

# Virtual Property Trade in Online Games

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## Abstract

Industry revenue for online games is surging. A unique phenomenon in this market is the significant volume of trade among game users, who exchange real money for virtual goods that do not exist in the real world. The market of virtual goods has two distinctive characteristics: 1) the resold virtual items normally involve no quality loss since the attributes are digitally defined, and 2) the game companies can impose measures to control the trade. Intuitively, game providers should restrict competition from potential traders of virtual goods. However, in practice game providers adopt dramatically different trade policies, ranging from strict prohibition to explicit permission. This paper attempts to analyze the optimal trade policy from a game provider's perspective. We find that, despite the reduction in unit revenue, the firm may benefit from abandoning a strict trade policy. This is because the players' decision to participate in the game is influenced by their estimate of the firm's optimal behavior, which in turn can be strategically affected by the trade policy.

**Keywords:** Online Games, Virtual Property, Game Theory, Pricing.

# 1 Introduction

## 1.1 Background

On May 15, 2012, Blizzard Entertainment launched Diablo 3, a computer game played through Internet. The game is priced at \$60 and within 24 hours of its launch, more than 3.5 million copies were sold. Diablo 3 is one of the thousands of the games played by massively multiple players on an interconnected platform, mostly via the Internet. These games are generally called online games. Thanks to advances in information technology in the past decade, the world has seen an explosive growth in the online gaming industry, both in terms of the number of players and in revenue. In 1999, the total number of active online game subscriptions in the world was 220,000.<sup>1</sup> By mid 2007, the online gaming population reached 217 million (comScore 2007). In the United States, while the Internet audience increased 4% in 2008, the number of online gamers grew 27% from 65 million to 86 million (comScore 2009). In terms of revenue, OECD reported that online games earned \$6.5 billion worldwide in 2007, an increase of 28% from 2006. In comparison, global revenue from online music was \$4.7 billion and from online advertising it was \$31 billion (OECD 2008). A recent analysis by iResearch (2009), an Internet consultancy, found that the global revenue for the online gaming industry in 2008 reached \$ 11.3 billion, with the U.S., China, and Korea as the top three markets, representing about 29%, 27%, and 21% of the global market, respectively.

These data indicate the rise of an important and fast growing industry that influences the leisure activities of millions of consumers. While many economic and managerial theories apply to this industry, a unique phenomenon distinguishes it from other traditional industries and thus deserves more studies: the trading of virtual properties. For example, virtual gold in the game “World of Warcraft” is indispensable (like real money in our real life), and to acquire the virtual gold a player has to sit in front of a computer and repeatedly instruct her avatar to dig in the mining spots. This item acquisition process is both boring and time consuming, and many players do not enjoy doing it. A 28-year-old manager may only have two hours everyday to play

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<sup>1</sup>See [www.mmogchart.com](http://www.mmogchart.com)

her favored game and she may like to spend the two hours in other more enjoyable activities such as finishing quests or interacting with other players. Therefore, she might be willing to spend some real money to buy the virtual gold from other players. In fact, it is shown that, in the U.S., more than half of ordinary gamers intend to pay real world money for virtual items (Wi 2009).

The real money trade of virtual properties is becoming increasingly significant. Apart from transactions that occur among individual players, there are companies established solely to profit from acquiring and reselling virtual goods (Castronova 2006), and there emerged designated online trading platforms (e.g., ItemBay and ItemMania in Korea, ItemBank in Japan). While we do not have the worldwide data, the volume of virtual properties traded in Korea was approximately \$800 million as early as 2004 and was expected to reach \$1.5 billion in 2008 (Wi 2009). According to 5173.com, one of China's major virtual goods transaction platforms, its trade volume of virtual properties reached \$1.37 billion in 2007, which is equivalent to more than half of the gaming industry's annual revenue in the country.

The market of virtual goods differs from that of second-hand durable goods in two important dimensions: 1) while a used durable good usually involves uncertain quality, a virtual item purchased from another party is identical to the one acquired directly from the game spot since the virtual item's attributes are digitally defined, and 2) game companies can impose measures to control the trade of virtual properties. The first difference suggests that game companies may potentially face intense competition from virtual property trade, while the second difference means that this competition can be controlled by the game companies. Intuitively, firms should avoid harsh price competition and prohibit the trade of virtual goods. However, surprisingly, gaming companies have adopted dramatically different policies. For example, *Second Life*, a 3D simulated world, allows its users to profit from trading virtual goods. On the other extreme, *World of Warcraft* — a role-playing game with 11.5 million subscribers worldwide — strictly prohibits the buying and selling of virtual items with real world money.

This paper builds on a game-theoretical model to formally examine a game provider's optimal policy toward the control of virtual property trade. Our analysis indicates that, when early

players are uninformed of the characteristics of the game prior to costly participation, the game provider may actually benefit from allowing the trade even though that would bring up harsh competition. The underlying economic mechanism lies in the role of a loose virtual trade policy in credibly committing the firm to (privately) set game characteristics that would lead to higher ex post player surplus and thus encourage ex ante game participation.

The rest of the paper is organized as follows: we first describe online games and virtual property trade, followed by a brief review of relevant literature. The model is then presented and we analyze the impact of the virtual property trade on equilibrium firm behavior. We then study optimal trade control. We also extend the model in several dimensions and examine the robustness of our results under alternative model specifications. The paper ends with concluding remarks and directions for future research.

## 1.2 Online Gaming

In Adams and Rollings (2006), online gaming “is a technology rather than a genre; a mechanism for connecting players together rather than a particular pattern of game-play.” Thus, online games can take a variety of forms. They can be browser based, or require a client end software on the players’ game devices (computers, game consoles, mobile phones, etc). Currently, online game generally refers to Massively Multiplayer Online Game (MMOG), where thousands of players interact with each other simultaneously in a synthetic world. This only became possible with the recent availability of broadband Internet connection, and MMOG has been a massive hit among gamers. In this dynamic market, *World of Warcraft*, developed by Blizzard Entertainment (a subsidiary of Vivendi), claims 11.5 million subscribers worldwide and enjoys a dominating position. NCsoft from South Korea, with several famous titles such as *Lineage*, is the second largest player in the global market. They are followed by a dozen fast growing companies from around the world.

Compared with traditional games played with consoles or PCs, the most important feature of online games is the vast amount of interaction among players. In the simulated world, players

can develop various communities, play in teams, make friends with other avatars, get married, or even encounter a virtual death.<sup>2</sup> There are various genres, including casual games, first person shooting, real time strategy, and role playing games.

In the virtual world, depending on the game design, a player may need to finish quests, accumulate experience points to advance her avatar toward upper levels, and cooperate/compete with other players. When conducting these essential activities, the performance of a player is based on two factors: the skill level of the avatar and the virtual resources, or properties, that the avatar owns. Depending on the particular game, these resources may be virtual gold, a sword with huge damage power, a fast machine-gun, or even a nice looking skirt. A player needs weapons to fight against monsters, armor to protect her avatar, and battle aids to replenish energy. More gold means the player can buy more virtual gears, a good sword means she is better equipped to fight against monsters and other players in peer-to-peer competition, and a proper pair of boots means the avatar can run faster. In short, virtual resources are as indispensable in the game as cash or shoes are in our daily life.<sup>3</sup>

### 1.3 Virtual Property Trading

Before the advent of online games (i.e., in the person-vs-machine era), game players could search for virtual items only within the game realm, which can be considerably time consuming. Nevertheless, the players can gain unlimited access to game-play with a one-time purchase of the game software. The business model of online games has been qualitatively changed, where time can be a commodity. The online nature of these games means that the active time during which a player stays in the game realm can be measured and charged accordingly. Thus, if consumers spend more time searching for and acquiring virtual items, they end up paying more to the game provider.

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<sup>2</sup>In general, players can revive their dead avatars at a possible loss of experience value or certain virtual items. However, in some cases, they may permanently lose their avatars and game accounts. For example, in *Diablo*, a role-playing game, there is a ‘Hardcore’ mode where the avatar is mortal and one can lose everything in the game if death occurs to the character.

<sup>3</sup>Playing the game without any virtual resources is possible, but the experience would be much less enjoyable.

Alternatively, they can save on this time and buy the items with real life money from other users. We shall define virtual property trade as the transaction of virtual properties between any two parties in the game world using real world money. For example, a player in *World of Warcraft* may need a hammer specially blessed for Dark Knights, but may find it too time consuming to acquire it from within the game. The player can then go to eBay, or even Google the item, look for a seller in the real world, and purchase the hammer with real world money. The seller will then deliver the hammer to the buyer's avatar in the game. Obviously, the current transaction process is far from sophisticated and leaves plenty of room for fraud. Nevertheless, many consumers keep participating in virtual property trade, and consequently numerous individuals and entities emerged who specialize in this kind of virtual business. According to Woodcock (2008), in the game "World of Warcraft", almost half of subscribers are those whose sole objective is to profit from selling virtual items.

To the gaming firms, the prevalence of virtual property trade implies direct revenue loss. For example, let us assume that a minimal of 60 gram of virtual gold is needed to finish certain quest in the "World of Warcraft." Suppose also that it take about 1 hour for an ordinary player such as the 28-year-old manager to mine 1 gram of the virtual gold. If the young manager decides to acquire the virtual gold by herself, she would then have to incur at least 60-hour game time before finishing the quest. On the other hand, a specialized trader can be more efficient in gold mining (e.g., with higher mining skill level or better knowledge of the mining spots), and can acquire the same amount of virtual gold in, say, 30 hours. As a result, if the manager buys the virtual item from the trader, the game provider's revenue from her item acquisition, which comes from the users' in-game time, would be decreased by half.

Therefore, one may naturally expect that the gaming firms would exercise some control over virtual property trade. Unlike second-hand markets for durable goods, the market of virtual goods can be controlled by the gaming firm. Generally, game subscription requires the players to agree to certain "Terms of Use" or "End User Licensing Agreement," basically allowing the game company to claim ultimate ownership of the virtual property. In practice, firms can choose

whether to exert this ownership power. When they decide to do so, they can adopt a variety of measures, ranging from monitoring in-game transfer of the virtual property to suspending the violating players' account. For example, *Final Fantasy XI* and *Warhammer Online* both have task forces dedicated to removing real money trading from the game. *World of Warcraft* players even face the risk of permanent suspension of their account if they are caught trading virtual gold. At the other extreme, however, some firms explicitly or implicitly allow the trade of virtual properties. For instance, in July 2005, Sony Online Entertainment introduced a new service in the U.S. called Station Exchange in which users on the *Everquest II* server can buy and sell the game's virtual items. In Korea, ItemBay openly lists virtual items of *Lineage II* for trade.

#### 1.4 Related Literature

The market for virtual goods differs from that of second-hand durable goods in two important aspects. First, the resale of durable goods usually assumes a certain degree of uncertainty on the quality of the used goods. This uncertainty creates information asymmetry between the sellers and the buyers, yielding the issues of adverse selection (Akerlof 1970, Genesove 1993, Hendel and Lizzeri 1999, Rust 1985, Samuelson 1984).<sup>4</sup> However, in the virtual world where the attributes are expressed in numbers (e.g., the amount of gold, the damage index of an axe), the quality level is usually public knowledge and the resold virtual items are usually considered identical to the “newly-acquired” ones. Moreover, there is typically no obsolescence of the virtual item as long as the game realm is still in business. Therefore, we will treat the traded virtual goods as perfect substitute, without depreciation, for those directly acquired during the game-play.

Second, once a consumer purchases a durable good, the ownership of the product is transferred. It is up to her to decide whether to sell the product again, and the previous seller has no control over the buyer's post purchase behavior. In other words, the market of second-hand durable goods are not controlled by the sellers. However, although the legal ownership of virtual goods is not yet clearly defined (Duranske 2008), in practice gaming firms can impose various control

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<sup>4</sup>There is also a stream of research that examines how the sales of used products influence sellers' decisions on optimal durability and new product launch (Bulow 1986, Dhebar 1994, Liebowitz 1982, Rust 1986, Swan 1972).



measures over the trading activities.

Essentially, virtual items are digital goods (i.e., products that can be digitized) and the item trade happens only in the online game realm. In this regard, our model is related to research on information goods and online trading. Bakos and Brynjolfsson (1999) examine the pricing and profit implication of bundling multiple unrelated information products. Hitt and Chen (2005) and Wu et al. (2008) study customized bundling, where prices depend on the number of goods (not the actual content) in the bundle, and pricing of information goods when consumers do not place positive values on all products. Sundararajan (2004) studies the pricing of information goods under incomplete information and found the relative independence between the optimal usage-based pricing schedule and the value of the fixed fee. Apart from price and bundling, Chen and Seshadri (2007) study the optimal quality limit and versioning design of information goods when customers have heterogeneous reservation utilities. Parker and Alstyne (2005) study the optimal design of information products under network externalities. Towards online trading of information goods, Dewan et al. (2000) study the impact of Internet and e-commerce on information providers. Dellarocas (2005) examine the optimal rating mechanism in online trading platforms such as eBay. Bakos et al. (2005) study the competitive interactions between an online broker with only trade execution capacity and a traditional full service broker. Aron et al. (2006) examine the impact of intelligent agents in the e-marketplaces. While these pioneering research provide insightful knowledge on the markets of information goods, our paper focuses on the secondary market of the virtual items and how firms should control the trade.

Our approach to understanding the trade of the virtual goods also resembles that of ticket resales in the advanced selling literature (Courty 2003a,b, Xie and Shugan 2001, Geng et al. 2007). However, in these studies the profitability of allowing ticket resale hinges on demand uncertainty and/or capacity constraint from the suppliers, which are absent in our context. In particular, virtual items are indispensable in online games and the supply can be unlimited. Therefore, our model assumes that the virtual item is a must to play the game and there is no limit in the gaming firm's production function. Moreover, this paper is related to the literature on costly

pre-purchase consumer participation (Villas-Boas 2009, Wernerfelt 1994).

To our best knowledge, this research is the first formal economic analysis on the influence of virtual property trade on firm profitability. One perspective in previous studies simply treats virtual property trade as pollution in the real world (Castronova 2006). The readers can resort to Evans et al. (2006) for a discussion of the online game market.

## 2 The Model

The model involves three types of economic agents: the ordinary players, the traders, and the gaming firm. We consider a monopolist firm with zero marginal cost who can provide an online game to potential users. There are two kinds of users: the ordinary players and the traders. Both kinds of users need to pay for the gaming firm's service but their objectives are completely different. We describe the parties' incentives and decisions in three building blocks.

### 2.1 The Ordinary Players

The typical life span for a game title is around two years and could be even longer if the title is popular. For example, while the *World of Warcraft* was released in 2004, it was still the most popular online role playing game in the world in 2008. In contrast, a player may stay with a game for a much shorter period. Usually, a player will stay with a single game for an average of six months.<sup>5</sup> In this regard, we assume that a consumer needs one period to play the game and she quits the game after one period. This means that the game provider faces a stream of potential players and needs to consider multi-period profit maximization. To capture this, we assume that the game lasts two periods and within each period, the market size (i.e., total number of potential players) is normalized to 1.

The players place a value of  $V$  to the gaming experience. They are assumed to be heterogeneous in their valuation of the game. More specifically, the valuation  $V$  follows a uniform

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<sup>5</sup>This figure is obtained from an interview with one of the largest game providers in the world. In a more recent survey (iResearch 2009), the time a player stays with a single title varies from 3 months to more than 5 years.

distribution between 0 and 1, i.e.,  $V \sim U[0, 1]$ .<sup>6</sup>

Within each period, a player realizes the value  $V$  of the game through her gaming experience and she pays to the gaming firm for the time she spends with the game. We distinguish two kinds of gaming time a player may spend: the time that is needed to acquire the necessary virtual items, denoted by  $\tau$ , and the game-play time to finish quests, to level-up the avatar, or to interact with other players and so on, denoted by  $t$ . Because virtual items are indispensable for the gaming experience, we assume that every player needs one piece of virtual item in order to realize the value  $V$  of the game. For example, a player may need to dig in a mine to gather the virtual gold, or defeat a certain number of monsters to find a wanted item. This process of searching and hunting for the virtual item is repetitive and can be boring and less enjoyable than game-playing. For example, the term “farming” is widely used in online games such as *World of Warcraft* and *Diablo 3*, and refers to the activity of collecting gold by repeatedly searching the same game map. Apart from item hunting, a player also needs to spend time on the game-play itself, e.g., finishing quests or interacting with other players. A game usually has a certain number of quests and/or virtual communities where the players can interact with each other. In some games where quests and virtual item acquisition are deeply intertwined,  $t$  and  $\tau$  may not be fully distinguishable from a consumer’s point of view. Later in Section 4.2, we extend the model and explore such case of inseparable  $t$  and  $\tau$ .

Both the item acquisition time  $\tau$  and the game-play time  $t$  represent financial costs to the players. We assume that the unit price of the gaming time is 1 and it is homogeneous across the players. This setting reflects industry practice. Generally, the unit price of gaming time, either based on monthly subscription fee or on fixed tariff for a certain block of effective usage, remains unchanged over time. For example, the monthly fee for *World of Warcraft* has been \$25 since its

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<sup>6</sup>This representation of consumer heterogeneity can also capture the different opportunity costs of time on the consumer side. For example, a 30-year old manager may have the same valuation for *World of Warcraft* as a MBA student of the same age, but the manager may have much less time available for the game. The higher opportunity cost of time for the manager can be qualitatively translated to a lower  $V$ .

launch in 2004, and the price for 30-hour gaming time is \$15. It follows that a player's utility is:

$$u = V - \tau - t. \tag{1}$$

Equation (1) essentially suggests that  $t$  and  $\tau$  are the financial costs to the players. In some games with excellent storyline, a consumer's utility may actually increase with her time spent in the game-play. In Section 4.1, we extend the model where the valuation  $V$  can be an increasing of  $t$ .

While the item acquisition time  $\tau$  is a must, it does not necessarily follow that it is a priori known to the players. When a new game is released, there is usually an in-game step-by-step guide provided by the game developer or the publisher. However, typically it is unknown how difficult the game is or how much time it will take to acquire a virtual item. In other words, the value of  $\tau$  is not revealed to the players at the early stage of a game's lifespan. However, as more and more players participate in the game and explore the virtual world, item-acquiring strategy guides can be gradually developed by early players and thus the difficulty level for item acquisition can be revealed to late comers. One example is the recently launched game Diablo 3. The game has no free trial but just a beta test offered to selectively invited fans. In the beta test, only 1/3 of the episodes were revealed and the difficulty of item acquisition has changed dramatically since its official launch.

To capture this dynamic information flow, we assume that the potential players in the first period do not know the actual value of  $\tau$  prior to participating in the game. Moreover, we assume that the first-period game participation requires a cost  $e > 0$ . This cost could represent the players' efforts to set up the game account, explore the game realm, search for game-playing strategy guides, etc. Once this cost is incurred, the players will be able to find out the actual time  $\tau$  for item acquisition. In the second period, however, the players can discern the value of  $\tau$  before joining the game. In addition, to simplify matters and without loss of insights, we assume that the second-period game participation is costless. This captures the notion that normally gaming information, in terms of both item-acquiring and game-playing strategy guides, will be readily available from the Internet or popular game magazines when it comes to the advanced

stage of the game.

## 2.2 The Traders

The objective of the traders is completely different from that of the players. In particular, the traders do not derive any value from playing the game itself. Rather, their sole purpose is to profit from acquiring the virtual item and reselling it to the players for real world cash. We make two basic assumptions about trader behavior. First, they are more efficient in item acquisition than the players. In particular, the traders only need to spend  $\alpha\tau$  of time to acquire the virtual item, where  $\alpha \in (0,1)$  captures the item acquisition inefficiency for the traders. Second, the traders do not spend any time on the game-play itself. Nevertheless, a player who buys the virtual good from a trader will still spend a time of  $t$  on game-play. Therefore, the direct revenue loss for the game provider, for each participating player involved in virtual property trading, is  $(1 - \alpha)\tau$ .

To simplify matters, we assume that the resale price for the virtual item is negligibly lower than what the players would pay should they acquire it by themselves from the game realm. This would be the case if, for instance, the traders are price followers to the game provider. As a result, if feasible, a player would buy the virtual item from the traders rather than acquire it directly. Admittedly, this simplified resale price  $\tau - \epsilon$  does not fully capture the demand-supply interactions in the virtual item market. Our focus in this paper is the gaming firm's optimal trade policy, which will be used to control the market size of the virtual goods. We also considered a more general setup in which the resale price of the virtual item is positively related to  $\tau$  and is somewhere between the traders' item acquisition cost (i.e.,  $\alpha\tau$ ) and the price charged by the game provider (i.e.,  $\tau$ ). We find that our major results remain robust in this general setting. A further assumption we make is that the virtual goods market is always cleared. Additionally, we assume that virtual property trading can occur only in period 2. In practice, at the early stage of a game title, there is normally a general lack of knowledge about the game and thus the traders may not have higher efficiency in terms of item acquisition than the players. That is, if a trader wanted to do business in period 1, he would have to spend the same amount  $\tau$  of time for item

acquisition, which renders the trade unprofitable.

### 2.3 The Game Provider

The game provider can gain revenue from the players' item acquisition and game-play. We assume that the time for game-play,  $t$ , is fixed and exogenously determined. In practice, the average time the players spend on playing a game is largely influenced by the fundamental nature and design of the game (e.g., roles, quests, and virtual maps). As a result, this playing time normally varies less across players within a game title than across different titles. In addition, the exogeneity of  $t$  essentially captures the idea that the fundamental nature and design of the game is decided long before its launch to the public, and reconfiguration is relatively difficult or costly. For example, Diablo 3 was announced on June 28, 2008 and launched on May 15 of 2012. The game took its developer - Blizzard - four years to finish the storyline and episodes. On another note, the firm that designs and develops the game (i.e., the one who decides on  $t$ ) can be different from the game distributors/operators who runs the game servers in certain territories (e.g., World of Warcraft in China is operated by Netease). In these cases, the game operator has much less control over  $t$  than the game developer.<sup>7</sup> Therefore, in studying the influence of the trade of virtual goods, we focus on the scenario where  $t$  is a pre-set parameter that influences the consumers' and the game operator's decisions. Later in the paper, we examine the alternative settings where  $t$  is an endogenous variable decided by consumers (in Section 4.1) or by the gaming firm (in Section 4.2).

In contrast, the game provider can and do control the time required for item acquisition (i.e.,  $\tau$ ). For example, game companies can, in a relatively easy manner, change the drop rate of virtual items, e.g., the probability that a player can seize a specific sword through slaying certain monsters or collect a particular quantity of virtual gold from a mine. If the game has a high drop rate, then the players can easily acquire the item within a short time duration (i.e., small  $\tau$ ). If the game has a low drop rate instead, then it would be hard to acquire the item and the

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<sup>7</sup>The gaming industry is deeply intertwined and the game publisher may also operate the game and has the market power to influence the development choices. In such cases, the gaming firm can also decide on  $t$ , albeit less easy than changing  $\tau$ .

item acquisition time would be lengthened (i.e., large  $\tau$ ). Within 6 weeks of its initial launch, Diablo 3 has changed its item drop rate through various patches. Another practice to effectively change  $\tau$  is to increase/decrease the randomness of the location of the virtual item in the game realm. Recall also that the price per unit gaming time is typically constant in practice, and is normalized to 1 in the current setup. Thus, the game provider can effectively adjust its revenue for each unit of item acquisition, and at the same time determine the number of participating players, by simply varying the item acquisition time  $\tau$  without changing the price of each unit of time. This represents a popular tactic in practice.<sup>8</sup> It is easy to implement, which involves only a few technical parameter adjustments in the server. It can also potentially ease consumers' fairness concerns about inter-temporal price fluctuation.

Moreover, the game provider can make strategic moves to cope with the trade of the virtual item. These moves can reduce its direct revenue loss from virtual item trade, and can be implemented through, for example, announcing that such transactions are illegal,<sup>9</sup> monitoring suspicious in-game transactions, suspending game accounts of violating players, or blocking transaction channels in the real world. For example, Blizzard Entertainment has taken significant steps to prevent real money trade and made it clear that the company has zero tolerance for such trade (Duranske 2008). In January 2007, eBay announced a ban on virtual property trade citing a prohibition by the gaming companies (Dibbell 2007). The lawsuit of Marc Bragg vs. Linden Lab is another example on how game providers can monitor and punish traders. In this regard, we assume that the gaming firm has a policy decision variable, denoted by  $\theta \in [0, 1]$ , which reflects its monitoring and preventive measures against the virtual property trade. With an increasing  $\theta$ , tighter policing measures are adopted and the trade of the virtual property is thus less likely

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<sup>8</sup>In China and South Korea, there are online games that are free to play but the providers profit from selling the virtual property themselves. This kind of practice can be considered as another means of charging the consumers based on their gaming time.

<sup>9</sup>Whether the trade of virtual items is legitimate is still subject to debate under the current legislative system. The question of who owns the virtual property, should it be the game provider or the actual players, is not yet answered by the current property law. The legitimacy issue addressed here simply represents the perspective that may be taken by a game provider, which may not correspond to any current property law.

to happen, potentially because real money transactions are more difficult to conduct, or fewer players are able to match with the traders. More specifically, this amounts to assuming that, a proportion  $\theta$  of the players have to acquire the virtual item by themselves, while the other proportion  $1 - \theta$  of them are able to pay real cash to buy from the traders, which will be referred to as the “item-acquiring” and the “item-buying” players, respectively. Moreover, to concentrate on the strategic consequence of this policy decision, we assume that it is costless to adopt any trade policy. We intend to show that, it can be in the game provider’s best interest not to stringently prevent virtual property trade, even if it is completely free to do so.

We assume that the game provider makes its decisions when the game title is launched. This leads to the following sequence of moves.

1. The game provider decides on the item acquisition time  $\tau$  and on the trade policy  $\theta$ ;
2. The potential players in the first period observe  $\theta$  (but not  $\tau$ ) and decide whether to join the game by incurring a cost  $e$ ;
3. In the second period, the potential buyers observe both  $\tau$  and  $\theta$  and decide whether to join the game, and the traders can supply the virtual item to a proportion  $1 - \theta$  of the participating players.

### 3 Analysis and Results

We start with a simple static model in which the game provider’s decisions are known to the players. This can be considered as a simplified version of the full model with only the second period. This simplified model can serve as a benchmark to illustrate the direct (negative) impact of virtual property trade on the game provider’s revenue. We then move to the analysis of the full model in which the trade policy can influence the first-period players’ participation decision.



### 3.1 A Simple Static Model

In this simple setting, the game provider's decisions (i.e., both  $\tau$  and  $\theta$ ) are commonly known and the potential players' game participation decision does not involve any fixed cost. That is, it is as if all the parties make their decisions in the second period. Note first that for all potential players, both those who have to acquire the virtual property from the game realm and those who can buy it from the traders, the effective price for item acquisition is (around)  $\tau$ . Therefore, a player will join the game if and only if  $V - \tau - t > 0$ . Given that  $V \sim U[0, 1]$ , the firm has a total demand of  $1 - \tau - t$ .

For each of the item-acquiring players, the total time spent in the game realm would be  $\tau + t$ , representing the item acquisition and the game-play time, respectively. This leads to a revenue of  $\pi_a = (1 - \tau - t)(\tau + t)$  from the item-acquiring players. In contrast, for each of the players buying the virtual property from the traders, the unit revenue of item acquisition comes only from the traders, which is  $\alpha\tau$ . Nevertheless, the item-buying players continue to spend an amount  $t$  of time on game-play. This suggests that the game provider's revenue, arising either directly or indirectly from the item-buying players, is  $\pi_b = (1 - \tau - t)(\alpha\tau + t)$ . Taken together, if the firm implements a trade control measure  $\theta$ , its total profit would be:

$$\pi_s = \theta\pi_a + (1 - \theta)\pi_b, \quad (2)$$

where the subscript  $s$  represents the static setting. For ease of exposition, we shall focus on the case where  $t < \frac{\alpha}{1+\alpha}$ . This ensures the existence of interior optimum without loss of insight.

**Proposition 1.** *In the static model, the gaming firm will strictly prohibit the trade of virtual goods (i.e.,  $\theta^* = 1$ ), and the optimal item acquisition time is  $\tau^* = 1/2 - t$ .*

*Proof:* We first calculate the firm's optimal item acquisition time  $\tau$  at any given  $\theta$ :

$$\tau^* = \frac{1}{2} \left( 1 - t \left( \frac{1 + \alpha + \theta - \alpha\theta}{\alpha + \theta - \alpha\theta} \right) \right). \quad (3)$$

We then substitute  $\tau^*$  into Equation (2). This yields:

$$\pi_s^* = \frac{(\alpha + \theta - \alpha\theta + t(1 - \alpha)(1 - \theta))^2}{4(\alpha + \theta - \alpha\theta)}. \quad (4)$$

It can be easily checked that  $\frac{\partial \pi_s^*}{\partial \theta} > 0$ , which means that  $\theta^* = 1$ . Substituting  $\theta^* = 1$  into Equation (3), we have  $\tau^* = 1/2 - t$ . **Q.E.D.**

The key observation here is that, intuitively, the firm can benefit from mitigating the virtual property trade. This is because the trade of virtual property effectively reduces the game provider's unit revenue of item acquisition from  $\tau$  to  $\alpha\tau$ . Meanwhile, both the item-acquiring and the item-buying players incur the same effective payment to obtain the virtual property, and thus have the same demand. In fact, it can be readily checked that, for any given  $\tau$ ,  $\pi_a \geq \pi_b$ . This means that, in this static setting, the virtual property trade is necessarily detrimental to the game provider. This captures the direct effect of virtual property trade on the firm's profitability.

Another interesting question arises naturally: how will the firm's optimal item acquisition time ( $\tau^*$ ) change in response to its trade policy ( $\theta$ )? From Equation (3), we have  $\frac{\partial \tau^*}{\partial \theta} > 0$ . This means that a tighter trade control is best accompanied by a longer item acquisition time. This reflects the firm's incentive to balance the two sources of revenues: from the game players who acquire the virtual item by themselves (i.e.,  $\pi_a$ ) and from those who can purchase it from the traders (i.e.,  $\pi_b$ ). For either  $\pi_a$  or  $\pi_b$ , the firm's revenue consists of the players'/traders' time spent on item acquisition and game-play. All else being equal, the relative revenue contribution of item acquisition versus game play is higher for  $\pi_a$  than for  $\pi_b$ . This is because, the traders are more efficient and thus spend less time on item acquisition than the players. As a result, relatively speaking, for the item-acquiring players the firm should focus more on raising the unit revenue from item acquisition, whereas for the item-buying players the firm should concern more about increasing the demand by cutting the item acquisition time. This is why, as the trade policy becomes more stringent and hence the firm's revenue from the item-acquiring players increases relative to that from the item-buying players, it is better off to increase the optimal item acquisition time  $\tau^*$ .

### 3.2 The Full Model

In this section we analyze the full model in which the game provider needs to consider revenues from both periods. We start with an analysis of the consumers' behavior. While the behavior of the players in period 2 remains unchanged as in the simple model in the previous section, the players in period 1 have a different decision making process. Note that the first-period players do not know the actual value of the item acquisition time  $\tau$  when deciding whether to join the game. Their participation decision has to be based on their rational estimate,  $\hat{\tau}$ , of the item acquisition time that would be set by the firm. This means that a first-period player will decide to incur the cost  $e$  to participate in the game if and only if her expected surplus,  $V - \hat{\tau} - t - e$ , is non-negative. Moreover, conditional on participation, the first-period players will find out the actual item acquisition time  $\tau$  set by the game provider. Given that the players' estimate is the rational expectation of the firm's behavior (i.e.,  $\hat{\tau} = \tau^*$ ) and the participation cost  $e$  is sunk, the first-period participating players' ex post surplus in equilibrium,  $V - \tau^* - t$ , would be strictly positive. This implies that, upon participation, the first-period players would necessarily spend  $\tau^*$  on item acquisition and  $t$  on game-play.

Let us then consider the game provider's optimization problem. The game provider needs to decide on  $\tau$  and  $\theta$  by maximizing its total profit across both periods, while taking into account the players' optimal decisions. Given  $\tau$  and  $\theta$ , the firm's revenue in period 2 shall remain the same as in the previous section, i.e.,  $\pi_2 = \pi_s$ . In addition, as discussed above, the first-period demand will be  $1 - \hat{\tau} - t - e$  and the unit revenue will be  $\tau + t$ . This yields the first-period revenue  $\pi_1 = (1 - \hat{\tau} - t - e)(\tau + t)$ , which is a function of the players' estimate  $\hat{\tau}$ . Note that the game provider cannot directly observe  $\hat{\tau}$ . As a result, the firm would maximize the total profit  $\Pi = \pi_1 + \pi_2$ , while taking  $\hat{\tau}$  as given. Taking the first-order derivative of  $\Pi$  with respect to  $\tau$  and imposing the rational expectation constraint  $\hat{\tau} = \tau^*$ , we will be able to solve the equilibrium item acquisition time as a function of the trade policy  $\theta$ :

$$\tau^* = \frac{1}{2} \left( 1 - t + \frac{1 - 2e - 3t}{1 + 2\alpha + 2\theta - 2\alpha\theta} \right). \quad (5)$$

Note that the players in period 1 know the firm's trade policy  $\theta$ , and they also understand the

firm's optimal decision process as elaborated above. Therefore, the above solution  $\tau^*$  also gives rise to the equilibrium value of the players' rational estimate  $\hat{\tau}^*$ .

We are then ready to study the interaction of the firm's decision variables  $\tau$  and  $\theta$ .

**Proposition 2.** *When  $t < \frac{1-2e}{3}$ , the optimal item acquisition time  $\tau^*$  decreases with  $\theta$ , but when  $t \geq \frac{1-2e}{3}$ ,  $\tau^*$  increases with  $\theta$ .*

The influence of the trade policy on the optimal item acquisition time at  $e \rightarrow 0$  is shown in Figure 1. Recall that in the simple static model in the previous section, the firm's optimal item acquisition time  $\tau^*$  always increases with  $\theta$ . However, here in the presence of the period 1 consumers,  $\tau^*$  may either increase or decrease with  $\theta$ . In particular, when the game play time  $t$  is sufficiently short, the firm should optimally set a lower item acquisition time as the trade control becomes tighter, whereas the reverse is true when  $t$  is sufficiently large.

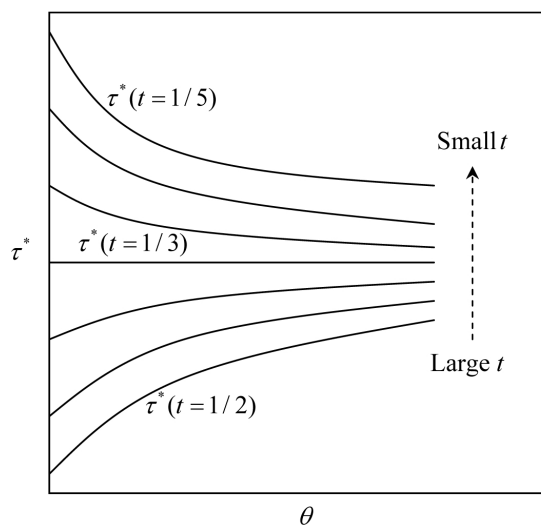


Figure 1: The Influence of the Trade Policy on the Optimal Item Acquisition Time when  $e \rightarrow 0$

To understand this, note that in setting the profit-maximizing item acquisition time, the firm takes into account the revenues from both periods. As discussed previously in the static model, the second-period tradeoff involves 1) raising the unit revenue (primarily for the item-acquiring players) by increasing the item acquisition time, and 2) enlarging total demand (primarily for the

item-buying players) by reducing the item acquisition time. Moreover, the firm needs to consider the revenue from the first-period players as well. As discussed earlier, the consumers in period 1 will participate in the game only when their expected surplus,  $V - \hat{\tau} - t - e$ , is non-negative. This means that the first-period demand is given by  $1 - \hat{\tau} - t - e$ , which is not *directly* influenced by the actual item acquisition time  $\tau$ . As a result, all else being equal, the firm has an incentive to raise  $\tau$  in order to increase the first-period profit.<sup>10</sup> This incentive is magnified as the equilibrium first-period demand increases.

Therefore, when the time for game-play is sufficiently short (i.e.,  $t < \frac{1-2e}{3}$ ) and thus the equilibrium first-period demand is sufficiently large, the firm's incentive to raise the item acquisition unit revenue from the first-period players outweighs its incentive to balance the second-period tradeoff between unit revenue and demand. In this case lowering the item acquisition time can increase the second-period profit. In particular, when the equilibrium item acquisition time is sufficiently high, reducing  $\tau$  and thus increasing the second-period demand can lead to a larger improvement in the second-period revenue from the item-acquiring players than from the item-buying players (i.e.,  $\frac{\partial \pi_a}{\partial \tau} < \frac{\partial \pi_b}{\partial \tau}$ ). This is because, all else being equal,  $\pi_a$  involves a higher unit revenue than  $\pi_b$ . As a result, as  $\theta$  increases and thus the firm cares more about the revenue from the second-period item-acquiring players, the firm would like to set a lower item acquisition time (i.e.,  $\frac{\partial \tau^*}{\partial \theta} < 0$ ).

On the other hand, when the game-play time is long enough (i.e.,  $t > \frac{1-2e}{3}$ ) and thus the equilibrium first-period demand is sufficiently small, the equilibrium item acquisition time  $\tau^*$  would not be too high. In this case raising the item acquisition time from the equilibrium point  $\tau^*$  results in a smaller reduction (or a larger increase) in the second-period revenue from the item-acquiring players than from the item-buying players (i.e.,  $\frac{\partial \pi_a}{\partial \tau} > \frac{\partial \pi_b}{\partial \tau}$ ), but can increase the first-period revenue. As a result, when the revenue from the second-period item-acquiring players becomes more important (i.e.,  $\theta$  increases), the firm would be better off increasing the item acquisition time (i.e.,  $\frac{\partial \tau^*}{\partial \theta} > 0$ ).

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<sup>10</sup>This issue of consumer hold up is reminiscent of Wernerfelt (1994) and Villas-Boas (2009).

We can also use the implicit function theorem to examine the influence of the trade policy on the optimal item acquisition time. Formally, we have

$$\frac{\partial \tau^*}{\partial \theta} = -\frac{\partial^2 \Pi / \partial \theta \partial \tau}{\partial^2 \Pi / \partial \tau^2} = -\frac{(1 - \alpha)(1 - 2\tau^* - t)}{\partial^2 \Pi / \partial \tau^2},$$

which is negative when the game-play time  $t$  is sufficiently short and thus the equilibrium item acquisition time  $\tau^*$  is sufficiently high, but positive when  $t$  is sufficiently high and thus  $\tau^*$  becomes sufficiently low.

Next, we study the influence of the virtual property trade policy on the game provider's profit and characterize the optimal trade policy.

**Proposition 3.** *There exists a threshold point  $\bar{t}$  such that, the firm's optimal trade policy is  $\theta^* = 1$  if  $t < \bar{t}$ , and  $\theta^* = 0$  if otherwise.*

This proposition establishes the central result of the paper. It suggests that the game provider may not always want to exert perfect control to completely remove virtual property trade, even though the cost of doing so is zero. Instead, it may be in the firm's best interest to adopt a loose policy accommodating the transactions between the traders and the players. In other words, the firm may optimally exercise self restriction in its prevention of virtual property trade. This stands in contrast to the static model in which perfect trade prohibition is the dominant firm strategy. Moreover, this proposition suggests that the firm's optimal trade policy takes an extreme form (i.e., either  $\theta^* = 1$  or  $\theta^* = 0$ ), and is influenced by the game-play time (i.e.,  $t$ ). In particular, the firm should encourage all second-period players to buy the virtual item from the traders, if and only if the game-play time is sufficiently long.

To understand this result, we need to investigate the effects exerted by the trade policy on the firm's equilibrium profit. Note first that the main effect on the second-period revenue, as identified in the static model in the previous section, still takes place here. All else being equal, a more strict trade policy can increase the number of second-period players who spend time acquiring the virtual item in the firm's game realm, thus improving the firm's profitability. In addition, there exists an indirect, strategic effect on the first-period players' participation behavior. Recall

that when the first-period players decide whether to join the game, they are informed of the trade policy  $\theta$  but not of the actual item acquisition time  $\tau$ . As a result, the first-period players' demand is influenced by their estimation on the item acquisition time ( $\hat{\tau}$ ). Fewer first-period players would participate if they expect that the firm would set a higher  $\tau$ . Moreover, the players understand the firm's optimization process regarding how the trade policy  $\theta$  may influence the firm's optimal setting of the item acquisition time. Therefore, to the extent that the firm's optimal decision on  $\tau$  indeed responds to  $\theta$ , as we demonstrate in Proposition 2, the trade policy can exert a strategic effect on the first-period demand.

In particular, when the game-play time is sufficiently shorter (i.e.,  $t < \frac{1-2e}{3}$ ), Proposition 2 suggests that a more strict trade policy implies that the firm would optimally set a shorter item acquisition time. This in turn induces more first-period players to participate in the game. In other words, the strategic effect of trade control on the first-period demand is positive in this case. As a result, the beneficial effect on the second-period revenue can be strengthened, and the firm will necessarily benefit from imposing a perfect control over virtual property trade.

However, when the time to play the game is medium (i.e.,  $\bar{t} > t \geq \frac{1-2e}{3}$ ), the strategic effect of trade control on player participation in the first period will become negative. This is because, as shown in Proposition 2, now a more strict trade policy implies that it is optimal for the firm to increase the time on item acquisition. Knowing this, less first-period players would like to join the game. Consequently, trade control becomes a “double-edged sword” and the firm's optimal trade policy hinges on the relative importance of the strategic versus the direct effects. As we show in the Appendix, in this case the firm's equilibrium profit is convex in  $\theta$ , which means that no intermediate trade policy can be optimal. In addition, a more strict trade policy can be harmful when  $\theta$  is relatively small, but beneficial when  $\theta$  is relatively large. This is because the relative importance of the main effect on the second-period revenue (i.e., the difference in the unit revenue between the item-acquiring and the item-buying players) is positively related to the equilibrium item acquisition time, which is in turn positively influenced by the trade policy  $\theta$ . For example, when  $\theta$  is relatively small, the equilibrium item acquisition time is relatively short as well, which

in turn means that the positive main effect is less important than the negative strategic effect.

Furthermore, when the game-play time is sufficiently long (i.e.,  $t \geq \bar{t}$ ), the firm would be better off if all second-period players are allowed to buy from the traders (i.e.,  $\theta^* = 0$ ). This is because a higher  $t$  reduces the importance of the main effect but increases the importance of the strategic effect. Note that the loss arising from a lower second-period unit revenue (due to a lower  $\theta$ ) becomes smaller, if the game-play time is longer and thus the second-period demand is lower (i.e.,  $\frac{\partial^2 \pi_2}{\partial \theta \partial t} = \frac{\partial(\pi_a - \pi_b)}{\partial t} = -(1 - \alpha)\tau < 0$ ). On the other hand, a longer game-play time means a larger improvement in the first-period revenue, as more first-period players choose to participate in the game in response to a less strict trade policy. Therefore, when  $t$  is sufficiently large, the firm would optimally set a perfectly loose trade policy.

The main message here is that a game provider should look into the nature and design of the individual games to decide on its optimal policy on virtual property trade. For example, if a game contains only limited quests and requires a limited playing time, the firm should strictly prohibit the trade of virtual property. If on the other hand the characteristics of another game requires a significant amount of time to play, with huge number of quests and maps, then virtual goods trade should be allowed for that game.

Another interesting implication is regarding the extreme form of the optimal trade policy (i.e.,  $\theta^* \in \{0, 1\}$ ). This finding helps us understand the dramatically distinctive trade control measures adopted by different game providers. Although in principle firms can exert varying levels of control over virtual property trade through, for example, imposing appropriate clauses in the ‘Terms of Use’ or the ‘End User Licensing Agreement,’ the actual use and implementation of these procedures differ greatly across game titles. For example, some games such as *RuneScape* and *World of Warcraft* strictly prohibit virtual property trade, while others such as *Lineage II* by NCsoft and *Everquest II* by Sony either tacitly or publicly allow it.



## 4 Extension

In this section, we extend our model in several dimensions and relax some of the assumptions regarding consumer utility function, game-play time, item acquisition time, and consumer information.

### 4.1 Game-play Time Decided by Players

In the previous sections we assume that a player derives an overall valuation  $V$  from playing the game and a player's time spent in the game-play ( $t$ ) represents only a financial cost to her. In this setup, the parameter  $t$  essentially captures a lump sum pay from the player to the game provider. Consequently,  $t$  decreases player utility. We also assumed that  $t$  is an exogenous parameter and is homogeneous across players. We now relax these assumptions. Instead of assuming player utility a decreasing function of  $t$ , we now model the player valuation as a concave function of  $t$ :  $V = \omega t - \frac{1}{2}t^2$ . This suggests that, when starting to play a game (i.e.,  $t$  is small) the longer a player stays in a game, the higher utility she receives. As  $t$  increases, she may reach a satiation point and playing the game may become boring and even decrease her utility. Furthermore, we also make the parameter  $t$  a player decision variable. Such a setup allows each individual consumer to optimally decide the time she will spend in the game, thus the payment to the gaming firm. This is also closely linked to games where the storyline is considered relatively short by the players and the gaming experience involves certain repetitive but enjoyable activities. With such changes, the player's utility function becomes:

$$u = \omega t - \frac{1}{2}t^2 - t - \tau, \tag{6}$$

where  $\omega \sim U[0, W]$ . To ensure the existence of internal solution, we focus on the case of  $W > \frac{1+\alpha}{\alpha}$ . Consistent with the previous specification, we also focus on the case of the participation cost  $e$  for period 1 consumers being sufficiently small, and total number of players in each period is normalized to 1.

Each player optimizes her game-play time  $t$ , and if her optimized utility  $u(t^*) > e$ , she will

participate in the game. Obviously,  $t^* = \omega - 1$  and  $u(t^*) = \frac{(\omega - 1)^2}{2} - \tau$ . Therefore, only those consumer with  $\omega$  larger than  $\sqrt{2\tau} + 1$  will participate in the game. This means that 1) in each period the number of players in the game is  $\frac{W - \sqrt{2\tau} - 1}{W}$ , and 2) in each period, the demand of game-play time  $t$  is

$$\int_{\sqrt{2\tau}+1}^W \frac{\omega - 1}{W} d\omega = \frac{(W - 1)^2 - 2\tau}{2W}.$$

Consequently, the firm's total profit becomes:

$$\begin{aligned} \Pi &= \pi_1 + \theta\pi_a + (1 - \theta)\pi_b, \\ \pi_1 &= \frac{(W - 1)^2 - 2(\hat{\tau} + e)}{2W} + \frac{W - \sqrt{2(\hat{\tau} + e)} - 1}{W}\tau, \\ \pi_a &= \frac{(W - 1)^2 - 2\tau}{2W} + \frac{W - \sqrt{2\tau} - 1}{W}\tau, \\ \pi_b &= \frac{(W - 1)^2 - 2\tau}{2W} + \frac{W - \sqrt{2\tau} - 1}{W}\alpha\tau. \end{aligned} \tag{7}$$

The equilibrium item acquisition time  $\tau^*$  for the firm can be calculated as in the previous sections and when  $e \rightarrow 0$ :

$$\tau^* = \frac{2(2 - W + \alpha - W\alpha + \theta - W\theta - \alpha\theta + W\alpha\theta)^2}{(2 + 3\alpha + 3\theta - 3\alpha\theta)^2}. \tag{8}$$

We now examine their relationship in equilibrium.

**Proposition 4.** *In the case of consumers deciding their own game-play time  $t$ , when  $W > 4$ , the optimal item acquisition time  $\tau^*$  decreases with  $\theta$  ( $\frac{\partial\tau^*}{\partial\theta} < 0$ ); and when  $W < 4$ ,  $\tau^*$  increases with  $\theta$  ( $\frac{\partial\tau^*}{\partial\theta} > 0$ ).*

Proposition 4 is strikingly similar to Proposition 2. Recall that in Proposition 2, when  $t < \frac{1-2e}{3}$  ( $t \geq \frac{1-2e}{3}$ ), the optimal item acquisition time decreases (increases) with  $\theta$ . In that setup with fixed  $V$  and exogenous  $t$ , a lower  $t$  means a higher potential demand for the game. In the current specification with  $V$  being an increasing function of  $\omega$ , and  $\omega \sim U[0, W]$ , a higher  $W$  essentially means players value the game more, and this translates into a higher number of players participating in the game.<sup>11</sup> Thus a large  $W$  is qualitatively similar to the case of a small  $t$  in the previous model specification.

<sup>11</sup>The number of players in the game is  $\frac{W - \sqrt{2\tau} - 1}{W}$  and this obviously increases with  $W$ .

Essentially, what Proposition 4 establishes is also consistent with that of Proposition 2: when the potential demand for the game is high, then a tight trade control should be accompanied by a lower item acquisition time. The intuition behind Proposition 4 is also similar to that of Proposition 2, relating to the issue of consumer hold up in period 1 and the relative importance of period 1 and period 2 demand.

Proposition 4 shows that our findings in the main model is robust under alternative specifications regarding the nature of online game-play and consumer utility. More importantly, it also confirms the strategic effects of trade control. The game provider will then take this into account and may strategically adopt a loose trade policy to signal a low item acquisition cost and to attract potential players.

**Proposition 5.** *When  $W < \frac{31}{14}$ , the firm's optimal trade policy  $\theta^* < 1$ .*

Proposition 5 qualitatively confirms the robustness of the findings in Proposition 3. It again suggests that when the potential demand is low ( $W$  being small), the game provider may want to have a more tolerating policy towards virtual goods trade. Such a tolerating trade policy can serve as a signal of a low item acquisition time ( $\tau$ ) to consumers, and consequently encourages participation for those in period 1.

## 4.2 Consumers Cannot Distinguish between $t$ and $\tau$

In the main model, we implicitly assumed that consumers can distinguish the game-play time  $t$  from item acquisition time  $\tau$ . We now relax this assumption and examine the case where consumers cannot distinguish between  $t$  and  $\tau$ . Furthermore, we also make the game-play time  $t$  an endogenous variable decided by the game provider.

We use the same utility function as in the main model (Equation 1). Since consumers cannot distinguish between  $t$  and  $\tau$ , the expected surplus for those in period 1 becomes:  $V - \hat{t} - \hat{\tau} - e$ . And they will participate in the game if and only if this expected surplus is non-negative. Compared to the main model, the game-play time  $t$  now becomes unknown to the consumers in period 1 and they have to rely on their rational expectation of  $t$  to make a participation decision. However,

from the gaming firm's point of view, because the traders do not spend time in game-play and they are more efficient in item acquisition,  $t$  and  $\tau$  represent two different revenue streams. To ensure interior solutions, we assume that the firm faces a quadratic cost structure with respect to  $t$  and  $\tau$ :  $\frac{c}{2}t^2$ , and  $\frac{c}{2}\tau^2$ .<sup>12</sup> With this modified model structure, the firm's total profit becomes:

$$\begin{aligned}\Pi &= \pi_1 + \theta\pi_a + (1 - \theta)\pi_b - \frac{c}{2}t^2 - \frac{c}{2}\tau^2, \\ \pi_1 &= (1 - \hat{\tau} - \hat{t} - e)(\tau + t), \\ \pi_a &= (1 - \tau - t)(\tau + t), \\ \pi_b &= (1 - \tau - t)(\alpha\tau + t).\end{aligned}\tag{9}$$

Following the analytical procedure as in the main model, we can solve the firm's optimal  $t$  and  $\tau$  as functions of  $\theta$ :

$$\begin{aligned}t^* &= \frac{c(2 - e) + (1 - \alpha)(1 - \theta)(\alpha + \theta + e - \alpha\theta)}{c^2 - (1 - \alpha)^2(1 - \theta)^2 + 2c(2 + \alpha + \theta - \alpha\theta)}, \\ \tau^* &= \frac{-(1 + e)(1 - \alpha)(1 - \theta) + c(1 + \alpha + \theta - e - \alpha\theta)}{c^2 - (1 - \alpha)^2(1 - \theta)^2 + 2c(2 + \alpha + \theta - \alpha\theta)}.\end{aligned}\tag{10}$$

**Proposition 6.** *When consumers can not distinguish between  $t$  and  $\tau$ , the firm's optimal game-play time  $t^*$  decreases with  $\theta$  ( $\frac{\partial t^*}{\partial \theta} < 0$ ), while its optimal item acquisition time  $\tau^*$  increases with  $\theta$  ( $\frac{\partial \tau^*}{\partial \theta} > 0$ ).*

Similar to the main model and the first extension, Proposition 6 establishes the strategic effect of the trade control: a loose trade control (lower  $\theta$ ) can signal a lower item acquisition time  $\tau$  for the consumers in period 1. Because the game-play time  $t$  is a firm decision now, a lower  $\theta$  also signals a higher  $t$ . As the trade control becomes more stringent ( $\theta$  increases), the firm has more incentive to raise  $\tau$  than to raise  $t$ . This is because the trade control in period 2 only affects the unit revenue from  $\tau$ , not  $t$ . Since consumers cannot distinguish between  $t$  and  $\tau$ , their participation decisions depend on the sum of  $t$  and  $\tau$ . The overall signaling effect of trade control

<sup>12</sup>With linear cost structure or zero marginal cost,  $t^*$  may approach 0 and consequently mixed strategy equilibrium may emerge.

$\theta$  can be obtained:

$$\frac{\partial(\tau^* + t^*)}{\partial\theta} = \frac{c(1-\alpha)(c^2 + 2c(2e - \alpha - (1-\alpha)\theta) + (1-\alpha)(1-\theta)(3 + 4e + \alpha + \theta - \alpha\theta))}{(c^2 - (1-\alpha)^2(1-\theta)^2 + 2c(2 + \alpha + \theta - \alpha\theta))^2}. \quad (11)$$

Depending on the value of  $c$ ,  $\alpha$ , and  $\theta$ , the above partial derivative can be either positive or negative, corresponding to qualitatively different strategic effect of  $\theta$ . For example, when  $\alpha = \frac{1}{8}$ ,  $\theta = \frac{1}{2}$ , and  $c = \frac{1}{32}$ , Equation (11) is positive. When  $\alpha = \frac{63}{64}$ ,  $\theta = \frac{1}{2}$ , and  $c = \frac{1}{32}$ , Equation (11) is negative.

Admittedly, when the game-play time is an endogenous variable, the gaming firm has to take into account the signaling effects of  $\theta$  on both  $t$  and  $\tau$ . This will moderate our results in certain conditions. For example, when  $c = 1$ , strict policy ( $\theta = 1$ ) is always optimal. On another note, our main insight with respect to the strategic effect of trade control remain robust.

When  $\frac{\partial(\tau^* + t^*)}{\partial\theta} > 0$ , a smaller  $\theta$  signals that a lower overall payment is required in period 1. Thus more consumers will join the game if they observe a loose trade policy. This, in turn, may encourage the gaming firm to adopt a loose trade policy in some cases. For example, when  $c = 1/8$  and  $\alpha = 1/2$ , the firm's profit at  $\theta = 1$  is strictly less than that at  $\theta = 0$ . In other words, a strict trade policy is at least inferior to zero trade control.<sup>13</sup>

### 4.3 Period 1 Consumers Having Some Knowledge of $\tau$

We now address the issue that consumers in period 1 have no knowledge of  $\tau$  prior to participation. More specifically, we assume that with probability  $\beta$ , consumers know  $\tau$  before participation, and with probability  $1 - \beta$ , they do not. Similar to the main model, all consumers can figure out  $\tau$  upon participation. In this extended model, for those who know  $\tau$ , their decision is similar to that in the static model. For those who do not know  $\tau$ , their behaviors are identical to that in the main model. From the gaming firm's point of view, this means that  $\beta$  portion of consumers in period 1 will behave similarly to the ordinary players in period 2 and acquire items by spending

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<sup>13</sup>The optimal trade policy may lie in somewhere between 0 and 1.

$\tau$  (since there is no trader in period 1). Consequently, the firm's profit becomes:

$$\begin{aligned}
\Pi &= \pi_1 + \theta\pi_a + (1 - \theta)\pi_b, \\
\pi_1 &= \beta(1 - \tau - t)(\tau + t) + (1 - \beta)(1 - \hat{\tau} - t - e)(\tau + t), \\
\pi_a &= (1 - \tau - t)(\tau + t), \\
\pi_b &= (1 - \tau - t)(\alpha\tau + t).
\end{aligned} \tag{12}$$

Following the analytical path as in the main model, we have:

$$\tau^* = \frac{1 + \alpha + \theta - \alpha\theta - e - t(2 + \alpha + \beta + \theta - \alpha\theta)}{1 + \beta + 2(\alpha + \theta - \alpha\theta)}. \tag{13}$$

**Proposition 7.** *When  $t < \frac{1 - 2e - \beta}{3 + \beta}$ ,  $\tau^*$  decreases with  $\theta$ ; when  $t \geq \frac{1 - 2e - \beta}{3 + \beta}$ ,  $\tau^*$  increases with  $\theta$ .*

Notice that when  $\beta \rightarrow 0$ , Proposition 7 is identical to Proposition 2. More interestingly, the threshold where  $\frac{\partial \tau^*}{\partial \theta}$  changes sign (i.e.,  $\frac{1 - 2e - \beta}{3 + \beta}$ ) decreases with  $\beta$ . This means, as the number of informed consumers ( $\beta$ ) increases,  $\tau^*$  is more likely to decrease with  $\theta$ , which suggests that a loose trade control (lower  $\theta$ ) tends to signal a rather higher item acquisition time  $\tau$ . In other words, a loose trade control can hurt the potential demand in period 1, and the gaming firm may be better off applying a tight trade policy. Not surprisingly, better knowledge of  $\tau$  at the consumer side will alleviate, but not remove, the results in the main model.

## 5 Discussion

The surging growth of the online game industry has created a new concept: the virtual economy, where there is real life value and real money trade for properties that exist only in a virtual world. Industry practitioners have adopted dramatically different attitudes toward the second hand market of such properties, ranging from strict prohibition to outright permission. While legislators are still debating over legal issues on this concept, until now no formal economic analysis has been done, in particular on the impact of virtual property trade on firm profitability. We attempt to fill this gap in this paper.

We concentrate on a monopolist firm providing an online game to potential players who enter the market sequentially and pay to spend time in the game realm. There is a virtual item that is indispensable to playing the game. Prior to incur a cost to sign up for the game, the first-period players are uncertain about the time they have to spend in order to acquire the virtual item in the game realm. The second-period players are certain about the item acquisition time before participation, and they can either acquire it from within the game realm or buy it with real world money from traders who are more efficient in item acquisition. The traders effectively reduce the game provider's unit revenue, and common sense therefore suggests that the firm should prohibit the trade. However, we identify another economic effect of virtual property trade that can be beneficial to the firm. In particular, the firm's trade policy can act as a signal to the early players, prior to making their decision on game participation, about the item acquisition cost they will have to pay. It is due to this signalling effect that a less strict trade policy may induce more players to join the game. Consequently, the firm may be better off strategically giving up strict control over virtual item trade, despite the potential revenue loss given to the traders.

Our stylized model is built on several assumptions that may not fully capture the complex nature of the fast evolving gaming industry. We also consider three extended setups where some key assumptions are relaxed. In the first extension, we study the case where consumers decide the amount of time they will spend in playing the game, and their valuation of the game may increase with the game-play time. In the second extension, we examine the case where consumers cannot distinguish between game-play time and item acquisition time, and we make the game-play time a decision variable by the gaming firm. In the third extension, we explore the possibility of consumers having some knowledge about item acquisition time. In all three extensions, we show that the strategic effect of trade control on player participation can counteract and even dominate the direct effect. That is, because of the strategic effect, the game provider can benefit from abandoning a strict trade policy in situations when the existence of the traders decreases its unit revenue.

While the extensions confirm the robustness of the results in the main model, our findings

can be limited in two important aspects. The first is the lack of competition in our model. We have only examined the incentives of a monopolist gaming firm. While the gaming industry is dominated by a few major players (Electronic Arts, Blizzard, NCSoft, Sony, Tencent) depending on the geographical locations, competition is certainly an important factor that may affect firms' decision on virtual trade. The second is that we do not explicitly model the second-hand market of the virtual goods. While our main results will hold as long as the item price is positively correlated to the acquisition cost, a more detailed examination of the supply-demand interaction in the virtual goods market would help understand the in-depth, structural impact of the trade policy.

The insights in this paper can go beyond online games and be applied to other contexts where player interaction is important and virtual property is valuable. For example, with advances in mobile broadband technology, games on cell phones are becoming increasingly interconnected. Connected console games are also growing fast in popularity. In these cases, player interaction is gaining increasing importance, and one may thus expect that the trade of virtual goods will surge in the near future. Moreover, similar to online games, these markets are commonly characterized by player uncertainty and costly participation. As a result, this paper can also shed light on realm management and virtual property trade in these industries.

Our stylized model provides directions for future research. We have assumed that each player needs one unit of the virtual item to play the game. An interesting issue for future investigation is regarding sequential item acquisition when, as is normally the case, there are multiple items to acquire. It can be interesting to explicitly consider the strategic behavior of the traders (e.g., item acquisition and price competition). In addition, to focus on the issue of virtual property trade, we have abstracted away from modeling the interaction in game-play between the players. Last but not least, social and psychological issues on game players (e.g., addition, cooperation, violence), both in online games and in the real world, deserve more research. We hope that this paper can inspire more formal economic analysis on this line of inquiry.



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## Appendix

**Proof of Proposition 2:** From Equation (5), calculating the first-order derivative of  $\tau^*$  with respect to  $\theta$ , we have:

$$\frac{\partial \tau^*}{\partial \theta} = \frac{(1 - 2e - 3t)(\alpha - 1)}{(1 - 2\alpha\theta + 2\alpha + 2\theta)^2}. \quad (\text{A-14})$$

Obviously, the above equation is negative if  $t < \frac{1-2e}{3}$ , and greater than zero if otherwise. **Q.E.D.**

**Proof of Proposition 3:** Substituting Equation (5) into the firm's profit function (i.e.,  $\Pi = \pi_1 + \pi_2$ ) and then taking the first-order derivative of the equilibrium profit  $\Pi^*$  with respect to  $\theta$ , we have:

$$\frac{\partial \Pi^*}{\partial \theta} = \frac{1 - \alpha}{4} \left( (1 - t)^2 + \frac{2(1 - 2e - 3t)^2}{(1 - 2\alpha\theta + 2\alpha + 2\theta)^3} + \frac{(1 - 2e - 3t)(1 + 2e + 5t)}{(1 - 2\alpha\theta + 2\alpha + 2\theta)^2} \right), \quad (\text{A-15})$$

which is positive when  $t < \frac{1-2e}{3}$ . This means that the optimal trade policy is  $\theta^* = 1$  if  $t < \frac{1-2e}{3}$ .

Consider then the case when  $t \geq \frac{1-2e}{3}$ . Taking the second-order derivative of the equilibrium profit  $\Pi^*$  with respect to  $\theta$ , we have:

$$\frac{\partial^2 \Pi^*}{\partial \theta^2} = \frac{-2(1 - 2e - 3t)(1 - \alpha)^2(2(1 - e - t) + (1 + 5t + 2e)(\alpha + \theta - \alpha\theta))}{(1 - 2\alpha\theta + 2\alpha + 2\theta)^4}, \quad (\text{A-16})$$

which is positive for  $t \geq \frac{1-2e}{3}$ . This implies that  $\Pi^*$  is convex in  $\theta$  in this case.

The above analysis suggests that the optimal  $\theta$  is either 0 or 1. Therefore, we just need to compare  $\Pi^*(\theta = 0)$  and  $\Pi^*(\theta = 1)$  to find out the optimal trade policy. The first-order derivative of  $\Pi^*(\theta = 0) - \Pi^*(\theta = 1)$  with respect to  $t$  is:

$$\frac{\partial(\Pi^*(\theta = 0) - \Pi^*(\theta = 1))}{\partial t} = \frac{(1 - \alpha)(1 + 2\alpha(1 + e + t + \alpha - \alpha t))}{(1 + 2\alpha)^2} > 0. \quad (\text{A-17})$$

This means that as  $t$  increases,  $\Pi(\theta = 0) - \Pi(\theta = 1)$  becomes greater. In addition, solving  $\Pi^*(\theta = 0) - \Pi^*(\theta = 1) = 0$  with respect to  $t$  yields two roots:

$$t = \begin{cases} \frac{3 + 6\alpha(1 + e + \alpha) - (1 + 2\alpha)\sqrt{9 + 4\alpha(1 + e)(4 + e)}}{-6\alpha(1 - \alpha)} \\ \frac{3 + 6\alpha(1 + e + \alpha) + (1 + 2\alpha)\sqrt{9 + 4\alpha(1 + e)(4 + e)}}{-6\alpha(1 - \alpha)}. \end{cases} \quad (\text{A-18})$$

Obviously, the second root is negative and we therefore have:

$$\bar{t} = \frac{3 + 6\alpha(1 + e + \alpha) - (1 + 2\alpha)\sqrt{9 + 4\alpha(1 + e)(4 + e)}}{-6\alpha(1 - \alpha)}. \quad (\text{A-19})$$

**Q.E.D.**

**Proof of Proposition 4:** Differentiating Equation (8) with respect to  $\theta$ , we have:

$$\frac{\partial \tau^*}{\partial \theta} = \frac{4(W - 4)(\alpha - 1)(2 - W + \alpha - W\alpha + (1 - W)(1 - \alpha)\theta)}{(-2 - 3\alpha(1 - \theta) - 3\theta)^3}.$$

Notice the expression in the right parenthesis of the numerator:

$$2 - W + \alpha - W\alpha + (1 - W)(1 - \alpha)\theta = 1 + (1 - W)\phi, \text{ where } \phi = 1 + \alpha + \theta - \alpha\theta > \alpha.$$

Because  $W > \frac{1 + \alpha}{\alpha}$ , we have  $W > \frac{1 + \phi}{\phi} \implies 1 + (1 - W)\phi < 0$ . Therefore, when  $W > 4$ ,  $\frac{\partial \tau^*}{\partial \theta} < 0$ , and when  $W \leq 4$ ,  $\frac{\partial \tau^*}{\partial \theta} \geq 0$ . **Q.E.D.**

**Proof of Proposition 5:** To show that  $\theta^* < 1$ , we just need to prove that at  $\theta \rightarrow 1$ , the firm's total profit decreases with  $\theta$ , i.e.,  $\frac{\partial \Pi^*(\tau, \theta)}{\partial \theta} < 0$ . From the envelope theorem, we have:

$$\frac{\partial \Pi^*(\tau, \theta)}{\partial \theta} \Big|_{\theta \rightarrow 1} = \frac{\partial \Pi(\tau, \theta)}{\partial \theta} \Big|_{\theta \rightarrow 1, \tau = \tau^*} = \frac{2(1 - \alpha)(2W - 3)(14W^2 - 17W - 31)}{625W}.$$

Because  $W > \frac{1 + \alpha}{\alpha} \geq 2$ , it can be easily checked that when  $W < \frac{31}{14}$ , the above expression is negative. **Q.E.D.**

**Proof of Proposition 6:** Differentiating  $t^*$  and  $\tau^*$  in Equation 10 with respect to  $\theta$ , we have:

$$\begin{cases} \frac{\partial t^*}{\partial \theta} = \frac{-(1 - \alpha)((1 - \alpha)^2(1 - \theta)^2 + c^2(3 + 2\alpha(1 - \theta) + 2\theta) + 2c(\alpha^2(1 - \theta)^2 + \theta(2 + \theta) + 2\alpha(1 - \theta^2)))}{(c^2 - (1 - \alpha)^2(1 - \theta)^2 + 2c(2 + \alpha + \theta - \alpha\theta))^2}, \\ \frac{\partial \tau^*}{\partial \theta} = \frac{(1 - \alpha)((1 - \alpha)^2(1 - \theta)^2 + 3c^2 + c^3 + c(3 + \alpha^2(1 - \theta)^2 + \theta(2 + \theta) + 2\alpha(1 - \theta^2)))}{(c^2 - (1 - \alpha)^2(1 - \theta)^2 + 2c(2 + \alpha + \theta - \alpha\theta))^2}. \end{cases} \quad (\text{A-20})$$

Given that  $0 < \alpha < 1$  and  $0 \leq \theta \leq 1$ , the signs in the above expressions can be easily checked.

**Q.E.D.**

**Proof of Proposition 7:** Differentiating  $\tau^*$  with respect to  $\theta$  in Equation 13, we have:

$$\frac{\partial \tau^*}{\partial \theta} = \frac{(1 - \alpha)(\beta + t(3 + \beta) + 2e - 1)}{(1 + \beta + 2\alpha(1 - \theta) + 2\theta^2)^2}.$$

The sign of the above expression depends on the sign of  $\beta + t(3 + \beta) + 2e - 1$ , and we can obtain the threshold of  $t$  in the Proposition. **Q.E.D.**