Price Manipulation in Peer-to-Peer Markets

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Abstract

Should a peer-to-peer platform set prices for the products on the platform, or should it let sellers set their own prices while providing price recommendations? Centralized prices allow the platform to use demand information it observes, while price recommendations allows for competition in which sellers set prices based on their private information.

We investigate the implications of each pricing regime for the profits of platform, buyers and sellers. When the platform recommends prices, it effectively plays the role of a sender in a multi-receiver cheap talk game.

We find that if the range of possible quality levels of sellers is small, the platform should centralize pricing. Price recommendations can be sustained in equilibrium only when the variance of aggregate demand is large. Otherwise, a price recommendation is not credible and the platform should let the sellers set their own prices. High (low) quality sellers have a stronger (weaker) preference for centralized pricing than the platform. Buyers, however, prefer centralized pricing only if the range of quality levels is large and the variance of aggregate demand is small. Otherwise, buyers prefer competing sellers who do not receive a price recommendation.

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1 Introduction

Platforms that operate peer-to-peer (P2P) markets can influence the interaction between sellers and buyers through their platform design. Among many examples, Lyft controls pricing to facilitate rides between riders (buyers) and drivers (sellers), Airbnb controls search results and recommends pricing to influence matching between hosts (sellers) who rent their houses to guests (buyers), and LendingClub assigns a credit worthiness score to borrowers (sellers) who are asking for a loan from investors (buyers).

Since the P2P structure is prevalent in many industries, it is not surprising that there is no single “one size fits all” market design of a P2P market. Platforms differ in many aspects including how search results are presented to buyers, their fee structure, how much choice buyers and sellers have and whether sellers can promote their offerings for an additional fee. Many of these differences arise from the choices platforms make when using information about consumer demand and seller competition to maximize their profits.

In many P2P markets, sellers often find it difficult to select prices for their products because of uncertainty about demand and competition. Equilibrium price levels, however, have a dramatic impact on the profits of the platform. Higher prices will lead to less transactions but with a higher margin, while low prices may increase the number of purchases but erode profit. For example, when Airbnb initially launched their platform, sellers were setting high prices that lowered the number of transactions, user satisfaction and platform revenue. Consequently Airbnb introduced a price recommendation tool for hosts in 2013 which they later improved in 2015 (Hill 2015). Because sellers set their prices based on beliefs they have about buyer demand, the platform can influence competition and price levels through supplying information to sellers or through coordinating prices directly.

A second factor that impacts the long-term profitability of the platform is the quality of matches achieved in realized transactions. Low-quality matches will lead to long-term churn of buyers who switch to competing options. The platform can use its information about buyer preferences and seller differentiation to influence the quality of matches offered to buyers, alongside the prices achieved in these transactions. In this paper we analyze how the information design of the platform can maximize its profits through manipulating the realized price levels, levels of competition and match quality.
Researchers recently devoted substantial attention to analyzing selling mechanisms on P2P platforms, with particular focus on using auctions vs. posted prices (Hammond 2010; 2013, Bauner 2015, Einav et al. 2015; 2018, Waisman 2017). Less attention was given to which party sets the prices on the platform. While Lyft’s algorithm sets the price for each ride (centralized pricing), Airbnb hosts and eBay sellers are free to set their own prices (competitive pricing). However, even in competitive markets, platforms sometimes participate by providing a price recommendation to sellers/hosts. Following its introduction of “price tips” in 2015, for example, Airbnb developed the recommendation tools further by introducing “smart pricing” in 2017, which lets the host set the maximum and the minimum price of a stay and the platform adjusts the price of the listing in response to predicted changes in demand.

When choosing whether to centralize pricing or let sellers compete, the platform has to consider two key factors. First is the amount of demand information the platform possesses relative to the information sellers have. In the case of Airbnb, for example, how good is the algorithm at predicting demand compared to the hosts themselves? If the platform decides to centralize pricing, the prices will not reflect the private information that sellers have. But if the platform chooses to let sellers compete without providing them with information, the sellers cannot use demand-related data available to the platform to assist their pricing decisions. Recommending prices may alleviate this trade-off partially; it lets the platform share some of its information with the sellers, while maintaining the flexibility of sellers to compete. As we will show, these choices may alter the level of competition in the market, influencing the equilibrium price levels and match quality. The second factor the platform needs to consider is that in a competitive market sellers will set relatively low prices, while centralized pricing allows the platform to extract more of the consumer surplus. A possible solution to this tradeoff is to let sellers compete while providing them with price recommendations that align with the platform’s goals. However, as we will show, price recommendations constitute cheap talk by the platform, posing a challenge to the usefulness of this strategy if the recommendations are not credible and ignored by the sellers.

Our goal in this paper is to describe when a platform would prefer to centralize or decentralize pricing and the implications of this decision for the platform, sellers and buyers. Our results can help platform designers make informed decisions regarding pricing regimes and information design. Because we also consider the effects of these choices on consumer surplus, our results can also help
guide policymakers and regulators considering the regulation and impact of P2P markets.

To study these questions, we construct a theoretical model of segmented competition between two differentiated sellers who sell imperfect substitutes on a platform to buyers. Buyers choose whether to buy a product in the market or pick an outside option. The choice that buyers have can be limited by the platform to buying from only one seller, or choosing from both. The platform also chooses whether to set prices for the sellers or to let the sellers compete with or without a price recommendation. If the sellers are allowed to compete, they can set prices for their product while taking the recommendation of the platform into account.

There are two sources of uncertainty in our model. First, each seller has private information about the quality of her product, which affects the utility buyers receive from the product. Second, there is an aggregate market-level shock to willingness to pay (e.g., how many visitors to a certain city are budget-conscious tourists and how many are business travelers with expense accounts). This shock is observed by the platform, but not by the sellers.

Like most real-life platforms, we assume that the platform receives a fixed-percentage fee of the sellers’ revenues. This means that while the platform wants to maximize the joint revenue of the sellers, each seller seeks to maximize their own profit and does not internalize the sales they take away from their competitor by lowering their price. Because of this misalignment of incentives between the sellers and the platforms, the sellers will not follow a price recommendation by the platform blindly. The sellers will form rational expectations of the platform’s strategy, i.e., in which state of the world the platform will choose to recommend each price. This messaging game where the platform recommends prices and the sellers have misaligned incentives is an instance of classic cheap talk (Crawford and Sobel 1982). However, unlike Crawford and Sobel (1982) and most of the cheap talk literature, our model has multiple receivers (sellers) that interact with each other (i.e., compete) and the outcome of this interaction determines the payoff of the sender. In a standard cheap talk game, the misalignment in incentives between the sender and the receiver is exogenous, while in our model it is endogenous and stems from the difference in market power between the platform and the sellers, as well as the level of competition among the sellers. Both of these are influenced by the price recommendations of the platform.

We find that the platform should choose to centralize pricing if there is little uncertainty about the quality of sellers. On the other hand, if this uncertainty is large and the variance of the
aggregate shock observed by the platform is large, the platform should recommend a price. If the variance of the aggregate shock is small, price recommendations cannot be credible in equilibrium and the platform should let the sellers set their own prices. The intuition is that the agents that posses the more valuable demand information should set the prices that reflect that information. As an example, consider an Airbnb market in a particular city. The willingness to pay of buyers depends on how many business travelers are looking to book in this market, which the platform can observe. The private information each seller (host) has is the (unobserved by the platform) quality of apartment offered. For example the quality of the view is often hard for platforms to assess even with sophisticated photo analysis algorithms. If both sources of uncertainty are small, e.g., it is either a place where people only come for business that has no good views at all or a place where all views are great and that attracts primarily tourists, the platform should centralize pricing to leverage its position as a monopolist. If both sources of uncertainty are large, e.g., there are different kinds of travelers in this market and some apartments can have extremely nice views while others are facing a brick wall, the platform should recommend a price and allow sellers to use both sources of information for pricing.

High quality sellers surprisingly exhibit a stronger preference for centralized pricing, while low quality sellers have a stronger preference for recommendation and competition. These differences stem from how competition affects the levels of demand and prices when a high quality seller faces a low quality seller.

We also find that buyer surplus is almost always maximized under competitive pricing by the sellers. Unlike sellers, buyers do not benefit when prices adjust with the state of the demand. If the price is too high, buyers can always take an outside option with a limited downside. But if the price is low when demand is high, they will enjoy a large surplus. Therefore, buyers are better off when prices respond the least to changing demand. In most cases, this happens under competitive pricing, as the sellers only take into account the information about their own quality. Price recommendations always hurt buyers compared to the fully competitive case, as it increases price variance without lowering the average price. Under centralized pricing, the average price is higher, but in extreme cases when the variance of the aggregate shock is small (all visitors are tourists), but the range of possible qualities is large (the views are superb or terrible), then the buyers prefer centralized pricing.
Our paper has implications for the design of platforms and analysis of the impact of platforms on competition and market prices. Unlike much of the previous research, we find that centralizing prices through a platform is not always profit maximizing for both the platforms and sellers. Moreover, there are cases where price recommendations from a platform may not be credible, and the platform might be better off not offering them at all. These choices of platform design can have substantial impact on the long term success of firms.

2 Contribution to Literature

Our work contributes to three streams of literature. First, our paper adds to the growing literature on P2P platforms and their design (Einav et al. 2016, Horton and Zeckhauser 2016, Zervas et al. 2017, Ke et al. 2017, Fradkin 2017, Fradkin et al. 2018). Within this literature, the research on pricing decisions have mostly looked at using auctions vs. posted prices. Auctions, however, are not a natural choice for many platforms, which is why we focus on two common mechanisms - centralized pricing where the platform sets pricing for all sellers vs. competitive pricing where the platform allows sellers to set their own prices, with or without a price recommendation.

Second, we add to the theoretical work on strategic communication (Milgrom 1981, 2008, Crawford and Sobel 1982, Sobel 2013), which has recently been applied in marketing contexts on persuasive communication (Gardete 2013, Chakraborty and Harbaugh 2014). Interestingly, cheap talk has rarely been applied to the analysis of a market with many competing receivers. The work of Kim and Kircher (2015), for example, has many senders who send cheap talk messages, while our work looks at a sender trying to coordinate a market using cheap talk. We prove that the results of Crawford and Sobel (1982) are robust to introducing competition in our model: we find that all possible equilibria have “coarse” communication, i.e., the platform recommends a range of prices instead of a single price and the platform (the sender) and the sellers (receivers) benefit when more fine-grained communication is possible.

Finally, our paper is related to the literature on oligopolistic competition under uncertainty (Klemperer and Meyer 1986, 1989, Gardete 2016). While in these works the level of competition is determined by exogenous uncertainty competitors face, our research extends these works to a scenario in which a market designer can control the level of uncertainty facing competitors and thus influence the level of the competition to her benefit.
3 Model and Centralized Pricing Benchmark

3.1 Setup

A mass of buyers who are distributed uniformly on the entire real line visit a P2P platform to buy a product from two potential sellers. Seller 1 is positioned at $-1$ and seller 2 is positioned at 1. A buyer located at $x \in \mathbb{R}$ has demand for up to one product. If they choose to buy a product from seller $i$, similar to a classic Hotelling model, their utility is:

$$u_i(x) = v + q_i - p_i - td_i(x)$$  \hspace{1cm} (1)

Two factors affect the utility that the buyer receives net of price and distance: the average willingness to pay $v$, and the quality difference of the product $q_i$. The willingness to pay $v$ captures the average utility buyers receive for the product, and is drawn from a uniform distribution $U[v, \bar{v}]$ with $v > 0$. If buyers are business travelers, for example, their willingness to pay might be higher than budget conscious tourists. A platform will be able to observe if searches for listings in a specific city, for example, come mostly from business travelers, or if a big conference is planned to be held in town. Buyers will also know their realization of $v$ before buying the product. Sellers, in contrast, are not exposed to the search process on the platform, and hence will have less information about $v$. For simplicity, we assume that the platform observes the realization of $v$ and the sellers do not.

We assume that the quality difference $q_i \in \{-q, q\}$ with $q > 0$ is private information of seller $i$, known to buyers, and has a-priori equal probability of being $q$ or $-q$. We will call a seller with $q_i = q$ an $H$-type seller, and with $q_i = -q$ an $L$-type. The quality difference captures various factors that distinguish products for buyers and sellers that can’t be observed by the platform. For example, Airbnb’s algorithm may find it hard to estimate the quality of the view of a listing, or to infer issues in a listing mentioned in textual reviews. Higher values of $q$ imply that there is more potential heterogeneity in quality in the products available for purchase.

The factor $t$ captures the disutility of mismatch between seller and buyer preferences and is common knowledge. $d_i(x)$ is the distance from seller $i$ to the buyer’s location $x$. Because we assumed, w.l.o.g., that seller 1 is located at $-1$ and seller 2 is at 1, the distances equal $d_1(x) = |x+1|$ and $d_2(x) = |1-x|$. Finally, we assume that the outside option is the same for all buyers with utility normalized to 0. The outside option can capture the utility from going to a competitor (taxi,
Before visiting the platform, buyers are not aware of the sellers and hence cannot buy from them. Once on the platform, buyers use search tools to find their preferred product. We assume a simple search technology: a share \( \alpha < 1/2 \) of buyers discover seller 1 only, another group of size \( \alpha \) discover seller 2 only, and the remaining mass of \( 1 - 2\alpha \) buyers discovers both sellers. We call the first two groups “captives” as they can only buy from one seller or pick the outside option. We call the buyers who are aware of both sellers “comparison shoppers” (or shoppers). Each buyer picks the option that gives them the highest utility from the sellers they are aware of and the outside option.

We assume that sellers have zero marginal costs, and that the platform receives a fixed share \( \phi \) of the sellers’ profits that is set exogenously. This revenue-sharing arrangement mimics many of the contracts in the P2P universe. Hence seller \( i \)’s profit is \((1 - \phi)p_iD_i(p_1, p_2)\) and the platform receives payoff \( \phi(p_1D_1(p_1, p_2) + p_2D_2(p_1, p_2)) \), where \( D_i(p_1, p_2) \) is the realized demand—the total mass of consumers buying from seller \( i \).

Because sellers maximize their individual profit, while the platform maximizes the joint profit, the platform’s payoff is maximized if both sellers set the monopoly price. The sellers, in contrast, have incentives to lower their prices in order to capture a larger share of the market. Hence the incentives of the platform and the sellers are not perfectly aligned with respect to setting prices. A second factor that affects the profit of the sellers and the platform is information asymmetry. If the platform sets the prices, it cannot use the information that the sellers have about \( q_i \). If the sellers set prices, they are uncertain about the willingness to pay \( v \) unless they receive information from the platform.

Our goal is to analyze the different strategies the platform can take with respect to pricing and information sharing. Specifically, we will analyze the following three cases in terms of platform and seller profits, consumer surplus, and quality of matching:

1. Centralized Pricing - The platform sets prices for both sellers.
2. Competition - The platform lets the sellers set their own prices without providing them information.
3. Recommendation - The platform recommends a non-binding price to the sellers, and each
seller sets their own price.

As we discuss in detail later, any message the platform sends to sellers is effectively a signal about the value of $v$, because $v$ is the only payoff-relevant private information that the platform has. In other words, all price recommendations that the platform can provide in our model are functions of $v$ and are therefore isomorphic to a “direct” message communicating the value of $v$.

The three cases above cover the full range of actions the platform can take to influence the sellers in our model. In some cases, however, the platform may be reluctant or unable to adopt one of these strategies. One example is regulatory constraints on employment. If sellers are service providers (for example, in a freelance market such as Upwork.com), setting project rates might increase the liability of the platform by turning it into an employer. Thus, even in cases when one of the above strategies is not profit maximizing for the platform, it might be the most profitable under specific constraints.

To finalize the model, the timing of the game is as follows:

1. The platform selects a pricing and signaling strategy.

2. Nature draws $v$ (observed by the platform and buyers), $q_1$ (observed by seller 1) and $q_2$ (observed by seller 2).

3. The platform gives a price recommendation to the sellers if it decided to do so.

4. Prices are set (by the platform or by the sellers).

5. Buyers visit the platform. A random set of mass $\alpha$ buyers learn $q_1$, $d_1$ and $p_1$. A second random non-overlapping set of mass $\alpha$ learn similar details about seller 2. The remaining mass of $1 - 2\alpha$ buyers learn $q_i$, $d_i$ and $p_i$ for both sellers.

6. Buyers make their purchase decisions and payoffs are realized.

At step 3, if the platform elects to not recommend a price, we can assume that it is giving an uninformative recommendation (e.g., a random price independent of the state of the world $v$ or the same price in every state of the world).
3.2 Centralized pricing benchmark

We begin the analysis with the benchmark case of a platform that sets centralized prices for sellers. The analysis allows us to illustrate the mechanics of the model and the loss of efficiency from lack of communication. The results will serve as a benchmark compared to platforms that let sellers set their own prices. We first find the buyers’ demand for each awareness segment, and then calculate the optimal prices and equilibrium profit given this demand. All proofs in the paper are relegated to the Appendix.

For captive consumers, each buyer chooses between seller $i$ and the outside option. There are two buyers who are indifferent between buying and not buying, equidistant to the left and to the right of $-1$ (seller 1) or 1 (seller 2). The demand from captives is then the mass of buyers between these two points:

$$D^c_i(p_i) = \frac{2(v + q_i - p_i)}{t}$$  \hspace{1cm} (2)

To find the demand from comparison shoppers, we first make an assumption that facilitates parsimony of the analysis:

**Assumption A1.** (i) $v > 2q + (3 - 2\alpha)t$ and (ii) $q < t$

The first part of Assumption A1 is a standard full coverage assumption for shoppers in the $[-1, 1]$ interval, and makes sure these shoppers buy from either firm 1 or 2. The second part implies that the difference in quality between sellers is not so high that comparison shoppers always buy only from one seller. Effectively, it guarantees that if the prices set by the sellers are equal, each seller will receive some demand from comparison shoppers even if qualities are different. Relaxing these assumptions will not change the results qualitatively, but will make the analysis less tractable.

Assumption A1 implies that comparison shoppers to the left of $-1$ or the right of 1 will never buy from the seller farthest from them, while shoppers in the interval $[-1, 1]$ will never pick the outside option. The resulting demand of shoppers is:

$$D^s_i(p_i, p_{-i}) = \frac{q_i - q_{-i} - p_i + p_{-i} + 2t}{2t} + \frac{v + q_i - p_i}{t}$$  \hspace{1cm} (3)

where the left hand side of the sum is the demand from shoppers in the $[-1, 1]$ interval, and the
right hand side are shoppers outside this interval.

Combining the demands from both segments the total demand for seller \( i \) is:

\[
D_i(p_i, p_{-i}) = \frac{v + q_i - p_i}{t} + (1 - 2\alpha) \frac{q_i - q_{-i} - p_i + p_{-i} + 2t}{2t} \tag{4}
\]

We can gain additional insight about the structure of demand by collecting terms and rewriting it as:

\[
D_i(p_i, p_{-i}) = \frac{2v + (3 - 2\alpha)q_i - (1 - 2\alpha)q_{-i} + 2t(1 - 2\alpha)}{2t} - \frac{3 - 2\alpha}{2t} p_i + \frac{1 - 2\alpha}{2t} p_{-i} \tag{5}
\]

This is a special case of the linear differentiated demand model often used in research on oligopolies under uncertainty [Klemperer and Meyer [1986] 1989]. Our model provides a micro-foundation for this reduced form demand model. The differentiation in the model stems from the difference in price sensitivities of consumers buying from a specific firm. Because each firm’s own price influences also the captive segment, the demand each firm sees is more elastic with respect to changes in its own price compared to changes in the competitor’s price.

Using the demand of each firm, the platform’s expected profit \( \mathbb{E}(\pi_P(p_1, p_2)) \) is:

\[
\mathbb{E}(\pi_P(p_1, p_2)) = \phi \mathbb{E}_{q_1, q_2}(p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2)) \tag{6}
\]

Solving for the profit-maximizing price results in the following:

**Proposition 1.**

- **The unique profit-maximizing centralized price is:** \( p_1^* = p_2^* = p^*(v) = \frac{v + (1 - 2\alpha)t}{2} \).

  The optimal centralized price increases with \( v \) and \( t \), but decreases with \( \alpha \).

- **The maximum centralized profit is:** \( \pi_P^{CP} = \phi \frac{(v + (1 - 2\alpha)t)^2}{2t} \).

  The optimal centralized profit increases in \( v \) and decreases with \( t \) and with \( \alpha \).

- **The maximum expected centralized profit is:**

\[
\mathbb{E}(\pi_P^{CP}) = \phi \frac{(\bar{v} + t(1 - 2\alpha))^3 - (v + t(1 - 2\alpha))^3}{6t(\bar{v} - v)}
\]


The expected centralized profit increases in $\bar{v}$ and $v$, and decreases with $t$ and with $\alpha$.

- The ex ante expected profit of a high type and low type seller is:

$$E(\pi_{CP}^H) = (1 - \phi) \left( \frac{E(\pi_{CP}^P)}{2\phi} + \frac{q(3 - 2\alpha)(\bar{v} + v + 2t(1 - 2\alpha))}{8t} \right)$$  \hspace{1cm} (7)

$$E(\pi_{CP}^L) = (1 - \phi) \left( \frac{E(\pi_{CP}^P)}{2\phi} - \frac{q(3 - 2\alpha)(\bar{v} + v + 2t(1 - 2\alpha))}{8t} \right)$$  \hspace{1cm} (8)

Proposition 1 shows that prices and profits increase, as expected, with the average willingness to pay $v$. It is interesting that the optimal prices increase with the disutility of mismatch $t$, but profits decrease with $t$. The intuition is that as $t$ increases, the demand from captive buyers may decrease as they shift to the outside option, but the demand from shoppers remains stable. An increase in $t$ allows to better monetize shoppers by increasing prices, but the overall effect is a decrease in profit because of the demand effect on captives.

A surprising result is that profit decreases with $\alpha$, as intuition might suggest that the platform could gain the most by exposing each buyer to only one product, and use monopoly pricing for each product. This intuition breaks when the platform has uncertainty over $q_i$. When the sellers differ in quality, if most buyers are captives (high $\alpha$), those aware of the low quality seller will only buy if the price is low. The platform, however, is constrained to setting the same price for both products. If these buyers are made aware about the other seller, however, they may be willing to make a purchase at a higher price from a high quality seller. Hence, it is in the interest of the platform to make more buyers informed and decrease $\alpha$.

4 Competition Manipulation with Cheap Talk

We now turn to the main analysis of the paper — a platform that is designed to allow sellers to set their own prices and compete. As mentioned before, the literature on platforms has generally found that coordinating prices through a central point of sale is often beneficial for the platform as it softens competition between sellers and increases sale prices.

In order to set their own prices, sellers will need to integrate over their beliefs about $v$ to maximize their own profits (Equation (5)). The platform can try to influence the sellers’ decision by providing them with a price recommendation that the sellers will incorporate into their decisions.
Providing a price recommendation and providing information about the value of \( v \) are equivalent because setting prices is the only action sellers can take, and \( v \) is the only missing piece of information sellers need from the platform. If the platform chooses to provide (possibly inaccurate) information about \( v \), the sellers can back-out the real value of \( v \) consistent with an equilibrium strategy of the platform, and make a pricing decision. Similarly, if the platform provides a price recommendation (and not a direct message about the value of \( v \)), the sellers will infer the values of \( v \) which are consistent (in equilibrium) with the platform’s recommendation.

We therefore assume that the platform’s strategy is a (possibly non-deterministic) mapping from the interval of possible realizations of \( v \in [v_1, v_2] \) to a message space on the same interval. In other words, the platform observes the realization of \( v \) and reports to the sellers some plausible value \( m(v) \in [v_1, v_2] \), which may or may not coincide with the actual realization.

An important feature of our model is that the message \( m(v) \) is costless for the platform (the Sender) to send, and that the platform’s incentives are misaligned with the sellers (the Receivers). Sellers have an incentive to lower prices to respond to competition and maximize their own profits, while the platform would like sellers to maximize their joint profit, which often means increasing their prices from a competitive level. This is an instance of a cheap talk game (Crawford and Sobel 1982), but unlike the extant cheap talk literature, our model features multiple receivers who interact strategically with each other. Our analysis also tries to answer whether cheap talk can be both a credible and a profitable equilibrium strategy with competing receivers.

A second interesting insight is that a “babbling equilibrium”, which is one of the possible equilibria of cheap talk games exists in our model. In such an equilibrium, the message sent by the platform is uninformative, i.e., it is statistically independent of the realization of \( v \). Examples of such strategies would be to always recommend the same price, or to report a random value of \( v \) to senders. The sellers will then ignore the message and rely on their prior beliefs over \( v \) when setting prices. This set of strategies is an equilibrium in our model since given this strategy of the sellers, any of the platform’s strategies will lead to the same payoff.

The babbling equilibrium is equivalent to the case when the platform refrains from giving a price recommendation and lets the sellers set their prices based on their private information and the prior information that they have. Effectively, letting sellers compete on prices without providing price recommendations is nested within the cheap talk setup as the babbling equilibrium. An example of
such strategy was AirBnB before the introduction of price tips and smart pricing. In that market, hosts were asked to set prices without being provided any guidance by the platform.

Because the babbling equilibrium is a unique case in our setup, we first analyze it, and then analyze the full set of cheap talk equilibria.

4.1 Competition without Price Recommendation

Suppose the platform provides a message $m \in [v, \overline{v}]$ to the sellers. We assume that the message is not informative about $v$, hence it is equivalent to not recommending any price.

Seller $i$ with type $\tau$ will set a price $p_{i\tau}$ to maximize their expected profit. We look for a symmetric subgame perfect equilibrium, and hence can denote the equilibrium strategy of sellers as $p_{NR\tau}$ for type $\tau \in \{H, L\}$ and drop the subscript $i$. The profit of a seller of type $\tau$ who sets a price $p_\tau$ (not necessarily equal to $p_{NR\tau}$) is:

$$E(\pi_\tau|m) = \frac{1}{2}(1 - \phi)p_\tau \mathbb{E}(D_{\tau}(p_\tau, p_{HNR}^\tau) + D_{\tau}(p_\tau, p_{LNR}^\tau))|m)$$

where the expectation is taken over the distribution of $v$ given the message $m$, and $D_{i\tau}$ denotes the demand of a seller with type $\tau$.

To solve for the equilibrium strategies, we note that the demand is linear in $v$, and hence we can replace each occurrence of $v$ with $E(v|m)$ in the profit functions. Because the message $m$ is uninformative, sellers will not update their beliefs about $v$. Hence $E(v|m) = E(v) = \frac{\overline{v} + \underline{v}}{2}$.

We then impose the restriction that $p_\tau$ is equal to $p_{\star\tau}$ on the first order conditions, and find the following:

**Proposition 2.** In an equilibrium where the platform sends an uninformative signal to the sellers:

- Equilibrium prices are:

  $$p_{NRH} = \frac{q}{2} + \frac{2E(v) + 2(1 - 2\alpha)t}{5 - 2\alpha}$$

  $$p_{NRL} = -\frac{q}{2} + \frac{2E(v) + 2(1 - 2\alpha)t}{5 - 2\alpha}$$

The prices increase with $t$. The price $p_{NRH}$ increases with $q$ and $p_{NRL}$ decreases with $q$. Both prices increase in $\alpha$ if $v_H + v_L > 8t$ and decrease in $\alpha$ otherwise. The prices $p_{NRL}$ and $p_{NRH}$
are higher than the centralized price when \( v_H \) is high enough and \( v \) is low enough.

- **The platform’s profits are**

\[
\pi_{NR}^P = \phi \frac{q^2(3 - 2\alpha)}{4t} - \phi \frac{16(\mathbb{E}(v) + (1 - 2\alpha)t)(2\mathbb{E}(v) - (5 - 2\alpha)v - t(3 - 8\alpha + 4\alpha^2))}{4t(5 - 2\alpha)^2} \tag{12}
\]

The profits of the platform are higher than under centralized pricing when

\[
q > \sqrt{\frac{2}{3 - 2\alpha}} \left( v + \frac{t(1 - 2\alpha)^2 - 2(\bar{v} + \underline{v})}{5 - 2\alpha} \right)
\]

- **The platform’s ex ante profits are:**

\[
\mathbb{E}(\pi_{NR}^P) = \phi(3 - 2\alpha)(4(\bar{v} + \underline{v} + 2t(1 - 2\alpha)) + q^2(5 - 2\alpha)^2) \\
\quad - \frac{8t(5 - 2\alpha)^2}{4t(5 - 2\alpha)^2} \tag{13}
\]

There is a \( \bar{q} \) such that the platform makes a higher ex-ante profit without a price recommendation compared to centralized pricing if and only if \( q > \bar{q} \).

- **The sellers’ ex ante profits are:**

\[
\mathbb{E}(\pi_{NR}^H) = \frac{(1 - \phi)(3 - 2\alpha)(2(\bar{v} + \underline{v} + 2t(1 - 2\alpha)) + (5 - 2\alpha)q)^2}{8t(5 - 2\alpha)^2} \tag{14}
\]

\[
\mathbb{E}(\pi_{NR}^L) = \frac{(1 - \phi)(3 - 2\alpha)(2(\bar{v} + \underline{v} + 2t(1 - 2\alpha)) - (5 - 2\alpha)q)^2}{8t(5 - 2\alpha)^2} \tag{15}
\]

There is a \( \bar{q}_H \) (\( \bar{q}_L \)) such that a high (low) quality seller makes a higher ex-ante profit without a price recommendation compared to centralized pricing if and only if \( q > \bar{q}_H \) (\( q > \bar{q}_L \)).

Proposition 2 shows that prices are linear in the beliefs of the sellers about the expectation of \( v \). When we analyze cheap talk next, this feature will come into play as the platform will want to influence the resulting equilibrium prices through influencing the beliefs of sellers.

The proposition has two interesting findings. First, prices under competition may be higher than prices set by a centralized planner. When \( v_H \) is high, and the realization of \( v \) is low, sellers will set too high prices because they expect higher demand than what is realized, and as a result will lower the platform’s profit. If the platform could affect the beliefs of sellers about \( v \), it might be able to better influence this competition to its benefit.
The second interesting finding is that in scenarios where \( q \) is high, and there is substantial uncertainty about the difference in seller quality, it is more beneficial for the platform to let sellers compete than to set prices for them. Effectively, the pricing power should lie with the players that hold the most uncertain information. In the next section we turn to analyze price recommendations, through a platform can influence the amount of information sellers have about \( v \), and thus control when it would be beneficial to let sellers compete.

4.2 Cheap Talk Recommendation

How can a platform utilize price recommendations profitably? Suppose the platform discloses a message \( s(v) : [\underline{v}, \overline{v}] \rightarrow [\underline{v}, \overline{v}] \) to sellers. As discussed before, messages about the value of \( v \) are equivalent to price recommendations. Because the relevant information being transmitted is the realization of \( v \), sellers will then map any message \( s(v) \) into a value in \([\underline{v}, \overline{v}]\), and hence any equilibrium with a different message space is equivalent to an equilibrium with the message space \([\underline{v}, \overline{v}]\).

To understand what actions the platform should take, we first analyze the response of sellers to a message \( m \) in the pricing subgame. When receiving a recommendation \( m \), sellers will update their beliefs about the distribution of \( v \) and set prices as shown in the following Lemma:

**Lemma 1.** Follows a message \( m \), the unique equilibrium prices sellers set are:

\[
p_R^H = \frac{q}{2} + \frac{2\mathbb{E}(v|m) + 2(1 - 2\alpha)t}{(5 - 2\alpha)}
\]

\[
p_R^L = -\frac{q}{2} + \frac{2\mathbb{E}(v|m) + 2(1 - 2\alpha)t}{(5 - 2\alpha)}
\]

where \( \mathbb{E}(v|m) = \frac{\int_{v:s(v)=m} vdv}{\int_{v:s(v)=m} dv} \)

The Lemma shows that the prices set by the sellers depend on the message \( m \) only through the belief about the expectation of \( v \). These updated beliefs follow from Bayesian updating. When updating from \( \mathbb{E}(v) \) to \( \mathbb{E}(v|m) \), sellers will take into account the equilibrium strategy \( s(v) \) used by the platform to narrow the possible values of \( v \) to those consistent with the message \( m \).

Because the platform influences the decision of the sellers by setting \( v \), we can calculate the the
profit of the platform as a function of the true state $v$ and the seller’s expectations induced by $m$:

$$\pi^R_{P}(v,m) = \frac{q^2(3-2\alpha)}{4t} - \frac{16(E(v|m) + (1-2\alpha)t)(2E(v|m) - (5-2\alpha)v - t(3-8\alpha + 4\alpha^2))}{4t(5-2\alpha)^2}$$

$\pi^R(v,m)$ is quadratic in $E(v|m)$ and linear in $v$. Consequently every state $v$ has a value $E^*(v)$ that maximizes the payoff of the platform:

$$E^*(v) = \frac{v(5-2\alpha) + t(1-2\alpha)^2}{4}$$  \hspace{1cm} (18)

This expectation does not equal to $v$ itself and is in fact always larger than $v$, hence the platform would like to inflate the sellers’ expectations of $v$ through the recommendation. However, as sellers are rational and anticipate this strategy of the platform, that is impossible.

Given this limitation, we show in the next Lemma (based on Crawford and Sobel (1982)’s Lemma 1) that only a finite set of of beliefs can be induced in equilibrium, which implies that the true value of $v$ cannot be communicated, and only an indication of ranges of values of $v$ can be sent as a message:

**Lemma 2.** If for every message $m$ the values $v \neq E^*(v|m)$, then there exists an $\varepsilon > 0$, such that for any two equilibrium messages $m_1$ and $m_2$ that induce different beliefs $E(v|m_1)$ and $E(v|m_2)$, the difference is at least $\varepsilon$, i.e., $|E(v|m_1) - E(v|m_2)| > \varepsilon$. Moreover, the set of expectations that can be induced in equilibrium is finite.

*Proof.* See Appendix.

Lemma 2 shows that whenever two messages induce different equilibrium beliefs, those beliefs will have at least some minimal distance between them. In order words, the platform cannot induce a continuous set of beliefs and will have “jumps” between them. The intuition behind this result is that because the platform’s incentives and the seller incentives differ, the platform will want to deviate from revealing the value of $v$ and send a message that induces an expectation closer to $E^*(v)$. To induce these higher beliefs, the platform needs a large enough jump from the true value. Because the message space is bounded and because there are jumps between beliefs, this means that there is a finite number of induced expectations possible equilibrium. The consequence of Lemma 2 is that the true value of $v$ cannot be communicated in equilibrium.
Given that there is no full revelation, we follow Crawford and Sobel (1982) to find an equilibrium in which the state space is partitioned into $n$ subintervals $[v, v_1], [v_1, v_2], \ldots, [v_{n-1}, \bar{v}]$ and the platform reveals to the sellers in which interval the realization of $v$ lies. Suppose that the realized state is $v \in [v_k, v_{k+1}]$. Let $m_k$ denote the message sent by communicating a random value drawn from $U[v_k, v_{k+1}]$. Hence, the message $m_k$ can be any value from the interval it represents, which rules out possible out-of-equilibrium beliefs.

Using Lemma 1, the equilibrium belief that determines the prices will be $E(v|m_k) = \frac{v_{k-1} + v_k}{2}$. To find the boundaries $v_k$ between the subintervals of the message space, we notice that if the true value is $v = v_k$, the platform should be indifferent between sending the messages $m_k - 1$ and $m_k$. We can write this indifference condition as:

$$\pi^R(v_k, m_{k-1}) = \pi^R(v_k, m_k), k = 1, \ldots, n - 1 \quad (19)$$

which can be rewritten as the following difference equation:

$$v_k = \frac{v_{k+1} + v_{k-1} - t(1 - 2\alpha)^2}{3 - 2\alpha} \quad (20)$$

with boundary conditions $v_0 = \underline{v}$ and $v_n = \bar{v}$.

The unique solution of equation (20) is:

$$v_k = C_1\lambda_1^k + C_2\lambda_2^k + v^* \quad (21)$$

where

$$v^* = -t(1 - 2\alpha)$$

$$\lambda_{1,2} = \frac{3 - 2\alpha \pm \sqrt{(3 - 2\alpha)^2 - 4}}{2}$$

$$C_1 = \frac{\bar{v} - v^* - \lambda_2^n(v - v^*)}{\lambda_1^n - \lambda_2^n}$$

$$C_2 = \frac{\lambda_1^n(v - v^*) - (\bar{v} - v^*)}{\lambda_1^n - \lambda_2^n}$$

This unique solution determines the interval boundaries $v_k$ for messages sent by the platform to reveal information about the value $v$ and recommend a price.
Once we know how to find the boundaries that determine messages, a second value that determines the equilibrium is the number of intervals \( n \). How large can \( n \) be? As \( n \) becomes larger, we approach full revelation, which was ruled out by Lemma 2. The fact that \( v_{k+1} \) has to be greater than \( v_k \) for every \( k \) allows us to write a condition that determines the maximum \( n \) possible:

\[
\frac{\overline{v} - v^*}{\underline{v} - v^*} (\lambda_1 - \lambda_2) > \lambda_1^\alpha (1 - \lambda_2) + \lambda_2^\alpha (\lambda_1 - 1)
\]  

(22)

These results are summarized in the following proposition.

**Proposition 3.** When there is a natural number \( n^* > 1 \) such that condition (22) holds, then there is a price recommendation equilibrium. In this equilibrium \([v, \overline{v}]\) is divided into \( n^* \) subintervals \([v, v_1], [v_1, v_2], \ldots, [v_{n^* - 1}, \overline{v}]\), where \( v_k \) is defined by equation (21). When \( v \in [v_k, v_{k+1}] \), the platform draws a value from \( U[v_k, v_{k+1}] \) and sends that value as a message to the sellers.

In the price recommendation equilibrium:

- **Equilibrium prices with a message from the subinterval \([v_k, v_{k+1}]\) are:**

\[
p^R_H|k = \frac{q}{2} + \frac{v_k + v_{k+1} + 2(1 - 2\alpha)t}{(5 - 2\alpha)}
\]

(23)

\[
p^R_L|k = -\frac{q}{2} + \frac{v_k + v_{k+1} + 2(1 - 2\alpha)t}{(5 - 2\alpha)}
\]

(24)

- **The ex ante expected equilibrium profits are:**

\[
\mathbb{E}(\pi^R_H(v)) = \phi \sum_{k=1}^{n^*} \frac{v_k - v_{k-1} (3 - 2\alpha)(4(v_k + v_{k-1} + 2t(1 - 2\alpha)) + q^2(5 - 2\alpha)^2)}{4t(5 - 2\alpha)^2}
\]

(25)

\[
\mathbb{E}(\pi^H_H) = (1 - \phi) \sum_{k=1}^{n^*} \frac{v_k - v_{k-1} (3 - 2\alpha)(2(v_k + v_{k-1} + 2t(1 - 2\alpha)) + (5 - 2\alpha)q)^2}{8t(5 - 2\alpha)^2}
\]

(26)

\[
\mathbb{E}(\pi^R_L) = (1 - \phi) \sum_{k=1}^{n^*} \frac{v_k - v_{k-1} (3 - 2\alpha)(2(v_k + v_{k-1} + 2t(1 - 2\alpha)) - (5 - 2\alpha)q)^2}{8t(5 - 2\alpha)^2}
\]

(27)

- **When \( n^* \geq 2 \), the platform and the sellers prefer price recommendation to no recommendation. There exists a \( \hat{q} \), such that the platform is better off under price recommendation compared to centralized pricing if and only if \( q > \hat{q} \). There also exists \( \hat{q}_H \) (\( \hat{q}_L \)) such that a high (low) type seller is better off under recommendation than under centralized pricing if and**
only if \( q > \hat{q}_H \) (\( q > \hat{q}_L \)).

Proposition 3 shows that whenever there is a natural number larger than 1 for which the inequality in (22) holds, it is more profitable for the platform to give recommendations, although it is an equilibrium not to give a recommendation. Moreover, when the uncertainty \( q \) is high enough, recommendations are more profitable to the platform (and the sellers) compared to centralized pricing. The intuition is that as \( n \) increases, the profit of the platform also increases, which makes recommendations preferable. When \( q \) is high enough, similarly to the no recommendation case, profits might also increase above the centralized pricing case.

When \( n^* \geq 3 \), there are multiple price recommendation equilibria. These equilibria differ by how coarse the partition of values of \( v \) is. Theorems 3 and 5 of Crawford and Sobel (1982) establish that in a cheap talk game, both the sender and receiver are ex ante better off in an equilibrium with a larger \( n \). Since the conditions of these Theorems hold in our model, the profit of the platform and the sellers increases with \( n^* \).

5 Market Outcomes

In this section we compare the benefits for sellers and consumers, as well as the equilibrium demand in the different pricing regimes. We start with illustrating the regions of parameters for which the platform or the sellers are better off in the different pricing regimes.

Figure 1 shows the regimes in which each player achieves maximum profit, as a function of \( q \) and \( \overline{v} \). We can see a common pattern emerge: when \( \overline{v} \) is high and \( q \) is low, all players prefer centralized pricing (top left); when \( \overline{v} \) is low and \( q \) is high, all players prefer no recommendation (bottom right); when both \( q \) and \( \overline{v} \) are large, recommendation leaves all players better off. The intuition is that \( \overline{v} \) captures the amount of information the platform has while \( q \) captures the amount of information the sellers have. If \( \overline{v} \) is small, i.e., close to \( \underline{v} \), there is little variation in the aggregate demand level and demand is consistent. Consequently there is little value to the platform’s information. Because, in addition, low values of \( \overline{v} \) cannot sustain the recommendation equilibrium, all players are better off if the sellers are allowed to price based on the information they posses. In contrast, if \( \overline{v} \) is high and \( q \) is low, there is little value to the sellers’ private information and the platform can safely ignore it and centralize pricing. Finally, if both sources of uncertainty are relatively strong,
the platform should recommend a price, so that the sellers can combine their private information with the platform’s information.

The second interesting feature to observe are in the differences between the three figures. High type sellers prefer centralized pricing more strongly than low types and even more than the platform. This result is formally stated in the following proposition:

**Proposition 4.** For \( \tilde{q}_H \), \( \tilde{q}_L \) and \( \tilde{q} \) as defined in Proposition 2 and \( \hat{q}_H \), \( \hat{q}_L \) and \( \hat{q} \) as defined in Proposition 3:

- \( \tilde{q}_H > \tilde{q} > \tilde{q}_L \)
- \( \hat{q}_H > \hat{q} > \hat{q}_L \)

The intuition is that when pricing is centralized, prices are equal across types. If one of the sellers is a high type and the other is low, the high type will obtain a large market share and a substantially larger profit than the low type. When the pricing is decentralized, the high type’s advantage is mitigated by the fact that the low type can lower their price to attract more buyers.

Next, we consider the expected total size of the market, i.e., the expected mass of buyers served in equilibrium. Total demand is equal to

\[
TD(p_1, p_2) = 2(1 - 2\alpha) + \frac{2v + q_1 + q_2 - p_1 - p_2}{t}
\]

The first summand comes from the shoppers that are located between the two sellers and always purchase due to Assumption A1. The second summand comes from the captives and the shoppers located to the left of seller 1 and to the right of seller 2. Using symmetry and integrating over \( q \) and \( v \), the expected total demand is:

\[
\mathbb{E}(TD(p_1, p_2)) = 2(1 - 2\alpha) + \frac{\bar{v} + \bar{v} - 2\mathbb{E}(p)}{t}
\]

The differences in total expected market coverage turns out to depend only on expected prices, which allows us to find the following result:

**Proposition 5.**

- \( \mathbb{E}(TD^R) = \mathbb{E}(TD^{NR}) > \mathbb{E}(TD^{CP}) \).
Figure 1: Pricing regimes that the platform and the sellers prefer depending on the values of $q$ and $\bar{v}$. Other parameters: $t = 1$, $\alpha = 0.45$, $\underline{v} = 5$. 
• The expected distance between a buyer and the seller they purchase from is \( E\left(\frac{TP}{4}\right) \).

Proposition 5 shows that total demand is higher when sellers compete on prices. It does not change if recommendations are feasible or not. This is because the only differences between prices under recommendation and no recommendation is that under recommendation the expectation over \( v \) is conditional on the message from the platform. As the sellers have rational expectations and the prices are linear in those expectations, summing over all possible messages and weighting by the message probability yields the same ex ante expectation and hence the same expected price. Under centralized pricing, the prices are higher on average, as the platform internalizes the substitution patterns between the two sellers and therefore faces less elastic overall demand than each seller individually.

Now we consider the consumer surplus (expected utility) of buyers, which we illustrate in Figure 2.

**Proposition 6.** When comparing the consumer surplus of buyers under the three pricing regimes:

• \( CS_{NR} > CS_R \).

• There exists a \( q' \) such that \( CS_{CP} > CS_{NR} \) if and only if \( q > q' \).

The first item of Proposition 6 emphasizes the contradicting preferences of the platform and the sellers with those of buyers. Similarly, buyers prefer centralized pricing only when \( q \) is large, which is exactly when the platform and the sellers prefer no recommendation. This result underscores the potential tradeoffs between the two sides of the market that platform designers have to consider. Because buyers have an outside option, they are shielded from some of the risk of experiencing a low realization of \( v \) or receiving \(-q\). In other words, the downside of participating in the market is limited. In this case, from an ex ante perspective, buyers prefer a payoff that varies more, as they can capture more of the upside. The more prices reflect the realizations of \( v \) and \( q \), the less variation there is in the buyers’ payoff. Therefore, buyers prefer those pricing regimes that attenuate the uncertainty the most when translating from realizations of \( v \) and quality to prices. Buyers always prefer no recommendation to recommendation, since then prices do not vary with \( v \). They also prefer centralized pricing when \( \bar{v} \) is small and therefore \( v \) matters little, while \( q \) is large.
Figure 2: Pricing regimes that the buyers prefer depending on the values of $q$ and $\bar{v}$. Other parameters: $t = 1$, $\alpha = 0.45$, $\bar{v} = 5$.

6 Conclusion

In this paper, we consider a platform that can use three different pricing regimes: (i) competitive pricing by sellers; (ii) centralized pricing by the platform; (iii) recommending prices to sellers. We find that from the platform’s and the seller’s perspective, the optimal choice of the pricing regime depends on the type of uncertainty prevalent in the market. If the aggregate demand uncertainty is more important than the uncertainty about the sellers’ quality, the platform should set prices in a centralized fashion. If the quality uncertainty is larger than the aggregate uncertainty, the platform should let the sellers set their own prices. A major advantage that the platform can utilize in markets when both types of uncertainties are high are price recommendations. In this case, providing sellers with some information, but not fully revealing it, may increase the profits of the platform above the centralized and the no recommendation case. This increase in profits in not always feasible, as there are cases when price recommendations will not be credible in equilibrium, and sellers will ignore them. Another interesting finding is who among the sellers prefers centralized pricing. We found that sellers with high quality prefer centralized pricing, although intuition would suggest that they would have stronger pricing power and would prefer pricing autonomy.
From the perspective of buyers, competitive decentralized pricing is almost always the best regime. Only when the aggregate uncertainty is small and the quality uncertainty is large, do buyers prefer centralized pricing.

Our analysis is not without limitations. In order to achieve a tractable solution, we assumed a specific simple demand form. Although we believe the results would hold in more generalized cases, this is still an open question. A second limitation of our model, which would be interesting to explore in future work is the amount of information buyers have, compared to the platform and the sellers. In our model buyers have full knowledge of all relevant model parameters, and relaxing this assumption may be important.

In terms of future work, there are two interesting questions that arise naturally from our model and we are considering to focus on. The first is a case when the platform can design the search technology and pick \( \alpha \) to maximize its profit. Platforms often change the amount of search results they display to customers strategically. Second, platforms can encourage sellers to disclose more information to buyers and enhance their seller profile. This would effectively increase the differentiation between sellers, which in our model would change the value of \( t \). Our analysis uncovers a tradeoff between maximizing the platform profit and consumer surplus which may inform platform designers and managers. Even though we do not model entry of buyers and sellers explicitly, often higher expected consumer surplus will lead to more buyers using the platform and a higher expected seller profit will encourage more sellers to join. Consequently, a growth-stage platform that is willing to sacrifice some profits for larger market share should let sellers set their own prices. A mature platform, in contrast, should use the profit-maximizing pricing regime. In fact, we might interpret the changes in Airbnb’s pricing strategy as following this rule. At first, while the company was growing, Airbnb let the hosts set their own prices. Later they introduced Price Tips, which is a price recommendation service. The introduction of smart pricing takes Airbnb even closer to a centralized pricing system.

For policymakers, our paper suggests that price recommendation systems may soften competition and potentially harm buyers, compared to not recommending prices. A critical part of many online platforms’ business model is the status of sellers or service providers as independent contractors, rather than employees. This allows the platforms to avoid, e.g., labor regulation. One criterion for determining the status of an employee vs. a contractor is their ability to set their own
price. Our paper shows that platforms do not always have to centralize pricing to achieve profits that are above competitive. Price recommendations allow platforms to extract large profits while avoiding the need to set prices for sellers. Regulators should therefore consider the impact of price recommendations and its influence on equilibrium outcomes when they consider the employment status of individuals.

References


A Proofs

Proof of Proposition 1

Because the profits of sellers 1 and 2 are ex-ante symmetric from the viewpoint of the platform, the optimal prices will have \( p_1^* = p_2^* \). The solutions to the first order conditions on price yield the expressions in the proposition. When setting \( p = p_1 = p_2 \), the expected profit is concave in price, hence the solution is a unique equilibrium.

The comparative statics analysis on prices is straightforward, as the optimal prices are linear in all parameters.

For \( t \), \( \frac{\partial \pi_{CP}}{\partial t} = \frac{1}{2} \left( (1 - 2\alpha)^2 - v^2 \right) < 0 \) when Assumption A1 holds; Integrating over \( v \) and using Leibniz’s integral rule also proves that \( \frac{\partial E(\pi_{CP})}{\partial t} < 0 \).

Finally, for \( \alpha \), \( \frac{\partial p_{NR}}{\partial \alpha} = -2(v + t(1 - 2\alpha)) < 0 \) because \( \alpha < 1/2 \). The same argument as above for integration results in \( \frac{\partial E(\pi_{CP})}{\partial \alpha} < 0 \).

Proof of Proposition 2

To find the equilibrium prices, the FOC of a seller of type \( \tau \) is:

\[
2E(v) + q(3 - 2\alpha) + t(2 - 4\alpha) - p^*_{NR}(2\alpha - 3) + \left( p^*_{LR} + p^*_{HR} \right) \frac{1 - 2\alpha}{4t} = 0 \tag{28}
\]

Imposing \( p^*_{\tau} = p^*_{NR} \) results in the equilibrium prices as the solution. Comparative statics with respect to \( t, q \) are straightforward because of linearity.

For \( \alpha \), \( \frac{\partial p^*_{NR}}{\partial \alpha} = \frac{4(\mathbb{E}(v) - 4t)}{(5 - 2\alpha)^2} > 0 \) when \( \mathbb{E}(v) > 4t \).

Comparing \( p^*_{LR} \) to \( p^* \), we find that \( p^*_{NR} > p^* \) when \( v < -q + \frac{2(v_H + v_L) - t(1 - 2\alpha)^2}{3 - 2\alpha} \) and \( v_H > \frac{1}{2} \left( q(5 - 2\alpha) + t(1 - 2\alpha)^2 + (3 - 2\alpha)v_L \right) \).

For the second item, the platform’s profit \( \pi^*_{NR} \) is compared to \( \pi^*_{CP} \). Because \( \pi^*_{NR} \) is quadratic and increasing in \( q \), and because \( \pi^*_{CP} \) does not depend on \( q \), finding the \( q \) for which \( \pi^*_{NR} \) gives the solution in the proposition. Finally, because \( \pi^*_{NR} \big|_{q=0} \leq \pi^*_{CP} \), there is a crossover of profits as described in the proposition.

For the third item, we follow a similar approach to the second item. \( \mathbb{E}(\pi^*_{NR}) \) is increasing and quadratic in \( q \) and \( \mathbb{E}(\pi^*_{CP}) \) is a constant, therefore, to show the existence of a crossing point, we only need to show that there is a point such that \( \mathbb{E}(\pi^*_{NR}) < \mathbb{E}(\pi^*_{CP}) \) for some \( q \). We take the lowest
possible value, \( q = 0 \). Then the inequality can be reduced to the following:

\[
\frac{4(3-2\alpha)(\mathbb{E}(v) + t(1-2\alpha))^2}{(5-2\alpha)^2} < \mathbb{E}\left(\frac{(v + t(1-2\alpha))^2}{2}\right)
\]

Note that (i) \( \frac{4(3-2\alpha)}{(5-2\alpha)^2} < \frac{1}{2} \) and (ii) \( (\mathbb{E}(v) + t(1-2\alpha))^2 < \mathbb{E}((v + t(1-2\alpha))^2) \) by Jensen’s inequality.

For the fourth term, first observe that \( \mathbb{E}(\pi^{CP}_H) \) is a linear increasing function of \( q \) and \( \mathbb{E}(\pi^{NR}_H) \) is a quadratic increasing function of \( q \). Moreover, if \( q = 0 \), \( \mathbb{E}(\pi^{CP}_H) = (1-\phi)\frac{\mathbb{E}(\pi^{CP}_H)}{2\phi} > \frac{\mathbb{E}(\pi^{NR}_H)}{2\phi} = \mathbb{E}(\pi^{NR}_H) \).

Second, we can rewrite \( \mathbb{E}(\pi^{NR}_L) = (1-\phi)(\frac{\mathbb{E}(\pi^{NR}_L)}{2\phi} - \frac{(\bar{v}+\bar{v}+2t(1-2\alpha))q}{2t(5-2\alpha)}) \). As \( \frac{3-2\alpha}{4} > \frac{1}{5-2\alpha} \), \( \mathbb{E}(\pi^{CP}_L) \) is decreasing in \( q \) faster than \( \mathbb{E}(\pi^{NR}_L) \). At \( q = 0 \) the profits are again proportional to the platform’s expected profit and therefore there is a crossing point. This concludes the proof.

**Proof of Lemma** We follow the steps from Lemma 1 in Crawford and Sobel (1982). Assume WLOG that \( \mathbb{E}(v|m_1) < \mathbb{E}(v|m_2) \). First, there exists a state \( \bar{v} \), such that \( \pi^P(\bar{v}, m_1) = \pi^P(\bar{v}, m_2) \). This state is

\[
\bar{v} = \frac{2(\mathbb{E}(v|m_1) + \mathbb{E}(v|m_2)) - t(1-2\alpha)^2}{5-2\alpha}
\]

and the optimal induced expectation in that state for the platform is \( \mathbb{E}^*(\bar{v}) = \frac{\mathbb{E}(v|m_1) + \mathbb{E}(v|m_2)}{2} \in (\mathbb{E}(v|m_1), \mathbb{E}(v|m_2)) \).

Second, it follows that \( \mathbb{E}(v|m_1) \) is not induced in equilibrium in any state \( v > \bar{v} \), as it is more profitable to induce \( \mathbb{E}(v|m_2) \) and vice versa, \( \mathbb{E}(v|m_2) \) is not induced in any state \( v < \bar{v} \).

Third, since \( \mathbb{E}(v|m_1) \) is a rational expectation over in which states the platform would choose to induce such expectation, \( \mathbb{E}(v|m_1) \leq \bar{v} \) and similarly \( \mathbb{E}(v|m_2) \geq \bar{v} \).

Fourth, we know that \( \mathbb{E}^*(v) \neq v \forall v \), then \( |\bar{v} - \frac{\mathbb{E}(v|m_1) + \mathbb{E}(v|m_2)}{2}| > \epsilon \), which means that \( \mathbb{E}(v|m_2) + \mathbb{E}(v|m_1) > \epsilon \). Since the state space is bounded, this means that there can only be a finite number of induced expectations.

**Proof of Proposition** First, we prove that price recommendation always gives a higher payoff than no recommendation to both the platform and the sellers. Given that \( \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - v} \frac{v_k + v_{k-1}}{2} = \mathbb{E}(v) \), the condition \( \mathbb{E}(\pi^{R}_i) > \mathbb{E}(\pi^{NR}_L) \) for \( t \in \{ P, H, L \} \) can be reduced to

\[
\sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - v} \left( \frac{v_k + v_{k-1}}{2} \right)^2 > \left( \frac{\bar{v} + v}{2} \right)^2
\]
which is true by convexity of the function \( f(x) = x^2 \).

Second, we prove that when \( q = 0 \), \( \mathbb{E}(\pi^R_H) < \mathbb{E}(\pi^C_P) \). This inequality reduces to

\[
\sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} \frac{4(3 - 2\alpha)((v_k + v_{k-1})/2 + t(1 - 2\alpha))^2}{(5 - 2\alpha)^2} < \mathbb{E}\left(\frac{(v + t(1 - 2\alpha))^2}{2}\right)
\]

Note that (i) \( \frac{4(3 - 2\alpha)}{(5 - 2\alpha)^2} < \frac{1}{2} \) and (ii) \( (\mathbb{E}(v|k) + t(1 - 2\alpha))^2 < \mathbb{E}((v + t(1 - 2\alpha))^2|k) \) by Jensen’s inequality. Then the inequality holds. The rest of the proof follows the same steps as the proof of proposition 2.

**Proof of Proposition 2** First consider the high type. Note that \( \mathbb{E}(\pi^R_H) = (1 - \phi)\left(\frac{\mathbb{E}(\pi^R_H)}{2\phi} + \frac{(\mathbb{E}(v) + t(1 - 2\alpha))q}{(5 - 2\alpha)t}\right) \).

If \( q = \hat{q} \), \( \mathbb{E}(\pi^R_H) < \mathbb{E}(\pi^C_P) \) as the first summands are equal, but the second linear part is larger for \( \mathbb{E}(\pi^C_P) \). As the profit in case of recommendation is quadratic and increasing, the crossing point has to be further to the right than \( \hat{q} \). Then \( \hat{q}_H > \hat{q} \).

Second, consider the low type. Again we can write it as a combination of quadratic increasing function of \( q \) and a linear decreasing function of \( q \). \( \mathbb{E}(\pi^R_L) = (1 - \phi)\left(\frac{\mathbb{E}(\pi^R_L)}{2\phi} - \frac{(\mathbb{E}(v) + t(1 - 2\alpha))q}{(5 - 2\alpha)t}\right) \).

By the same logic as before, we get that at \( q = \bar{q} \), \( \mathbb{E}(\pi^R_L) > \mathbb{E}(\pi^C_P) \). We also know that at \( q = 0 \) \( \mathbb{E}(\pi^R_L) < \mathbb{E}(\pi^C_P) \). Both functions are monotonic, so it has to be the case that \( \hat{q}_L < \hat{q} \).

The proof for no recommendation regime follows the same steps.

**Proof of Proposition 3** As established in the text, the relationship between the expected total demands is determined by the expected prices. They are the following:

\[
\mathbb{E}(p^{CP}(v)) = \frac{\bar{v} + \underline{v} + 2(1 - 2\alpha)t}{4}
\]

\[
\mathbb{E}(p^{NR}) = \frac{p^{NR}_H + p^{NR}_L}{2} = \frac{\bar{v} + \underline{v} + 2(1 - 2\alpha)t}{5 - 2\alpha}
\]

\[
\mathbb{E}(p^R) = \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} \frac{v_k + v_{k-1} + 2(1 - 2\alpha)t}{5 - 2\alpha} = \frac{\bar{v} + \underline{v} + 2(1 - 2\alpha)t}{5 - 2\alpha}
\]

Noting that \( 5 - 2\alpha > 4 \) completes the proof.

**Proof of Proposition 4** First we need to derive the expressions for expected consumer surplus. Consider loyalists. They purchase from the seller that they are aware of if and only if their distance
from that seller is below $\frac{v-p_i+q_i}{t}$. The maximum utility a loyal buyer can achieve is $v - p_i + q_i$. The utility of loyal buyers decreases linearly with distance. Then the total surplus of loyal consumer is an area under a triangle with base $2 \frac{v-p_i+q_i}{t}$ and height $v - p_i + q_i$. Then $CS^l = \left(\frac{v-p_i+q_i}{t}\right)^2$.

Now consider shoppers. The shoppers that are located at $(-\infty, -1) \cup (1, +\infty)$ act essentially as loyals (by A1). Then their surplus is $\frac{(v-p_i+q_i)^2}{2t}$. The shoppers in the interval $[-1, 1]$ choose between the two sellers. Denote by $\tilde{x} = \frac{q_1 - q_2 - p_1 + p_2}{2t}$ the location of the shopper indifferent between the two sellers. Then the consumer surplus of the buyers in $[-1, \tilde{x}]$ is a trapezoid with area $\frac{2(v+q_1-p_1)(\tilde{x}+1)}{2t}$ and for the buyers in $[\tilde{x}, 1]$ it is $\frac{2(v+q_2-p_2)(1-\tilde{x})}{2t}$. Combining everything together, we get the following expression for consumer surplus:

$$CS(p_1, p_2) = \frac{(v + q_1 - p_1)^2}{t} + \frac{(v + q_2 - p_2)^2}{t} + (1 - 2\alpha) \left( \frac{2(v + q_1 - p_1)(\tilde{x} + 1) + 2t(\tilde{x}^2 + 1) + 2(v + q_2 - p_2)(1 - \tilde{x})}{2} \right)$$

Expected consumer surplus under centralized pricing is:

$$E(CS^{CP}) = \frac{\mathbb{E}(v^2) + q^2(5 - 2\alpha) - t^2(3 - 8\alpha + 4\alpha^2)}{2t} \quad (30)$$

Expected consumer surplus under no recommendation regime is

$$E(CS^{NR}) = \frac{q^2(5 - 2\alpha)}{8t} + \frac{1}{t(5 - 2\alpha)^2} \left( 2\mathbb{E}(2\mathbb{E}(v) - v(5 - 2\alpha))^2 \right) + 2t(1 - 2\alpha)^2(3 - 8\alpha)\mathbb{E}(v) - t^2(37 - 126\alpha + 124\alpha^2 - 40\alpha^3))$$

Expected consumer surplus under recommendation regime is

$$E(CS^R) = \frac{q^2(5 - 2\alpha)}{8t} + \frac{1}{t(5 - 2\alpha)^2} \left( 2\sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - v} \mathbb{E}((2\mathbb{E}(v|k) - v(5 - 2\alpha))^2|k) + 2(1 - 2\alpha)^2(3 - 2\alpha)\mathbb{E}(v) - t^2(37 - 126\alpha + 124\alpha^2 - 40\alpha^3) \right)$$
Second, we need to show that \( \mathbb{E}(CS^{NR}) > \mathbb{E}(CS^{R}) \). This reduces to

\[
\mathbb{E}((2\mathbb{E}(v) - v(5 - 2\alpha))^2) > \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} \mathbb{E}((2\mathbb{E}(v|k) - v(5 - 2\alpha))^2|k)
\]

\[-8(2 - \alpha)(\mathbb{E}(v))^2 > -8(2 - \alpha) \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} (\mathbb{E}(v|k))^2
\]

\[-8(2 - \alpha) \left( \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} \mathbb{E}(v|k) \right)^2 > -8(2 - \alpha) \sum_{k=1}^{n} \frac{v_k - v_{k-1}}{\bar{v} - \underline{v}} (\mathbb{E}(v|k))^2
\]

which is true by concavity of \( f(x) = -8(2 - \alpha)x^2 \).

Finally, we need to show that there exist a \( q' \) such that \( \mathbb{E}(CS^{NR}) > \mathbb{E}(CS^{CP}) \) if and only if \( q < q' \). Both are quadratic and increasing in \( q \), but \( \mathbb{E}(CS^{CP}) \) is increasing faster. Therefore, when \( q \) is large enough, \( \mathbb{E}(CS^{CP}) > \mathbb{E}(CS^{NR}) \). Then we need to show that if \( q = 0 \), \( \mathbb{E}(CS^{NR}) > \mathbb{E}(CS^{CP}) \).

The difference between the two expressions is

\[
\mathbb{E}(CS^{NR}) - \mathbb{E}(CS^{CP}) =
\]

\[
= \frac{(1 - 2\alpha)^4 t^2 + 2(3 - 2\alpha)(1 - 2\alpha)^2 t(\bar{v} + \underline{v}) + (3 - 2\alpha)^2 \bar{v}^2 + (4\alpha^2 - \alpha - 7)\bar{v} \underline{v} + (3 - 2\alpha)^2 \underline{v}^2}{2(5 - 2\alpha)^2 t} > 0
\]

This completes the proof.