Sponsored Content Advertising in a Two-sided Market

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Abstract

A sponsored content advertisement is a new ad format in which the brand’s content takes the same form and qualities of a publisher’s original content. While many advertisers have largely embraced this new advertising format, consumers seem to react negatively towards it if they correctly identify it as a promotional message and find it poorly integrated within the editorial context. In this paper, we present an analytical model that studies the strategic role of sponsored content advertising in a two-sided media market. We first demonstrate that a monopolist media platform’s profits decrease in consumers’ likelihood of identifying the sponsored content ads as mere promotional messages. Then we show that the opposite pattern occurs in the competitive environment. We identify conditions under which competing platforms would choose sponsored content advertising over traditional advertising. Despite consumers’ sentiment towards sponsored content ads, they can be better off together with the advertisers when platforms choose this ad format. However, both competing platforms offering sponsored content ads may result in a Prisoner’s Dilemma equilibrium outcome. Finally, we show that two symmetric media platforms can choose different advertising strategies, leading to an asymmetric equilibrium outcome in which one platform adopts sponsored ads and the other one adopts traditional ads.

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1 Introduction

According to American press institute, sponsored content advertising (often also referred to as native advertising) is defined as a type of advertising where a brand’s content takes “the same form and qualities of a publisher’s original content.” The definition here refers to an advertising platform like New York Times or BuzzFeed as the ‘publisher.’ The definition also suggests that this content provides useful information to the readers so that their perception about the sponsored brand tends to be more favorable. While some forms of sponsored content advertising (e.g., advertorial) have been there for more than a hundred years, recent innovations in digital media have expanded the scopes of sponsored content advertising. With more primitive forms of sponsored content advertising, consumers were always urged to take concrete actions. The contemporary version of sponsored content advertising, however, never asks a consumer to buy a product. Instead, it portrays a favorable picture of the corresponding brand. As a result, the content looks more convincing and authentic, as if the publishing platform itself has developed the material, instead of the brand.

From an advertiser’s perspective, the development of sponsored content advertising sounds quite promising, but at the same time it can have negative impact. In particular, although sometimes readers may accept sponsored content as useful and relevant as any other editorial content, a recognition of sponsored content advertising as a mere promotional message makes the readers more upset (Wojdynski and Evans 2016). If for whatever reason readers identify an editorial-like product review as a brand’s promotional message, they may feel that the brand has tried to mislead them. Of course, one simple way brands can alleviate this problem is to clearly label the content as promotional material. In reality though, many brands do not take such actions fearing that a clear labeling would transform the sponsored content advertising back into traditional advertising (equivalent to banner advertising) and thus the whole purpose of developing sponsored content advertising will not be served. A recent survey undertaken by a content marketing platform shows that across different platforms about sixty percent of the readers on average fail to identify a

\[1\text{https://www.americanpressinstitute.org/publications/reports/white-papers/the-definition-of-sponsored-content/}\]
sponsored content article as a promotional activity of a brand (they believe that the article was written by the staff reporters of the publishing platform). This survey also shows that about forty eight percent of these readers felt deceived once they were told that the article was an example of sponsored content advertising.²

A publishing platform too may lose credibility once its readers realize that the editorial-like article is neither relevant nor well integrated with actual editorial content. As a result, consumers may decide to stop visiting the platform or visit it less frequently. In a two-sided market, this decision may affect a platform in two ways. The first one comes as a direct effect for the platform which charges a price to their readers for accessing news content - the lower the number of visitors is, the lower the gross revenue will be (from readers’ side) of the platform. Additionally, if only a handful of consumers visit the platform, even the advertisers will not be happy. In that case, a platform may lose a substantial number of advertisers too, which in effect will further reduce this platform’s revenue. This potential threat, however, does not necessarily dampen the spirit of either advertisers or publishing platforms in developing sponsored content advertisements. According to a recent market intelligence study performed by Business Insider, spending on sponsored content advertising is expected to grow to 21 billion US dollar, rising from 4.7 billion in 2013.³ In another latest survey, the Association of National Advertisers reports that about sixty three percent of American marketers plan to substantially increase their budget for sponsored content advertising.⁴ While on average the share of budget for sponsored content advertising is yet to surpass the share of banner or other forms of traditional advertising, sponsored content advertising is increasingly gaining popularity.

In a recent article, Forbes has identified competition, transparency, and content creation as three most crucial factors in the context of sponsored content advertising.⁵ The article also mentions that the names of the sponsored content ad products (such as BrandSpeak, BrandConnect, BrandPost, etc.) are “maddeningly similar” and often leave the audience confused about the real intentions of the platforms. This article suggests that as competi-

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²https://contently.com/strategist/2015/09/08/article-or-ad-when-it-comes-to-native-no-one-knows/
⁴https://www.ana.net/content/show/id/33530
tion for ad dollars has become rife, the platforms are still learning about creating effective advertising strategies.

A key question remains – why do marketers and publishing platforms embrace sponsored content advertising even when they face a risk of losing a readership base? When Yahoo CEO Marissa Mayer gave a presentation at Cannes Lions in 2014, the biggest advertising festival in the world, she explained some of the reasons. According to Ms. Mayer, the viewers of sponsored content ads are 3.6 times more likely (compared to the viewers of traditional ads) to perform a brand related search and 6 times more likely to perform a generally related search. Since sponsored content advertising provides a detailed story and thorough information, viewers of the ad take much more interest in pursuing the content. Traditional advertising like banner ads cannot influence their readers in the same way. Industry experts also argue that with the rise of ad blocking technology, sponsored content advertising now seems like the only reliable form of online advertising which can still get the attention of the viewers. As some of the biggest media platforms in the world, including New York Times, Wall Street Journal, The Atlantic, etc., have been accepting sponsored content ads (with exclusive contracts to the reputed brands) more than ever, the future prospects of sponsored content advertising look brighter. Yet, almost every consumer survey shows that majority of the readers do not like the concept of sponsored content advertising and feel disappointed with those platforms which allow brands to publish irrelevant and ill-integrated content in an editorial set-up. In 2015, Federal Trade Commission of America also echoed viewers’ concerns by imposing a set of regulations on sponsored content advertising techniques. In the absence of proper academic and industrial research on sponsored content advertising, it is thus not clear whether the supply side enthusiasm has a concrete basis. Even the revenue and profit numbers do not always justify the surge in spending on sponsored content ads. A recent study in 2016 shows that for most of the marketers sponsored content only gives a sell-through rate of 5 percent or less.

“At their best, native ads are a seamless part of the reading experience. Depending on who you are, that’s either great or horrible.” (Keller, BusinessWeek, 2013) From advertis-

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6http://adage.com/article/digitalnext/effective-native-ads-a-solution-ad-blockers/302476/
7https://www.ftc.gov/tips-advice/business-center/guidance/native-advertising-guide-businesses
ers’ perspectives, sponsored content advertising seems to be more engaging and enjoyable. Unlike the advertisers though, consumers are less enthusiastic since the experience with sponsored content advertising can vary to a great extent. If the advertisements provide useful information to the consumers or are well integrated with other editorial content, a consumer may accept these sponsored content advertising. In contrast, when sponsored content is not well integrated with the actual journalistic content, consumers experience a sense of annoyance. Given the different views between consumers and advertisers/platforms surrounding sponsored content advertising, we have three objectives for this paper – (a) to understand under what conditions and market structures a media platform would adopt sponsored content advertising instead of traditional advertising, (b) to examine whether sponsored content advertising is necessarily a more profitable strategy (for a media platform) than traditional advertising, and (c) to explore the situations when sponsored content advertising can offer higher surplus to both consumers and advertisers. To answer question (a) we derive the complete equilibrium conditions for monopoly as well as duopoly. For question (b), we provide conditions under which sponsored content as an equilibrium advertising strategy can lead to lower payoffs for the platform (i.e., we show the existence of the prisoner’s dilemma outcome). Lastly, to address question (c) we analyze whether, in contrast to the conventional wisdom, consumers can be better off in the presence of sponsored content advertising.

We organize the remainder of the paper as follows: in the next section we present the literature review. In section 3, we explain our model. Following that, we discuss equilibrium results. In section 5 we discuss the managerial implications and draw our concluding remarks. All proofs can be found in the appendix.

2 Literature Review

Because sponsored content advertising is a recent phenomenon, the academic research on this topic is currently at a nascent stage. Early papers on sponsored content advertising such as Becker-Olsen (2003) have experimentally identified the benefits of sponsored content ads for the advertiser as well as the platform. Becker-Olsen (2003) also explains
how the informational context in sponsored content ads forces the readers (consumers) to engage in higher levels of information processing, and in turn affects a consumer’s attitude towards advertising in general. Carlson (2015) provides a balanced critique of sponsored content advertising and explains how this new form of advertising may be eroding the boundaries between editorial and advertising, and changing the normative understandings of journalistic autonomy.

Recent experimental studies such as Wojdynski and Evans (2016) have also suggested that while a higher transparency level certainly helps the consumers to identify the message as an advertisement, most of the time this ad recognition leads to negative evaluations. Lee et al. (2016) alternatively suggests that if consumers have strong information-seeking motivation (as opposed to socializing motivation), then they would positively evaluate sponsored content advertising, and would also show a higher willingness to share the content. While a stream of research such as Campbell and Marks (2015) as well as Conill (2016) has qualitatively and thoroughly discussed the pros and cons of this new ad format in digital advertising, Sahni and Nair (2016) provides the first empirical paper to show that sponsored content advertising specially pays off when advertisers and platforms disclose the sponsored nature of the content to the consumers. Their main findings suggest that the advertisers benefit from native (sponsored content) advertising and the loss of welfare for the consumers are likely to be minimal. We too find that the consumers and advertisers can be better off with sponsored content advertising as long as consumers’ disutility towards sponsored content is not excessively high. Unlike Sahni and Nair (2016), we model competition between media platforms and show that when the competing platforms are highly (or rarely) differentiated, offering sponsored content would not be the optimal strategy. As a result, the platforms in these cases will continue with traditional advertising even when advertisers may benefit more from sponsored content advertising. As far as we know, our paper is the first analytical paper to study the strategic impact of sponsored content advertising in the two-sided market under different market structures.

We follow the basic tenets of the two-sided market model with network effects as laid out in the seminal papers such as Rochet and Tirole (2003) and Armstrong (2006). Rochet and Tirole (2003) provides a general framework of a two-sided market by illustrating that
any market with network externalities can be considered as a two-sided market as long as
a platform can effectively cross-subsidize between different user groups. Armstrong (2006)
on the other hand offers the primary structure of a two-sided market where at least one
group of economic agents opt for single-homing (i.e., they only choose one platform). We
follow Armstrong (2006) by assuming every reader (as well as every advertiser) chooses only
one platform. Unlike Armstrong’s paper, our paper formulates the cross side externality
as a potentially probabilistic negative externality. Following Katz and Shapiro (1985) that
analyze the role of consumer’s expectations and network externalities, we assume that
rational consumers and rational advertisers’ expectations are both correct in equilibrium.
More recent papers like Ellison and Ellison (2005) as well as Tucker and Zhang (2010)
suggest that almost all online markets show strong evidence of network externalities. Tucker
and Zhang (2010) specifically finds that many online retail websites get heavier traffic
of sellers when there exists a broad base of buyers and vice versa. In comparison, we
have incorporated the asymmetric externality across the two sides of the market in our
model (advertisers prefer consumers while consumers dislike ads), which is more consistent
with the context of media markets. Furthermore, Chen and Xie (2007) finds that due to
cross market network effects, an important factor like customer loyalty in one market may
actually reduce the profit in a secondary market when the two markets are interdependent.
In contrast, we show that even when consumers’ sensitivity towards sponsored content
advertising is higher than the sensitivity towards traditional advertising, due to cross-side
externalities the consumers may in fact be better off under sponsored content advertising.

Our paper also contributes to the growing literature on media markets. Gal-or and
Dukes (2003) and Dukes (2004) finds that when two media producers are more differen-
tiated, their volume of informative advertising increases which in turn intensifies price
competition between the producers. Similarly, our paper shows that as the media plat-
forms become more differentiated, they start to allow advertisers to post sponsored content
which are certainly more informative than the traditional advertisements. Dukes (2004)
further shows that when the extent of differentiation in the media market is too high then
the levels of advertising exceed the socially optimal level of advertising. In our paper, the
extreme differentiation between two media producers does not offer a suitable landscape
for offering sponsored content advertising and thus the media platforms in this case only allow traditional advertising. Gal-or and Dukes (2003) additionally finds that an increase in volume of informative advertising intensifies the price competition among the producers. Because of the two-sided nature of the marketplace in our model, we however find that there is no monotonic relationship between change in advertising strategy and degree of price competition. In our case, the relative sizes of negative externalities under traditional and sponsored content advertising play a crucial role in shaping a platform’s advertising strategy and the equilibrium outcome.

Godes, Ofek and Sarvary (2009) suggests that media firms competing in a duopoly would set higher prices for media content than a monopolist firm. Similarly, our paper shows that competition between two media producers does not necessarily make the consumers better off. Zhu and Dukes (2015) also finds that in the context of factual content, consumers may not benefit from the competition among media producers. Recently, Amaldoss, Du and Shin (2016) finds that in a two-sided media market competing platforms adopt asymmetric pricing strategies when all the platforms target single-homing consumers and advertisers. On the other hand, when consumers and advertisers are multi-homing, relative sizes of cross-side network effects would determine whether the platforms should opt for asymmetric or symmetric (customized or uniform) pricing strategies. Their assumption of different groups of consumers having different sensitivity towards advertising drives this interesting result of asymmetric pricing strategies in the single-homing case. In contrast, in our model all the consumers have the same sensitivity towards advertising, we still see that the competing platforms may adopt asymmetric pricing strategies. This result takes place when the two symmetric media platforms choose asymmetric advertising strategies in equilibrium.
3 Model

3.1 Platforms

Two competing platforms, 1 and 2, are horizontally differentiated and located on the two extremes of each of the two Hotelling lines (faced by the readers (henceforth consumers) and the advertisers, respectively). Each platform offers media content to the consumers and allow advertisers to post either traditional ads, denoted by $T$ (traditional advertisements) or sponsored content ads, denoted by $S$ (sponsored content/native advertisements). A traditional ad can be perceived as a banner ad which does not leave any scope of confusion as the consumers instantaneously recognize it as a promotional message. However, a sponsored content ad may look like an editorial or some other form of media content provided by the platform staff and thus can potentially provide informational or entertainment value to a consumer.

Platform $i$ ($i = 1, 2$) charges prices, $p_{iC}^{υω}$ and $p_{iA}^{υω}$, to the consumers and the advertisers, respectively. Subscripts $C$ and $A$ denote for consumers and advertisers, whereas superscripts $υ$ and $ω$ represent the advertising strategies of the platforms 1 and 2, respectively. A platform has three decision variables – price for the consumers, price for the advertisers, and the ad type (i.e., whether to adopt traditional ads or sponsored ads). To focus on platforms’ advertising choice and the strategic interaction between the platforms and both sides of the market, we assume that each platform chooses only one advertising format. In addition, the marginal cost of offering the media content or creating advertising messages is 0. Thus, the profits of a platform $i$ are given by

$$\Pi_{i}^{υω} = p_{iC}^{υω} x_{iC}^{υω} + p_{iA}^{υω} x_{iA}^{υω}, \ i \in \{1, 2\}. \quad (1)$$

$x_{iC}^{υω}$ and $x_{iA}^{υω}$ are respectively consumers’ demand and advertisers’ demand for platform $i$. We further assume that both markets for the consumers and the advertisers are fully covered. The table below explains all available strategies and profits for the two platforms.

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9 We analyze the situation in which both sides of the market are incompletely covered in Section 3.4. In this case, a platform has monopoly power on both sides of the market.
### Table 1: Platforms’ Strategies and Profits

<table>
<thead>
<tr>
<th>Platform 1 / Platform 2</th>
<th>Traditional Ad</th>
<th>Sponsored Content Ad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Ad</td>
<td>Case TT ($\Pi_1^{TT}, \Pi_2^{TT}$)</td>
<td>Case TS ($\Pi_1^{TS}, \Pi_2^{TS}$)</td>
</tr>
<tr>
<td>Sponsored Content Ad</td>
<td>Case ST ($\Pi_1^{ST}, \Pi_2^{ST}$)</td>
<td>Case SS ($\Pi_1^{SS}, \Pi_2^{SS}$)</td>
</tr>
</tbody>
</table>

#### 3.2 Consumers

Consumers are uniformly distributed along the Hotelling line. We assume that the consumers obtain an intrinsic utility $u_0$ from consuming either platform’s media content. However, depending on her location on the Hotelling line, a consumer may incur a mismatch cost $t_C$ per unit of distance traveled (for example, the presentation style of the media content differs from that of her preferred style). Put differently, $t_C$ captures the strength of consumers’ brand preferences towards the two platforms. Consistent with prior research on advertising in the media market (e.g., Anderson and Gabszewicz 2006), we assume that the traditional ads are perceived as nuisance and thus create negative externalities for the consumers. In particular, $\gamma$ captures the extent of negative externalities to consumers when they see the traditional ads. When $\gamma$ increases, a consumer’s dislike for an ad also increases. A consumer’s expected total amount of disutility is given by $\gamma x_{iT}^T \text{e}^T$ (superscript $e$ denotes the expected value, $T$ denotes traditional ads), where $x_{iT}^T$ is consumers’ expected total number of the advertisements on platform $i$. Therefore, when the platform $i$ adopts the traditional ads, a consumer with the location of $x_{iC}^T$ obtains the following utility from this platform:

$$U_{iC}^T = u_0 - t_C x_{iC}^T - p_{iC}^T - \gamma x_{iT}^T. \quad (2)$$

In contrast, when a platform adopts sponsored content ads, there are two possible outcomes. The first possibility is that the sponsored content ads are nicely integrated within the context and present useful or entertaining information to consumers. In this situation, consumers do not incur any disutility. The second possibility is that these sponsored content ads are not well integrated with the editorial content, and thus consumers identify them as mere promotional messages and face negative externalities. We assume that the second
outcome occurs with a probability of \( \beta \). To some extent, \( \beta \) can be seen as 1 in the case of traditional advertising, i.e., consumers never fail to identify traditional ads as promotional messages and they always incur a disutility upon seeing these ads. To ease exposition, we denote \( \beta \) as the likelihood for consumers to identify sponsored ads as irrelevant and poorly-integrated promotional messages in the rest of this paper.

To further clarify, \( (1 - \beta) \) refers to the first possibility discussed in the previous paragraph. There are two distinct situations within this outcome: i) consumers are completely unaware of the fact that the underlying message is a sponsored content ad, or ii) consumers become aware of the sponsored content ad but find it well-integrated within the editorial context. In this paper, we do not differentiate between these two situations. We only highlight that the total probability for the previous two situations to occur is \((1 - \beta)\). Again to ease exposition, we just denote \((1 - \beta)\) as the probability that consumers do not identify sponsored ads.

Importantly, because sponsored content ads come with minimal transparency with the intention of being perceived as editorials, consumers react more negatively to them as compared to seeing a traditional ad if they correctly identify the nature of the ads and feel the incongruence within the context (Wojdynski and Evans 2016). Specifically, we assume that consumers’ disutility for this type of ad, conditional on correct identification, is given by \( \gamma_S \), which is greater than the negative externality parameter \( \gamma \) in the case of traditional ads (\( \gamma_S > \gamma \)). On the other hand, when a sponsored content ad integrates well with the organic content and remains “unidentified” with the probability of \((1 - \beta)\), consumers would treat the ad as a regular piece of media content and thus would not incur any disutility. Therefore, consumers’ expected disutility is given by \( \beta \gamma_S x_{iA}^{es} \), where \( x_{iA}^{es} \) is consumers’ expected total number of the sponsored ads on platform \( i \). To summarize, when the platform \( i \) adopts the sponsored content ads, a consumer with the location of \( x_{iC}^S \) obtains the following expected utility (superscript \( e \) denotes the expected value):

\[
U_{iC}^S = u_0 - t_C x_{iC}^S - p_{iC}^S - \beta \gamma_S x_{iA}^{es}.
\] (3)

The parameter \( \beta \) can be interpreted in different ways. In contrast to being the proba-
bility of sponsored ads’ being integrated poorly with the editorial content and thus being identified by consumers, we can also interpret \( \beta \) as the proportion of sponsored ads that consumers can correctly recognize as promotional messages and thus they incur a greater disutility of \( \gamma_s \). \( \beta \) can also be considered as a proxy variable for the quality of sponsored content advertising. As the quality of sponsored content improves, identifying the content as a special form of promotional activity becomes more difficult (\( \beta \) decreases).

### 3.3 Advertisers

Similar to consumers, advertisers are also uniformly distributed on a Hotelling line. Advertisers’ transportation cost of per unit of distance is given by \( t_A \). In other words, \( t_A \) captures the strength of advertisers’ brand preferences towards the two platforms. In making such assumptions we follow the existing literature (e.g., Kaiser and Wright 2006, Rasch 2007, Esteban and Harnandez 2012) on two-sided markets which explains that the media platforms are horizontally differentiated from advertisers’ perspective. One can interpret advertisers’ horizontal preference towards platforms as their preference to consumers with certain demographics (closer to the positioning of a certain platform). Advertisers obtain an intrinsic utility of \( A_0 \) from simply appearing on a platform, and the presence of consumers creates a positive externality for them. We assume that an advertiser gets a marginal utility of \( A \) for each consumer’s exposure to the traditional ad. In other words, an advertiser’s gross utility from displaying a traditional ad on a platform with an expected number of \( x^e_{iC} \) consumers is given by \( A_0 + Ax^e_{iC} \). Therefore, when a platform \( i \) adopts the traditional ad, an advertiser with the location of \( x^T_{iA} \) obtains the following expected utility (the price this advertiser pays, \( p^T_{iA} \), can be seen as the total payment for \( x^e_{iC} \) number of impressions.)

\[
U^T_{iA} = A_0 - t_A x^T_{iA} - p^T_{iA} + Ax^e_{iC}.
\] (4)

Given the characteristics of the sponsored ads, advertisers’ utility depends on whether or not consumers identify them as a promotional message. We assume that an advertiser gets a marginal utility of \( A \) for each consumer’s exposure to an identified sponsored content ad. This is because consumers will not gain additional positive impressions for the
advertiser upon identifying the nature of the promotional message. In contrast, in case of an unidentified sponsored content ad, the advertiser obtains a higher marginal utility $\alpha$ per consumer exposure ($\alpha > A$). Recall that $\beta$ is the likelihood for consumers to correctly identify the sponsored content ads. As a result, an advertiser’s expected gross utility from displaying a sponsored content ad on the platform with an expected number of $x_{iC}^S$ consumers is given by $A_0 + \beta A x_{iC}^S + (1 - \beta) \alpha x_{iC}^S$. To summarize, when the platform $i$ adopts sponsored content ads, an advertiser with the location $x_{iA}$ obtains the following expected utility

$$U_{iA}^S = A_0 - t_A x_{iA}^S - p_{iA}^S + \beta A x_{iC}^S + (1 - \beta) \alpha x_{iC}^S.$$  

Note that because a sponsored ad is more valuable to the advertiser if it is not identified by the consumers, $\alpha > A$, advertisers’ expected utility decreases when consumers are more likely to identify sponsored content ads (as $\beta$ increases).

### 3.4 Monopoly Platform

Although the main objective of this paper is to analyze the platform’s ad choice under competition, as a starting point we explore a monopolistic setting where there is only one media platform. The main difference from duopoly is that the outside option for both consumers and advertisers in the monopoly context is no consumption, yielding a utility of zero (in a duopoly, both groups’ outside option is to consume or purchase from the other platform and thus would enjoy weakly positive utility). The expected utilities of consumers and advertisers with the monopoly platform are thus given by

$$U_C^T = u_0 - t_C x_C^T - p_C^T - \gamma x_A^T,$$  

consumers in the presence of $T$,  

$$U_C^S = u_0 - t_C x_C^S - p_C^S - \beta \gamma x_A^S,$$  

consumers in the presence of $S$,  

$$U_A^T = A_0 - t_A x_A^T - p_A^T + A x_C^T,$$  

advertisers in the presence of $T$,  

$$U_A^S = A_0 - t_A x_A^S - p_A^S + \beta A x_C^S + (1 - \beta) \alpha x_C^S,$$  

advertisers in the presence of $S$.  

13
The monopoly platform’s profits under traditional and sponsored content ads are given by:

\[ \Pi^v = p_C^v x_C^v + p_A^v x_A^v, \quad v \in \{T, S\}. \] (10)

### 3.5 Two-sided Market and Assumptions

The two-sidedness of the market has been defined in the early literature on two-sided platforms (i.e., [Rochet and Tirole 2003](#)), [Armstrong 2006](#). The most important aspect of this two-sidedness is the existence of the inter-group externalities. In our context, \( \gamma \) and \( \gamma_S \) (\( \gamma_S > \gamma \)) capture the negative externalities consumers experience with traditional ads and identified sponsored ads, respectively. On the other hand, \( A \) and \( \alpha \) (\( A < \alpha \)) represent the positive externalities advertisers enjoy from the presence of consumers, with traditional ads and unidentified sponsored content ads, respectively.

To focus on the impact of sponsored content ads on the competition between platforms, we assume that either a consumer or an advertiser can choose only one platform. In other words, we analyze the “single-homing” situation. As [Armstrong (2006)](#) points out, both sides doing multi-homing is less prevalent and we leave that to future research.

To focus on the more interesting analysis, we make the following assumptions:

\[ A \geq \gamma_S \geq \gamma, \] (11)

\[ (1 - \beta)(\alpha - A) + \gamma - \beta \gamma_S > 0, \] (12)

\[ t_C > \max\{t_{C1}, t_{C2}\} \] (13)

The first assumption means that advertisers’ marginal utility of accessing an additional reader should be greater than readers’ marginal disutility of being exposed to an additional ad (otherwise no ads will be observed in equilibrium). It also means that consumers’ disutility of a sponsored ad, conditional on correctly identifying it, is greater than their disutility of a traditional ad. The second assumption states that the marginal utility of

\[ t_{C1} = \frac{2\beta^2 \gamma_S^2 (\alpha + \beta - \alpha - A - \beta - t_A) + \beta \gamma_S (\gamma ((\alpha - A)(\beta - 1) + 2t_A) + (\alpha + A + \beta + A - \alpha)(\alpha(2\beta - 1) - 2(A - 1) - t_A)) - (\gamma (\alpha + A + \beta + A - \alpha - A)(\beta - 1) - t_A)}{2A^2(\beta - 1) - 2A + 2\beta A - \gamma + 6t_A + \alpha(2\beta - 1) - 2(2A - 1) - \beta \gamma_S - \gamma)} \]

\[ t_{C2} = \frac{2\beta^2 \gamma_S^2 (\alpha + \beta - \alpha - A - \beta - t_A) + \beta \gamma_S (\gamma ((\alpha - A)(\beta - 1) + 2t_A) + (\alpha + A + \beta + A - \alpha)(\alpha(2\beta - 1) - 2A - 1) - t_A)) - (\gamma (\alpha + A + \beta + A - \alpha - A)(\beta - 1) - t_A)}{2A^2(\beta - 1) - 2A + 2\beta A - \gamma + 6t_A + \alpha(2\beta - 1) - 2(2A - 1) - \beta \gamma_S - \gamma)} \]
sponsored ads to an advertiser, if unidentified, is significantly higher than that of a traditional ad. The third assumption states that the two platforms are sufficiently horizontally differentiated from consumers’ perspective and ensures a unique pure strategy equilibrium.

The following table summarizes all the notations in our model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0$</td>
<td>Consumer’s intrinsic utility from accessing a platform</td>
</tr>
<tr>
<td>$t_C$</td>
<td>Consumer’s transportation cost</td>
</tr>
<tr>
<td>$T$</td>
<td>Traditional Advertising</td>
</tr>
<tr>
<td>$S$</td>
<td>Sponsored Content Advertising</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Identifying probability of a sponsored content ad</td>
</tr>
<tr>
<td>$p_{i\omega}^{\upsilon C}$</td>
<td>Price charged by platform $i$ (platform 1’s ad strategy is $\upsilon$) to its consumer, $\upsilon, \omega \in {T, S}$</td>
</tr>
<tr>
<td>$x_{iC}^{\upsilon \omega}$</td>
<td>Consumers’ demand for platform $i$ when 1 has ad strategy $\upsilon$, 2 has ad strategy $\omega$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Consumers’ marginal disutility towards a traditional ad</td>
</tr>
<tr>
<td>$\gamma_S$</td>
<td>Consumers’ marginal disutility towards a sponsored content</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Advertiser’s intrinsic utility from a platform</td>
</tr>
<tr>
<td>$A$</td>
<td>Advertiser’s marginal utility from showing a recognized sponsored ad</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Advertiser’s marginal utility from showing an unrecognized sponsored ad</td>
</tr>
<tr>
<td>$t_A$</td>
<td>Advertiser’s transportation cost</td>
</tr>
<tr>
<td>$e$</td>
<td>Expected value</td>
</tr>
<tr>
<td>$p_{i\omega}^{\upsilon A}$</td>
<td>Price charged by platform $i$ to its advertisers (1 has ad strategy $\upsilon$, 2 has ad strategy $\omega$)</td>
</tr>
<tr>
<td>$x_{iA}^{\upsilon \omega}$</td>
<td>Advertisers’ demand for platform $i$ (1 has ad strategy $\upsilon$, 2 has ad strategy $\omega$)</td>
</tr>
</tbody>
</table>

4 Equilibrium Results

In this section, we analyze the platforms’ advertising and pricing decisions. We start our discussion with a monopolist platform, and then move on to the case of the duopoly platforms.

4.1 Analysis of Monopoly

When the monopoly platform offers traditional ads, we assume that markets for both consumers and advertisers are incompletely covered. In this case, the platform’s demand
from consumers and advertisers is given by

\[ x^T_C = \frac{u_0 - p^T_C - \gamma x^e_T}{t_C} \in (0, 1), \]  
\[ x^T_A = \frac{A_0 - p^T_A + A x^e_T}{t_A} \in (0, 1). \]  
\( (14) \) \( (15) \)

Not surprisingly, given consumers’ disutility from seeing the ads, their demand decreases in the externality parameter \( \gamma \) and their expected number of ads on the platform, \( x^e_T \). By contrast, given advertisers’ utility from reaching consumers, their demand increases in the externality parameter \( A \) and their expected number of consumers on the platform, \( x^e_T \). The platform’s profit is then given by \( \Pi^T = p^T_C x^T_C + p^T_A x^T_A \).

We assume that both consumers and advertisers have rational expectations about the number of players on the other side of the market. Furthermore, these expectations are correct in equilibrium. Such assumptions on rational expectation are fairly common in the literature (see, for example, Becker 1991, Katz and Shapiro 1985). This leads to the following equations:

\[ x^T_C = x^e_T, \quad \text{and} \quad x^T_A = x^e_T. \]  
\( (16) \)

Solving the platform’s optimization problem, we obtain the following prices, demand and profits when the platform offers traditional ads.

\[ p^T_C = \frac{2t_A u_0 + A u_0 \gamma - A_0 t_C \gamma - A A_0 t_C - A^2 u_0}{4t_A t_C - (A - \gamma)^2}, \]  
\[ p^T_A = \frac{t_A u_0 (A + \gamma) + A_0 (2t_A t_C + (A - \gamma) \gamma)}{4t_A t_C - (A - \gamma)^2}, \]  
\[ x^T_C = \frac{A A_0 + 2t_A u_0 - A_0 \gamma}{4t_A t_C - (A - \gamma)^2}, \]  
\[ x^T_A = \frac{2A_0 t_C + u_0 (A - \gamma)}{4t_A t_C - (A - \gamma)^2}, \]  
\[ \Pi^T = \frac{A_0^2 t_C + t_A u_0^2 + A_0 u_0 (A - \gamma)}{4t_A t_C - (A - \gamma)^2}. \]  
\( (17) \) \( (18) \) \( (19) \) \( (20) \) \( (21) \)

Next, we analyze the optimal strategies for the monopoly platform when it offers
sponsored content ads. Similarly, we assume that markets for consumers and advertisers are incompletely covered. Recall that there is a probability $\beta$ with which consumers can identify a sponsored content ad as a mere promotional message. Once this happens, consumers will incur a disutility, $\gamma_S$, and advertisers will not be able to enjoy a higher utility from exposing consumers to the “organic content”. In this case, consumers’ utility is given by $u_0 - t_C x_C - p_{CS} - \beta \gamma_S x^e_S$, and advertisers’ utility is given by $A_0 - t_A x_A - p_{AS} + \beta A x^e_C + (1 - \beta) \alpha x^e_S$. Therefore, the platform’s demand from consumers and advertisers is given by (again superscript $e$ denotes the expected value)

$$x^S_C = \frac{u_0 - p^S_C - \beta \gamma_S x^e_A}{t_C} \in (0, 1),$$

$$x^S_A = \frac{A_0 - p^S_A + (\beta A + (1 - \beta) \alpha)x^e_C}{t_A} \in (0, 1).$$

Note that when consumers are more likely to identify sponsored content ads as ill-integrated promotional messages, i.e., when $\beta$ increases, demand from consumers and advertisers will both decrease. The latter occurs because advertisers prefer sponsored ads to traditional ads if consumers cannot identify the former (and thus incorrectly believe that these ads are organic content): $\alpha > A$. The platform’s profit is then given by

$$\Pi^S = p^S_C x^S_C + p^S_A x^S_A.$$  

Given our assumptions on rational expectations, we again have $x^e_C = x^S_C$, and $x^e_A = x^S_A$. Solving the platform’s optimization problem, we obtain the following prices, demand and
profits when the platform offers sponsored content ads.

\[ p^*_C = \frac{2t_A t_C u_0 + \alpha(1 - \beta)(\alpha(\beta - 1)u_0 - A_0 t_C)}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2} \]
\[ - \frac{A^2 \beta^2 u_0 + A \beta (A_0 t_C - u_0(2\alpha(\beta - 1) + \beta \gamma_S)) + \beta \gamma_S (A_0 t_C + \alpha(\beta - 1)u_0)}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2} \]

\[ p^*_A = \frac{t_A u_0(\alpha + \beta(-\alpha + A + \gamma_S) - A_0(\beta \gamma_S(\beta(\alpha - A + \gamma_S) - \alpha) - 2t_A t_C)}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2}, \]

\[ x^*_C = \frac{2t_A u_0 - A_0(\beta(\alpha - A + \gamma_S) - \alpha)}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2}, \]

\[ x^*_A = \frac{2A_0 t_C - u_0(\beta(\alpha - A + \gamma_S) - \alpha)}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2}, \]

\[ \Pi^*_C = \frac{A_0 u_0((1 - \beta)\alpha + \beta A - \beta \gamma_S) + A_0^2 t_C + t_A u_0^2}{4t_A t_C - ((1 - \beta)\alpha + \beta A - \beta \gamma_S)^2}. \]

Given the importance of the brand preference parameter (i.e., the transportation cost parameter), \( t_C \) and \( t_A \), in the literature of product differentiation, we discuss their impact on prices for both sides of the market below.

**Lemma 1.** The price for consumers increases in \( t_C \) and \( t_A \), whereas the price for advertisers decrease in \( t_C \) and \( t_A \).

This result highlights the opposite impact of transportation costs on prices for consumers and advertisers, and is unique to the two-sided market context. Recall that the monopolist would charge \( u_0/2 \) in a one-sided market (standard Hotelling model with incomplete coverage), which is independent of the transportation cost \( t \). The monopolist’s profit would have been \( u_0^2/(4t) \). The intuition of the different impact of transportation costs on consumers and advertisers is as follows. First, note that the market is incompletely covered on both sides. When the transportation cost for consumers, \( t_C \), increases, holding everything else constant, demand from consumers will decrease. As a result, the platform also becomes less attractive to advertisers, which leads to a price reduction for advertisers. On the other hand, the platform increases price for consumers, because now there are fewer advertisers so that consumers’ disutility from seeing ads would decrease. Similarly, when the transportation cost for advertisers \( t_A \) increases, holding everything else constant, demand from advertisers will decrease. Fewer advertisers lead to a lower disutility from consumers, therefore enabling the platform to charge a higher price for consumers.
After discussing prices for consumers and advertisers, we next examine the profitability of sponsored ads compared to traditional ads. First, it is easy to see that regardless of the platform’s advertising strategies, the platform’s profit always decreases in consumers’ marginal disutility of viewing ads: \( \frac{\partial \Pi_T^*}{\partial \gamma} < 0, \frac{\partial \Pi_S^*}{\partial \gamma_S} < 0 \). Intuitively, a greater disutility of consumers towards either type of ads would reduce consumers’ demand, which in turn decreases the platform’s appeal to advertisers. Hence, the platform’s profit suffers when \( \gamma \) or \( \gamma_S \) increases. Next, we compare the profitability of the two different advertising strategies in the following proposition.

**Proposition 1.** The platform’s profits with sponsored ads decrease in \( \beta \). When consumers’ probability of identifying sponsored ads is below a threshold, i.e., \( \beta < \beta^* \), it is more profitable for the platform to adopt sponsored ads. Furthermore, when consumers’ brand preference is beyond a threshold, i.e., \( t_C^* \geq t^*_C \), it is more profitable to offer sponsored ads.\(^{11}\)

The first part of Proposition\(^1\) states that the platform suffers from consumers’ increased tendency to identify sponsored ads. It will offer sponsored ads in equilibrium (because it is more profitable) when consumers are relatively unlikely to identify sponsored ads. This happens because as \( \beta \) decreases, consumers are less likely to incur the greater disutility from seeing the sponsored ads compared to the traditional ads (recall \( \gamma_S > \gamma \)). In addition, advertisers will enjoy a greater expected cross-side externality as \( (\beta A + (1 - \beta)\alpha) \) increases as \( \beta \) decreases. As a result, more consumers would be willing to purchase the product and more advertisers would be attracted to the platform. When \( \beta \) is below a threshold, i.e., \( \beta < \beta^* \), the increased demand from consumers and advertisers make the sponsored ads more profitable compared to the traditional ads.

It is worth pointing out that sponsored ads can be more profitable even when \( \beta \gamma_S > \gamma \), i.e., when consumers’ expected disutility from sponsored ads is greater than their disutility from the traditional ads. This result is interesting because the platform’s profits always decrease in \( \gamma \) or \( \gamma_S \), so a greater \( \beta \gamma_S \) cannot be treated simply as an increase in \( \gamma \). Since the necessary condition for sponsor ads being more profitable is \( (1 - \beta)(\alpha - A) + \gamma - \beta \gamma_S > 0 \) (recall our assumption Equation \(^{11}\)), a high value of \( \beta \gamma_S \) must be outweighed by a high value of \( (\alpha - A) \). The key intuition here is that even if consumers dislike sponsored ads...

\(^{11}\)\( \beta^* \) and \( t_C^* \) are given in the appendix.
more than traditional ads, as long as advertisers prefer sponsored ads significantly more (recall $\alpha > A$), the net effect on the platform’s profits can still be positive. As $(\alpha - A)$ increases, the cutoff value $\beta^*$ approaches 1; as a result, a sponsored content ad is now more likely to be profitable than a traditional ad even when $\beta$ takes a very high value.

The second part of Proposition 1 states that the platform will offer sponsored ads in equilibrium when consumers’ transportation cost is sufficiently high. When $t_C$ increases, consumers have a stronger brand preference so it becomes more challenging to attract those who are located further away from the platform. Recall from Lemma 1 (earlier discussion in this section) that prices for consumers will go up but prices for advertisers will decrease, regardless of the platform’s advertising strategies. This means either way the revenue from advertisers will decrease. Given that advertisers prefer sponsored ads when consumers do not identify them, the platform then has a stronger incentive to offer sponsored ads and try to compensate for the loss of revenues from advertisers.

### 4.2 Analysis of Duopoly

In this section, we begin our analysis by laying out all possible outcomes in the game between platforms 1 and 2 when they are competing for consumers and advertisers. Recall that when it comes to the advertising decisions, the options available to each platform are to offer traditional ads (T), or to offer sponsored content ads (S). Each platform then chooses the prices for its consumers and advertisers. As shown in Table 1, there can be three possible outcomes: both platforms offer traditional ads (denoted as TT), both firms offer sponsored content ads (denoted as SS), and one firm offers traditional ads whereas the other offers sponsored ads (denoted as TS or ST). After solving each of these subgames, we analyze whether and when each of them is an equilibrium.

To capture the competition between the two platforms on both sides of the market, we assume that the market is fully covered both on consumers’ side and on advertisers’ side. This structure is appealing because it allows one platform’s strategy on one side (say platform 1’s price for consumers) to influence not only its own other side (platform 1’s demand from advertisers), but also indirectly affect the demand of consumers and advertisers from platform 2. To simplify the exposition, in the main text we present the
analysis of two symmetric cases, TT and SS (refer to the appendix for details). The analysis of the asymmetric case TS/ST is given in the appendix.

### 4.2.1 Both Platforms Offer Traditional Ads

When both platforms offer traditional ads, platform 1’s demand from consumers and advertisers are given by (those of platform 2’s are symmetrically defined)

\[
x_{TT1C} = \frac{p_{TT2C} - p_{TT1C} + t_C - \gamma x_{eTT1C} + \gamma x_{eTT2C}}{2t_C},
\]

\[
x_{TT1A} = \frac{p_{TT2A} - p_{TT1A} + t_A + Ax_{eTT1A} - Ax_{eTT2A}}{2t_A}.
\]

Intuitively, platform 1’s demand from consumers decreases in its price, \( p_{TT1C} \), and consumers’ expected number of ads on it, \( x_{eTT1C} \). By contrast, it increases in platform 2’s price, \( p_{TT2C} \), and in consumers’ expected number of ads on platform 2, \( x_{eTT2C} \). Platform 1’s demand from advertisers increases in platform 2’s price for advertisers, \( p_{TT2A} \), and the expected number of consumers on its own platform, \( x_{eTT1C} \). Each platform’s profit is then given by \( \Pi_{TTi} = p_{TTiC} x_{TTiC} + p_{TTiA} x_{TTiA}, \ i \in \{1, 2\} \).

With the assumption of rational expectations, \( x_{TTiC} = x_{eTTiC} \), \( x_{TTiA} = x_{eTTiA} \), we obtain the following prices and profits when both platforms offer traditional ads after solving the platforms’ optimization problem:

\[
p_{TTiC}^* = t_C - A,
\]

\[
p_{TTiA}^* = t_A + \gamma,
\]

\[
\Pi_{TTi}^* = \frac{t_C + t_A - A + \gamma}{2}.
\]

It is interesting to note that each platform’s profit increases in consumers’ disutility from seeing the ads, \( \gamma \), and decreases in advertisers’ utility from reaching consumers, \( A \): \( \frac{\partial \Pi_{TTi}^*}{\partial \gamma} > 0, \frac{\partial \Pi_{TTi}^*}{\partial A} < 0 \). This occurs because of the opposite cross-side externality in this context: the former raises advertisers’ prices and the latter pushes down consumers’ prices. It is worth contrasting this comparative statics with the situation in the monopoly platform in Section 4.1 where \( \gamma \) always negatively affects the platform’s profit: \( \frac{\partial \Pi_{TTi}^*}{\partial \gamma} < 0 \). This
opposite pattern with respect to $\gamma$ occurs because the marginal consumers face different outside options across different market structures. The marginal consumer in monopoly trades off between buying from the only platform and not buying at all. An increase in $\gamma$ leads to a decrease in the platform’s demand from consumers (refer to Equation (14)). By contrast, the marginal consumer in duopoly trades off between buying from platform 1 and buying from platform 2 (refer to Equation (30)). In other words, the negative impact on one platform’s consumer demand from $\gamma$ is alleviated by the number of advertisers on the other platform. When the two competing platforms are symmetric, $\gamma$’s negative impact on consumers can be completely mitigated from raising the price for advertisers in the duopoly situation.

4.2.2 Both Platforms Offer Sponsored Ads

When both platforms offer sponsored content ads, platform 1’s demand from consumers and advertisers are given by (those of platform 2’s are symmetrically defined)

\begin{align*}
x_{1C}^{SS} &= \frac{p_2^{SS} - p_1^{SS} + t_C - \beta \gamma_s x_{1A}^{eSS} + \beta \gamma_s x_{2A}^{eSS}}{2t_C}, \quad (35) \\
x_{1A}^{SS} &= \frac{p_2^{SS} - p_1^{SS} + t_A + (x_{1C}^{eSS} - x_{2C}^{eSS})(\beta A + (1 - \beta)\alpha)}{2t_A}. \quad (36)
\end{align*}

Similar to the discussion in Section 4.2.1, platform 1’s demand from consumers decreases in its price, $p_{1C}^{SS}$, and consumers’ expected number of ads on it, $x_{1A}^{eSS}$. It increases in platform 2’s price, $p_{2C}^{SS}$, and in consumers’ expected number of ads on platform 2, $x_{2A}^{eSS}$. More importantly, platform 1’s demand from consumers decreases in $\beta$, the likelihood for consumers to correctly identify sponsored ads, if and only if consumers expect to see more ads on platform 1 compared to platform 2, $x_{1A}^{eSS} > x_{2A}^{eSS}$. In other words, consumers’ probability of identifying sponsored ads only hurts the focal platform if consumers expect this platform to be less desirable in terms of the amount of ads on it. Similarly, due to the fact that advertisers prefer sponsored ads if consumers do not identify them (recall $\alpha > A$), platform 1’s demand from advertisers decreases in $\beta$ when the expected number of consumers is higher on its own platform: $x_{1C}^{eSS} > x_{2C}^{eSS}$. In this case, each platform’s profit is then given by $\Pi_i^{SS} = p_{iC}^{SS} x_{iC}^{SS} + p_{iA}^{SS} x_{iA}^{SS}$, $i \in \{1, 2\}$. 

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With the assumption of rational expectations, \( x_{iC}^{SS} = x_{iC}^{eSS} \), \( x_{iA}^{SS} = x_{iA}^{eSS} \), we obtain the following prices and profits when both platforms offer sponsored ads after solving the platforms’ optimization problem:

\[
p_{iC}^{SS*} = t_C - \beta A - (1 - \beta)\alpha
\]

\[
p_{iA}^{SS*} = t_A + \beta \gamma_S,
\]

\[
\Pi_i^{SS*} = \frac{t_C + t_A - \beta A - (1 - \beta)\alpha + \beta \gamma_S}{2}.
\]

It is interesting to note that each platform’s profit increases in consumers’ disutility from seeing the ads, \( \gamma_S \), and decreases in advertisers’ utility from reaching consumers, \( A \) and \( \alpha \). This result occurs because of the opposite cross-side externality in this context: the former raises advertisers’ prices to compensate for consumers and the latter pushes down consumers’ prices to attract more advertisers. Again it is worth noting that consumers’ disutility from seeing the sponsored ads, \( \gamma_S \), actually helps to improve the profits of the competing platforms: \( \frac{\partial \Pi_i^{SS*}}{\partial \gamma_S} < 0 \), whereas in the case of the monopoly platform in Section 4.1, \( \gamma_S \) always negatively affects the platform’s profit: \( \frac{\partial \Pi^*_S}{\partial \gamma_S} < 0 \). The intuition is discussed in details in Section 4.2.1 and is thus not repeated here.

Recall that Proposition 1 states that a monopolist platform’s profits with sponsored ads always decrease in consumers’ likelihood of correctly identifying sponsored ads, \( \beta \). This outcome occurs because a higher \( \beta \) leads to a reduction in both consumers’ demand (see Equation (22)) and advertisers’ demand (see Equation (23)). Next, we assess the impact of \( \beta \) on competing platforms’ profits in duopoly.

**Proposition 2.** When both platforms offer sponsored ads (SS), their profits increase when consumers are more likely to identify the sponsored ads: \( \frac{\partial \Pi_i^{SS*}}{\partial \beta} > 0 \).

Proposition 2 highlights the importance of accounting for the market structure when assessing the impact of consumers’ likelihood to identify sponsored ads on platforms’ profits. Contrary to the situation of a monopolist platform offering sponsored ads, the two competing platforms actually benefit if consumers have a greater likelihood of identifying sponsored ads.

\[^{12}\text{When } A \text{ is larger than } t_C, \text{ readers have free access to the media content.}\]
the sponsored ads. This result arises for three reasons. First, advertisers’ expected utility of reaching one more consumer, \( \beta A + (1 - \beta)\alpha \), decreases when \( \beta \) increases because consumers are less likely to mistake sponsored ads as organic content (the latter of which gives advertisers a higher marginal utility: \( \alpha > A \)). This leads to a stronger incentive for the platforms to raise the price for consumers because their appeal with sponsored ads to advertisers has reduced. Indeed, the price for consumers, \( p_{SC}^{SS} = t_C - \beta A - (1 - \beta)\alpha \), increases in \( \beta \). Second, consumers’ expected disutility from sponsored ads, \( \beta \gamma_S \), increases in \( \beta \). Therefore, a higher \( \beta \) motivates platforms to charge a higher price for advertisers to compensate for increased consumers’ disutility. This can be easily seen from the price for advertisers, \( p_{SA}^{SS} = t_A + \beta \gamma_S \). Third, recall that both the marginal consumer and the marginal advertiser in the duopoly context trades off between buying from platform 1 and buying from platform 2. As long as the other platform has the same incentives to raise prices when \( \beta \) increases, the focal platform can increase prices for both consumers and advertisers until the optimal points. Put differently, the platforms’ demand from both sides of the market does not suffer from a higher price like it did in the monopoly situation. In fact, because the two competing platforms are symmetric, their actual demand remains the same even after the price increase. To summarize, the combination of the three factors leads to a positive impact of \( \beta \) on competing platforms’ profits.

4.2.3 Equilibrium in Duopoly

After analyzing all the subgames, we next discuss the equilibrium outcomes. Proposition 4 summarizes the equilibrium outcomes in terms of three cutoff values of advertisers’ transportation cost – \( t_{A1}, t_{A2}, \) and \( t_{A3} \).\(^{13}\)

**Proposition 3.** When \( t_A < \min\{t_{A1}, t_{A3}\} \) or \( t_A > \max\{t_{A1}, t_{A2}\} \), both platforms offering traditional ads (TT) is the unique equilibrium. When \( t_{A3} < t_A < \min\{t_{A1}, t_{A2}\} \), there exist two equilibria: one in which both platforms offer traditional ads (TT) and the other where both platforms offer sponsored ads (SS). Lastly, when \( t_{A1} < t_A < t_{A3} \) or \( t_{A2} < t_A < t_{A1} \), both platforms offering sponsored ads (SS) is the unique equilibrium.

Our results show that when advertisers have either very low or very high transportation costs, both platforms offering traditional ads is the unique equilibrium outcome. When

\(^{13}\)The thresholds, \( t_{A1}, t_{A2} \) and \( t_{A3} \) are defined in the appendix.
advertisers’ transportation cost is very low \( (t_A < \min \{ t_{A1}, t_{A3} \}) \), regardless of their advertising strategy platforms can only ask for a low price from the advertisers. However, when the platforms offer sponsored content instead of traditional ads, even the consumers’ price is also lower \( (p^*_{iC}^{SS} < p^*_{iC}^{TT}) \) and cannot compensate for the loss from advertiser’s side. As a result, both platforms offer traditional ads (TT) in this case.

As \( t_A \) increases, a platform (say platform 1) may contemplate offering sponsored content ads to boost advertisers’ utilities. In this scenario, the deviation from traditional advertising to sponsored content advertising would conditionally allow a platform to charge a higher price to the advertisers (compared to the case of TT): \( p^*_{1A}^{ST} > p^*_{1A}^{TT} \). The advertisers will be willing to pay a higher price and expect that sponsored content advertising will help them to mitigate the adverse effect of an increase in \( t_A \). In particular, when consumers’ transportation cost is high enough \( (t_C > \frac{(\gamma S - \gamma) (A(1+\beta)+\alpha(1-\beta)-2(\beta S + \gamma))}{6(1-\beta)(\alpha - A)}) \) and the size of negative network externality for consumers from traditional advertising is not too low \( (\gamma > 2(A(1+\beta)+\alpha(1-\beta))-\beta \gamma S) \), platform 1 that shifts to sponsored content advertising will charge a higher price to its advertisers \( (p^*_{1A}^{ST} > p^*_{1A}^{TT}) \) if advertisers’ transportation cost exceeds a critical value \( (t_A > \frac{(2A(1\beta) + 2a(1-\beta) - \beta S - \gamma) (A\beta(\gamma S - \gamma) - \beta^2 S + \gamma (1-\alpha(1-\beta)))}{A((1+\beta)(\gamma - \beta S) - 6(1-\beta) t_C) + (\beta S - \gamma)(-\alpha(1-\beta) - \beta \gamma S - \gamma) + 6C(\alpha - \beta + 3 \beta S - 3 \gamma)} \)). In addition, an increase in advertiser’s price is not the only factor that raises platform 1’s profit.

When the deviating platform 1 (sponsored content advertising) charges a higher price to the advertisers, a lower number of advertisers post promotional messages on that platform: \( x^*_{1A} < 1/2 \). As a result, a consumer’s disutility from experiencing ads also shrinks. Thus platform 1 can now charge a higher price to its consumers as well: \( p^*_{1C}^{ST} > p^*_{1C}^{TT} \). In total, as \( t_A \) increases, by deviating from T to S, a platform can charge higher prices to both its consumers and advertisers. Eventually, an increase in advertisers’ transportation costs may give both platforms enough incentives to switch from traditional ads to sponsored content ads. In fact, when \( t_{A3} < t_A < \min \{ t_{A1}, t_{A2} \} \), there exist two equilibria simultaneously: SS and TT. Given our assumption \( ((1-\beta)(\alpha - A) + \gamma - \beta \gamma S) > 0 \), TT equilibrium is the more profitable one. In this case, it is up to the platforms to coordinate in order to achieve a mutually better outcome.

When the advertisers’ transportation costs further increase to \( t_{A1} < t_A < t_{A3} \) or \( t_{A2} < t_A < t_{A1} \), SS becomes the unique equilibrium. The even higher value of \( t_A \) now ensures
a higher advertisers’ price for a platform which wants to switch from traditional ads to sponsored ads: $p^{ST\ast}_{1A} > p^{TT\ast}_{1A}$. At the same time, this value of $t_A$ prevents any sponsored content offering platform from deviating to traditional ads – not only the price for the advertisers is reduced but also the consumers’ price is so high that very few consumers would participate in this platform: $p^{TS\ast}_{1A} < p^{SS\ast}_{1A}$ and $x^{TS\ast}_{1C} < 1/2$.

When $t_A$ is sufficiently high, i.e., $t_A > \max\{t_{A1}, t_{A2}\}$, regardless of their advertising strategies the platforms can charge a very high price to the advertisers. Now, a platform which currently offers traditional ads (given that the rival also offers traditional ads) will find that deviating to sponsored ads may not result in a positive change in advertisers’ price, and even if there is a positive change the magnitude is negligible. However, deviating to sponsored ads will certainly lead to a significant negative change in consumers’ price: $p^{ST\ast}_{1C} < p^{TT\ast}_{1C}$. Unlike the previous case, this time the deviating platform can not charge higher price to the consumers because a higher number of advertisers post promotional messages on that platform: $x^{ST\ast}_{1A} > 1/2$. More advertisers would flock to the sponsored content offering platform because of a higher $t_A$ which has reduced their utility. Thus, there are no incentives for any platform to deviate to sponsored content advertising strategy.

On the other hand, if both platforms are currently offering sponsored ads, then they will find that for a deviating platform, a positive change in consumers’ price will outweigh the small negative change in advertisers’ price. As a result, in equilibrium both platforms offer traditional ads.

One might intuit that platforms are more likely to offer sponsored content ads in equilibrium when consumers are less likely to identify sponsored ads. After all, a lower likelihood for consumers to identify sponsored ads implies a lower expected disutility (a higher expected utility). Thus, one important question is whether both platforms would still offer traditional ads when consumers are completely unable to identify the sponsored content ads, i.e., when $\beta = 0$. The next corollary answers this question.

**Corollary 1.** *When consumers cannot identify sponsored ads at all ($\beta = 0$), both platforms offering traditional advertising (TT) can still be the unique equilibrium outcome.*

Focusing on the parameter range where SS is the unique equilibrium ($t_{A1} < t_A < t_{A3}$ or $t_{A2} < t_A < t_{A1}$), we next discuss whether sponsored content advertising necessarily makes
both platforms better off compared to the traditional advertising.

**Proposition 4.** When advertisers’ utility from sponsored ads is sufficiently higher than traditional ads, i.e., $\alpha > A + (\beta \gamma_S - \gamma)/(1 - \beta)$, both platforms offering sponsored content advertising is always a Prisoner’s Dilemma outcome.

Although the popularity of sponsored content advertising has grown in the past few years, this proposition shows that it can in fact be a Prisoner’s Dilemma outcome. Ironically, even if advertisers strongly prefer sponsored ads to traditional ads, $\alpha > A + (\beta \gamma_S - \gamma)/(1 - \beta)$, the two competing platforms would still have been better off if they both choose traditional ads: $\Pi_{SS}^* < \Pi_{TT}^*$. The intuition is explained in two steps as follows. First, note that the price for consumers is lower when both platforms offer sponsored ads: $p_{IC}^{SS*} < p_{IC}^{TT*}$. However, this is not because consumers dislike sponsored ads more than they dislike the traditional ads (though it is true that $\gamma_S > \gamma$). Instead, price is lower for consumers because advertisers prefer sponsored ads to traditional ads, as long as consumers cannot identify sponsored ads perfectly, i.e., $\beta A + (1 - \beta)\alpha > A$ if $\beta < 1$. Advertisers’ stronger preference towards sponsored ads gives platforms a stronger incentive to cut prices for consumers to better capitalize on advertisers. This is unique to the context of the two-sided media market. Given that the price for advertisers when sponsored ads are offered, $p_{IA}^{SS*} = t_A + \beta \gamma_S$, is not significantly higher than that when traditional ads are offered, $p_{IA}^{TT*} = t_A + \gamma$, (in fact, the former might even be lower), the overall profits for the platforms are lower when they choose sponsored ads compared to traditional ads: $\Pi_{i}^{SS*} < \Pi_{i}^{TT*}$.

Second, suppose both platforms are offering traditional ads now. By unilaterally deviating to sponsored ads, one platform can lower its price for consumers and gain a bigger market share on this side of the market. As a result, this focal platform becomes more appealing to advertisers for two reasons: more consumers and a more attractive advertising format (recall sponsored ads are strongly preferred by advertisers when $\alpha > A + (\beta \gamma_S - \gamma)/(1 - \beta)$). Therefore, this platform can raise its price for advertisers and improve its total profits. By a similar logic, the other platform has incentives to follow suit by shifting to sponsored ads and cut prices for consumers as well. This leads to the Prisoner’s Dilemma outcome in which both platforms offer sponsored ads even if offering traditional ads is a more profitable outcome.
After comparing platforms’ profitability between the two advertising formats, the natural question is whether consumers and advertisers are better off with sponsored ads. We answer this question and summarize the welfare implications in the next proposition.

**Proposition 5.** When both platforms offer sponsored content advertising,

1. consumers are better off when \( \gamma_S \leq \gamma_1 = \frac{[2(1 - \beta)(\alpha - A) + \gamma]}{\beta}; \)
2. advertisers are also better off when \( \gamma_S \leq \gamma_2 = \frac{[(1 - \beta)(\alpha - A)]}{2} + \gamma]/\beta. \)

At first blush, it may seem that when consumers dislike sponsored ads more than they dislike traditional ads (because they feel misled by the platform), they should be worse off when platforms offer sponsored ads in equilibrium. However, the first part of Proposition 5 states otherwise as long as consumers’ disutility towards sponsored ads is not excessively high (\( \gamma_S \leq \gamma_1 \)). This result arises for three reasons. First, recall from the discussion of Proposition 4, the price for consumers is lower when both platforms offer sponsored ads: \( p^*_C iA < p^*_C iA \), because now the platforms have stronger incentives to extract more surplus from the advertisers who prefer sponsored ads. Second, in the sponsored ads equilibrium, consumers are not exposed to more ads compared to the situation of the traditional ads. In fact, the number of ads appearing in the platform stays the same across the two advertising formats: \( x^*_iA iA = x^*_iA iA = 1/2 \). Third, consumers cannot always identify sponsored ads perfectly as long as \( \beta < 1 \). In other words, there are situations where consumers will mistake sponsored ads as organic content and whenever this happens, consumers do not experience disutility at all. The combination of the three reasons leads to a welfare increase for consumers.

The second part of Proposition 5 states that advertisers are also better off in the presence of sponsored ads, as long as consumers’ disutility towards sponsored ads does not exceed their disutility towards traditional ads by too much. This result can be better understood by comparing an advertiser’s utility across the two advertising formats: \( u_A(SS) = A_0 - t_A x - p^*_A + \beta A x_i^SS + (1 - \beta)\alpha x_i^SS \) and \( u_A(TT) = A_0 - t_A x - p^*_A + A x_i^TT \). On the one hand, because consumers cannot perfectly identify sponsored ads (\( \beta < 1 \)), this advertiser enjoys a higher utility from accessing the same amount of consumers (recall in both equilibria consumers’ demand is 1/2): \( \beta A + (1 - \beta)\alpha > A \). On the other hand, because consumers
dislike sponsored ads more, the platforms have to adjust advertisers’ price accordingly: 
\[ p_{SA}^{SS} = t_A + \beta \gamma_S, \] which increases in both \( \beta \) and \( \gamma_S \). When consumers’ disutility towards sponsored ads is below a threshold, i.e., when \( \gamma_S < \gamma_2 = \left( \gamma + \frac{(1-\beta)(\alpha-A)}{2} \right) / \beta \), advertisers’ benefit of exposing consumers to sponsored ads outweighs the price they have to pay. Therefore, advertisers are overall better off with sponsored ads.

Finally, it is worth pointing out that consumers are more likely to be better off than advertisers in the presence of sponsored content ads (because \( \gamma_1 > \gamma_2 \)). In other words, when platforms shift from traditional ads to sponsored ads and \( \gamma_2 < \gamma_S < \gamma_1 \), consumers are better off but advertisers are worse off. Again this happens because advertisers have to pay to indirectly compensate for consumers’ disutility towards sponsored ads, \( p_{SA}^{SS} = t_A + \beta \gamma_S \), whereas consumers now enjoy a lower price, \( p_{SC}^{SS} < p_{TC}^{TT} \).

4.3 Asymmetric Equilibrium

From the discussion on Proposition 3 we already know that as \( t_A \) increases, an individual platform would have stronger incentives to offer sponsored content ads. Furthermore, our analysis demonstrates that for a particular range of \( t_A \), only one of the platforms will be better off by deviating to sponsored content advertising. In particular, when \( \max\{t_{A1}, t_{A3}\} < t_A < t_{A2} \), the asymmetric equilibrium occurs in which one platform adopts sponsored content advertising but the other platform offers traditional advertisements. The next proposition summarizes this result and compares the pricing of the two platforms across consumers and advertisers.

**Proposition 6.** (a) When \( \max\{t_{A1}, t_{A3}\} < t_A < t_{A2} \), one platform offering sponsored ads whereas the other platform offering traditional ads is the unique asymmetric equilibrium.

(b) In the asymmetric equilibrium when both consumers and advertisers’ transportation costs are sufficiently high (\( t_{C1} > t_{C3}, t_{A1}^{**} > t_A > t_{A1}^{*} \)), the platform which offers traditional ads will charge a higher (compared to the platform which offers sponsored ads) price to the consumers whereas the platform which offers sponsored ads will charge a higher price to the advertisers: \( p_{1C}^{TS} > p_{2C}^{TS}, p_{1A}^{TS} < p_{2A}^{TS} \).

When the transportation cost of the consumers increases, one might expect a lower participation from consumers on both platforms. As a result, a platform may want to
increase the price for its advertisers so that it reduces the participation of advertisers and in turn encourages consumers to join the platform in spite of their higher transportation cost (since consumers consider advertisements as negative externalities). However, when platforms compete on both price and advertising strategy, this conventional wisdom will not hold. The platform which offers traditional advertising (say platform 1) will charge a lower price for the advertisers compared to the platform which offers sponsored ads: $p_{TS1}^* < p_{TS2}^*$. Meanwhile, the platform with traditional advertising will charge a higher price to the consumers: $p_{TS1}^* > p_{TS2}^*$. To some extent, this result has the flavor of “tacit collusion” between the two competing platforms that allows each of them to focus on one side of the market.

The platform (say platform 2) that adopts sponsored content ads certainly wants more participation from the advertisers’ side because it is the advertisers who show a more favorable attitude towards sponsored content advertising. Fearing that a lower participation from consumers may discourage the (more lucrative) advertisers from joining, this platform offers a lower price (compared to the platform which offers traditional ads) to the consumers, $p_{TC2}^* < p_{TC1}^*$, and thus encourages them to join platform 2. In contrast, platform 1 focuses on extracting surplus from consumers’ side and hence it charges a lower price (compared to the other platform’s price) for advertisers. Overall, competition on multiple dimensions (price and advertising) eventually leads to the directionally opposite effects on the pricing strategies of the two platforms.

Our results also suggest when consumers’ expected disutility from sponsored content ads is less than their disutility from the traditional ads (i.e., $\beta \gamma_s < \gamma$), the platform which offers sponsored content ads will have higher consumer market share if $t_A > max\{t_{A1}, t_{A*}\}$  
When $\beta \gamma_s > \gamma$ but the other condition involving $t_A$ remains intact, the platform that offers traditional ads will have a higher consumer market share.

$^{15}$ $t_{A*}$ is given in the appendix.
5 Conclusion

Over the last few years, sponsored content advertising has become more popular with both media platforms and advertisers. The main advantage of this advertising format is that readers are likely to view these ads as editorial content by the media platform, and thus may be more engaged and form a more positive impression about the underlying brands (advertisers). However, if consumers identify sponsored content ads as imposed promotional messages, they are likely to react more negatively to them. As a result, it becomes an important question to understand the impact of sponsored content advertising in the context of two-sided media markets.

Our analysis shows that whether or not a platform should adopt the sponsored content ads depends on how likely consumers can identify them as promotional messages, the extent of differentiation between the competing platforms, and the inter-group externalities across consumers and advertisers. In particular, both platforms should offer sponsored content ads when advertisers view them as moderately differentiated. We have confirmed industry experts’ intuition that advertisers can be better off with sponsored content ads compared to the traditional ads, as long as consumers are not too unhappy with this new ad format. Interestingly, although consumers dislike sponsored ads more than they dislike the traditional ads, their surplus can increase when both platforms offer sponsored ads, because now they can enjoy a lower price.

We demonstrate that it is possible for competing platforms to end up in a Prisoner’s Dilemma outcome by both offering sponsored content ads. This happens because either of the two platforms has incentives to undercut its rival’s price for consumers in order to attract more advertisers. This result demonstrates that even if one side of the market strongly prefers a particular instrument, it might not be beneficial for the platform in the two-sided market to offer such instrument under competitive environment. This result also contributes to the advertising literature – while existing wisdom suggests that Prisoner’s Dilemma outcome in advertising emerges because of competition over ad budget (Corfman and Lehmann 1994), we find that the same outcome can be seen even when firms compete over advertising types.
Managerially, we highlight the result that depending on the market structure, consumers' awareness of the sponsored content ads and their ability to identify these ads have different impact on media platforms' profits. While a monopoly platform is worse off when consumers become more sophisticated and have a higher success rate of identifying sponsored ads, the opposite is true for competing platforms in duopoly. It is also important to note that the existence of asymmetric equilibrium confirms that mere imitation of the rival platform's advertising technique may adversely affect the profitability of a media platform. Also, regardless of the naivety of the consumers or the willingness of the advertisers, while deciding on the optimal advertising strategy media platforms must consider the extent of differentiation. Our result confirms that too low or too high of a differentiation between the competing platforms may not offer the platforms a suitable landscape in promoting sponsored content ads. Lastly, we show that contrary to the popular wisdom consumers may benefit from sponsored content ads as under specific conditions their consumer surplus increases. Thus, an effective integration of sponsored content with regular editorial content can help the advertisers to strengthen their relationships with the consumers which in turn will make the consumers more accepting of sponsored content ads.

This paper takes a first stab at understanding the strategic impact of sponsored content advertising on media platforms, advertisers, and consumers. To simplify analysis, we assumed that platforms can either adopt traditional advertising or sponsored ads. In reality, the transparency of a sponsored content ad can be a continuous strategic variable. As a result, consumers may react differently depending on the presentation style and the content of the sponsored ads as well as the disclosure policy of the platform. Future research can explore the implications of these characteristics of sponsored ads. This paper also does not consider the potentially important role of dynamics in this context. Over time, as some consumers become more sophisticated and are more likely to identify sponsored ads, it will be interesting and helpful to incorporate consumers’ heterogeneity in responding to these ads and analyze its impact. Finally, the collaboration between media platforms and advertisers to create high-quality sponsored ads can be very costly, and may affect the eventual ad choices. Future research can study the co-production of ads and organic content, and sheds insights on when and how it is optimal to engage in co-creation of advertising.
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Appendix

Analysis of Asymmetric Strategies of Platforms

When platform 1 offers sponsored content ads and platform 2 offers traditional ads (denoted by $ST$), their demand from consumers and advertisers are given by

$$x_{1C}^{ST} = \frac{p_{2C}^{ST} - p_{1C}^{ST} + t_C - \beta \gamma S x_{1C}^{ST} + \gamma x_{2A}^{ST}}{2t_C},$$  (40)

$$x_{1A}^{ST} = \frac{p_{2A}^{ST} - p_{1A}^{ST} + t_A + x_{1C}^{ST}(\beta A + (1 - \beta)\alpha) - A x_{2C}^{ST}}{2t_A},$$  (41)

$$x_{2C}^{ST} = 1 - x_{1C}^{ST},$$  (42)

$$x_{2A}^{ST} = 1 - x_{1A}^{ST}.$$  (43)

In this case, each platform’s profit is then given by

$$\Pi_i^{ST} = p_i^{ST} x_i^{ST} + p_i^{ST} x_i^{ST}, \ i \in \{1, 2\}.$$  (44)

With the assumption of rational expectations, $x_{1C}^{ST} = x_{1C}^{ST}, \ x_{1A}^{ST} = x_{1A}^{ST},$ we obtain the following prices and profits when both platforms offer sponsored ads after solving the platforms’ optimization problems:

$$p_{1C}^{ST*} = \{-\alpha^3 + 3 \alpha^2 \gamma - 2 \alpha \gamma^2 - A^3(\beta + 1)^3 + A^2(\beta + 1)((\beta + 1)(3 \alpha (\beta - 1) + 2 \beta \gamma_S + 3 \gamma) + 3 \beta t_C + t_C) + A((\beta + 1)(3 \alpha (\beta - 1)^2 + 2 \alpha(\beta - 1)(2 \beta \gamma_S + 3 \gamma) + 2 \gamma(\beta \gamma_S + \gamma)) + 18(\beta + 1)t_A t_C - 2t_C(\alpha(\beta - 1)(3 \beta + 2) + (2 \beta + 3)(\beta \gamma_S + \gamma))) + \alpha^3 \beta^3 - 3 \alpha^3 \beta^3 + 3 \alpha^2 \beta^2 \gamma + 3 \alpha^2 \beta^2 t_C + 2 \alpha^2 \beta^2 t_C + 3 \alpha^3 \beta - 6 \alpha^2 \beta \gamma + 2 \alpha \beta \gamma^2 - 6t_A t_C(3 \alpha(\beta - 1) - \beta \gamma_S + 3 \gamma + 6 t_C) - 6 \alpha^2 \beta t_C + 2 \beta \gamma_S(\alpha(\beta - 1) + \gamma)(\alpha(\beta - 1) + 2t_C) + 4 \alpha \beta t_C + 3 \alpha^2 t_C - 4 \alpha \gamma t_C + 2 \gamma^2 t_C\}/\{(2 \alpha + 2 A(\beta + 1) - \beta(2 \alpha + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha \beta - 2(\beta \gamma_S + \gamma)) - 36 t_A t_C\},$$

$$p_{1A}^{ST*} = \{2 A^2(\beta + 1)(\beta \gamma_S + \beta t_A + \gamma + t_A) + A(\beta \gamma_S + \gamma)(2 \alpha(\beta - 1) + (2 + 3)(\beta \gamma_S + \gamma)) - 2 t_A((\beta + 1)(2 \alpha(\beta - 1) + 2 \beta \gamma_S + 3 \gamma) + 3(\beta - 1)t_C)) + (\beta \gamma_S + \gamma)^2(2 \alpha(\beta - 1) + \beta \gamma_S + \gamma) + t_A(2 \alpha^2(\beta - 1)^2 + 2 \alpha(\beta - 1)(2 \beta \gamma_S + 3 \gamma) + (\beta \gamma_S + \gamma)(3 \beta \gamma_S + \gamma) + 6t_C(\alpha(\beta - 1) - 3(\beta \gamma_S + \gamma)) - 36 t_A t_C\}/\{(2 \alpha + 2 A(\beta + 1) - \beta(2 \alpha + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha \beta - 2(\beta \gamma_S + \gamma)) - 36 t_A t_C\},$$

$$p_{2C}^{ST*} = \{-\alpha^3 + 2 \alpha^2 \gamma - A^3(\beta + 1)^3 + A^2(\beta + 1)((\beta + 1)(3 \alpha(\beta - 1) + 3 \beta \gamma_S + 2 \gamma) + (\beta + 3)t_C) +$$
\[ A(-\alpha(\beta^2 - 1)(3\alpha(\beta - 1) + 4\gamma) - 2\beta^2(\beta + 1)\gamma_S^2 - 2t_C(\alpha(\beta^2 + \beta - 2) + (3\beta + 2)(\beta\gamma_S + \gamma)) - 2\beta(\beta + 1)\gamma_s(3\alpha(\beta - 1) + \gamma) + 18(\beta + 1)t_At_C) + a^3\beta^3 - 3a^3\beta^2 + 2a^2\beta^2\gamma_S + a^2\beta^2t_C + 2\beta^2\gamma_S^2(\alpha(\beta - 1) + t_C) + 3a^3\beta - 4a^2\beta_S - 6t_At_C(3a(\beta - 1) + \beta\gamma_S - \gamma + 6t_C) - 2a^2\beta_S + \beta\gamma_S(3a(\beta - 1) + 2\gamma)(\alpha(\beta - 1) + 2t_C) + 6a\beta t_C + a^2t_C - 6a\gamma t_C + 2\gamma^2t_C\} / \{2(a + 2A(\beta + 1) - \beta(2a + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha - 2(\beta\gamma_S + \gamma)) - 36t_At_C\}, \]

\[ p_{2A}^{ST*} = \{2A^2(\beta + 1)\beta(\beta\gamma_S + \gamma) + (\beta + 1)t_A) + A(-\beta\gamma_S + \gamma)(2a(\beta - 1)(2\beta + 1) + (3\beta + 2)(\beta\gamma_S + \gamma)) - 2t_A((\beta + 1)(2a(\beta - 1) + 3\beta\gamma_S + 2\gamma) - 3(\beta - 1)t_C)) + (\beta\gamma_S + \gamma)(\alpha(\beta - 1) + \beta\gamma_S + \gamma)(2a(\beta - 1) + \beta\gamma_S + \gamma) + t_A(2a^2(\beta - 1)^2 + 2a(\beta - 1)(3\beta\gamma_S + 2\gamma) + (\beta\gamma_S + \gamma)(\beta\gamma_S + 3\gamma) - 6t_C(\alpha(\beta - 1) + 3(\beta\gamma_S + \gamma)) - 36t_A^2t_C\} / \{2(a + 2A(\beta + 1) - \beta(2a + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha - 2(\beta\gamma_S + \gamma)) - 36t_At_C\}, \]

\[ \Pi_1^{ST*} = \{-A^3(\beta + 1)^2 + A^2((\beta + 1)(2a(\beta - 1) + (\beta + 3)\gamma + \beta(\beta + 2)\gamma_S) + (\beta + 1)^2t_A + 3\beta t_C + t_C) + A(-\alpha^2 + 4a\gamma - \alpha^2\beta^2 - \beta\gamma_S(\alpha(2\beta^2 + \beta - 3) + (3\beta + 4)\gamma) - 2a\beta^2\gamma_S - (\beta + 1)\beta^2\gamma_S^2 + 2a^2\beta - 2a\beta\gamma - 2\beta\gamma^2 + t_A(3(\beta + 5)t_C - (\beta + 1)(2a(\beta - 1) + 3\beta\gamma_S + 2\gamma)) - t_C(3a(\beta - 1) + (3\beta + 2)(\beta\gamma_S + \gamma)) - 3\gamma^2 + t_A((\alpha(\beta - 1) + \gamma)(\alpha(\beta - 1) + 3\beta\gamma_S + \gamma) - 3t_C(\alpha(\beta - 1) + \beta\gamma_S + 5\gamma) - 18t_C^2) + (\beta\gamma_S + \gamma)((\alpha(\beta - 1) + \gamma)(\alpha(\beta - 1) + \beta\gamma_S + \gamma) + t_C(3a(\beta - 1) + \beta\gamma_S + \gamma)) - 18t_A^2t_C) / \{2(a + 2A(\beta + 1) - \beta(2a + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha - 2(\beta\gamma_S + \gamma)) - 36t_At_C\}, \]

\[ \Pi_2^{ST*} = \{-A^3(\beta(\beta + 1)^2 + A^2(-\alpha + 3\beta^3(\alpha + \gamma_S) + \beta^2(\alpha + 2\gamma + 4\gamma_S + t_C) + (\beta + 1)^2t_A + \beta(\gamma_S + 3(-\alpha + \gamma + t_C)) + \gamma) + A(t_A(3(5\beta + 1)t_C - (\beta + 1)(2a(\beta - 1) + 2\beta\gamma_S + 3\gamma)) - (\alpha(\beta - 1) + \beta\gamma_S + \gamma)(\alpha(\beta - 1)(3\beta + 2) + \beta((3\beta + 2)\gamma_S + \gamma) + (2\beta + 3)t_C + \gamma)) + t_A((\alpha(\beta - 1) + \beta\gamma_S)(\alpha(\beta - 1) + \beta\gamma_S + 3\gamma) - 3t_C(5a(\beta - 1) + 5\beta\gamma_S + \gamma) - 18t_C^2)) + (\alpha(\beta - 1) + \beta\gamma_S + \gamma)^2(\alpha(\beta - 1) + \beta\gamma_S + t_C) - 18t_A^2t_C) / \{2(a + 2A(\beta + 1) - \beta(2a + \gamma_S) - \gamma)(\alpha + A(\beta + 1) - \alpha - 2(\beta\gamma_S + \gamma)) - 36t_At_C\}. \]
Proofs of Propositions

Proof of Lemma 1

Proof. When the monopolist offers traditional advertising, we obtain the following comparative statics (recall \( A \geq \gamma_S \geq \gamma \)).

\[
\frac{\partial P^T_C}{\partial C} = \frac{(A-\gamma)(A+\gamma)}{(A-\gamma)^2 - 4At_C} > 0, \quad \frac{\partial P^T_A}{\partial C} = -\frac{2t_A(A+\gamma)(A0 - A0\gamma + 2At_u)}{(A-\gamma)^2 - 4At_C} < 0,
\]

\[
\frac{\partial P^S_C}{\partial C} = \frac{2t_C(A+\gamma)(u_0(A-\gamma)+2At_u)}{(A-\gamma)^2 - 4At_C} > 0, \quad \frac{\partial P^S_A}{\partial C} = -\frac{(A-\gamma)(A+\gamma)(u_0(A-\gamma)+2At_u)}{(A-\gamma)^2 - 4At_C} < 0.
\]

When the monopolist offers sponsored content advertising, we obtain the following comparative statics.

\[
\frac{\partial P^{S*}_C}{\partial C} = \frac{(\alpha+A\beta-\alpha\beta)^2 - \beta^2 \gamma_S^2}{(A+\beta(-A-\gamma_S))^2 - 4At_C} > 0, \quad \frac{\partial P^{S*}_A}{\partial C} = -\frac{2t_A(A+\beta(-A-\gamma_S)(A_0(\alpha+\beta(-A-\gamma_S)) + 2At_u)}{(A+\beta(-A-\gamma_S))^2 - 4At_C} < 0,
\]

\[
\frac{\partial P^{S*}_C}{\partial A} = \frac{2t_C(A+\beta(-A-\gamma_S)(u_0(A+\beta(-A-\gamma_S)) + 2At_u)}{(A+\beta(-A-\gamma_S))^2 - 4At_C} > 0, \quad \frac{\partial P^{S*}_A}{\partial A} = -\frac{(\alpha+A\beta-\alpha\beta)^2 - \beta^2 \gamma_S^2}{(A+\beta(-A-\gamma_S))^2 - 4At_C} < 0.
\]

\[
\square
\]

Proof of Proposition 1

Proof. \( \frac{\partial \Pi^{S*}}{\partial \beta} = -\frac{(A-A+\gamma_S)(A_0((1-\beta)A+\beta(A-\gamma_S)+2At_u)u_0((1-\beta)A+\beta(A-\gamma_S)+2At_u)}{(A+\beta(-A-\gamma_S))^2 - 4At_C} < 0. \) Because \( \Pi^{N*} \) does not depend on \( \beta \), it is easy to see that when \( \beta < \beta^* = \frac{A-A+\gamma_S}{A-A+\gamma_S} < 1, \Pi^{S*} > \Pi^{N*}. \)

When \((1-\beta)(A-A)+\gamma-\beta\gamma_S > 0, \) \( \frac{\partial \Pi^{S*} - \Pi^{N*}}{\partial t_C} = A_0((1-\beta)(A-A)+\gamma-\beta\gamma_S)(A_0((1-\beta)(A-A)+\gamma-\beta\gamma_S)+4At_u) > 0. \) This means that the profit difference is an increasing function of \( t_C. \)

Denote \( t^{*}_C = \frac{u_0(-A^2\beta A_0+A_0\beta(A+\gamma_S)+((1+2)A_0A_0)(A+\beta(A-\gamma_S))+(A_0A_0(\alpha(\alpha-1)(\beta+\gamma_S)+4At_u) - A_0A_0(1-\beta)(A+\gamma_S)+4At_u)}{A_0(1-\beta)(A-A+\gamma-\beta\gamma_S)+4At_u} \), so that \( \Pi^{S*} = \Pi^{N*} \) when \( t_C = t^*_C. \)

Therefore, when \( t_C \geq t^*_C, \Pi^{S*} \geq \Pi^{N*}. \)

\[
\square
\]

Proof of Proposition 2

Proof. \( \frac{\partial \Pi^{SS*}}{\partial \gamma_S} = (A - A + \gamma_S) > 0 \) because \( \alpha > A. \) \( \frac{\partial \Pi^{SS*}}{\partial \gamma_S} = \beta/2 > 0. \)

\[
\square
\]

Proof of Proposition 3

Proof. \( \Pi^{TT} - \Pi^{ST} = \)

\[
A(2-\beta)A(3A - (1-2A)\alpha - (t_A(1+\beta) + t_A(1-A)(1-\beta)\gamma - (1-\beta)\gamma^2 + \beta((1+\beta)t_A - (1-\beta)(t_A + A-A(1+\gamma)))\gamma_S + 2\beta^2 \gamma_S^2)) - \\
\frac{2t_A(2A+\gamma - (2A+\gamma))(A+\gamma - 2(2A+\gamma)) - 2t_A(2A+\gamma - (2A+\gamma))(A+\gamma - 2(2A+\gamma))}{A(1+\gamma - 2(2A+\gamma)) - 2t_A(2A+\gamma)} > 0 \) if \( t_A < \min\{t_A1, t_A2\} \)

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or \( t_A > \max\{t_{A1}, t_{A2}\} \) where
\[
t_{A1} = \frac{(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{36A C}, \quad t_{A2} = \frac{(1-\beta)(\alpha-A)(2a+b(-2a+2A+\gamma S))}{6(\alpha+A(\beta-1)-b(\alpha+\gamma S))}
\]

Similarly, \( \Pi_S^T - \Pi_T^S = \)
\[
\frac{\alpha^2((1+\beta)\beta^2-2(\beta\gamma S-\gamma))(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{2(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}-36A C C
\]
\[
\frac{\alpha(\alpha-A)(2A+\beta\gamma S+\gamma)}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))} > 0 \text{ if } t_{A2} < t_A < t_{A1} \text{ or } t_{A1} < t_A < t_{A3}
\]

where
\[
t_{A3} = \frac{(1-\beta)(\alpha-A)(2A+\beta\gamma S+\gamma)}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))}.
\]

Given our assumption \((1-\beta)(\alpha-A) + \gamma - \beta\gamma S > 0\), \( t_{A2} \) is always higher than \( t_{A3} \). Thus, \( TT \) will be the only equilibrium when \( t_A < \min\{t_{A1}, t_{A3}\} \) or \( t_A > \max\{t_{A1}, t_{A2}\} \). On the other hand, \( SS \) will be the unique equilibrium if \( t_{A2} < t_A < t_{A1} \) or \( t_{A1} < t_A < t_{A3} \). Lastly, multiple equilibrium will exist when \( t_{A3} < t_A < \min\{t_{A1}, t_{A2}\} \).

Also,
\[
P_{AT}^{ST} - P_{AT}^{TT} = \frac{\alpha^2((1+\beta)\beta^2-2(\beta\gamma S-\gamma))(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{2(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}-36A C C
\]
\[
\frac{\alpha(\alpha-A)(2A+\beta\gamma S+\gamma)}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))} > 0 \text{ if } t_A > \]
\[
t_{A4} \text{ given that } t_{C3} < t_C \text{ and } \gamma > \gamma_1
\]
\[
P_{AT}^{ST} - P_{AT}^{TT} = \frac{\alpha^2((1+\beta)\beta^2-2(\beta\gamma S-\gamma))(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{2(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}-36A C C
\]
\[
\frac{\alpha(\alpha-A)(2A+\beta\gamma S+\gamma)}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))} > 0 \text{ if } t_{A3} < t_C,
\]
\[
t_A < t_{A1} \text{ and } \gamma > \gamma_2
\]
\[
P_{AT}^{ST} - P_{AT}^{TT} = \frac{\alpha^2((1+\beta)\beta^2-2(\beta\gamma S-\gamma))(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{2(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}-36A C C
\]
\[
\frac{\alpha(\alpha-A)(2A+\beta\gamma S+\gamma)}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))} < 0 \text{ if } t_{C3} < t_C
\]

where
\[
t_{C3} = \frac{(\beta\gamma S-\gamma)(A(1+\beta)+\alpha(1-\beta)-2(\beta\gamma S+\gamma))}{6(\alpha-A(1-\beta))},
\]
\[
\gamma_1 = 2(A(1+\beta) + \alpha(1-\beta)) - \beta\gamma S,
\]
\[
\gamma_2 = 3(\alpha(1-\beta) - A(1-\beta)) + \beta\gamma S,
\]
\[
t_{A4} = \frac{2(2a+2A(\beta+1)-\beta(2a+\gamma S)S-\gamma)(\alpha+A(\beta+1)-\alpha\beta-2(\beta\gamma S+\gamma))}{6(\alpha-A(1-\beta)-b(\alpha+\gamma S))}.
\]

Thus as \( t_A \) increases (and \( t_{A2} > t_A > t_{A1} \) or \( t_{A1} > t_A > t_{A2} \), deviating from traditional
advertising strategy is profitable for a platforms when \( t_{C3} < t_C \) and \( \gamma_1 < \gamma < \gamma_2 \). The above analysis however shows that both the platforms will deviate from traditional advertising when \( t_{A2} > t_{A3} > t_A > t_{A1} \) or \( t_{A1} > t_A > t_{A2} \). When \( t_{A2} > t_A > t_{A3} > t_{A1} \), we instead have an asymmetric equilibrium (proved below).

\[
C < C \quad \text{and} \quad \gamma_1 < \gamma < \gamma_2.
\]

Proof of Corollary 1

Proof. When \( \beta = 0 \), the valid range for TT equilibrium is still non-empty based on the cutoff thresholds in Proposition 3. As a result, TT can still be the equilibrium.

Proof of Proposition 4

Proof. We know from Proposition 3 that when \( \max \{ t_{A2}, t_{A3} \} < t_A < t_{A1} \), SS is the unique equilibrium outcome. \( \Pi_i^{SS^*} - \Pi_i^{NN^*} = -[(1 - \beta)(\alpha - A) + \gamma - \beta \gamma_S]/2 < 0 \) when \( \alpha > A + (\beta \gamma_S - \gamma)/(1 - \beta) \). Therefore, SS is a prisoner’s dilemma outcome when this condition holds.

Proof of Proposition 5

Proof. Consumer surplus when both platforms offer traditional ads is \( CS(TT) = 2 \int_0^{1/2} (u - t_C x - p_{CT} - \gamma/2) dx = A - 5t_C/4 + u - \gamma/2 \). By contrast, consumer surplus when both platforms offer sponsored ads is \( CS(SS) = 2 \int_0^{1/2} (u - t_C x - p_{CS} - \beta \gamma_S/2) dx = \beta A + (1 - \beta)\alpha - 5t_C/4 + u - \beta \gamma_S/2 \).

Therefore, the difference in consumer surplus across the two advertising formats is \( CS(SS) - CS(TT) = (1 - \beta)(\alpha - A) + (\gamma - \beta \gamma_S)/2 \). This is positive when \( \gamma_S \leq [2(1 - \beta)(\alpha - A) + \gamma]/\beta \).

Advertiser surplus when both platforms offer traditional ads is \( AS(TT) = 2 \int_0^{1/2} (A_0 - t_A x - p_{AT} + A/2) dx = A_0 + A/2 - 5t_A/4 - \gamma \). By contrast, consumer surplus when both platforms offer sponsored ads is \( AS(SS) = 2 \int_0^{1/2} (A_0 - t_A x - p_{AS} + \beta A/2 + (1 - \beta)\alpha/2) dx = A_0 + \beta A/2 + (1 - \beta)\alpha/2 - 5t_A/4 - \beta \gamma_S \).

Therefore, the difference in consumer surplus across the two advertising formats is \( CS(SS) - CS(TT) = (1 - \beta)(\alpha - A)/2 + (\gamma - \beta \gamma_S) \). This is positive when \( \gamma_S \leq [(1 - \beta)(\alpha - A)/2 + \gamma]/\beta \).

It is easy to see that \( [2(1 - \beta)(\alpha - A) + \gamma]/\beta > [(1 - \beta)(\alpha - A)/2 + \gamma]/\beta \), so that it is more likely for consumer surplus to improve when platforms shift from traditional ads to sponsored ads.

Proof of Proposition 6
Proof. From proof of proposition 4, we can conclude that $\Pi^{\text{ST}}_i - \Pi^{\text{TT}}_i > 0$ if $t_{A2} > t_A > t_{A1}$. Similarly, we also see that $\Pi^{\text{TS}}_i - \Pi^{\text{SS}}_i > 0$ if $t_A > \max\{t_{A1}, t_{A3}\}$. Once we combine these two conditions, the resulting condition is $t_{A2} > t_A > \max\{t_{A1}, t_{A3}\}$. Therefore, as long as this condition is satisfied, in equilibrium one of the platforms will offer sponsored ads whereas the other platform will offer traditional ads.

Below we show why any other resulting condition will not hold.

(i) $\Pi^{\text{ST}}_i - \Pi^{\text{TT}}_i > 0$ if $t_{A1} > t_A > t_{A2}$ and $\Pi^{\text{TS}}_i - \Pi^{\text{SS}}_i > 0$ if $t_A < \min\{t_{A1}, t_{A3}\} =>$ $\min\{t_{A1}, t_{A3}\} > t_A > t_{A2} =$ this is not possible given our assumption $((1-\beta)(\alpha-A)+\gamma-\beta \gamma S > 0$, this assumption ensures that $t_{A2}$ is always greater than $t_{A3}$.

(ii) $\Pi^{\text{ST}}_i - \Pi^{\text{TT}}_i > 0$ if $t_{A1} > t_A > t_{A2}$ and $\Pi^{\text{TS}}_i - \Pi^{\text{SS}}_i > 0$ if $t_A > \max\{t_{A1}, t_{A3}\} =>$ this is not possible because of the inherent logical inconsistency.

(iii) $\Pi^{\text{ST}}_i - \Pi^{\text{TT}}_i > 0$ if $t_{A2} > t_A > t_{A1}$ and $\Pi^{\text{TS}}_i - \Pi^{\text{SS}}_i > 0$ if $t_A < \min\{t_{A1}, t_{A3}\} =>$ this is not possible because of the inherent logical inconsistency.

Also,

\[
P^{\text{ST}}_i - P^{\text{TT}}_i = \frac{2A^2(1-\beta^2)(\beta \gamma S + \gamma) + A(-1-\beta)(\beta \gamma S + \gamma)(4\alpha + \beta \gamma S + \gamma) + 2\beta(\beta + 1)\gamma S t_A - 2t_A(\beta \gamma S - 6(1-\beta)t_C)}{(2a+2A(\beta+1)-\beta(2a+\gamma S)-\gamma)(\alpha + A(\beta+1) - \alpha-2(\beta \gamma S + \gamma) - 36t_ATC)} \]

\[
+ \frac{\alpha(1-\beta)(\beta \gamma S + \gamma)(2a(1-\beta)+\beta \gamma S + \gamma)+2A((\beta \gamma S)(\alpha(1-\beta) + \beta \gamma S - \gamma) + 6(1-\beta)t_C)}{(2a+2A(\beta+1)-\beta(2a+\gamma S)-\gamma)(\alpha + A(\beta+1) - \alpha-2(\beta \gamma S + \gamma) - 36t_ATC)} > 0 \text{ if } t_A > t^*_A \text{ given that } t_{C3} < t_C.
\]

\[
P^{\text{ST}}_i - P^{\text{TT}}_i = \frac{A(1+\beta)((1+\beta)\gamma \beta S + 2(1-\beta)t_C)+2A(\beta \gamma S + \gamma)(4\alpha + \beta \gamma S + \gamma) + 2\beta(\beta + 1)\gamma S t_A - 2t_A(\beta \gamma S - 6(1-\beta)t_C)}{(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)(\alpha + A(1+\beta) - \alpha-2(\beta \gamma S + \gamma) - 36t_ATC)} \]

\[
- \frac{\alpha(1-\beta)(\gamma - \beta \gamma S)(-\alpha(1-\beta) + 2(\beta \gamma S + \gamma)) + 2t_C(\alpha(1-\beta)^2 + 4\beta \gamma S t_C - 6t_A)}{(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)(\alpha + A(1+\beta) - \alpha-2(\beta \gamma S + \gamma) - 36t_ATC)} < 0 \text{ if } t_A < t^*_A \text{ given that } t_{C3} < t_C.
\]

\[
x^{\text{ST}*}_i - x^{\text{TT}*}_i = \frac{2A^2(1-\beta^2) - A(1-\beta)(\beta \gamma S + \gamma) + 2a(1-\beta)^2 + 2\alpha(1-\beta)(\beta \gamma S + \gamma) + 6(1-\beta)t_C}{(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)(\alpha + A(1+\beta) - \alpha-2(\beta \gamma S + \gamma) - 36t_ATC)} > 0 \text{ if } t_A > \max\{t_{A1}, t^*_A\} \text{ given that } \beta \gamma S < \gamma.
\]

where,

\[
t^*_A = \frac{(1-\beta)(A+\alpha)(\beta \gamma S + \gamma)(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)}{2A(1+\beta)((1+\beta)\gamma \beta S - 6(1-\beta)t_C)+\beta(\beta \gamma S - \gamma)(\alpha(1-\beta) - \beta \gamma S - \gamma) - 6(1-\beta)t_C}
\]

\[
t^{**}_A = \frac{A^2(1+\beta)((1+\beta)\gamma \beta S - 2(1-\beta)t_C)+\beta(2(\beta \gamma S + \gamma) - \gamma - \beta \gamma S)(\alpha(1-\beta) + 2(\beta \gamma S + \gamma))}{2A^2(1+\beta)((1+\beta)\gamma \beta S - 2(1-\beta)t_C)+\beta(2(\beta \gamma S + \gamma) - \gamma - \beta \gamma S)(\alpha(1-\beta) + 2(\beta \gamma S + \gamma)) + 12(\gamma - \beta \gamma S)}
\]

\[
t^{***}_A = \frac{(1-\beta)(A+\alpha)(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)}{6(1-\beta)(A+\alpha)(2a+2A(1+\beta)-\beta(2a+\gamma S)-\gamma)}
\]