse. We show ISAs maybe attractive even when students are risk neutral. Moen (1998) focuses on how human-capital investments are financed and shows that if the investments are financed by contingent loans, which carry interests only if the borrower is employed, optimal investments can be obtained in a wide range of situations. We explicitly consider the role of student's financial difficulty in the case of unfavorable employment outcome and show that ISAs may not be optimal from the university's and social-welfare perspective, particularly if the student's financial difficulty is small. As such, our findings challenge the Chapman (2006) conclusion (on the basis of the extent literature) that ISAs are in general welfare increasing compared to either bank loans or up-front fees. We contribute to the literature on ISAs by explicitly considering students' financial difficulties and generating new insights including about when ISAs may be welfare enhancing.

A large stream of literature following Becker (1964) examines implications of consumers' borrowing and repayment decisions for investment in human capital (more generally, in projects with notably poor collateral).[7] Christen and Morgan (2005) show income inequality contributes to increased consumer borrowing to finance the purchase of durable goods. We show that a higher income inequality can lead universities to primarily focus on high-income

[7]Please refer to Lochner and Monge-Naranjo (2016) for a comprehensive overview of literature on student loans and repayments.

students for profits by raising tuition. In the context of collateral-free loans, Zhang and Liu (2012) show lenders use peer-lending decisions to infer creditworthiness of borrowers. In our model, the creditworthiness of students is decided by the offer of admission to the university. Bao, Ni, and Singh (2018) develop a theory model to explain near-zero default rates even in the absence of any collateral in informal lending markets. Our focus is on studying the student-loan and ISA contract design to induce effort by students and maximize payoffs for stakeholders.

This paper is related to the literature on consumer moral hazard. In a variety of contexts, consumers decide how much effort they put in using the product which in turn dictates consumers' as well as firms' payoffs. Shavell (1979) and Ma and Riordan (2002) consider consumer's effort choice after buying insurance. Lutz (1989) and Padmanabhan and Rao (1993) consider consumer effort in response to product warranties. Iyer and Singh (2018) study the effect of consumer moral hazard in the context of product safety. In this paper, students decide their effort level after accepting the university's offer of admission. The level of effort directly affects the likelihood of students' job market success.

Our work also contributes to the evolving research on education in the recent marketing literature. Grewal, Meyer, and Mittal (2022) argue "education is an important decision and consumption domain for consumers" and encourage the discipline to "embrace education as a substantive topic, a research setting, and a central field of inquiry." Among this literature, Yoon, Yang, and Morewedge (2022) and Zhou, Gill, and Liu (2022) are most closely related to our work. Yoon, Yang, and Morewedge (2022) find that students psychologically realize the financial costs of loans long before the repayments begin. This early cost realization plays an important role in students' college choice. We examine how expected loan and ISA repayments influence rational students' effort choices during college. Zhou, Gill, and Liu (2022) explore the effect of public education crowdfunding on the academic achievements of students. We focus on the effect of student loans and ISAs on the job market success of students. In addition, we examine student surplus, university profit, and social welfare implications of these education financing options.

The rest of this paper is organized as follows. The next section introduces the base model, separately analyzes the student loan financing and ISA financing options, and compares them to generate insights. Section 3 presents the main analysis in which the university can offer loan-ISA hybrid financing. Section 4 presents four extensions of the hybrid financing model. Section 5 concludes the paper.

2 Base Model

Consider a university that makes an offer of admission to a prospective student. The offer of admission also includes a fee contract that must be signed by the student at the time of accepting the offer of admission. In our base model, we consider two separate types of fee contracts a university might offer to the student: a conventional upfront-fee contract and an ISA contract. In the case of an upfront-fee contract, the university simply sets a price p for university education. The student must pay the entire amount p at the time of admission. In the alternative ISA contract, there is no upfront-payment requirement. The student does not pay anything until completing her university education. However, at the time of admission, the student must agree to pay a fraction ρ of her after-graduation income y. The fraction ρ is the university's stake in student's future income. The student's income y is either 1, which represents that the student is successful, or zero, which represents that the student is not successful, on the job market.

The student has no past savings at the time of university admission. The student decides whether to accept the university's offer of admission. We assume the student's outside option is zero. If the university offers an upfront-fee contract, accepting the contract requires the student to finance her education by obtaining funds from a competitive debt market. In this case, the student borrows an amount p and pays her university fee. For simplicity, we assume the competitive debt market offers funds at an interest rate of zero. The student agrees to fully repay her debt at the end of her university education. If the student fails to repay her debt at the time it is due, she incurs an additional disutility δ for every unit of borrowed funds. This disutility δ captures the effect of debt refinance or cost associated with insolvency and we refer to it as the level of student's financial difficulty. In markets where debt refinancing is readily available or where government may offer debt-forgiveness programs to students, we can expect the level of student's financial difficulty δ to be relatively small.

Note, in the main analysis, we assume students are homogeneous in their financial difficulty and the level of financial difficulty δ is common knowledge. The effect of heterogeneity in δ and information asymmetry (students may be privately informed about δ) is formally examined and presented in section 4.1.

If the university offers an ISA contract, which requires the student to pay a fraction ρ of her after-graduation income y, she decides whether to accept the offer. If the student is successful in the job market (i.e., earns an after-graduation income $y = 1$), she pays a fraction ρ of her income to the university. However, if the student is not successful in the job market (i.e., her after-graduation income y is zero), she is not requires to pay anything back to the university. The fraction ρ is essentially the university's stake in the student's income.

If the student accepts university's offer of admission, she enrolls in the university to acquire education. However, university enrollment alone does not guarantee success. A student must study hard and acquire skills in order to succeed on the job market. We represent the student's effort by $e \in [0, 1]$. The student incurs a cost e^2 when putting effort e. A higher effort e increases the likelihood that the student will be able to find a job after graduation. Specifically, the student's after-graduation income y is given by

$$y = \begin{cases} 1 & \text{with prob } a + ke, \\ 0 & \text{with prob } 1 - (a + ke), \end{cases}$$

where a captures factors that are independent of the student's effort (such as value of university brand name or different majors) and k the effectiveness of student's effort. Because the effort e directly affects the likelihood of student's success on the job market, it may include student's non-academic activities (e.g., extra-curricular activities and effort during job) in addition to how hard the student studies while attending the university. The parameters a and k are such that both the equilibrium effort e^* and probability $a + ke^*$ of student's success on the job market are bounded between 0 and 1.

2.1 Student Loan Financing

First, consider that the university offers a conventional upfront-fee contract together with its offer of admission to the student. In this case, the student finances her education (i.e., the upfront fee p_D) using a student loan. We use subscript D to represent that the student financed her education using a debt. The student's expected utility from accepting the university's offer of admission is

$$U_D = (a + ke_D)(1 - p_D) - [1 - (a + ke_D)](1 + \delta) p_D - e_D^2. \tag{1}$$

The first term represents that with probability $a + ke_D$ the student expects an income $y = 1$, out of which she repays p_D to her lender. The second term represents that the student expects to not succeed on the job market with probability $1 - (a + ke_D)$ in which case she earns $y = 0$ and is unable to repay her debt. In this case, she incurs an additional disutility δ for every unit of borrowed funds. The third term is simply the student's cost of effort. We assume the discount factor is one. In addition, the condition $0 < k < \sqrt{\frac{2(1-a)-\delta(1-a)^2}{1+\delta}}$ ensures (1) the probability of student's success on the job market is bounded between a and 1, and (2) both student's and university's maximization problems have real solutions.[8]

If the student accepts the university's offer of admission, the university receives the entire fee p_D upfront from the student. Therefore, the university expects to receive the fee p_D that it sets for education if the student accepts the offer of admission, and nothing otherwise. The university's payoff does not depend on whether the student is actually successful on the job market. The university maximizes its expected payoff π_D by setting a fee p_D that leaves the student indifferent between accepting and rejecting the university's offer of admission. We assume the student accepts the offer of admission if she is indifferent between accepting and rejecting the offer. Note that in our main analysis, we assume the university's objective is to maximize its expected profit. In a model extension, presented in Section 4.2, we assume the university cares about both its expected monetary payoff and student surplus.

The timing of actions (also shown in Figure 1) is as follows. The university moves first

[8]The lower bound of a for the probability of student's success on the job market in needed to ensure the student's equilibrium effort $e_D^* > 0$.

and sets the price p_D that is to be paid in full by the student when accepting its offer of admission. The student decides whether to accept the university's offer. If the student accepts the offer, she borrows funds from the debt market and pays the entire fee p_D to the university right away. Next, the student decides her costly effort e_D. In the next stage, job market outcome is realized. If the student is successful, she repays her debt. However, if unsuccessful, she incurs an additional default-related disutility. Finally, payoffs are realized.

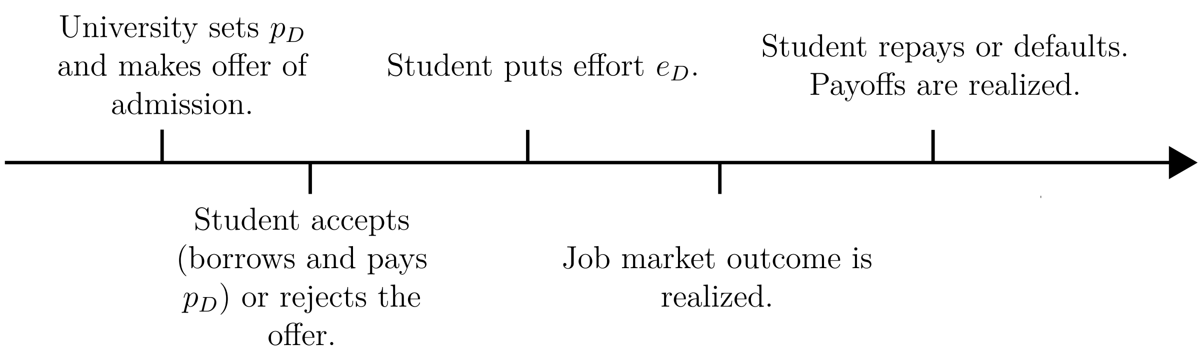

Figure 1: Timing: student loan financing

We solve this sequential-move game by backward induction. The following proposition summarizes equilibrium fee p_D^* set by the university, effort e_D^* chosen by the student, the resulting university profit, student surplus, and overall welfare. All proofs are in the Appendix.

Proposition 1 *If the university offers an upfront fee contract, in the equilibrium,*

(a) it sets fee $p_D^* = \frac{2-\left(k^2+2a-2\right)\delta-2\sqrt{1+\delta\left[1-a^2+(1+\delta)\left((1-a)^2-k^2\right)\right]}}{k^2\delta^2}$ *, and*

(b) the student's effort level $e_D^* = \frac{1+\delta-a\delta-\sqrt{1+\delta\left[1-a^2+(1+\delta)\left((1-a)^2-k^2\right)\right]}}{k\delta}$.

(c) The university's equilibrium expected profit is p_D^**, expected consumer surplus is zero, and the overall welfare is* p_D^**.*

First, we discuss the effect of parameters a, k, and δ on the equilibrium price p_D^*. (These effects are also presented graphically in Figure 2.) The student's effort level, which the university can perfectly anticipate in equilibrium, plays an important role in driving firm's pricing decision. If the student's effort is more effective (i.e., k is higher), the university expects the student to study harder and likely succeed on the job market. Therefore, the

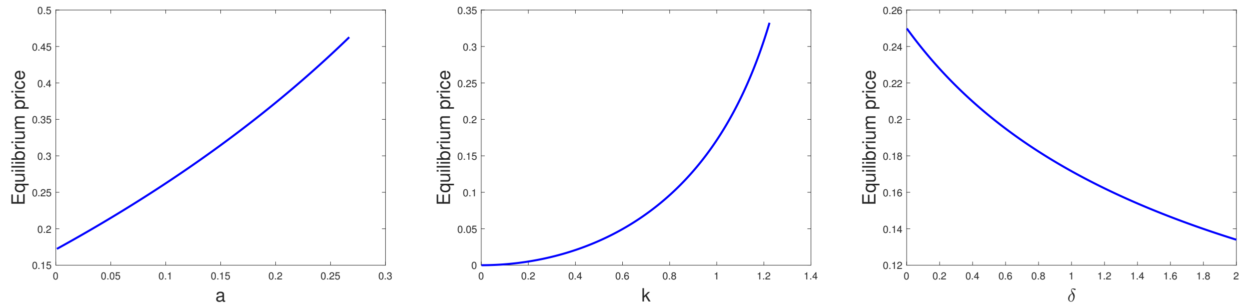

Figure 2: Student loan financing: Equilibrium price p_D^* with a (using $k = 1$, $\delta = 1$), k (using $a = 0$, $\delta = 1$), and δ (using $a = 0$ and $k = 1$).

university sets a higher fee, which extracts all the expected surplus away from the student. If the university is reputed or the major is attractive (i.e., a is higher), the university understand students are more likely succeed. Therefore, the university sets a higher fee. This higher fee induces the student to study harder because failure on the job market after paying a higher fee imposes a larger disutility on the student. A failure on the job market is also more painful if default is more costly for the student (δ is higher). Therefore, a higher δ induces the student to study harder. However, because a higher δ makes borrowing p_D to acquire university education less rewarding for the student, the university sets a lower fee.

Next, we discuss student surplus, university profit, and overall welfare. The fee p_D^* set by the university leaves the student indifferent between accepting and rejecting the university's offer of admission. Therefore, the expected student surplus is zero and expected university profit, which equals the fee paid by the student, is strictly positive. If the student succeeds on the job market, she receives a positive payoff. However, if the student is not successful, her payoff is negative. Because expected student surplus is zero, the overall welfare equals the university's expected profit. The effect of a change in parameters a, k, or δ on the firm's expected profit (and overall welfare) is the same as that on the fee p_D^* set by the university.

2.2 Income Share Agreement

Next, consider the university offers an ISA contract with its offer of admission to the student. The ISA contract does not require any upfront payment by the student. Therefore, in this case, the student does not need to borrow any funds from the debt market. If the student accepts the contract and is eventually successful in the job market, she pays a fraction ρ (specified by the university) of her income $y = 1$. However, if the student is not successful on the job market and her income is $y = 0$, she does not pay anything back to the university. We use the subscript S to represent that the student financed her education through an ISA contract. Because the student succeeds with probability $a + ke_S$, her expected utility (accounting for the cost e_S^2 of her effort) is

$$U_S = (a + ke_S)(1 - \rho_S) - e_S^2. \tag{2}$$

The university does not receive any upfront payment from the student. In addition, the university's payoff depends on whether the student is actually successful on the job market. The university is essentially invested in the student's future success (the probability of which is $a + ke_S$) and owns a stake ρ_S in the student's income. Therefore, the university's expected payoff is

$$\pi_i = (a + ke_S)\,\rho_S. \tag{3}$$

The student chooses the level of effort e_S to maximize her expected utility U_S and the university sets the stake ρ_S to maximize its expected payoff π_S. We assume $0 < a < \frac{2(1+\delta)}{\delta} - \sqrt{\frac{4+6\delta+3\delta^2}{\delta^2}}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, which jointly ensure that both optimal effort e_S^* and optimal stake ρ_S^* are feasible (i.e., are bounded between 0 and 1).

The university moves first and sets the stake ρ_S in the student's after-graduation income. At the time of accepting the university's offer of admission, the student must agree to transfer this stake ρ_S in her income to the university. The student decides whether to accept the university's offer of admission. If the student accepts the offer, she does not pay anything upfront to the university. Next, the student decides her costly effort e_S. In the next stage, job market outcome is realized and the student transfers a proportion ρ_S of her income to the university. Finally, payoffs are realized. A summary of the timing of actions is shown in Figure 3.

Similar to the previous section, we solve the game by backward induction. The following proposition summarizes equilibrium stake ρ_S^* set by the university, effort e_S^* chosen by the student, the resulting university profit, student surplus, and overall welfare.

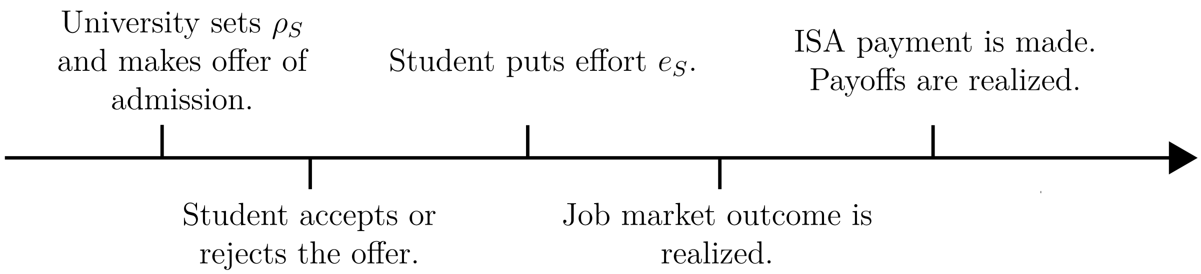

Figure 3: Timing: income share agreement

Proposition 2 *If the university offers an ISA contract, in the equilibrium,*

(a) it sets stake $\rho_S^* = \frac{1}{2} + \frac{a}{k^2}$*, and*

(b) the student's effort level $e_S^* = \frac{k}{4} - \frac{a}{2k}$.

(c) The university's equilibrium expected profit is $\pi_S^* = \frac{\left(2a+k^2\right)^2}{8k^2}$*, expected consumer surplus is* $\frac{1}{16}\left(4a + k^2 - \frac{12a^2}{k^2}\right)$*, and the overall welfare is* $\frac{1}{16}\left(12a + 3k^2 - \frac{4a^2}{k^2}\right)$.

First, we discuss the effect of university's stake ρ_S on the student's effort decision. An admitted student understands the likelihood of her success in the job market depends on how hard she studies while acquiring her university education. Although the student puts all the effort (incurs the entire cost e_S^2), she does not enjoy all the benefit: a proportional ρ_S of her future income goes to the university. This reduces the student's incentive to study hard. The larger the stake ρ_S the smaller the student's incentive to study and smaller the likelihood of student's success on the job market. Therefore, if the university wants to incentivize the student to study hard (so the student can be successful at the job market) it must set a lower stake. But a lower stake reduces the payment the university receives from a successful student. The university essentially faces a trade-off between value creation and value extraction.

Next, we examine the effect of change in parameters a and k on the student's effort and the university's stake. (See Figure 4 for a graphical presentation of the effect of parameters a and k on stake ρ_S.) A larger a indicates the student's success in the job market is largely determined by exogenous factors, e.g., major popularity or university reputation, whereas a larger k indicates it is largely determined by student's effort. If a is large, exogenous factors can ensure student success and there is really no need for the university to incentivize the student to study hard. The university sets a higher ρ_S and the student responds to it by

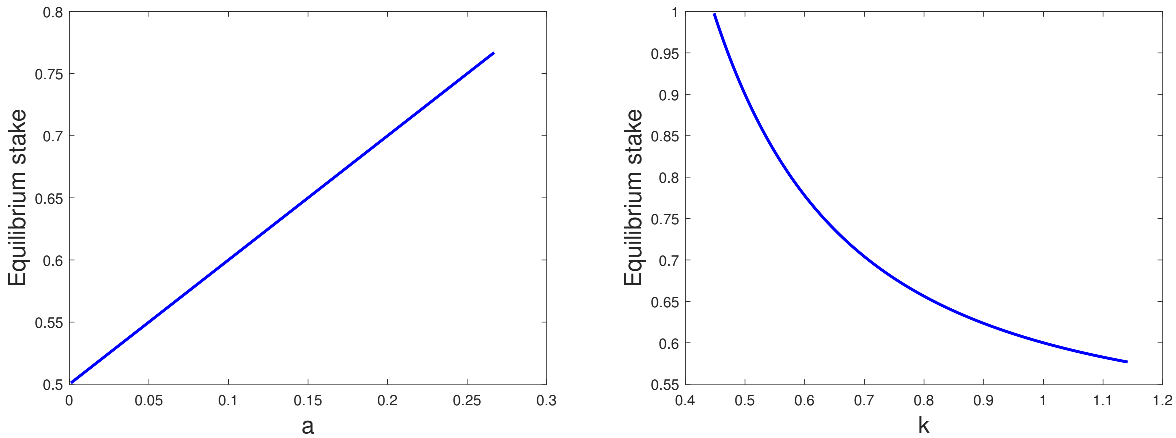

Figure 4: Income share agreement: Equilibrium stake ρ_S^* with a (using $k = 1$ and $\delta = 1$) and k (using $a = 0$ and $\delta = 1$).

studying less. In contrast, a larger effort effectiveness k makes the student success highly responsive to her effort. The university sets a smaller stake to incentivize the student to study harder and the student responds by studying harder.

Finally, we examine the effect of change in parameters a and k on expected profit, student surplus, and welfare. The student's equilibrium likelihood of success on the job market $(\frac{a}{2} + \frac{k}{4}^2)$ and the equilibrium stake ρ_S^* are both increasing in a. Therefore, the university's expected profit increases in a. Note the student's equilibrium likelihood of success on the job market is increasing in k as well. Although the university sets a lower ρ_S^* when k is larger, the decision is driven by the university's desire to receive higher profit, by incentivizing the student to study harder. Therefore, the university's expected profit is increasing in k as well.

A higher k leads to higher student surplus. If k is higher, the student works harder (and therefore the student is more likely to succeed on the job market) and the university sets a lower stake. Both effects contribute to a higher student surplus. Interestingly, the effect of parameter a on the expected student surplus is non-monotonic. Specifically, if $a < \frac{k^2}{6}$, expected student surplus increases in a. However, if $a > \frac{k^2}{6}$, it decreases in a. If a is small and k is large (i.e., $a < \frac{k^2}{6}$), the likelihood of student's success on the job market crucially depends on the student's effort. In this case, the university is primarily concerned about incentivizing the student to study hard (i.e., in value creation) than in value extraction through stake ρ_S. Thus, in response to a higher a, the university increases its stake ρ_S^* less aggressively and leaves some of the incremental surplus for the student. As a result, the expected student surplus increases. If a is large and k is small (i.e., $a > \frac{k^2}{6}$), the student's effort is less important in value creation (i.e., in determining student's chance of success). In this case, the university is not so concerned about incentivizing the student to study hard. Therefore, in response to a higher a, the university increases its stake ρ_S^* aggressively and extracts surplus from students beyond the incremental gain. As a result, the expected student surplus declines.

The total welfare depends on the likelihood of student success on the job market, which increases with both a and k. Therefore, expected welfare also increases with both a and k. A stake $\rho_S^* \in (0, 1)$ implies the student receives a strictly positive expected surplus from joining the university. Because the student does not pay anything if the job market outcome is unfavorable, ex-post surplus is never negative.

2.3 Comparison of Loan and ISA Financing

Student effort plays an important role in determining student success and value creation in the job market. Therefore, both the university and the policymaker care about whether the student works harder under loan financing or ISA financing. In the following proposition, we compare the student's effort for the two financing options.

Proposition 3 *The student puts higher effort toward her job market success if she finances her education through debt financing than ISA financing (i.e., $e_D^* > e_S^*$).*

Regardless of the level of financial difficulty (δ) of the student, debt financing induces a higher level of student effort than ISA financing. The reason is that, unlike in the case of debt financing, the student is not the full claimant of the returns to her effort in the case of ISA financing. Because a fraction ρ_S^* of the student's return goes to the university, the student is less motivated to put effort toward her job market success when she signs the ISA financing contract with the university. This result is similar in spirit to the well-known result in the context of distribution channels where a retailer provides lower customer service in a decentralized channel compared to a centralized channel because the retailer bears all the cost of providing service but does not receive all the benefit (the manufacturer extracts part of the extra surplus) (see Tirole, 1988). Consistent with our result, Madonia and Smith (2019) find individuals perform substantially worse in poker games when having sold a share of earning (ISA financing).

Although the student puts higher effort when financing her education using a loan, it is not obvious if she is actually better off under student-loan financing. In addition, the university's decision to offer an ISA contract may depend on how offering such a contract affects university's profit. The policymaker may wonder which financing scheme is optimal from the overall welfare perspective. In the following proposition, we compare the student surplus, university profit, and overall welfare between the loan financing and ISA financing. The comparison is also shown graphically in Figure 5. For ease of presentation, in the Appendix, we define thresholds $\hat{\delta}$ and $\tilde{\delta}$ for the student's financial difficulty δ above which the university and the planner, respectively, prefer the student's use of ISA financing.

Proposition 4 *In comparison to the debt financing, the student's use of ISA financing is preferred by*

(a) the student, unconditionally,

(b) the university, if $\delta > \hat{\delta}$, and

(c) the social planner, if $\delta > \tilde{\delta}$ (where $\hat{\delta} > \tilde{\delta}$).

In the case of debt financing, the university's profit maximizing price p_D^* extracts all the expected surplus away from the student. However, in the case of ISA financing, for any $\rho_S^* \in (0, 1)$, the expected student surplus is strictly positive. Therefore, the student prefers ISA financing to debt financing. It is useful to note that under debt financing if the student is not successful on the job market, the outcome can be devastating for students (i.e., ex-post surplus can be negative). In contrast, in the case of ISA contract, the ex-post surplus is never negative because the student is not required to pay anything if she is not successful on the job market.

The university prefers student's use of debt financing, if $\delta \leq \hat{\delta}$, and prefers ISA contract, otherwise. The intuition is the following. The student puts higher effort and is more likely to succeed on the job market under debt financing. In addition, unlike in the case of ISA contract, the university extracts all the surplus away from the student under debt financing.

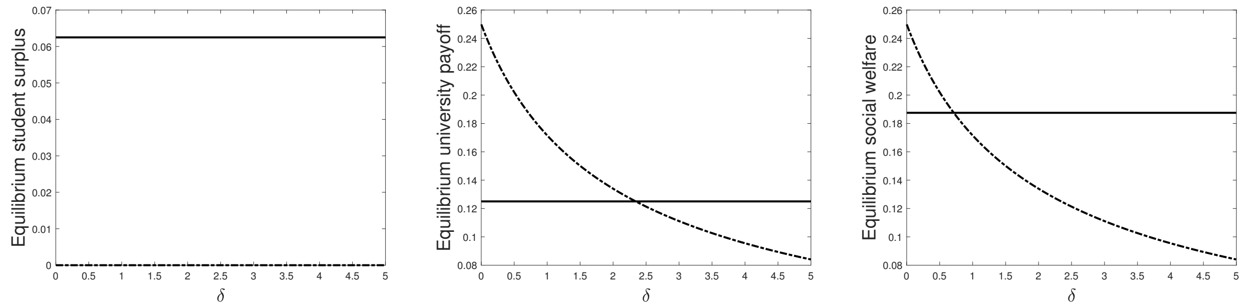

Figure 5: Expected student surplus, university profit, and social welfare for student loan financing (dotted line) and ISA financing (solid line) with δ (using $a = 0$, $k = 1$).

Therefore, the university prefers student's use of debt financing when δ is small. A higher δ discourages the student from opting for debt financing. The university responds by reducing the upfront fee p_D^* to make debt financing a viable option for the student. Therefore, an increase in δ reduces university's profit under debt financing. Because students cannot default in the case of ISA financing, the viability of the ISA contract for the student does not depend on the level of δ. As a result, university prefers debt financing over ISA financing if the financial difficulty δ faced by the student is sufficiently small but not otherwise.

The social planner cares about the overall welfare, which is the sum of the student surplus and university's profit. Because expected student surplus is always higher in the case of ISA financing, the planner prefers ISA financing over a larger set a parameter space (for $\delta > \tilde{\delta}$, where $\hat{\delta} > \tilde{\delta}$) than the university. Note, the university's and the planner's preferred financing option is the same if δ is sufficiently small (both prefer debt financing) and if δ is sufficiently small (both prefer ISA financing). However, if $\delta \in \left(\tilde{\delta}, \hat{\delta}\right)$, the university may be unwilling to offer an ISA contract to the student. In this case, the social planner may be able to increase the overall welfare by requiring the university to offer an ISA financing contract to students together with its offer of admission.

3 Student Loan-ISA Hybrid Financing

In the previous section, we considered the possibilities of a student financing her university education using either a loan or an ISA. However, as highlighted in the examples presented in the Introduction section, a university may also offer a hybrid financing contract containing both an upfront payment (price p_H) and an ISA components (stake ρ_H). In this case, if the student accepts the university's offer of admission, she finances her education using both a student loan and an ISA. The student puts an effort e_H toward her job market success at a cost e_H^2. This effort translates into a probability $a+ke_H$ of job market success. If the student is successful on the job market, she receives an income $y = 1$, out of which she uses p_H to repay her loan and a share ρ_H of her income (i.e., $y\rho_H = 1 \cdot \rho_H = \rho_H$) for the ISA payment. However, if the student is not successful on the job market, she receives no income, defaults on the loan payment (and therefore incurs a disutility of $(1 + \delta) p_H$), and is not requires to make any ISA payment. The other assumptions are the same as in the previous section. The expected utility of the student can be written as

$$U_H = (a + ke_H)(1 - p_H - \rho_H) - [1 - (a + ke_H)](1 + \delta) p_H - e_H^2 \tag{4}$$

and the university's expected profit as

$$\pi_H = p_H + (a + ke_H) \rho_H. \tag{5}$$

The student maximizes her expected utility in (4) by choosing her effort level e_H given university's price p_H and stake ρ_H decisions. The university sets p_H to extract all the surplus away from the student and ρ_H to maximize its expected profits in (5) anticipating student's subsequent effort choice. The expressions for equilibrium price p_H^*, stake ρ_H^*, and effort e_H^* levels are provided in the Appendix. A comparison of the university's and the student's equilibrium decisions and expected payoffs among the hybrid financing, loan financing, and ISA financing reveals the following insights.

Proposition 5 *If the university offers a hybrid financing contract to the student,*

(a) the university sets a stake ρ_H^* that is smaller than ρ_S^* and a price p_H^* that is smaller than p_D^*.

(b) the student's effort level e_H^* is larger than e_S^* but smaller than e_D^*.

(c) the university earns a higher profit than loan only or ISA only options but the expected student surplus is zero.

In the case of ISA contract, the university accumulates its payoff through a stake ρ_S^* in the student's future income. Although a higher stake earns the university a larger proportion of a successful student's income, it also reduces the student's incentive to put effort toward job market success causing the likelihood of student's success to decrease. The use of hybrid-financing contract allows the university to set a smaller stake ρ_H^*; thus, incentivizing the student to prepare harder for job market success than in the case of ISA contract.

Unlike in the case of upfront-fee contract where the student was the sole claimant of all her future income, a proportion of student's income is claimed by the university in the hybrid contract. Therefore, the student's expected after-graduation payoff is lower in the case of hybrid contract. The implication is that in the case of hybrid contract, the university sets a lower price p_H^* (the role of which is to extract all the surplus from the student) than the price p_D^* in the case of upfront-fee contract. In the case of hybrid financing, the student does not put as much effort as she does in the case of upfront-fee contract. The intuition is similar to that for the comparison of the student's effort between up-front fee and ISA contracts: hybrid contract requires the students to share a proportion of her after-graduation income which reduces the student's incentive to work hard toward job market success.

The hybrid-financing contract allows the university to benefit from both the price (upfront tuition fee) and ISA options. The ISA part of the hybrid contract creates value for the student by reducing the disutility associated with default and the up-front fee part of the hybrid contract allows the university to extract this value away from the student. As a result, from the university's perspective, offering a hybrid contract to the student is more desirable than price-only or ISA-only contracts. Because the up-front price part of the hybrid-financing contract allows the university of extract all the expected surplus from the student, the expected student surplus is zero.

An analytical comparison of social welfare is not tractable. We numerically compare social welfare among loan-financing, ISA-financing, and hybrid-financing contracts. Consistent with our intuition from Proposition (4), we find that ISA-financing contract may be socially desirable if the disutility δ associated with default is sufficiently high. However, if δ is small, hybrid financing results in the highest social welfare. Note, social welfare for upfront-fee contract cannot be higher than hybrid-financing contract. The reason is that in both cases expected consumer surplus is zero but university's expected profit is weakly higher in the case of hybrid financing.

Finally, we discuss comparative statics results with respect to our main parameter of interest δ. For tractability, we assume the probability of student's success on the job market is e_H (equivalently, we set $a = 0$ and $k = 1$) for the hybrid-financing model. We find the equilibrium p_H^* is decreasing in δ, whereas ρ_H^* is increasing in δ. The reason is that a higher financial difficulty δ makes the student less willing to borrow money for education. The university responds by reducing p_H^* to encourage student participation and relies on ρ_H^* to make profits. Both a lower p_H^* and a higher ρ_H^* (in response to a higher δ) induce a lower effort e_H^* by the student. In the equilibrium, the expected student surplus is zero. The university's expected profit (and therefore, social welfare) decreases with δ because a lower e_H^* reduces the likelihood of student's success on the job market.

4 Extensions

In this section, we demonstrate the robustness of our main results and generate new insights by examining four extensions of the hybrid-financing model presented in Section 3. To facilitate closed-form solutions, we make a simplifying assumption that the probability of student's success on the job market is simply e (i.e., we assume $a = 0$ and $k = 1$). Formal proofs of propositions and results in this section are presented in a Web Appendix.

4.1 Heterogeneous δ

In the models presented above, we assumed students are homogeneous in the level of their financial difficulty δ, which is common knowledge. In this section, we consider students are heterogeneous in their financial difficulty δ. Specifically, a proportion $\theta = \frac{1}{2}$ of students are type $t = h$ who face high financial difficulty ($\delta = \delta_h$) whereas the other half of students are type $t = l$ who face low financial difficulty ($\delta = \delta_l$, where $\delta_l < \delta_h$). Because, type-h students are often low-income students and type-l students are often high-income students, we can interpret the difference $\delta_h - \delta_l$ as a measure of income inequality in the market. In addition, a student's financial-difficulty type $t \in \{h, l\}$ is her private information. The university offers a menu of contracts: (p_h, ρ_h) and (p_l, ρ_l) such that in the equilibrium the type-h students accept the (p_h, ρ_h) contract and the type-l students accept the (p_l, ρ_l) contract. The other assumptions are the same as in section 3.

The expected payoff of a type-t student who accepts the (p_t, ρ_t) contract and puts effort e_t toward job market success is

$$u_t(p_t, \rho_t) = e_t(1 - p_t - \rho_t) - (1 - e_t)(1 + \delta_t)p_t - e_t^2 \tag{6}$$

whereas, the expected payoff of a type-t student, who deviates and accepts the (p_{-t}, ρ_{-t}) contract, where $-t$ represents "not type t", and puts effort e_{td} is

$$u_t(p_{-t}, \rho_{-t}) = e_{td}(1 - p_{-t} - \rho_{-t}) - (1 - e_{td})(1 + \delta_t)p_{-t} - e_{td}^2. \tag{7}$$

The university's expected profit is

$$\pi = \frac{1}{2}(p_h + e_h\rho_h) + \frac{1}{2}(p_l + e_l\rho_l). \tag{8}$$

In the equilibrium, both types $t \in \{h, l\}$ of students receive non-negative expected payoffs (i.e., $u_t(p_t, \rho_t) \geq 0$) and prefer the (p_t, ρ_t) contract than the (p_{-t}, ρ_{-t}) contract (i.e., $u_t(p_t, \rho_t) \geq u_t(p_{-t}, \rho_{-t})$). We introduce the shadow price for type-t students, λ_t^* (derived in the Web Appendix). The following proposition presents the equilibrium menu of contracts and the corresponding effort level chosen by the student.

Proposition 6 *In the equilibrium, the university offers a menu of contracts:* (p_h^*, ρ_h^*) *and* (p_l^*, ρ_l^*)*, where* $\rho_h^* = \frac{2(\delta_h+\delta_l)(2+\delta_l+2(2+\delta_h+\delta_l)\lambda_h^*)\lambda_l^* - 8\delta_l(1+\delta_l)\lambda_l^{*2} - 2\delta_h(1+2\lambda_h^*)(1+2(1+\delta_h)\lambda_h^*)}{\delta_h^2+4(\delta_h-\delta_l)(\delta_h\lambda_h^*-\delta_l(1+\lambda_h^*))\lambda_l^*}$,
$p_h^* = \frac{\delta_l(2(3+4\lambda_h^*)\lambda_l^*-8\lambda_l^{*2})-\delta_h(1+8\lambda_h^*(1+\lambda_h^*-\lambda_l^*)-2\lambda_l^*)-4(1+\lambda_h^*)(1+2\lambda_h^*)+4(3+4\lambda_h^*)\lambda_l^*-8\lambda_l^{*2}}{\delta_h^2+4(\delta_h-\delta_l)(\delta_h\lambda_h^*-\delta_l(1+\lambda_h^*))\lambda_l^*}$,
$\rho_l^* = -\frac{2(1+2\lambda_l^*)(1+2(1+\delta_l)\lambda_l^*)}{\delta_l}$, *and* $p_l^* = -\frac{\delta_l+8\delta_l\lambda_l^*(1+\lambda_l^*)+4(1+\lambda_l^*)(1+2\lambda_l^*)}{\delta_l^2}$. *The type-$t$ student accepts the* (p_t^*, ρ_t^*) *contract and puts effort* $e_t^* = \frac{1+\delta_t p_t^* - \rho_t^*}{2}$.

Because type-l students' disutility associated with a default is smaller, they self select into the high-fee-low-stake contract that includes a significant upfront fee p_l^* and a small stake ρ_l^*. A large p_l^* and a small ρ_l^* incentivizes type-l students to work hard towards their job market success. On the contrary, type-h students, intimidated by the possibility of default, opt for the low-fee-high-stake contract (i.e., p_h^* is small but ρ_h^* is large). The type-h students are less motivated to put costly effort towards their job market success because they recognize a large share of their job market reward will be lost to the university.

Next, we examine the effect of income inequality in the market on the university's tuition fee. If the level of income inequality is sufficiently large (i.e., δ_l is sufficiently small relative to δ_h), the university's profit from an individual type-l student is higher than that from an individual type-h student. The reason is that in this case type-l students study harder, are more likely to succeed on the job market, and are willing to pay a large fee for university education. However, if the level of income inequality is small (i.e., δ_l and δ_h are not too dissimilar), the university earns more from an individual type-h student. The university distorts type-l students' full-information contract to separate them from type-h students who are actually not that different from type-l students. The implication is that an increase in the income inequality can lead the university to rely primarily on high-income (type-l) students (by raising the tuition fee) for its profits.[9] The university's decisions may exaggerate income inequality because it induces the high-income students to put a higher effort and succeed on the job market with a higher probability.

Finally, we find the expected social welfare is higher in the asymmetric information case (when the university cannot distinguish between the two types of students a priori) than in the full-information case. The lack of information makes extracting surplus away from type-l students difficult for the university. Although university's expected profit drops, the gain in expected consumer surplus (driven by a lower p_l and a lower cost of effort e_l^2) is large and causes social welfare to increase. This welfare result suggests protecting students' financial-status-related information can help policymakers increase both expected student surplus and social welfare.

[9] Consistent with our results, Cai and Heathcote (2022) empirically show that the rise of income inequality in the US can explain more than half of the observed increase in average tuition fee.

4.2 University Cares about Student Surplus

In the models presented above, we assumed the university's objective is to maximize its profits. However, all universities are not entirely profit driven. They also care about the welfare of their students. In this section, we present an extension of the hybrid-financing model and assume the university cares not only about its profit but also student surplus. Specifically, the university puts a weight β on the expected student surplus and $1-\beta$ on its monetary profit. All the other assumptions are the same as in section 3. We derive the equilibrium in the entire range of β. The following proposition presents the firm's equilibrium price p^* and stake ρ^*, and the student's equilibrium effort e^* for all values of β. (The threshold $\hat{\beta}$ is defined in the Web Appendix.)

Proposition 7 *(a) If* $\beta < \hat{\beta}$, $\rho^* = \frac{2\left[-2+(-2+\delta)\delta+\sqrt{(2+\delta)^2(1+\delta+\delta^2)}\right]}{9\delta(1+\delta)}$, $p^* = \frac{2+\delta+\delta\rho^*-2\sqrt{(1+\delta)(1+\delta\rho^*)}}{\delta^2}$, *and* $e^* = \frac{1}{2}\left(1-p^*\delta-\rho^*\right)$.

(b) If $\hat{\beta} \leq \beta < \frac{1}{2}$, $\rho^* = \frac{1-2\beta}{2-3\beta}$, $p^* = 0$, *and* $e^* = \frac{1-\beta}{2(2-3\beta)}$.

(c) If $\beta \geq \frac{1}{2}$, $\rho^* = 0$, $p^* = 0$, *and* $e^* = \frac{1}{2}$.

The equilibrium price p^* and stake ρ^* depends on the extent β with which university cares about student surplus. If β is small ($\beta < \hat{\beta}$), the university sets an upfront fee $p^* > 0$ and a stake $\rho^* > 0$. Both p^* and ρ^* (and therefore e^* as well) are independent of β. As expected and consistent with results presented in section 3, a profit driven university sets strictly positive levels of upfront fee and stake. However, if β is sufficiently large ($\hat{\beta} \leq \beta < \frac{1}{2}$), the university does not ask for any upfront fee ($p^* = 0$). In this case, the university sets a stake $\rho^* = \frac{1-2\beta}{2-3\beta}$, which is declining in β. Therefore, student's incentive to prepare for the job market increases with β. Equivalently, the equilibrium effort $e^* = \frac{1-\beta}{2(2-3\beta)}$ is increasing in β. As a result, both expected consumer surplus and social welfare are also increasing in β. For $\beta \geq \frac{1}{2}$, the university's optimal stake ρ^* is also zero. The university provides free education and its expected payoff consists entirely of the expected student surplus. In this case, because both p^* and ρ^* are zero, e^* and consumer surplus are independent of β. The university's expected payoff and social welfare are both increasing in β simply because β is the weight that the university puts on student surplus.

4.3 Endogenous Education Quality

The assumption that the likelihood of student's success on the job market depends on the characteristics of the major/university and the student's effort helped us understand basic forces that play a role in driving the university's price and stake and the student's effort decisions. The likelihood of student's success may depend not only on the student's effort but also on the university's education quality, which may be endogenous to the contract offered to the student. Here, we present a hybrid-financing-model extension, in which the likelihood of student's success also depends on the university's investment in education quality. We assume the student succeeds on the job market with probability ϕe, where $\phi = 1$, if the university invests $c > 0$ in education quality, and $\phi \in (0, 1)$, otherwise. Therefore, the education quality captures various aspects of education, training, placement services, alumni services, etc. that increase the likelihood of student's success on the job market. The game starts with the university making the quality-investment decision. After the university's quality-investment decision, the stages in the hybrid-financing model follow. Other assumptions are the same as in section 3.

The university's equilibrium quality decisions (in $\delta - c$ space) are graphically presented in Figure 6. As expected, the university invests in enhancing education quality when the cost c is small. The university also has higher incentive to invest in education quality when the student's disutility δ from default is small. As discussed in proposition 5, a small δ induces the university to set a high upfront fee p and a low stake ρ. The student puts high effort e when she accepts a high p and low ρ contract. Because the impact of university's investment c is the most significant when e is large, the university sets high education quality when δ is small. A sufficiently large δ results in the university not investing in education quality. When the education quality is low, the student's willingness to pay for education is low, which results in a lower upfront fee p and a higher stake ρ contract offered by the university. The student is less willing to put effort towards her job market success. A lower quality of education and a lower effort level contribute to a lower likelihood of student's success on the job market when δ is large. The implication is that both university's expected profit and social welfare decrease when the university switches from high to low education quality due

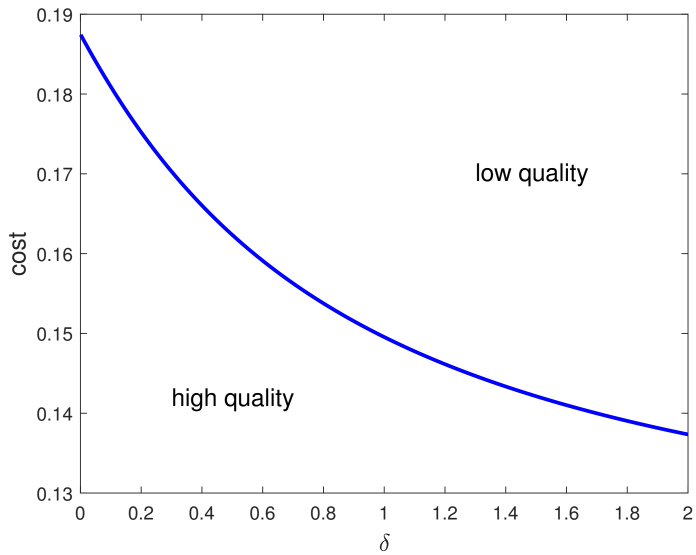

Figure 6: Endogenous university quality in $\delta - c$ space (using $a = 0$, $k = 1$)

to the increase in δ.

4.4 Competing Universities

In all the models and extensions presented above, we considered the university a monopoly. In this section, we examine an alternative market structure in which two competing universities ($j = 1, 2$) serve a unit mass of students distributed uniformly on a Hotelling line of unit length. Universities are located at the two ends of the line (university 1 on the left and university 2 on the right). Students incur a unit transportation cost of t. Universities simultaneously set their hybrid-financing contracts (p_j, ρ_j). A consumer's valuation v_j of university j's education, given p_j and ρ_j, is given by

$$v_j = e\,(1 - p_j - \rho_j) - (1 - e)\,(1 + \delta)\,p_j - e^2. \tag{9}$$

If the student at location x accepts university 1's contract, her expected utility u_1 is given by $v_1 - tx$. However, if the student accepts university 2's contract, her expected utility u_2 is $v_2 - t\,(1 - x)$. Each student compares u_1 and u_2 and decides which university to attend. We assume the market is fully covered. The expected profit π_j of university j can be written as $(p_j + e\rho_j)\,d_j$, where d_j is the demand for university j's education. An appropriate range of

δ ensures equilibrium solutions are feasible. Other assumptions are the same as in section 3.

Similar to the monopoly case, in the equilibrium, competing universities offer hybrid financing contracts ($p_j^* > 0$, $\rho_j^* > 0$) to the students. As expected and consistent with results presented section 3, p_j^* decreases in δ, ρ_j^* increases in δ, and e^* decreases in δ. (The comparative statics results are obtained numerically.) The implication is that the university's expected profit also decreases in δ. Because the intuition for these results is similar to those presented in section 3, they are not repeated here. Unlike in the monopoly case, competing universities are unable to extract all the surplus away from students. Therefore, students receive positive expected surplus. In addition, this expected consumer surplus increases with δ because a higher δ incentivizes universities to offer even lower prices to students. Driven by this increase in expected consumer surplus with δ and in contrast to the monopoly case, the social welfare also increases with δ.

5 Conclusion

Students typically finance their university education using student loans. A major challenge with student loans is that they may lead to default and cause additional financial difficulties for students who fail to achieve a favorable job market outcome after college. ISAs, which eliminate loans and therefore default risk for students, are being considered an innovative alternative to student loans. Recently, a number of universities and professional schools have started offering an ISA financing option to students. This paper examines a university's incentive to offer ISA financing to a student and the student's incentive to put effort toward job market success. The analysis links the roles of the student's financial difficulty level and features of the university education (e.g., the attractiveness of study major and the strength of university's brand name) in determining the university's contract and student's effort decisions.

Because ISA financing requires the student to pay a percentage of her income (unlike the income-independent repayment required for loan financing), it changes the student's incentive to put effort in an important way—the student faces moral hazard because she incurs the entire cost of effort while a percentage of her income goes to the university. As a result, the student is less motivated to put effort when financing her education using an ISA than a student loan. However, because the student is not only protected against any default but also receives a strictly positive payoff, she prefers ISAs to student loans. The same is not always true for the university. The university prefers student's loan financing when the level of financial difficulty related to the student's default is small. In this case, the student puts high effort and thus generates large surplus, which the university extracts using the upfront tuition fee. Of course, if the student's financial difficulty level is sufficiently large, the university also prefers student's ISA financing. The implication is that ISAs are socially desirable in a broader range of student's financial difficulty than the range in which they are profit maximizing for the university.

These findings have important policy implications. A policymaker may need to incentivize the university to offer ISA financing to students, for example, using a conditional subsidy or a mandate particularly when the student's disutility from default is too large. In addition, a policy can exploit the desirable features of both student-loan (incentivize high effort) and ISA-financing (protect against default) by offering subsidized loans (e.g., through a 529 plan), by imposing a ceiling on university tuition, or offering assistance to the student in the case of default.

We also examine the university's incentive to offer a hybrid loan-ISA financing contract and find that, consistent with anecdotal observations, the university prefers to offer the hybrid financing contract than the ISA or the upfront tuition fee contract. In doing so, the university exploits features of both the ISA contract and the upfront tuition fee contract to its advantage. However, a university that cares about the student surplus (in addition to its own profit) is more likely to offer ISA financing option to the student, particularly when the student's cost of default is high. If the market consists of both high- and low-financial-difficulty students, the university may offer a menu of contracts such that high-financial-difficulty (low-financial-difficulty) students finance their education primarily using ISA (student-loan) financing. Because universities are more likely to invest in education quality when offering upfront fee contracts, we expect higher quality universities to serve low-financial-difficulty students (i.e., affluent markets).

We expect a reasonable amount of data to be available from universities offering (or experimenting with) ISA financing in the near future. Future work can exploit these datasets to empirically examine the effect of university's decision to offer ISA financing on student outcomes and to test many of our predictions.

References

Bao, Weining, Jian Ni, and Shubhranshu Singh (2018): "Informal Lending in Emerging Markets," *Marketing Science*, 37(1), 123–137.

Becker, Gary S (1964): *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*. New York: National Bureau of Economic Research.

Cai, Zhifeng, and Jonathan Heathcote (2022): "College Tuition and Income Inequality," *American Economic Review*, 112(1), 81–121.

Chapman, Bruce (2006): "Income Contingent Loans for Higher Education: International Reforms," *Handbook of the Economics of Education*, 2, 1435–1503.

Christen, Markus, and Ruskin M. Morgan (2005): "Keeping Up With the Joneses: Analyzing the Effect of Income Inequality on Consumer Borrowing," *Quantitative Marketing and Economics*, 3, 145–173.

Friedman, Milton, and Simon Kuznets (1945): "Income from Independent Professional Practice," New York: National Bureau of Economic Research.

Grewal, Rajdeep, Robert Meyer, and Vikas Mittal (2022): "Education and Marketing: Decision Making, Spending, and Consumption," *Journal of Marketing Research*, 59(1), 1–10.

Grout, Paul A. (1983): "Education Finance and Imperfections in Information," *The Economic and Social Review*, 15(1), 25–33.

Iyer, Ganesh, and Shubhranshu Singh (2018): "Voluntary Product Safety Certification," *Management Science*, 64(2), 695–714.

Lochner, Lance, and Alexander Monge-Naranjo (2016): "Student Loans and Repayment: Theory, Evidence, and Policy," *Handbook of the Economics of Education*, 5, 397–478.

Lutz, Nancy A. (1989): "Warranties as Signals under Consumer Moral Hazard," *The RAND Journal of Economics*, 20(2), pp. 239–255.

Ma, Ching-To Albert, and Michael H. Riordan (2002): "Health Insurance, Moral Hazard, and Managed Care," *Journal of Economics & Management Strategy*, 11(1), 81–107.

Madonia, Greg, and Austin C. Smith (2019): "All-In or checked-out? Disincentives and selection in income share agreements," *Journal of Economic Behavior & Organization*, 161, 52–67.

Moen, Espen R. (1998): "Efficient Ways to Finance Human Capital Investments," *Economica*, 65(260), 491–505.

Nerlove, Marc (1975): "Some Problems in the Use of Income-contingent Loans for the Finance of Higher Education," *Journal of Political Economy*, 83(1), 157–183.

Padmanabhan, V., and Ram C. Rao (1993): "Warranty Policy and Extended Service Contracts: Theory and an Application to Automobiles," *Marketing Science*, 12(3), 230–247.

Shavell, Steven (1979): "On Moral Hazard and Insurance," *The Quarterly Journal of Economics*, 93(4), 541–562.

Tirole, Jean (1988): "The Theory of Industrial Organization," The MIT Press.

Yoon, Haewon, Yang Yang, and Carey K. Morewedge (2022): "Early Cost Realization and College Choice," *Journal of Marketing Research*, 59(1), 136–152.

Zhang, Juanjuan, and Peng Liu (2012): "Rational Herding in Microloan Markets," *Management science*, 58(5), 892–912.

Zhou, Chen, Manpreet Gill, and Qiang Liu (2022): "Empowering Education with Crowdfunding: The Role of Crowdfunded Resourcesand Crowd Screening," *Journal of Marketing Research*, 59(1), 97–117.

Appendix

Proof of Proposition 1

We use backward induction to solve the equilibrium. A student's expected payoff is

$$U_D = (a + ke_D)(1 - p_D) - [1 - (a + ke_D)](1 + \delta)p_D - e_D^2 \tag{A.1}$$

The student's pseudo-optimal effort decision e_D^o solves the following first order condition

$$\frac{\partial}{\partial e_D} U_D = 0$$

or equivalently,

$$e_D^o = \frac{k}{2}(1 + \delta p_D) \tag{A.2}$$

We substitute (A.2) into (A.1) and get the student's pseudo-optimal expected payoff

$$U_D^o = a(1 + \delta p_D) - (1 + \delta)p_D + \frac{k^2}{4}(1 + \delta p_D)^2 \tag{A.3}$$

When the the pseudo-optimal admission probability $a + ke_D^o$ is interior (i.e., between zero and one), we have

$$\frac{\partial}{\partial p_D} U_D^o = -1 - \frac{\delta(2(1 - a) - k^2(1 + \delta p_D))}{2} < 0 \tag{A.4}$$

Therefore, the university's optimal fee p_D^* makes the student be indifferent between matriculating and not matriculating.

$$a(1 + \delta p_D^*) - (1 + \delta)p_D^* + \frac{k^2}{4}(1 + \delta p_D^*)^2 = 0$$

or equivalently,

$$p_D^* = \frac{2 - (k^2 + 2a - 2)\delta - 2\sqrt{1 + \delta[1 - a^2 + (1 + \delta)((1 - a)^2 - k^2)]}}{k^2\delta^2} \tag{A.5}$$

We substitute (A.5) into (A.2) and get the student's equilibrium effort

$$e_D^* = \frac{1 + \delta - a\delta - \sqrt{1 + \delta[1 - a^2 + (1 + \delta)((1 - a)^2 - k^2)]}}{k\delta} \tag{A.6}$$

The validity of this equilibrium requires the following system of inequalities to hold

$$\begin{cases} e_D^* > 0 \\ 0 < a + ke_D^* < 1 \\ a, k, \delta > 0 \end{cases} \tag{A.7}$$

The first inequality ensures that the equilibrium effort is interior. The second inequality ensures that the equilibrium probability of student's success is between zero and one. Solving (A.7), we have $0 < k < \sqrt{\frac{2(1-a)-\delta(1-a)^2}{1+\delta}}$.

The university's equilibrium expected payoff is equal to the fee p_D^* collected from the student. In equilibrium, the student is indifferent between matriculating and not matriculating, so the expected student surplus is $CS_D^* = 0$. Then, the overall welfare is $TS_D^* = p_D^* + 0 = p_D^*$.

Proof of Proposition 2

We use backward induction to solve the equilibrium. A student's expected payoff is

$$U_S = (a + ke_S)(1 - \rho_S) - e_S^2 \tag{A.8}$$

The student's pseudo-optimal effort decision e_S^o solves the following first order condition

$$\frac{\partial}{\partial e_S} U_S = 0$$

or equivalently,

$$e_S^o = \frac{k}{2}(1 - \rho_S) \tag{A.9}$$

We substitute (A.9) into (A.8) and get the student's pseudo-optimal expected payoff

$$U_S^o = \frac{1 - \rho_S}{4}(4a + k(1 - \rho_S)) > 0 \tag{A.10}$$

The university's expected payoff is

$$\pi_S = (a + ke_S)\rho_S \tag{A.11}$$

We substitute (A.9) into (A.11) and get the university's pseudo-optimal expected payoff

$$\pi_S^o = (a + \frac{k^2}{2}(1 - \rho_S))\rho_S \tag{A.12}$$

The university's optimal stake ρ_S^* maximizes (A.12) subject to the student's expected payoff (A.10) being non-negative. Since the condition is always true, the university's optimal stake ρ_S^* solves the following first order condition

$$\frac{\partial}{\partial \rho_S}\pi_S^o = 0$$

or equivalently,

$$\rho_S^* = \frac{1}{2} + \frac{a}{k^2} \tag{A.13}$$

We substitute (A.13) into (A.9) and get the student's equilibrium effort

$$e_S^* = \frac{k}{4} - \frac{a}{2k} \tag{A.14}$$

The validity of this equilibrium requires the following system of inequalities to hold

$$\begin{cases} 0 < \rho_S^* < 1 \\ e_S^* > 0 \\ 0 < a + ke_S^* < 1 \\ a, k, \delta > 0 \end{cases} \tag{A.15}$$

The first inequality ensures that the equilibrium stake is interior. The second inequality ensures that the equilibrium effort is interior. The third inequality ensures that the equilibrium probability of student's success is between zero and one. Solving (A.15), we have $0 < a < 1$ and $\sqrt{2a} < k < \sqrt{2(2-a)}$. We combine these conditions with (A.7) and get the conditions for a and k, such that the equilibrium exists and is interior in both student-loan-financing case and income-share-agreement case.

$$\begin{cases} 0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta} \\ \sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}} \end{cases} \tag{A.16}$$

We substitute (A.13) and (A.14) into (A.12) and get the university's equilibrium expected payoff

$$\pi_S^* = \frac{(2a+k^2)^2}{8k^2} \tag{A.17}$$

We substitute (A.13) and (A.14) into (A.10) and get the consumer surplus

$$CS_S^* = \frac{1}{16}(4a + k^2 - \frac{12a^2}{k^2}) \tag{A.18}$$

Then, the overall welfare is

$$TS_S^* = \pi_S^* + CS_S^* = \frac{1}{16}(12a + 3k^2 - \frac{4a^2}{k^2}) \tag{A.19}$$

Proof of Proposition 3

By (A.6) and (A.14), we have

$$e_D^* - e_S^* = \frac{4 + (4 - 2a - k^2)\delta - 4\sqrt{1 + \delta(2 + (1 + a^2)\delta - (2a + k^2)(1 + \delta))}}{4k\delta} \tag{A.20}$$

If $0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, then we have

$$\frac{4 + (4 - 2a - k^2)\delta - 4\sqrt{1 + \delta(2 + (1 + a^2)\delta - (2a + k^2)(1 + \delta))}}{4k\delta} > 0 \tag{A.21}$$

The result immediately follows.

Proof of Proposition 4

(a) By Proposition 1, in the student-loan-financing case, the equilibrium consumer surplus is zero. By Proposition 2, in the income-share-agreement case, the equilibrium consumer surplus is $\frac{1}{16}(4a + k^2 - \frac{12a^2}{k^2}) > 0$. Thus, students unconditionally prefer ISA financing.

(b) By Proposition 1, in the student-loan-financing case, in the equilibrium university expected payoff is

$$\pi_D^* = \frac{2 - (k^2 + 2a - 2)\delta - 2A}{k^2\delta^2} \tag{A.22}$$

where $A = \sqrt{1 + \delta[1 - a^2 + (1 + \delta)((1 - a)^2 - k^2)]}$. If $0 < a < \frac{2(1+\delta) - \sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, then we have

$$\frac{\partial}{\partial\delta}\pi_D^* = \frac{4 + 2((1 - a)^2 - k^2)\delta^2 - 4A - (2 - 2a - k^2)(A - 3)\delta}{k^2\delta^3 A} < 0 \tag{A.23}$$

By Proposition 2, in the income-share-agreement case, the equilibrium university expected payoff is $\frac{(2a+k^2)^2}{8k^2}$, which is independent of δ. We solve for the cutoff $\hat{\delta}$ in the following equation

$$\pi_D^* = \frac{(2a + k^2)^2}{8k^2} \tag{A.24}$$

and we get

$$\hat{\delta} = \frac{8(2 - 2a - k^2) - 4\sqrt{16 - 8a(4 - 3a) - 8(2 - a)k^2 + 2k^4}}{(2a + k^2)^2} > 0 \tag{A.25}$$

Thus, the university gets a higher expected equilibrium payoff under debt financing if $\delta < \hat{\delta}$, and under ISA financing otherwise.

(c) By Proposition 1, in the student-loan-financing case, in the equilibrium social welfare is

$$TS_D^* = \pi_D^* + 0 = \frac{2 - (k^2 + 2a - 2)\delta - 2A}{k^2\delta^2} \tag{A.26}$$

where $A = \sqrt{1 + \delta[1 - a^2 + (1 + \delta)((1 - a)^2 - k^2)]}$. By (A.23), we have

$$\frac{\partial}{\partial \delta} TS_D^* = \frac{\partial}{\partial \delta} \pi_D^* < 0 \tag{A.27}$$

By Proposition 2, in the income-share-agreement case, the equilibrium social welfare is

$$TS_S^* = \pi_S^* + CS_S^* = \frac{1}{16}(12a + 3k^2 - \frac{4a^2}{k^2}) \tag{A.28}$$

We solve for the cutoff $\tilde{\delta}$ in the following equation

$$TS_D^* = TS_S^* \tag{A.29}$$

and we get

$$\tilde{\delta} = \frac{16(2 - 2a - k^2) - 8\sqrt{(4 - 2a - k^2)(4 - 6a - 3k^2)}}{3k^4 + 12ak^2 - 4a^2} \tag{A.30}$$

Thus, the social welfare is greater under debt financing if $\delta < \tilde{\delta}$, and under ISA financing otherwise.

Since $TS_D^* = \pi_D^*$ and $TS_S^* = \pi_S^* + CS_S^* > \pi_S^*$, by (A.23), we have $\tilde{\delta} < \hat{\delta}$.

Proof of Proposition 5

We first solve the equilibrium decisions and outcomes for the hybrid-financing model and then compare them with those in the debt-only / ISA-only model. We use backward induction to solve the equilibrium. A student's expected payoff is

$$U_H = (a + ke_H)(1 - p_H - \rho_H) - [1 - (a + ke_H)](1 + \delta)p_H - e_H^2 \tag{A.31}$$

The student's pseudo-optimal effort decision e_H^o solves the following first order condition

$$\frac{\partial}{\partial e_H} U_H = 0$$

or equivalently,

$$e_H^o = \frac{k}{2}(1 + \delta p_H - \rho_H) \tag{A.32}$$

We substitute (A.32) into (A.31) and get the student's pseudo-optimal expected payoff

$$U_H^o = a(1 + \delta p_H - \rho_H) - (1 + \delta)p_H + \frac{k^2}{4}(1 + \delta p_H - \rho_H)^2 \tag{A.33}$$

When the the pseudo-optimal admission probability $a + ke_H^o$ is interior (i.e., between zero and one), we have

$$\frac{\partial}{\partial p_H} U_H^o = -1 - \frac{\delta(2(1-a) - k^2(1 + \delta p_H - \rho_H))}{2} < 0 \tag{A.34}$$

Therefore, the university's pseudo-optimal fee p_H^o makes the student be indifferent between matriculating and not matriculating.

$$a(1 + \delta p_H^o - \rho_H) - (1 + \delta)p_H^o + \frac{k^2}{4}(1 + \delta p_H^o - \rho_H)^2 = 0$$

or equivalently,

$$p_H^o = \frac{2 - 2A + \delta(2(1-a) - k^2(1 - \rho_H))}{k^2\delta^2} \tag{A.35}$$

where $A = \sqrt{1 + \delta(2 + (1 + a^2)\delta - 2a(1 + \delta) - k^2(1 + \delta)(1 - \rho_H))}$.

Note that the university's expected payoff is

$$\pi_H = p_H + (a + ke_H)\rho_H \tag{A.36}$$

We substitute (A.32) and (A.35) into (A.36) and get the university's pseudo-optimal expected payoff

$$\pi_H^o = \frac{2 - 2B + \delta(2 - 2a + k^2((2 + \delta - B)\rho_H - 1))}{k^2\delta^2} \tag{A.37}$$

where $B = \sqrt{1 + \delta(2 + (1 + a^2)\delta - 2a(1 + \delta) - k^2(1 + \delta)(1 - \rho_H))}$.

The university's optimal stake ρ_H^* maximizes (A.37), so we have

$$\frac{\partial}{\partial \rho_H}\pi_H^o = 0$$

or equivalently,

$$\rho_H^* = \frac{2}{3} + \frac{2((2+\delta)C - 2 - \delta(5 - 6a + (2 - 3a(2-a))\delta))}{9k^2\delta(1+\delta)} \tag{A.38}$$

where $C = \sqrt{1 + \delta(3a^2\delta - 6a(1+\delta) + (4 - 3k^2)(1+\delta))}$.

We substitute (A.38) into (A.35) and get the university's optimal fee

$$p_H^* = \frac{2 - 2A^* + \delta(2(1-a) - k^2(1-\rho_H^*))}{k^2\delta^2} \tag{A.39}$$

where $A^* = \sqrt{1 + \delta(2 + (1+a^2)\delta - 2a(1+\delta) - k^2(1+\delta)(1-\rho_H^*))}$.

We substitute (A.38) and (A.39) into (A.32) and get the equilibrium student effort

$$e_H^* = \frac{k}{2}(1 + \delta p_H^* - \rho_H^*) \tag{A.40}$$

We substitute (A.38) and (A.39) into (A.32) and get the equilibrium student effort

$$\pi_H^* = p_H^* + (a + ke_H^*)\rho_H^* \tag{A.41}$$

Now we compare the equilibrium decisions and outcomes between the hybrid-financing model and the debt-only / ISA-only model.

(a) If $0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, by (A.5) and (A.39), we have

$$p_D^* > p_H^* \tag{A.42}$$

If $0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, by (A.13) and (A.38), we have

$$\rho_S^* > \rho_H^* \tag{A.43}$$

(b) If $0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, by (A.6) and (A.40), we have

$$e_D^* > e_H^* \tag{A.44}$$

If $0 < a < \frac{2(1+\delta)-\sqrt{4+6\delta+3\delta^2}}{\delta}$ and $\sqrt{2a} < k < \sqrt{\frac{2-2a+\delta-2a\delta+a^2\delta}{1+\delta}}$, by (A.14) and (A.40), we have

$$e_S^* < e_H^* \tag{A.45}$$

(c) By (A.5) and (A.35), we have

$$p_D^* = p_H^o(\rho_H)|_{\rho_H=0} \tag{A.46}$$

Therefore, the university's equilibrium expected payoff in the debt-only model π_D^* is equal to the university's pseudo-optimal expected payoff in the hybrid-financing model with $\rho_H = 0$.

$$\pi_D^* = \pi_H^o(\rho_H)|_{\rho_H=0} \tag{A.47}$$

By the optimally of ρ_H^*, we have

$$\pi_D^* = \pi_H^o(\rho_H)|_{\rho_H=0} < \pi_H^o(\rho_H)|_{\rho_H=\rho_H^*} = \pi_H^* \tag{A.48}$$

By (A.17) and (A.36), we have

$$\pi_S^* = \pi_H(p_H, \rho_H)|_{(p_H,\rho_H)=(0,\rho_S^*)} \tag{A.49}$$

By (A.18) and (A.33), there exists a small positive $\hat{p}_H$ such that

$$\begin{aligned} U_H^o(0,\rho_S^*) - U_H^o(\hat{p}_H,\rho_S^*) &= \hat{p}_H(1 + \tfrac{\delta}{4}(4 - 4a - k^2(2 + \delta\hat{p}_H - 2\rho_S^*)) \\ &< \tfrac{1}{16}(4a + k^2 - \tfrac{12a^2}{k^2}) \\ &= CS_S^* \end{aligned} \tag{A.50}$$

Therefore, the interior hybrid contract $(\hat{p}_H, \rho_S^*)$ is feasible. Moreover, we have

$$\pi_H(\hat{p}_H,\rho_S^*) - \pi_H(0,\rho_S^*) = \hat{p}_H + \frac{k^2\delta\hat{p}_H\rho_S^*}{2} > 0 \tag{A.51}$$

so we get

$$\pi_S^* = \pi_H(0, \rho_S^*) < \pi_H(\hat{p}_H, \rho_S^*) \tag{A.52}$$

By the optimally of (p_H^*, ρ_H^*), we have

$$\pi_H(\hat{p}_H, \rho_S^*) < \pi_H(p_H^*, \rho_H^*) = \pi_H^* \tag{A.53}$$

Finally, by (A.35), we know the equilibrium expected student surplus is zero in the hybrid-financing case.