The Emergence of Opinion Leaders in Social Networks

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Abstract

A major challenge in many word-of-mouth marketing campaigns is the cost-effective identification of opinion leaders, consumers who exert disproportionate influence on the purchase decisions of other consumers. However, empirical studies agree on few consumer characteristics that are strong predictors of opinion leadership. In the present research, we attempt to explain these null or weak findings through a game-theoretic perspective on opinion leader-follower relationships. Our model combines social network, word-of-mouth communication, consumer heterogeneity, and time preference in a setting in which each consumer makes once-in-a-lifetime choice between a new product and an outside option. We find that: (a) opinion leaders emerge in equilibrium whenever certain general conditions are met; (b) counter-intuitively, the most "patient" consumer with the highest time discount factor may become an opinion leader; and (c) equilibria in which opinion leaders are highly-connected and/or have low time discount factors may be neither efficient to consumers nor profit-maximizing to firms, even though they are likely to be focal outcomes.

Keywords: Word of mouth; opinion leaders; social networks; new-product diffusion models; game theory; consumer behavior

1. Introduction

The strategic use of word-of-mouth (WOM) communication as a marketing tool has gained increasing prominence in recent years. An industry report (PQ Media 2007) estimates that US companies' spending on WOM marketing jumped 35.9% in 2006 to $981.0 million, compared with an overall growth of 7.7% in marketing expenditure, and would reach $1 billion in 2007. Novel catchphrases such as "buzz advertising" and "viral marketing" are now well known to marketers, while industry players from established conglomerate Procter and Gamble to enterprising startup BzzAgent (with clients like Coca-Cola and Kellogg) continuously press ahead efforts in utilizing WOM as a medium for product promotion.

Yet, amidst "the buzz on buzz" (Dye 2000), much remains to be clarified as to how product recommendations are spread or shared among consumers, how they affect consumer purchase timing and decisions, and how they may help or hurt profits. A key issue regards opinion leaders (alternatively called "influentials"), consumers who have disproportionate influence on other consumers' purchase decisions. Opinion leaders, as uncovered in numerous studies from the 1940s to the present, are often ordinary consumers with direct influence limited to only a few fellow consumers who are described as followers or opinion seekers (Flynn et al. 1996; Rogers 2003). Firms interested in WOM marketing would naturally hope to find "universal opinion leaders" whom they can always focus resources on and urge to spread good words about their products efficiently; it would even be more helpful if opinion leaders have easy-to-observe traits that allow firms to identify them at little cost. But empirical studies have revealed opinion leadership to be largely domain specific; moreover, researchers can agree on few consumer characteristics that are strong predictors of opinion leadership (or, conversely, opinion seeking). Indeed, Tremor, Procter and Gamble's WOM unit, has gone on to use multi-step intensive screening procedures to single out influentials who then become the "seeds" of their WOM marketing campaigns (Walker 2004). Other firms, however, might not like to bear the financial implications of setting up such an elaborate screening system.

This paper is first and foremost an attempt to explain, from a theoretical perspective, the null or weak findings on opinion leadership characteristics in the empirical literature. We seek to demonstrate the impossibility of locating individual traits (apart from

definitional ones) that are strong predictors of opinion leadership or opinion seeking, by means of a game-theoretic framework under which consumption experience is shared directly or indirectly between heterogeneous consumers in a social network. We show that, under a range of parameters, multiple equilibria, each with different sets of opinion leaders, are possible under our framework, and it is also possible that the most "patient" consumer in the network with the highest time discount factor becomes an opinion leader in equilibrium. Through the notion of focal equilibria, our framework can also accommodate the relatively notable correlation between opinion leadership and network centrality/innovativeness that has been found in previous studies. And yet, as we demonstrate in a subsequent section (in which we also consider a firm that cultivates its own opinion leaders or "buzz agents" in the population), such focal equilibria are not necessarily efficient information-sharing mechanisms for consumers nor profit-maximizing WOM promotion structures for firms. Our research thus casts some doubt on the cost-effectiveness of WOM marketing campaigns as well as their benefits to consumer welfare, and calls for firms to deliberate with care when trying to rely on opinion leaders to promote products.

1.1 Opinion Leadership

Opinion leadership as an idea originated in communication studies in the 1940s and 1950s that led to Katz and Lazarsfeld's (1955) seminal work. These authors suggest, in a "two-step flow" framework, that an opinion leader, under the influence of the mass media, formed her opinion, which was then passed on to her followers or opinion seekers. In addition, they offered the following explication:

> "What we shall call opinion leadership, if we may call it leadership at all, is leadership at its simplest: it is casually exercised, sometimes unwitting and unbeknown, within the smallest grouping of friends, family members, and neighbors. It is not leadership on the high level of a Churchill, or of a local politico, nor even a social elite. It is at quite the opposite extreme: it is the almost invisible, certainly inconspicuous, form of leadership at the person-to-person level of ordinary, intimate, informal, everyday contact." (p. 138)

Subsequent studies on opinion leadership are consistent with this notion (examples in marketing include King and Summers 1970, Childers 1986, Flynn et al. 1996, Nair et al. 2006, and Godes and Mayzlin forthcoming), which means that an "opinion leader" in those studies can be, in many ways, the polar opposite of a more intuitive view of an opinion leader as "a Churchill", "a local politico", "a social elite", or media celebrity in general. While media celebrities certainly "lead opinions" in many cases, they can often be classified as "innovators" in the typical paradigm of diffusion of innovations,[1] because they often learn about new products and are invited to try them much earlier than all others. Their influence can also be considered part of the influence of the mass media, because of the high exposure of their opinions in the media. However, the opinion leaders that are studied in the literature as well as here are rather in line with Katz and Lazarsfeld's conceptualization. In fact, an opinion leader in this sense may not even be aware ("unwitting") that she is an opinion leader at all, nor may a follower be aware that her purchase decision is influenced by a fellow consumer ("unbeknown"). Researchers have also now recognized that flow of opinion may consist of more than two steps, so that an opinion leader in one opinion leader/follower relationship can be a follower in another such relationship.

Thus, in this study, we shall follow the literature in understanding opinion leaders as consumers "who exert an unequal amount of influence on the decisions on others" (Rogers and Cartano 1962; Flynn et al. 1996) at a person-to-person level. No other a prior notions about opinion leadership is assumed, in accordance with previous studies (in fact, one of our goals is to show that no other a priori notions about opinion leadership *can* be assumed). As Flynn et al. point out, opinion leadership must also be accompanied by opinion seeking: an opinion leader must have someone to "lead", and form an opinion leader/follower relationship with that other consumer. Thus what we shall focus on in our research (and establish formally in a later section) are consumer relationships in which there is a systematic, unidirectional influence of one consumer's purchase decision and recommendation on another's purchase decision; this influence can be direct (that is, between "neighbors" in a social network), as well as indirect. We shall be especially

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[1] Researchers on diffusion of innovations classify adopters into five categories: innovators, early adopters, early majority, late majority, and laggards (Rogers 2003). Note that what Bass (1969) originally called "innovators" is different from Rogers' (2003) sense of the word; see the discussion in Mahajan, Muller, and Srisvastava (1990), who adapt Rogers' classification to the Bass model.

interested in "pure" opinion leaders, who influence other consumers but are themselves not influenced by any other opinion leaders. Unless otherwise stated, the term "opinion leader" in subsequent sections is intended to denote these consumers.

1.2 Findings on Opinion Leader/Follower Characteristics

Empirical studies on opinion leaders have not led to very conclusive evidence regarding what makes them opinion leaders, or, conversely, what makes a follower *not* an opinion leader.[2] King and Summers (1970) and Childers (1986) only find low ($r < .3$) correlation between opinion leadership and measures representing innovativeness, risk perception, and product category experience.[3] Flynn et al. (1996) discover higher correlations between opinion leadership and a number of consumer characteristics, but, after controlling for product involvement (which, as we shall argue, poses a problem of causality with opinion leadership formation), only perceived knowledge ($r = .56$) and innovativeness ($r = .47$) have $r > .3$. Moreover, their study does not reveal opposite correlation between opinion seeking and the characteristics that are expected to correlate with opinion leadership. In short, while there may be some ground in expecting an innovative consumer to be an opinion leader, a follower is equally likely to be innovative or not.

In communication studies, opinion leaders are typically viewed as early adopters of new products, but not that early to be pioneering innovators. They are not particularly adventurous regarding new products, and exactly because of the fact that their taste is near to the norm, their recommendations are valued by other consumers. While Rogers (2003, p.282-283) describes innovator – who make up only about two percent of the population – as "venturesome" with new products, early adopters, who adopt a little later than the innovators, are characterized by their esteemed opinion and their network centrality (roughly, how "well-connected" one is in the social network). In fact, network centrality is found to be pivotal in predicting a consumer's influence in studies such as Godes and Mayzlin (forthcoming).

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[2] In survey studies, opinion leaders can be identified using self-report measures (e.g. King and Summers 1970, Childers 1986, Flynn et al. 1996), or, more accurately (but harder to implement), through sociometric methods tracing communication patterns among the members of the community under examination (Valente 1995).

[3] Childers (1986) uses an opinion leadership measurement scale that is modified from King and Summers (1970); later, Flynn et al. (1996) propose a third alternative.

As mentioned, it has also been found that opinion leadership is related to product involvement, knowledge of the category concerned, or, in the case of opinion leaders who are market mavens, cross-category knowledge (Feick and Price 1987). However, the direction of causality between involvement or knowledge and opinion leadership is uncertain. It can be intuited that consumers become highly involved or knowledgeable towards a category only after they emerge as opinion leaders in equilibrium in the repeated play of some "WOM game", and therefore often have to scout the environment for new products to try them before their followers.

1.4 Related Literature

Our research stands at the confluence of several streams of literature. To start with, a key feature of our framework is that consumers share their consumption experience through WOM communication with their network neighbors. WOM has long been recognized as an important influence on consumer decision. It has been demonstrated that face-to-face WOM has a greater impact on consumer choice over printed information because of its vividness and credibility (Borgida and Nisbett 1977, Herr, Kardes, and Kim 1991). In recent years, the rise of the Internet makes written WOM ("Word of Mouse") through online channels a subject of both theoretical and empirical research including Godes and Mayzlin (2004), Chevalier and Mayzlin (2006), and Chen and Xie (forthcoming).

Despite such progress, a closer look at how social networks influence WOM communication is relatively lacking in the marketing literature. Brown and Reingen (1987) is an early exception with a detailed empirical study of the effects of social networks on WOM communication; they notably find evidence for Granovetter (1973)'s "strength-of-weak-ties" theory, which states that rarely activated weak social ties are important bridges that bring information across tightly knit social groups. But otherwise, as has been remarked by Godes et al. (2005), the marketing literature has been relatively silent about the structure of one's network and the role the actor plays in it. Recent development along this line includes Mayzlin (2002), who considers simple chain and circular networks in her model of "buzz" advertising. But otherwise, social network studies are more common in economics and sociology than in marketing.

In economics, whenever social interactions are studied, researchers traditionally either assume that agents with private information make publicly observable decisions in

sequence (Banerjee 1992, Bikhchandani, Hirshleifer, and Welch 1992, Smith and Sørenson 2000), or agents randomly sample other agents in the population for their opinion (Banerjee 1993, Ellison and Fudenberg 1995, Banerjee and Fudenberg 2004). Çelen and Kariv (2004) also take on the Banerjee (1992) and Bikhchandani et al. (1992) type of social learning problem but limits each agent's information about the history of play to only the decision of the agent immediately in front of her in the queue (rather than the whole history, as in the earlier papers), and find a gradual convergence to herd-like behavior, which is commonly observed in this stream of literature. There is also an accumulating literature on social networks in economics. For example, there are studies on network formation (Jackson and Wolinsky 1996, Bala and Goyal 2000), or agents playing cooperative (Myerson 1977, Bolton, Chatterjee, and McGinn 2003) or dyadic non-cooperative games in exogenous networks (Corominas-Bosch 2003, Cassar 2007). Bala and Goyal (1998) and Gale and Kariv (2003) investigate how information flows in general networks and find conditions for long-run convergence of payoffs and behavior.

Social networks have been studied for many years in sociology, an early and representative example being Granovetter (1973). On the modeling front, random network is often taken as a benchmark in that field, though it has long been acknowledged that this needs be adjusted to accommodate real data (Rapoport 1953a, b). But network research in the past ten years highlights even more sharply the complexity of real-life networks. Empirical evidence has been found for Watts and Strogatz's (1998) and Watts' (1999) "small-world" model (a concept that can be dated back to Milgram 1967) as well as Barabási and Albert's (1999) model, which is most prominently characterized by a scale-free distribution of the number of neighbors to which an agent is connected.[4] However, despite the success of both models in many examples, they fail to characterize network structure satisfactorily in many cases too (Watts 2003). It is in the spirit of finding non-trivial conclusions for WOM communication with marketing implications in general networks that this research is conducted.

In subsequent sections, we describe our conceptual framework and then our formal model. We then set out our general propositions, including the finding that, under a range of parameters, there exist equilibria in which opinion leaders emerge. We also

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[4] Garber et al. (2004), building from Goldenberg, Libai, and Muller (2001, 2002), is a recent and rare attempt to apply such concepts in marketing problems through both simulation and analysis of real life data.

develop further generalizations of our model, and demonstrate that obvious candidates of focal equilibria – those in which the opinion leaders are well-connected and/or relatively "impatient" – are neither necessarily efficient to consumers nor profit-maximizing to firms. We conclude with an overall discussion of our results and scope for further development.

2. Conceptual Framework

In this study, we conceptualize opinion leaders as consumers who influence other consumers but are themselves not influenced by any consumers, and model their emergence under the following framework:

1. Consumers are connected through a social network consisting of WOM communication links;
2. There is a product category for which selling takes place over a finite number of time periods;
3. Every consumer demands at most one unit of the category during the selling horizon, and has preference over early rather than late purchase and consumption, all else being equal;
4. Over the selling horizon, the category consists of a new product and also a well-known outside option, both of which are available at fixed, constant prices;
5. Consumers are ex ante homogeneous but ex post heterogeneous regarding the consumption utility of every product; that is, the expected utility of a product before consumption is the same for every consumer, but its realized consumption (experience) utility can be different from one consumer to another;
6. All consumers hold the same prior information regarding the category;
7. An opinion leader purchases and consumes the new product at the beginning of the selling horizon;
8. After consumption, the opinion leader communicates with her immediate network neighbors about her consumption experience truthfully;
9. Consumption experience regarding the outside option has no informational value by itself, since the outside option is well known; consumption experience regarding the

new product has informational value by itself and can be used to reduce uncertainty over the new product (in a Bayesian manner to be specified in the formal model);

10. There is a positive ex ante probability that the follower's purchase decision will differ according to the opinion leader's consumption experience.

It needs be re-emphasized that, in our framework, an opinion leader needs not be directly communicating with her follower: they may be separated by one or more consumers in a chain of network links. But an opinion leader can still have an indirect influence (in the sense of feature 10 above) on the follower through a series of purchase decisions/consumption experience along the chain that links them up, while a follower may be influenced by more than one opinion leader. This second possibility justifies the clause of positive ex ante probability in feature 10: suppose, for example, a follower takes recommendation from r opinion leaders, and her belief structure is such that she purchases the new product if and only if at least $m < r$ opinion leaders offer positive recommendation for the product. Then, if it turns out that more than m opinion leaders offer positive recommendation, any single opinion leader's recommendation is ex post irrelevant given the all other opinion leaders' recommendations. But, ex ante, there is a positive probability that any single opinion leader's recommendation is pivotal.

We acknowledge that not all opinion leader/follower relationships in reality possess all the above features. For example, some opinion leaders merely give recommendations of products without having personally consumed them, or may give dishonest recommendations. Moreover, opinion leaders sometimes (though less so than innovators) introduce new product information into the population, while here we only examine the function of an opinion leader in reducing consumption risk for the follower by sharing her consumption experience of the new product with her network neighbors. Lastly, in reality, new information about the product over and above fellow consumers' consumption experience may appear after it becomes available. But we believe that our simplification, summarized above, do possess sufficient realism in capturing many opinion leader-follower relationships while allowing us to illustrate clearly our ideas about opinion leadership formation.

To help understand our conceptual framework, consider a stylized example of two new theatre shows being put up at the same time over the same duration or "run". One of

them (the "new product") is created by new talents, while the other (the "outside option") is created by a well-known production/acting team. Previews for media representatives were completed before the shows were open to the public, and thus media information about both shows is already widely circulated at the beginning of the run.[5] Assume that every member of the theatre-going public would choose to see at most one show during the run, and prefers to enjoy a show earlier rather than later. But the dilemma is that, while the show by the well-known creative team can offer a predictable expected quality, the show by the new talents may be refreshingly excellent or the opposite, and poses more uncertainty than the former. A theatre goer may thus like to wait for an acquaintance to go to the show created by the new talents first and give her recommendation (in addition to the media information that she already knows), before making a decision on how to spend her money and time on theatre-going during the run period. In this context, the opinion leaders would be theatre goers who see the show created by the new talents at the beginning of the run and then share their comments on it with their social contacts.[6] How might these opinion leaders emerge out of the theatre-going public to see a show always earlier than others in situations like the one described, and influence others through "ordinary, intimate, informal" contact?

3. The Model

We now present our formal model based on the conceptual framework described in the previous section. Consider a (finite) set C of M consumers. A network G is a graph or a set of ordered pairs defined over C, so that, for any consumers i and j in C, i is "linked to" j (or "is a neighbor of" j) iff $(i, j) \in G$. We shall assume that G is an undirected graph, that is, $(i, j) \in G \Leftrightarrow (j, i) \in G$. Moreover, G is connected, so that starting from every consumer i in C, it is possible to reach any other consumer j in C by traversing links in G. The links in G are "WOM communication links" indicating that consumers at two ends of a link (also called "nodes" or "vertices" in graph theory) meet and exchange information

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[5] We may also assume that avid theatre enthusiasts (the "innovators" in this context) see both shows as soon as the runs begin and quickly spread information about and opinions on the shows in blogs and forums, which then become absorbed by the rest of the population at the start of the run. The rest of the population is the target of our model.
[6] A recent example of a "new-talent" theatre show being marketed by an active WOM campaign involving attempted identification of opinion leaders is described in Cox (2006).

regularly regarding the product category to be modeled.[7] Note that, if two consumers are socially connected in some sense (say, family relationship) but never communicate about the product category over the selling horizon, they are effectively not linked in the present model. In other words, G is more precisely the subset of the social network that is relevant as communication channels to our model. The configuration of G is common knowledge among the consumers, as is the fact that each consumer knows her own position in the network and can identify the network position of each of her neighbors (we shall relax these assumptions later). It is also assumed that consumers are rational Bayesian decision makers, and that this fact is common knowledge among the consumers.

We formulate our analysis in the context of a multi-period "WOM game" to be played by the consumers in C. The game comprises T periods indexed by the label t, $t = 1, 2, 3 \ldots T$ (in the theatre show example, these periods cover the "run" of the two shows, each period lasting, say, a week). Time preference is modeled by a time discount factor $\delta_i \in [0,1)$ for consumer i,[8] which can be interpreted as a consumer's "impatience" with product consumption. This factor can be different from consumer to consumer, and we assume that each consumer's time discount factor is common knowledge among the consumers (we shall relax this assumption later). Within the duration of the game, each consumer needs to make at most one purchase, and she may only choose between two perfectly substitutable alternatives at fixed prices: a new product and an outside option. A consumer may only make a purchase at the beginning of a period. The utilities of purchase are moreover defined so that the no-purchase utility is zero.

It is common knowledge that, upon the purchase of a new product in period t by any consumer i, her consumption experience can be characterized as a resolution of uncertainty into either of two states, "good" or "bad", with utility $\delta_i^{t-1} u_g$ and $\delta_i^{t-1} u_b$ respectively ($u_g > u_b$ and have the same values among all consumers; the price of the product is also included in these utilities).[9, 10] The new product is supposed to be

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[7] We do not consider a consumer as being "linked to herself"; in other words, for any i in C, $(i,i) \notin G$.

[8] We shall also consider a model with linear time cost in a later section.

[9] The more general case would be when the realized utility has multiple or a continuum of possible values. Extension of our results along this direction will be discussed in a later section; as will be the shown, our assumption ultimately only relies on the WOM communication having a binary message space with elements "good" and "bad", while the real consumption utility can have numerous possibilities.

launched just before the start of the game; whether its consumption experience is "good" or not may differ among the consumers, and this heterogeneity is captured by a probability q that denotes the likelihood that it is "good" to any one consumer. The value of q is not known, but given the information released by its manufacturer, consumers form priors regarding the distribution of q. We assume that all consumers receive the same information and thus form the same common prior for q (Harsanyi 1968), which is supposed to be non-degenerate and can be described by a probability density function $f(q)$.

The expected utility of the outside option (inclusive of its price) is $\delta_i^{t-1} U_0$ for consumer i at the beginning of period t. In other words, for any consumer, the expected utility of the outside option before time discount is the same constant U_0, which we assume is common knowledge; in the theatre show example, U_0 can be seen as the expected quality of the well-known creative team's latest effort. There is still possibly heterogeneity among the population regarding the outside option, in that any individual consumer's realized consumption experience (ex post utility) before time discount will be U_0 plus a zero-mean i.i.d. random variable with commonly known distribution across the population; the realization of this random variable is not known to the consumer before consumption, and its zero-mean distribution may take any form such as continuous normal or discrete binomial. But, whatever form it takes, the distribution is assumed to be common knowledge, and thus any consumption experience of the outside option by a consumer has no information value by itself to another consumer. By contrast, the corresponding randomness of the new product's ex post utility, though i.i.d and known to be discrete binomial, has an underlying parameter, q, that is not known for sure by the consumers; it is because of this extra uncertainty that consumption experience of the new product by a consumer has information value by itself to another consumer.

Define $q_0 = (U_0 - u_b)/(u_g - u_b)$, so that $U_0 = q_0 u_g + (1 - q_0) u_b$. For convenience of exposition, we shall formulate our subsequent results in terms of q_0 rather than U_0, bearing in mind that it is not necessary that heterogeneity in consumption utility for the

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[10] We have not included network externalities either in the sense of critical mass or personal network threshold (e.g. Goldenberg, Libai, and Muller 2005, Jackson and Yariv 2007) to focus on the purely informational value of WOM.

outside option in exactly the same way as with the new product. Thus q_0 is not necessarily in [0,1] either. Assume $U_0 > 0$, so that the outside option is strictly preferable to no purchase, and consumers always only need to compare the prior or updated expected utility of the new product to that of the outside option – rather than the no-purchase utility – when making purchase decisions. Our assumptions thus imply that, by the end of the game, every consumer should have made a purchase of either product.

At the beginning of every period, a consumer who has not yet made a purchase makes one decision over three alternatives: (1) purchase the new product; (2) purchase the outside option; (3) defer purchase. If (1) or (2) is chosen by consumer i at the beginning of period t, her purchase outcome (good or bad) is resolved during t. The consumer does not have any more decision to make in subsequent periods. Thus, at the beginning of any period t, a consumer's action space is either the empty set (iff she made a purchase before period t), or the set $A =$ {"purchase new product", "purchase outside option", "defer purchase"}. If a consumer purchases either product, that is, if she picks either of the first two actions in A, we say that she has "made a purchase". As we model it, any purchase decision by a consumer is a once-in-a-lifetime decision for her. Hence, if a consumer's action space is the empty set at the beginning of period t, it remains so for all the periods afterwards.

[Insert Figure 1 around here]

To model WOM communication, at the end of every period, every consumer meets with every one of her network neighbors, denoted by the set $N(i) = \{ j : (i, j) \in G \}$, individually and in sequence according to some consistent matching scheme. If i made a purchase at the beginning of t, the product purchased as well as i's experienced utility/purchase outcome are truthfully reported to all her network neighbors during the meetings. [11] The time duration of all the meetings, which may be seen as "social interactions", is supposed to be short compared with the duration of a period – which

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[11] We have not assumed any "second-order" or "hearsay" WOM communication. Suppose j and k are not neighbors of each other, although they might have common neighbors; our model setting is such that k would not hold any information (except by inference from what k might observe from her neighbors) about j's actions at any time throughout the game. We may rephrase our assumption as follows: if any second-order WOM communication takes place at all, it is disregarded by the recipient of the information (presumably because "hearsay" is considered too error prone in the relay process). This assumption is more innocuous than it might seem at first sight, since if we in fact prefer to model highly regarded second-order WOM, it can be approximated by adding extra links to join up the "neighbors of neighbors" such as j and k in our example. We may then re-apply our analysis to the modified network.

corresponds to the average time interval between social interactions – and so we assume that all consumers meet with all their neighbors approximately "simultaneously". Figure 1 presents the timeline of the whole communication and decision-making mechanism.

4. Main Results

The WOM game is, in general, a game with imperfect and incomplete information. To start with, the real q, which is determined by nature before period 1, is concealed from all consumers throughout the game. Moreover, a consumer cannot observe directly the decisions and purchase outcomes of consumers who are not linked to her. But the decisions and purchase outcomes of all her network neighbors are completely observable to her (or, more precisely, are communicated to her truthfully along the network links); these are in fact the only observable information of the game environment for that consumer, and, together with her own history of play, completely specify her information state in the game. This leads to the following definition (recall that the set A = {"purchase new product", "purchase outside option", "defer purchase"}):

Definition 1. A consumer plays a pure strategy iff the observed history of play (including purchase decisions and outcomes) of her neighbors up to the beginning of any period t determines her choice of action over A at the beginning of period t with certainty, if she has not already made a purchase before that.[12]

This is, in fact, a special case of the more general and standard definition that a consumer's (behavioral) strategy is a mapping from all possible information states in which the consumer has to make a choice over the actions in A to probabilistic mixtures over the actions in A.[13] The consumer plays a pure strategy iff all these probabilistic mixtures are degenerate; otherwise, she plays a mixed strategy.

In the remaining part of this paper, we shall adopt the following tie-breaking rule:

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[12] A formal definition of the history of play of a consumer's neighbors is given in the Appendix (at the beginning of the proof of Proposition 2).

[13] Note that the strategy mapping needs be defined not only for information states that are realizable in equilibrium but also off-the-equilibrium ones too, as long as the consumer needs to "move" (choose over the actions in A) in those states.

Tie-breaking Rule 1. If a consumer is going to make a purchase but is indifferent in terms of expected utility between the two products, she buys the new product.[14]

In more detail, the purchase decision problem is a comparison between two probabilities: suppose that, at the beginning of period t, based on the information (if any) regarding the choice(s) and purchase outcome(s) of her neighbor(s) that she has received so far, consumer i's posterior mean for q is calculated to be $\hat{q}$ according to Bayes' rule. Then, if she decides to make a purchase right away, she would choose the new product iff:

$$\delta_i^{t-1}[\hat{q}u_g + (1-\hat{q})u_b] - \delta_i^{t-1}[q_0 u_g + (1-q_0)u_b] = \delta_i^{t-1}(\hat{q} - q_0)(u_g - u_b) \geq 0.$$

That is, she would choose the new product iff $\hat{q} \geq q_0$ and her decision problem becomes simplified to a comparison between her posterior mean for q and q_0.[15]   We then point out a lower bound on the "interesting" range of $\bar{q} = \int q f(q) dq$, given q_0:

PROPOSITION 1. *If $\bar{q} = \int q f(q) dq < q_0$, the only equilibrium is that all consumers purchase the outside option at the beginning of period 1.*

Proof. A priori, a consumer buys the outside option iff $\bar{q} < q_0$, and so any consumer who buys before all others will not provide any extra information about the new product. Thus no consumer attempts to wait for any other consumer to purchase first.

Before proceeding to a more detailed equilibrium analysis, we present an essential result. The proof of the following proposition is available in the Appendix:

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[14] If this tie-breaking rule is instead in favor of the outside option, some of the $\geq$ and $\leq$ signs in subsequent discussion need be changed to strict inequality signs, and vice versa. But our conclusions remain essentially valid; what is basically needed is that each consumer has a definite tie-breaking policy. What we have not considered is the possibility that a consumer randomizes her choice in a tie situation; this potentially adds to the complexity of the problem, which we will not deal with in this paper. This discussion also applies to tie-breaking rule 2.

[15] Note the use of Tie-breaking Rule 1 here. Note moreover that this tie-breaking rule only admits equilibria in which a consumer never randomizes over purchasing the new and the outside option. An equilibrium in which a consumer randomizes over purchasing the two products at some point may occur only if that consumer sees that purchasing both products incurs the same expected utility, which, according to the tie-breaking rule, should lead to her definitely purchasing the new product instead of randomizing.

PROPOSITION 2. *If consumer i knows the strategies of all other consumers, then, at the beginning of any period, i strictly prefers deferring purchase to making a purchase immediately if:*

(1) given the history of play of i's neighbors and all other consumers' strategies, there is a positive probability that, if i defers a sufficient number of periods that is permissible given that the game has T periods, she would receive WOM information that would reverse her purchase decision from the one she would have taken if she had purchased immediately; and

(2) i's time discount factor is sufficiently high.

This proposition can be roughly summarized as follows: it is always better to wait as long as (1) there might be more information coming in, (2) such information might be "useful" in the sense that it might reverse the consumer's decision, and (3) waiting is not too costly. The power of the proposition lies in the "might"s: as long as decision-reversing information comes with a positive probability, it is already worth deferring purchase, unless the time cost is too high. Note also that the proposition relies only on i knowing the strategies of all other consumers; it is applicable in, but does not require, equilibrium play.[16]

We now consider the possibility of opinion leadership emerging in equilibrium. Although the WOM game, being a well-defined game with regular strategy space, has at least one mixed-strategy equilibrium, we cannot be sure at this point that an equilibrium always exists that involves one or more consumers who make the earliest purchases as a pure strategy and, through their strategy, exert influence on other consumers' purchase decision. However, the explanatory power of the WOM game regarding the emergence of opinion leaders hinges on whether we can show that such consumer(s) possibly exist in equilibrium, who are then in the role of opinion leaders. This is what we attempt to do in the following propositions. We first define opinion leadership formally in accordance with our conceptual framework:

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[16] In fact, the proposition is true even when i does not know for sure the strategies of other consumers but hold probabilistic estimates over which strategies the other consumers might be using; the phrase "all other consumers' strategies" in Part (1) should then be modified to "i's probabilistic estimates of all other consumers' strategies". The proof of this generalization is very similar to the original proof and is available upon request.

Definition 2. A consumer i is an opinion leader if, under equilibrium play: (1) she purchases the new product at the beginning of period 1 with certainty; (2) there is a positive ex ante probability that her consumption experience reverses the purchase decision of another consumer, say j, from the decision that j would have made had she purchased at the beginning of period 1.

Together with the basic specifications of our model, an opinion leader who satisfies Definition 2 fulfills the requirements of our conceptual framework regarding opinion leader/follower relationship. Note that a consumer who randomizes between purchasing and deferring in period 1 but ends up purchasing in that period in realized play arguably becomes an "incidental" opinion leader. But our definition of opinion leadership is stronger: it requires a systematic direction of influence, so that an opinion leader according to our definition plays a pure strategy and behaves as an opinion leader with certainty. Only equilibria with such opinion leaders approximate the real-life emergence of consumers who are consistently opinion leaders in their networks.

PROPOSITION 3. (1) *In any equilibrium, there is a positive probability that at least one consumer purchases at the beginning of period 1;* (2) *In any pure strategy equilibrium under which not all consumers purchase at the beginning of period 1, there exists at least one opinion leader.*

Proof. Suppose that, in a certain equilibrium, (1) is not true, and no consumer plays a strategy that incurs a positive probability on making a purchase before period $s+1$ for some positive integer s. Suppose consumer i plays a strategy that incurs a positive probability, say α, on making a purchase at the beginning of period $s+1$. Then, given i's time preference and the fact that she will have received no WOM information from other consumers before the beginning period $s+1$, she can improve her payoff by playing a strategy that transfers the probability α from making a purchase at the beginning of period $s+1$ to doing that at the beginning of period 1 instead. This leads to a contradiction. For Part (2), first note that, using similar logic as with the proof of Part (1), at least one consumer purchases at the beginning of period t with certainty for $t = 1, 2$ (the proof for $t = 2$ needs to incorporate the premise of Part (2)). Then, for any consumer j who purchases at the beginning of period 2, j must have chosen this strategy (at least)

because she is thus better off than if she purchased at the beginning of period 1. In other words, ex ante, there must be a positive probability that j's purchase decision will be reversed because of the information (consumption experience) that a consumer who is directly linked to j, say i, communicates to j after making a purchase at the beginning of period 1. Consumer i's purchase must also be a purchase of the new product, since otherwise the information is useless to j. By Definition 2, i is an opinion leader.

We introduce another tie-breaking rule before presenting further results:

Tie-breaking Rule 2. If, at the beginning of period t, a consumer who has not yet purchased finds that, given the observed history of play of her neighbors and her conjectured strategies of all other consumers in the network, a purchase at the beginning of period t incurs the same expected utility as another strategy that involves deferring in period t, and both are best responses, the consumer purchases at the beginning of period t.

This rule is useful when the time discount factor(s) of some consumer(s) happen to be exactly the same as some critical time discount factor that determines the types of possible equilibria.[17]

Next, define the distance between any two consumers i and j as the smallest number of links needed to be traversed to travel from i to j along the network. For, example, if i and j are network neighbors, the distance between them is one. If they are not linked in the network but have one or more common neighbors, the distance between them is two.

[Insert Figure 2 around here]

Suppose a set L of consumers satisfies the premise that, given any $i \in L$, if $j \in N(i)$ but $j \notin L$, then $\forall l \in L \ j \in N(l)$. An example of a set L containing two consumers is shown in Figure 2. It is easy to see that any consumer j who is not in L is at the same distance from any consumer in L, and thus we can talk about "the distance

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[17] See footnote 14 for a discussion on tie-breaking rules. Tie-breaking Rule 2 should also not be abused. For example, if we push its application too far, then there can be no mixed strategy equilibrium in the WOM game. The argument is as follows: by Corollary 1, at least one consumer purchases with positive probability at the beginning of period 1; if this probability is less than one, then, by definition of a mixed strategy equilibrium, it must be that purchasing and deferring yield the same expected utility to that consumer. By Tie-Breaking Rule 2, she would then definitely purchase right away and the equilibrium would break down. We shall limit the use of this rule only on consumers who play pure strategy in an equilibrium.

between j and L" as a single-valued and well-defined number that is equal to the distance between j and any $i \in L$. We next present the major proposition of this paper, the proof of which is in the Appendix:

PROPOSITION 4. *There exists a non-increasing sequence* $\Theta = (\delta^*_1, \delta^*_2, \delta^*_3 \ldots)$, *which (a) is bounded by zero and one, (b) depends on f(q)*, q_0, u_g *and* u_b *only but not on M and the consumers' time discount factors, and (c) is strictly decreasing in* δ^*_k *if* $\delta^*_k \in (0,1)$, *such that the following are true:*

(1) If $\forall i \in C \; \delta^*_{d(i)} \geq \delta_i$, *where C is the set of consumers and d(i) is i's number of neighbors, then there exists a pure strategy equilibrium in which all consumers purchase at the beginning of period 1;*

(2) Suppose (a) the members of Θ *are not identically one, so that* $\omega = \min\{k : \delta^*_k < 1\}$ *exists; (b) there exists a set L of* ω *consumers such that, given any* $i \in L$, *if* $j \in N(i)$ *but* $j \notin L$, *then* $\forall l \in L \; j \in N(l)$; *and (c) T is sufficiently large. Then there exists* $\delta_L \in [0,1)$ *such that, if and only if* $\forall i \in C \setminus L \; \delta_i > \delta_L$, *there exists an equilibrium in which:*

(i) the consumers in L purchase the new product in period 1 and are the (only) opinion leaders;

(ii) every consumer who is not in L purchases only after at least one of her neighbors makes a purchase, so that a consumer who is at distance σ≥1 from L does not purchase earlier than in period σ+1; moreover, for every σ, at least one consumer at distance σ from L purchases earlier than all consumers at larger distances from L.

As a whole, Proposition 4 tells us that, under a range of parametric conditions, an equilibrium with at least one opinion leader is bound to exist. Part (1) implies that this is true when the consumer time discount factors are sufficiently low. Part (2) suggests that this is true when "most" of the time discount factors are sufficiently high, the game is sufficiently "long", and a condition concerning network topology is fulfilled; moreover, the game will last at least $\sigma_{max} + 1$ periods, where σ_{max} is the maximum possible distance between L and any consumer in the network who is not in L. A set of key quantities in

both parts is $\Theta = (\delta *_1, \delta *_2, \delta *_3 ...)$. As can be intuited from the proof, the interpretation of $\delta *_k$ is such that $(1/\delta *_k) - 1$ is the proportional increase in non-time-discounted ex ante expected utility when a consumer makes a purchase decision based on the consumption experience of k other consumers who purchase at the beginning of period 1, compared with when she makes a purchase decision based solely on prior information at the beginning of period 1. The number ω is therefore the smallest number of consumers with which their consumption experiences of the new product can have a collective influence on a consumer who is informed about those experiences. It is thus no surprise that L cannot have fewer than ω consumers: if there are fewer consumers, their consumption experiences will have no effect on any consumer who purchases at the beginning of period 2, and so a consumer who purchases at the beginning of period 2 with positive probability should transfer that probability to period 1, contradicting Part (2)(ii). Note that the time discount factors of the consumers in L are irrelevant in this equilibrium because the off-the-equilibrium strategies of the consumers (specified in the proof but omitted in the Proposition for the sake of brevity) are such that every consumer who is at distance one from L but not in L purchases only after all the consumers in L have purchased. Thus no consumer in L, however patient she might be, has an incentive to deviate: it is impossible that she will receive any decision-reversing information as a result of her deviation.

If $\omega = 1$ (it can be seen from the Appendix that this is true iff $\bar{q} \geq q_0 > \int q(1-q)f(q)dq / \int (1-q)f(q)dq$), so that condition (b) in Part (2) is satisfied for all consumers, and T is sufficiently large, then the equilibria are "degenerate" if the time discount factors are all sufficiently high – more precisely, if the time discount factors are all higher than $\max_{i \in C}\{\delta_{\{i\}}\}$ (note that $\delta_{\{i\}}$ is the δ_L mentioned in Proposition 4(2) when $L = \{i\}$; it is not necessarily equal to δ_i). In such a case, any $\{i\}$ can be the opinion leader set L, and any consumer can be the sole opinion leader in equilibrium. It can also happen counter-intuitively that this opinion leader is the most "patient" consumer, who has the highest time discount factor in the population.

When the time discount factors are all of intermediate ranges, Proposition 4 does not apply, and the identification of general equilibrium characteristics is seemingly difficult.

But we observe a couple of transitional patterns: (1) If we start with $\forall i \in C \; \delta^*_{d(i)} \geq \delta_i$ and let the time discount factor of consumer j, say δ_j, increase while keeping all other time discount factors constant, then, once δ_j becomes higher than $\delta^*_{d(j)}$, there is an equilibrium in which consumer j is the only consumer who ever defers from purchasing at the beginning of the game, and in fact purchases at the beginning of period 2; (2) Suppose $\omega = 1$ and $T \geq M$. Start with $\forall i \in C \; \delta_i > \max_{i \in C}\{\delta_{\{i\}}\}$, so that the one-opinion leader equilibria are degenerate. Then, select an arbitrary consumer, say j, and let δ_j decrease while keeping all other time discount factors constant; when δ_j dips below $\min_{i \in C \setminus \{j\}}\{\delta_{\{i\}}\}$, the only equilibrium with only one opinion leader has j – who is the most "impatient" consumer in the network – as that opinion leader. At the end of both of these transitions, the most "patient" consumer is not an opinion leader in the family of equilibria under examination; moreover, the most "impatient" consumer is (or remains) an opinion leader.

5. Generalizations

5.1 Relaxing the Common Knowledge Assumptions

In the development so far, we have assumed that consumer discount factors and the network structure are common knowledge, and every consumer knows her position in the network (which is itself a commonly known fact). Now, suppose that only the total number of consumers, M, is common knowledge, as well as the fact the network is connected. Apart from these, consumers share a common prior regarding the possible network structure as well as the consumer discount factor at any position in any realized network structure. Over and above this common prior, every consumer has her own additional information about her local network environment, which she can use to generate posterior beliefs on the network and discount factors. We shall not specify the type of additional information that any one consumer may hold, except that it at least includes the number of network neighbors she has (she can also track the history of play of her neighbors as the game proceeds through the modeled WOM communication). We shall also assume that: (1) every consumer acquires the common prior and all her additional information before the WOM game begins, and this fact is common

knowledge; (2) given a realized network and a realized set of discount factors in that network, the additional information that a consumer in any position of the network holds is common knowledge; what is not known by any consumer with certainty is which network and which set of discount factors have been realized. Otherwise, prior information about the products as well as other elements of the model remains common knowledge.

Given the above, it is meaningful to talk about a Bayesian-Nash equilibrium (Harsanyi 1968) under which every consumer chooses a strategy (in the sense described at the beginning of Section 4) at the beginning of the game conditioned on the common prior and her additional information about her local network environment. Moreover, under equilibrium, the strategy of every consumer as a function of the realized network and realized time discount factors becomes, effectively, common knowledge; another way of presenting this statement is that every consumer's strategy as a function of her additional information about her network environment is effectively common knowledge under equilibrium. The two presentations are equivalent because of assumption (2) in the previous paragraph. Note that any equilibrium strategy should be specified conditioned on the additional information that the consumer holds. Also, as the game proceeds, a consumer may update her beliefs about the network and the discount factors according to her neighbors' history of play; but the effect of this updating as manifested in that consumer's behavior can be incorporated into her strategy, which is always assumed to be conditioned on her neighbors' history of play.

We now examine whether previous definitions and results may remain appropriate or valid under the incomplete information framework. Definition 1, the tie-breaking rules, and Proposition 1 obviously do not need modification. Proposition 2 also needs little modification as long as the premise "If consumer i knows the strategies of all other consumers" is interpreted as "If consumer i knows the strategy of every other consumer as a function of that consumer's additional information about her local network environment." It can be seen from its proof that, all that Proposition 2 requires for a premise is there being a positive probability of decision reversal under any one specific realization of network structure and time discount factors that is consistent (i.e. occurs with positive probability) with i's most updated information about the network.

For Definition 2, the "ex ante probability" in its statement should be understood as ex ante with respect to q and the realized equilibrium play, but conditioned on the true network structure and discount factors; in other words, it is "ex ante" from the point of view of an observer at the beginning of period 1 who knows the true network and the time discount factors of every consumer in it but not the true q. However, by this definition of an opinion leader, Proposition 3 is in general not valid – an equilibrium play under incomplete information might end up having all consumers choosing to wait for a number of periods, because they do not have sufficient information about the network to know that other consumers are in fact also waiting.

Proposition 4 is also in general not valid; however, (1) still holds when the common prior is such that only discount factors that are lower than δ^*_1 have positive probability of being realized. On the other hand, it is straightforward to check that (2) still holds when $\omega = 1$ (note that Θ and ω are independent of the network structure and specific consumer discount factors) and the common prior is such that only discount factors that are higher than a threshold (namely, the supremum $\delta_{\{i\}}$ over all i and all possible networks given the prior) have positive probability of being realized. In other words, (1) still holds when consumers believe a priori that all consumers in the population are sufficiently impatient. And (2) still holds when $\omega = 1$ and consumers believe a priori that all consumers in the population are sufficiently impatient; an equilibrium satisfying (2) in this case would be one in which a single consumer, say i (there is still multiplicity in that any i can be an opinion leader), becomes the opinion leader and purchases the new product at the beginning of period 1, while other consumers, realizing this in equilibrium, at least do not purchase at the beginning of period 1, knowing that there is a positive ex ante probability that i's consumption experience may reverse their a priori decision, and they are patient enough to wait for information to seep to them through the network, whatever its structure. Common knowledge of the exact time discount factor of each consumer is not necessary either.

5.2 The New Product Having More Than Two Possible Outcome States

It is worthwhile to ask if our model can be generalized to cases when the realized utility, u, has multiple or a continuum of possible values. Suppose these values are bounded within some interval $[\underline{u}, \overline{u}]$ on which we can define a family of probability density

functions $\phi(u;q,\xi)$ where q is a scalar parameter, ξ is a row-vector parameter, and each member of the family is completely characterized by the combined row-vector parameter (q,ξ). Moreover, assume that $\phi(u;q_1,\xi_1)$ first-order stochastically dominates $\phi(u;q_2,\xi_2)$ iff $q_1 \geq q_2$ whatever the values of ξ_1 and ξ_2. This is satisfied, for example, with the probability q in the binary outcome scenario.[18] Consumers hold the common prior $f(q,\xi)$ regarding the new product, while for the outside option with commonly known U_0, assume that a solution exists for (q_0,ξ_0) in the equation $U_0 = \int_{\underline{u}}^{\bar{u}} u\phi(u;q_0,\xi_0)du$. Then the purchase decision of any consumer is solely dependent on whether her posterior mean for the new product's q is larger than or equal to q_0.

We need to maintain the "coarse communication" assumption that the message space regarding the consumption experience of a new product has only two elements: "good" and "bad". More specifically, after making a purchase decision, a consumer tells her network neighbors which product she has bought, and, if that is a new product, says that the product is "good" iff the realized utility u of the product is larger than or equal to some exogenously given reference value $u_0 \in (\underline{u},\bar{u})$;[19] her consumption experience message is "bad" otherwise. She relates no more information than that – or, if she relates more information, the message recipients disregard such extra information, considering it too error prone. Assume that all members of $\phi(u;q,\xi)$ have no mass point (zero measure) at u_0 and define the following:

$$R(q,\xi) = \int_{u_0}^{\bar{u}} \phi(u;q,\xi)du \text{, and so } 1 - R(q,\xi) = \int_{\underline{u}}^{u_0} \phi(u;q,\xi)du \text{.}$$

Note that, by definition and the assumption regarding q, $R(q,\cdot)$ is increasing in q. In fact, $R = q$ in the original binary outcome scenario, and it can be easily seen that R plays the same role as q in the updating process in the generalized scenario. For example, in a connected network with only two consumers (a dyad), say consumers a and b, if consumer a sees that consumer b has bought the new product and found it good, her posterior mean for q is:

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[18] In fact, the parameter in any single-parameter family of distributions with monotone likelihood ratio property satisfies this requirement (Milgrom 1981).

[19] To be more detailed, u_0 needs be at some "non-trivial" level such that: (1) there is positive probability, according to the prior $f(q,\xi)$, that the function $R(q,\xi)$ as defined later is non-constant; (2) $0 < R(q_0,\xi_0) < 1$.

$$\hat{q} = \frac{\int qR(q,\xi)f(q,\xi)dqd\xi}{\int R(q,\xi)f(q,\xi)dqd\xi} \; .$$

An examination of the proofs suggests that previous results are essentially valid under this general scenario (since only the fact that R is increasing in q is really necessary for the proofs to work), with adjustments that replace q by $\int R(q,\xi)d\xi$ in some cases.

5.3 Linear Time Cost

Up to now we have assumed a time discount model to represent consumers' time preference. Another popular model is to assume that each consumer i suffers a fixed time cost per period of deferral, say c_i, so that, if i purchases a new product at the beginning of period t, her ex-post payoff is $u_g - c_i(t-1)$ if the product that turns out to be "good", $u_b - c_i(t-1)$ if the product turns out to be "bad". Similar generalizations can be applied to the outside option (of which, to reiterate a point, only the expected utility U_0 minus time cost is relevant to decision making).

The only major difference between this model and the time discount model is that i then certainly makes a purchase no later than at the beginning of period $T_i *$, where $T_i *$ is the largest integer that is smaller than or equal to $1+\max\{U(q_{\mathrm{sup}}),U_0\}/c_i$ with $q_{\mathrm{sup}} = \sup\{q : f(q) > 0\}$, and we define the function $U(q) = qu_g + (1-q)u_b$. Moreover, if $T_i* \le 1$, she purchases at the beginning of period 1, implying that, under this time preference model, in contrast with the time discount model, there can be "natural" opinion leaders – especially those for whom $u_g < c_i$, so that $T_i* \le 1$ even when $q_{\mathrm{sup}} = 1$.

Examination of the proofs shows that all our results remain essentially valid upon some minor adjustments that include replacing $\delta *_r$ by the quantity $\pi_r = [\max\{U(\bar{q}),U_0\}](1-\delta *_r)/\delta *_r$. The transformed members of Θ then become a non-decreasing sequence that can be used to construct a proposition that is analogous to Proposition 4. There is otherwise little substantive difference in results between the two time preference models.

6. Focal Equilibria

6.1 Efficiency Issues

Our objective in previous sections is to show why empirical studies have not found strong correlations between opinion leadership and various consumer characteristics. That is, the question "who is highly likely to be an opinion leader?" is bound to have no sound theory-based answer. As such, multiplicity of equilibria is an essential to our findings.

But, on the other hand, the fact that opinion leaders are often (though not always) socially well-connected (see Rogers 2003) can be explained by considering possible focal equilibria. Similarly, the notion of focal equilibria can be used to explain the mild correlation between opinion leadership and innovativeness – which, we can assume, is positively related to time preference – in some empirical studies (Flynn et al. 1996). Take the simple example of a WOM game with $T = 2$ to be played in a "star" network, in which one central consumer is linked to M-1 neighbors each of whom is not connected to anyone else. Suppose the central consumer's time discount factor is δ_c while all other consumers have the same time discount factor δ_v. Suppose also that $\delta_c > \delta^*_{M-1}$ and $\delta_v > \delta^*_1$. Then two equilibria may arise: (1) the central consumer is an opinion leader while all others are followers; (2) the central consumer is a follower while all others are opinion leaders. Equilibrium (1) seems to be a natural focal outcome, especially when $\delta_c \leq \delta_v$ (i.e. the central consumer is no more "patient" than the other consumers); this concurs with empirical observations too.

However, efficiency consideration reveals more ambiguous results. It can be shown (see Appendix) that the equilibrium in which the central consumer is the sole opinion leader is less efficient (in an ex ante sense) than that in which she is the sole follower if:

$$\delta_c / \delta^*_{M-1} > (\delta_v / \delta^*_1)(M\text{-}1) - (M\text{-}2).$$

It is possible that the inequality is in a reverse direction while δ_c is higher than δ_v, in which case the efficient equilibrium is also focal in the sense that the more "patient" and well-connected central consumer is the opinion leader. But in general (unless $1/\delta^*_{M-1} \leq (\delta_v / \delta^*_1)(M\text{-}1) - (M\text{-}2)$), the inequality is satisfied as long as δ_c is sufficiently higher than δ_v, in which case a tradeoff having a highly connected opinion leader and having relatively "impatient" opinion leaders comes out in favor of the latter. Even more interestingly, the inequality may also be true when $\delta_c < \delta_v$, in which case the efficient equilibrium is the one in which consumers who are neither the most well-connected nor the most "impatient" become opinion leaders. This last possibility occurs, for example,

when $\delta_c / \delta^*_{M-1} > M (\delta_v / \delta^*_1)$ or $\delta_c / (M \delta_v) > \delta^*_{M-1} / \delta^*_1$. The left hand side of the last inequality is a number that compares the two equilibria in terms of the consumers' time preferences and network structure. The right hand side, meanwhile, compares the quality of information from M-1 opinion leaders (which is higher the smaller the value of δ^*_{M-1}) with the quality of information from just one opinion leader. The insight offered by the inequality is that, if the quality of information from M-1 opinion leaders sufficiently exceeds that from one opinion leader, it becomes efficient for the whole network if M-1 opinion leaders are to supply trial information about the new product to only one follower rather than the reverse – and even if the opinion leaders are more "patient" than the follower. We expect that this type of tradeoff also exists in WOM game of longer duration in general networks, so that the obvious candidates of focal equilibria – namely those with well-connected and/or "impatient" opinion leaders – are not necessarily efficient for the society. But, as supported by empirical studies, relatively highly-connected consumers are often opinion leaders, and the consumers as a whole may be at a loss because of their tendency to converge to such equilibria.

It is also worth noting that, in general networks, if there is high heterogeneity in network connectedness among the consumers while time preference heterogeneity is relatively low, the former may be the primary factor in determining which equilibria are focal, in which case the most "patient" consumer may become the opinion leader in a focal equilibrium. If consumer connectedness and time discount factors correlate negatively in a network, it can happen that no specific equilibrium becomes uniquely focal, and, counter-intuitively, an equilibrium with an opinion leader who is neither well-connected nor especially "impatient" is as likely to emerge as those in which the best-connected or most "impatient" consumer is the opinion leader.

6.2 Profit-maximization Issues

In recent years, new WOM marketing firms like BzzAgent are recruiting and cultivating opinion leaders among consumers with little or no screening. These opinion leaders, sometimes called "buzz agents", might have only little influence on fellow consumers before they were recruited, and might not be highly connected (in terms of WOM communication links). However, through incentive and motivation schemes, the WOM marketing firm that recruited them could turn them into opinion leaders who actively

communicated with other consumers regarding the product marketed. "Buzz agents" do not necessarily have to offer positive recommendations of the product, although it seems that they tend to do so (Walker 2004).

This type of marketing effort usually changes the structure of the social network (more precisely, the WOM communication network that is relevant to the product marketed) by adding communication links, not to mention changing equilibrium play. Even the more conventional WOM marketing as practiced by Tremor, Procter and Gamble's WOM unit, which aims at established opinion leaders, could have mistaken certain consumers as opinion leaders when they are indeed not , but nevertheless induced those consumers to become real opinion leaders, thus changing the network structure and equilibrium play.

It can be demonstrated that such campaigns are not always profitable to the firm. In the context of our model, consider a WOM game played by a group of M consumers for whom $\bar{q} < q_0$. Then the firm producing the new product reaps no profit from this group of consumers because of Proposition 1, if it does not do anything else. Now, suppose the firm selects one consumer out of the group by some process that we shall not model, and, at a net cost c, incentivize her to spread recommendation about the new product, perhaps also to consume it herself. Suppose the price of the new product is fixed at p (this would be an appropriate assumption in the theatre show example), and marginal cost is normalized to zero. The firm's estimation of the probability that the opinion leader would give a positive recommendation to other consumers is $\hat{q}$, while the other consumers' prior is such that only a positive recommendation persuades them to buy the new product. The expected number of other consumers the opinion leader would spread words to before they make a purchase decision is $1 \le r \le M - 1$. Lastly, the firm might have a time preference for profit, which is captured by a discount factor $\delta_F \in [0,1]$ for the expected duration over which words spread and persuaded consumers make purchase decisions. Then the ex ante net profit of this marketing campaign is $\pi = -c + \delta_F p\hat{q}r$. If $\pi < 0$, the firm is better off not carrying out the campaign. This is likely when the following are true:

1. The cost c of cultivating an opinion leader is high;

2. The firm's time discount factor δ_F is low;

3. The price p is high;

4. The estimated probability of successful persuasion, $\hat{q}$, is low; this may be due to the opinion leader, being known as having received incentives from the firm, is not considered trustworthy,[20] or may be simply because the product itself is of such low quality that the opinion leader is unlikely to spread positive recommendations (despite the incentives);

5. The number r is small i.e. the opinion leader is expected to talk to only a few other consumers.

It is beyond the scope of this paper to investigate details about how incentive structures might be designed to ensure that a WOM marketing campaign is worth running at all. But the above analysis already reveals that it requires much deliberation.

More importantly, such a campaign imposes an opinion leadership structure on the consumers that may hurt future profits. Suppose that the M consumers are connected by a star network that is not altered by the marketing campaign. However, prior to the campaign, the central consumer was the sole follower (perhaps because she happened to be relatively patient and a focal equilibrium emerged on this dimension); after the campaign, she becomes the sole opinion leader. Now, suppose that, some time after the campaign, the firm launches a different new product at price $p'>0$ (with marginal cost that is normalized to zero) to the same consumers in a two-period WOM game, but without WOM marketing effort. But now, the central consumer as the opinion leader purchases and consumes first. Lastly, assume that the prior for q is such that the new product is a priori preferable to the outside option, while the other consumers only purchase the new product if the central consumer finds that it is good; meanwhile, in the former equilibrium, at least m consumers, where $1 \leq m \leq M\text{-}1$, must find the new product to be good for the central consumer to purchase it. The change in profit due to the change in opinion leadership structure is then:

$$p'\left[1 + \delta_F q(M-1) - \left(M - 1 + \delta_F \sum_{m'=m}^{M-1} C_{m'}^{M-1} q^{m'} (1-q)^{M-1-m'}\right)\right],$$

where q is the true probability that a consumer finds the new product good. This quantity turns out to be negative whenever $M \geq 2$ and $0 \leq q \leq 1$ except in certain special cases when it is zero, such as when $M = 2$ and $q = 1$ (see Appendix). To sum up, the firm in general suffers due to the change in equilibrium structure caused by its previous WOM campaign. Our calculation thus also demonstrates that focal equilibria – in this case, the focal equilibrium with the central consumer as the opinion leader because of centrality and perhaps also time discount factor criteria (the latter is especially important when M is just two or three) – are not necessarily profit-maximizing to the firm.

7. Conclusion and Discussion

A central result of this paper is that opinion leadership is not necessarily linked to any consumer characteristics apart from definitional ones; this suggests that opinion leaders are not "born" but "made". The very possibility that useful product recommendation from another consumer is available, coupled with some or all consumers being sufficiently forward looking, could already lead to the emergence of opinion leaders and followers among a network of consumers. In fact, in the conceptualization of opinion leadership since Katz and Lazarsfeld (1955), opinion leader/follower relationship can be subtly and even unconsciously formed; opinion leaders can simply emerge endogenously among a population of consumers as a result of equilibrium play, and the particular equilibrium that results can be "selected" due to purely accidental initial conditions. Note that, even if the consumers and their networks remain the same, the structures of possible equilibria and their opinion leaders may vary from case to case depending on the relative risk and prices of the available alternatives. Thus firms trying to create effective word-of-mouth marketing should be careful not to take for granted that any opinion leaders identified by commonly used surveys are always the opinion leaders in every case of new-product diffusion, even within the same category of products – though it is possible that opinion leaders with focal characteristics such as high connectivity are indeed often the opinion leaders. But it has to be re-emphasized that these are likely, though not certain, scenarios; opinion leaders may even be among the most "patient" consumers in the network in some cases. Moreover, emergence of opinion leaders may occur even

when the consumers are completely homogeneous in terms of time preference. Also, focal equilibria with opinion leaders who are well-connected and/or have low time discount factors may be neither efficient to consumers nor profit-maximizing to firms.

Our results offer a parsimonious possible explanation of the appearance of opinion leaders in a society without having to resort to exogenous individual difference parameters to differentiate consumers. After all, as stated in the introduction, there have not been enough convincing individual difference explanations of the phenomenon of opinion leadership in reality. Many empirically identified opinion leaders are in fact ordinary consumers with a limited number of consumers under their influence; nevertheless, they do consistently purchase new products earlier than their followers. Although opinion leaders seem to know more about the product category than their followers, this can be a consequence of equilibrium play rather than an exogenous factor, as we have argued earlier. It should be noted that, if a consumer who has become an opinion leader is driven to acquire more product knowledge outside of her network in order to improve her decisions, this very fact, if known to other consumers, is sufficient to make the opinion leader more likely to stay in her position – what better for other consumers to do than to wait for someone who is in the "know" to offer recommendations whenever a new product in the relevant category appears in the future? The opinion leader then becomes differentiated from other consumers in having a possibly more "reliable" prior than others; for example, the opinion leader might purchase the outside option even when other consumers' priors indicate a preference for the new product, but since the opinion leader is trusted for her choice, other consumers might turn to the outside option too. The "WOM game" at this stage is admittedly different from our set up, but we believe that what we offer in this paper is how the initial "push" for opinion leadership may have come about.

7.1 Comparison with the Volunteer's Dilemma

There is some similarity between the characteristics of the equilibrium involved in Proposition 4 and the pure strategy equilibria in the classic volunteer's dilemma (e.g. Diekmann 1985, 1993). The typical setup of the volunteer's dilemma is a binary-action multi-player game in which a public good is provided if and only if one volunteer pays a cost for it. The value of the public good to any individual player is larger than her

volunteering cost; both the value of the public good and the volunteering cost can be symmetric or asymmetric between players. But, across all cases that are studied, to any player, there being a volunteer is preferable to no one volunteering, but everyone wants to free-ride and hopes that another person will pay the volunteering cost. Similarly, in the WOM game, if ω=1, the consumers' time discount factors are sufficiently high, and the game has a sufficiently large number of periods, then, at any point in the game, every consumer hopes that some other consumer will "volunteer" and try the new product before her, so that she can get more information about it to help her optimize her choice. An opinion leader is then a "volunteer" at the beginning of period 1.

But overall, it should be noted that the WOM game is more complex in almost every aspect than any volunteer's dilemma model. What we have emphasized in this study are cases with general networks in which information exchange between consumers is incomplete and localized, a domain that has never been investigated in research on the volunteer's dilemma. What we have shown is that opinion leaders may still emerge in these situations. The efficiency issues that we have uncovered are also unknown in volunteer's dilemma studies, in which payoff values are often rather arbitrarily assigned.

7.2 Limitations and Future Directions

Despite the generalizations that we have made, our study is limited in a number of dimensions. For example, we have not examined the possibility that each consumer may have her own idiosyncratic values for u_g and u_b (for instance, a consumer may derive pleasure in the act of trying new products, so her u_g and u_b are much larger than other consumers'); if these values are not known to other consumers, but it is common knowledge that the consumers' utility values are randomly determined according to some probability density function, then this heterogeneity is equivalent to the multiple outcome states scenario and can be incorporated into the present theory. But if they are commonly known throughout the game, then each consumer i will have her own sequence Θ_i, and the complications that result mean our findings are not necessarily valid.

Another limitation is that, in reality, consumers form social links proactively in case they want to seek out product information. A consumer may ask her friend to introduce herself to another friend who might already be an opinion leader. There is a stream of

literature in economics about network formation, such as Bala and Goyal (2000), and it might be a worthwhile direction for further exploration in the context of the present set up. An additional remark is that, if well-connected consumers are likely to be opinion leaders in focal equilibria, and consumers prefer to form links with opinion leaders, then well-connected people are likely to be even more well-connected as time passes. This is consistent with the network model from which Barabási and Albert (1999) deduce their scale-free power-law distribution of vertex connectivities.

As a whole, the theoretical framework and results in this paper can form the basis of further development in research on WOM communication in social networks in relation to new-product diffusion. First of all, questions may be asked regarding how pricing and other marketing mix variables may affect equilibrium play, as they all affect the utilities u_g, u_b, and U_0, as well as $f(q)$, which in turn potentially affects Bayesian inference processes in the population and equilibrium characteristics. For example, an increase in the price of the new product is expected to lead to a decrease in the effective u_g and u_b, and if this decrease is about the same with both utilities, q_0 will increase as a result, indicating that the outside option becomes more favored and deferring purchase for more information becomes more attractive, controlling for time preference (and supposing the new product is still a priori preferred over the outside option). Another example is that credible, informative advertising of the new product may make f less diffuse.

Also interesting is the possibility that the firm selling the new product indeed knows the true q (this is not necessarily the case; for example, the producers of a new film are often as much in the dark as anyone else as to whether movie goers will like their work), and its pricing can be used as a signal for q, but with a revenue function that is discontinuous because the number of opinion leaders in equilibrium (as well as other equilibrium characteristics) may change abruptly at various critical values of q_0, which affects Θ. A possible direction along these lines is to focus on simpler networks, such as complete graphs. Lastly, the firm manufacturing the new product may also influence consumers' prior through advertising. If advertising makes the prior more "optimistic", more consumers may emerge as opinion leaders in equilibrium; but this comes at the expense of advertising costs. A formal analysis on this dilemma may lead to new insights about the interaction between advertising, WOM, and diffusion efficiency.

Figure 1.

Timeline of the model. Note that, according to our model assumptions, all consumers should have made a purchase before the game ends.
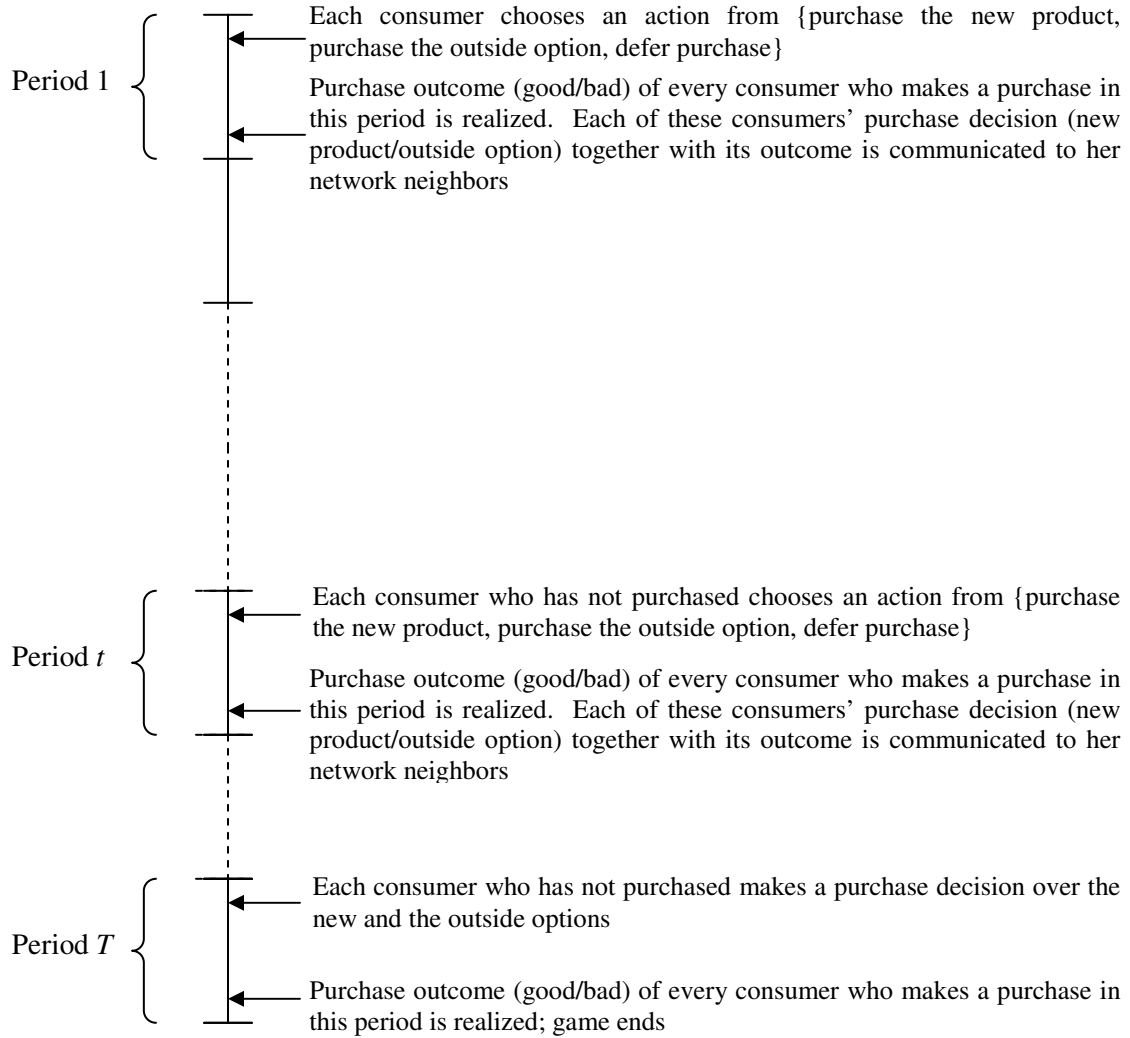
Period 1
- Each consumer chooses an action from {purchase the new product, purchase the outside option, defer purchase}
- Purchase outcome (good/bad) of every consumer who makes a purchase in this period is realized. Each of these consumers' purchase decision (new product/outside option) together with its outcome is communicated to her network neighbors

Period t
- Each consumer who has not purchased chooses an action from {purchase the new product, purchase the outside option, defer purchase}
- Purchase outcome (good/bad) of every consumer who makes a purchase in this period is realized. Each of these consumers' purchase decision (new product/outside option) together with its outcome is communicated to her network neighbors

Period T
- Each consumer who has not purchased makes a purchase decision over the new and the outside options
- Purchase outcome (good/bad) of every consumer who makes a purchase in this period is realized; game ends

Figure 2.

An example of a set *L* of two consumers that satisfy description (b) in Proposition 4(2), together with all their (four) network neighbors. Each thick dot represents a consumer, and each straight line represents a network link. The consumers inside the dotted ellipse form the set *L*. Note that the consumers in *L* satisfy the requirement of such a set as applied to Proposition 4 whether they are connected to each other or not. It is also irrelevant whether or how the consumers outside *L* are connected to each other.
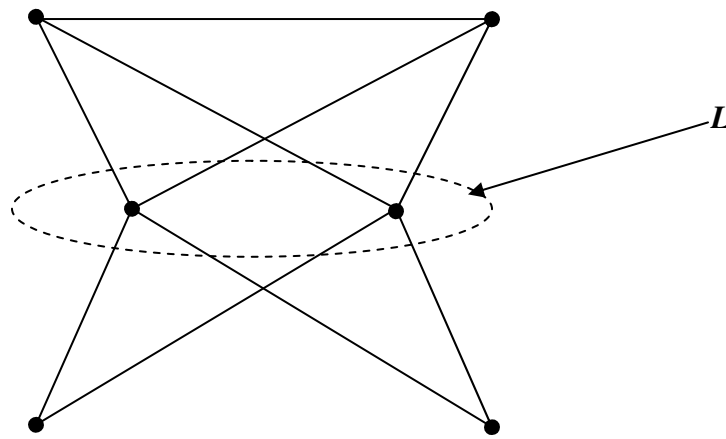
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APPENDIX

PROOF OF PROPOSITION 2. For any consumer i, formally define the history of play of her neighbors as follows. Let $d(i) = \#[N(i)]$ be i's number of neighbors. Order the consumers in $N(i)$ in some pre-specified way to form a row vector of $d(i)$ dimensions. Call this vector $\underline{N(i)}$. At the beginning of any period t, i might observe one and only one of the following possible purchase events with any consumer in $N(i)$: (1) the consumer purchases the new product which turns out to be good, an event denoted as ng; (2) the consumer purchases the new product which turns out to be bad, an event denoted as nb; (3) the consumer purchases the outside option, an event denoted as o. [21] Let $E = \{ng, nb, o\}$ be the set of possible purchase events. The local history of play as observed by i up to the end of period t (or up to the beginning of period t+1), called $h_i(t)$, is defined to be a row vector of $d(i)$ dimensions that is an element of the set $H_i(t) = [E \times \{1,2,3...t\} \cup \{(0,0)\}]^{d(i)}$; $h_i(t)$'s components are ordered in the same way as the components of $\underline{N(i)}$, and we can say that this characterization maps each component (consumer) of $\underline{N(i)}$ to the corresponding component in a vector belonging to $H_i(t)$. The component of $h_i(t)$ that corresponds to consumer $j \in N(i)$ is (0,0) iff j has not yet purchased by the end of period t; otherwise, the first element of the component indicates the purchase event of j and the second element indicates the period at the beginning of which j made that purchase. We also define the singleton $H_i(0) = \{(0,0)\}^{d(i)}$; its element $h_i(0)$ represents the (null) history of play of i's neighbors at the beginning of period 1. For consumer i and integers t and t+s with $t \geq 0$, $s > 0$, say that $h_i(t)$ and $h_i(t+s)$ are consistent with each other iff, for m = 1, 2, 3 … $d(i)$, if the mth component of $h_i(t)$ is not (0,0), then the mth component of both vectors are identical.

Given that i knows the strategies of all other consumers, let $\Pr(h_i(t)|q)$ be the probability that i observes history of play $h_i(t)$ from her neighbors if the probability of the new product being good is in fact q. Then i's posterior distribution of q upon observing history of play $h_i(t)$ is:

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[21] Given that any trial information of the outside option tells consumers nothing new in addition to what is already commonly known, the (independently) realized experienced utility of a purchase of this product is irrelevant to decision making; only the purchase itself is possibly relevant.

$$f(q \mid h_i(t)) = \frac{\Pr(h_i(t) \mid q) f(q)}{\int \Pr(h_i(t) \mid q') f(q') dq'}.$$

As can be expected, the prior $f(q)$ is equal to $f(q \mid h_i(0))$ since $\Pr(h_i(0) \mid q) = 1$. We also define, for any probability density function $g(q)$ of q, the mean $\hat{q}(g) = \int q g(q) dq$.

Suppose the game has come to the beginning of period t and i has observed history of play $h_i(t\text{-}1)$ from her neighbors. Suppose $\hat{q}(g) \geq q_0$ where $g(q) = f(q \mid h_i(t-1))$. For any positive integer $s \leq T\text{-}t$, define $RH_i(t+s-1) \subseteq H_i(t+s-1)$ such that $h_i(t+s-1) \in RH_i(t+s-1)$ iff it (a) is consistent with $h_i(t\text{-}1)$; (b) can be realized with positive probability given $h_i(t\text{-}1)$ and the consumer strategies; and (c) satisfies $\hat{q}(f(\cdot \mid h_i(t+s-1))) < q_0$, so that i would then purchase the outside option at the beginning of period $t + s$, thus reversing her decision had she made a purchase at the beginning of period t.

Next, let $\Pr(h_i(t+s-1) \mid q, h_i(t-1))$ be the probability of $h_i(t+s-1)$ happening given that the probability of the new product being good is in fact q and that $h_i(t\text{-}1)$ has happened. Then define:

$$RP(q, s, h_i(t-1)) = \sum_{\substack{h_i(s+t-1) \\ \in RH_i(s+t-1)}} \Pr(h_i(s+t-1) \mid q, h_i(t-1))$$

$$= \sum_{\substack{h_i(s+t-1) \\ \in RH_i(s+t-1)}} \frac{\Pr(h_i(s+t-1) \mid q)}{\Pr(h_i(t-1) \mid q)},$$

and also the function:

$$U(q) = q u_g + (1-q) u_b.$$

Note that $U_0 = U(q_0)$ by definition. Also, since $\hat{q}(g) \geq q_0$, $U(\hat{q}(g)) \geq U(q_0) > 0$. Now, if i decides to defer purchase by a further s periods, her expected utility is:

$$EU_{defer} = \delta_i^{s+t-1} \int \{[1 - RP(q, s, h_i(t-1))] U(q) + RP(q, s, h_i(t-1)) U(q_0)\} g(q) dq$$

$$= \delta_i^{s+t-1} \left\{ \int [U(q) g(q) dq + \sum_{\substack{h_i(s+t-1) \\ \in RH_i(s+t-1)}} \int \Pr(h_i(s+t-1) \mid q)(q_0 - q)(u_g - u_b) f(q) dq \right\}$$

$$= \delta_i^{t+s-1} \{U(\hat{q}(g)) + \pi(s, h_i(t-1))\},$$

where $\pi(s, h_i(t-1)) > 0$ because, by definition, if $h_i(t+s-1) \in RH_i(t+s-1)$, then

$$\frac{\int q \Pr(h_i(s+t-1)\mid q)f(q)dq}{\int \Pr(h_i(s+t-1)\mid q)f(q)dq} < q_0, \text{ that is,}$$

$$\int \Pr(h_i(s+t-1)\mid q)(q_0 - q)f(q)dq > 0.$$

On the other hand, the utility of purchasing at the beginning of period t is:

$$EU_{purchase} = \delta_i^{t-1}U(\hat{g}(q)).$$

This means that deferring s periods is strictly preferable to purchasing immediately as long as:

$$\delta_i^s > \frac{U(\hat{q}(g))}{U(\hat{q}(g)) + \pi(s, h_i(t-1))},$$

which can always be achieved by a sufficiently high time discount factor in $[0,1)$, since $\pi(s, h_i(t-1)) > 0$. Note the crucial assumption that there is a positive probability of decision reversal, that is, the assumption that $RH_i(t+s-1)$ is not an empty set. But as long as this set is not empty and i's time discount factor is sufficiently high, it is always strictly preferable for her to purchase. Note also that, it may happen that, with some equilibrium outcomes, i might find that, after deferring y periods, at the beginning of period $t + y < t + s$, deferring has become useless in that the probability is zero that she would reverse her decision anymore whatever new information she might receive from her neighbors. She would then purchase in period $t + y$ instead of period t. But as such possibility may only increase i's expected utility relative to the strategy "defer purchase by exactly s periods", and the proof just presented is still valid.

The proof for the case $\hat{q}(g) < q_0$ is similar.

PROOF OF PROPOSITION 4. We first present a useful lemma:

LEMMA 1. Let $g(q)$ and $v(q)$ be two non-negative, integrable functions defined over $q \in [0,1]$. Moreover, $\int g(q)dq > 0$, $\int v(q)g(q)dq > 0$ (all integrals are over $[0,1]$), $g(q)$ does not only have a single mass point (that is, its normalized form is a non-degenerate prior), and $v(q)$ is non-constant. Then:

(1) $\dfrac{\int qv(q)g(q)dq}{\int v(q)g(q)dq} > \dfrac{\int qg(q)dq}{\int g(q)dq}$ if $v(q)$ is monotonically increasing in q;

(2) $\dfrac{\int qv(q)g(q)dq}{\int v(q)g(q)dq} < \dfrac{\int qg(q)dq}{\int g(q)dq}$ if $v(q)$ is monotonically decreasing in q.

Proof. (1) follows from the fact that, given the monotonicity premise, the probability density functions $v(q)g(q)/\int v(q')g(q')dq'$ and $g(q)/\int g(q')dq'$ satisfy the monotone likelihood ratio property, so that the former first-order stochastically dominates the latter (Milgrom 1981). Since they are not identical, the strict inequality in expected value follows. Dominance and thus inequality hold in reverse direction for (2).

Next, we define the sequence $\Theta(g)$ with members $\delta*_r(g)$, $r = 1, 2, 3 \dots$, such that

$$\delta*_r(g) = \frac{U(q_m(g))}{U(q_m(g)) + \pi_r(g)},$$

where $q_m(g) = \max\{\hat{q}(g), q_0\}$, $U(q_m(g)) = q_m(g)u_g + (1 - q_m(g))u_b$, and

$$\pi_r(g) = \int P(q, r, g)(q_0 - q)(u_g - u_b)g(q)dq,$$

where $P(q, r, g) = 0$ if $\hat{q}(g) < q_0$, but otherwise is the probability that a consumer reverses her a priori purchase decision (of choosing the new product) upon directly observing the purchase outcomes of r other consumers, given that probability of the new product being good is in fact q and q is distributed according to the probability density function $g(q)$. Correspondence with notations in the main text are as follow: $\hat{q}(f) = \bar{q}$, $\delta*_r(f) = \delta*_r$, and $\Theta(f) = \Theta$.

LEMMA 2. $\pi_r(g)$ is non-negative, non-decreasing with r, and is strictly increasing in r if it is positive.

Proof. This is straightforward when $q_0 < \hat{q}(g)$ since then $\pi_r(g) \equiv 1$. Otherwise, the lemma is equivalent to showing that $U(\hat{q}(g)) + \pi_r(g)$ (i.e. the expected utility in making decision based on r other consumers) has the properties as described in the lemma. This can be deduced from Proposition 2. For any consumer i, the difference between observing the purchase outcomes of r consumers and $r+s$ consumers, $s > 0$, is equivalent to the difference between purchasing at the beginning of period 2 in a hypothetical WOM game with r opinion leaders, and purchasing at the beginning period 3 when, in addition to the r opinion leaders, it is known that another s consumers (who are not connected to the r opinion leaders) will purchase at the beginning of period 2; moreover, $\delta_i = 1$ in this

hypothetical game and all the $r+s$ consumers make their purchases based on $g(q)$ only. This setting is not possible in principle according to our general set up, but the mechanism behind the proof of Proposition 2 still applies, and so we know that deferring in order to observe the purchase outcomes of all the $r + s$ consumers is at least as good as not deferring. Hence $\pi_r(g)$ must be non-negative and non-decreasing with r.

Moreover, it is not difficult to show, by repeatedly applying the method in the proof of Proposition 2, that $U(\hat{q}(g)) + \pi_r(g)$, and hence $\pi_r(g)$, is strictly increasing when observing one more consumer in addition to r consumers incurs a positive ex ante probability, over all possible purchase outcome realizations of the r consumers, to "tip the balance" and reverse the purchase decision that is based only on the r consumers. To prove this, first note that, by Lemma 1, if decision reversal can happen with k out of r consumers finding the new product bad (while the remaining r-k consumers find it good), then reversal can also happen if s out of r consumers find it bad too, as long as $r \geq s \geq k$. So, suppose $\pi_r(g) > 0$ so that there exists a positive integer $k(r)$ satisfying $1 \leq k(r) \leq r$ and $\int q^{1+r-s}(1-q)^s g(q)dq / \int q^{r-s}(1-q)^s g(q)dq < q_0$ iff $s \geq k(r)$. Then:

$$\frac{\int q^{1+r-k(r)}(1-q)^{k(r)} g(q)dq}{\int q^{r-k(r)}(1-q)^{k(r)} g(q)dq} < q_0 \leq \frac{\int q^{2+r-k(r)}(1-q)^{k(r)-1} g(q)dq}{\int q^{r-k(r)+1}(1-q)^{k(r)-1} g(q)dq}.$$

Now, consider $R = r + 1$, where R is also a positive integer. We now look for $k(R)$ that satisfies:

$$\frac{\int q^{1+R-k(R)}(1-q)^{k(R)} g(q)dq}{\int q^{R-k(R)}(1-q)^{k(R)} g(q)dq} < q_0 \leq \frac{\int q^{2+R-k(R)}(1-q)^{k(R)-1} g(q)dq}{\int q^{R-k(R)+1}(1-q)^{k(R)-1} g(q)dq}.$$

It can be easily shown from the above inequalities and Lemma 1 that $k(R)$ exists and $1 = R - r \geq k(R) - k(r) \geq 0$. If $k(R) = k(r)$, adding one consumer potentially "tips the balance" when only $k(r)-1$ of the r consumers find the new product bad. This is because the decision based on those r consumers alone is still in favor of the new product, but if the additional consumer finds the new product bad, the decision becomes in favor of the outside option.. If $k(R) = k(r)+1$, adding one consumer potentially "tips the balance" when $k(r)$ of the r consumers find the new product bad. This is because the decision based on those r consumers alone is in favor of the outside option, but if the additional consumer finds the new product good, the decision becomes in favor of the

new product. Note that we have assumed $k(r)$ is positive, which is equivalent to saying that $\pi_r(g)$ is positive, as is stated as a sufficient condition in the lemma for it to be strictly increasing.

Lemma 2 leads to the characterization of $\Theta = (\delta *_1, \delta *_2, \delta *_3 ...)$ in Proposition 4.

To prove Part (1) of the proposition, observe that, for any consumer i, if every other consumer makes purchases (of the new product) at the beginning of period 1, she should purchase at the beginning of period 1 too, because, according to the premise of this proposition, the purchase outcomes of her $N(i)$ neighbors would not provide enough information to justify her deferring purchase by one period. Note that she has no incentive to defer purchase any further either, because all her neighbors purchase at the beginning of period 1 and would not, through purchasing at later periods, allow her to gather information by inference about the choices and purchase outcomes of consumers who are not her neighbors.

For Part (2), to prove the "if" part of the statement, it suffices to prove the existence of an equilibrium with the following strategic characteristics given the premises:

A. All consumers in L purchase the new product with certainty in period 1;

B. Every consumer who is at distance one from L purchases only after all consumers in L have purchased;

C. Every consumer who is not in L purchases only after at least one of her neighbors has purchased or when the game has come to period T;

D. Every consumer who is not in L purchases only when there is zero probability that deferring will lead to a reversal of her purchase decision.

Note first that premise (a) is satisfied only if $\bar{q} \geq q_0$, so that the new product is the a priori choice of purchase, and hence the consumers in L must purchase the new product if they purchase in period 1. It is also easy to see that characteristic D is guaranteed by premise (c) and the "if" clause of the statement because of Proposition 2 – and it is therefore necessary to state for the "if" clause that all time discount factors of consumers outside L must be higher than some threshold time discount factor, say $\delta_{if(L)}$, in order to satisfy premise (2) (the "sufficiently patient" premise) of Proposition 2; but see the end of this proof for the relationship between δ_L in the Proposition and any candidate $\delta_{if(L)}$.

Given this and characteristic C, and the premises (a) and (b) of Part (2), it is

straightforward to prove that characteristics A and B are mutually consistent equilibrium outcomes. It can be said that the ω consumers in L become opinion leaders as if they have been "coerced" to be so, since all other consumers – starting with the neighbors of the ω consumers who are not in L – will not purchase if not all the opinion leaders have purchased.

Characteristic B is required for this "coercion" mechanism to succeed. It is sustainable as an equilibrium strategy, given the "if" clause of the statement, if we can prove that, for every consumer outside L, there is a positive probability that, under equilibrium, waiting for her neighbors to purchase will reverse her a priori purchase decision – that is, will lead to her purchasing the outside option. We already know this must be true for consumers at distance one from L, since $\omega = \min\{k : \delta*_k < 1\}$. Now suppose this is true for all consumers at distances that are smaller than or equal to σ from L. Then, for any consumer i at distance $\sigma+1$ from L, there is a positive probability that deferring will lead to the observation of one of her neighbors, say j, purchasing the outside option in a certain period, say t. This fact, by itself, should lead to i purchasing the outside option as well, since, if she does not regard other components of the observed history of play of her neighbors, the best she can to is to follow j's decision of purchasing the outside option. Now, conditioned on observing j purchasing the outside option in period t, other components of the history of play of i's neighbors may be realized in different ways with different probabilities. Some of these realizations may lead to i purchasing the new product, while some may not. However, if all of the possible realizations are of the former type, there would be probability one that the fact of observing j purchasing the outside option alone will lead to i purchasing the new product, which is a contradiction. Hence we conclude that some of the possible realizations must lead to i purchasing the outside option. This in turn implies that there is positive probability that a consumer at distance $\sigma+1$ from L purchases the outside option, and characteristic B is justified by induction.

To prove the "only if" clause, observe that the time discount factors are lower bounded by zero; hence $\delta_L = \inf\{\delta_{if(L)}\}$ exists, i.e. the set of time discount factors that can be used as threshold for the "if" clause of Part (2) must have an infimum. Therefore, if $\exists i \in C \setminus L \ \delta_i \leq \delta_L$, there cannot be any equilibrium satisfying characteristics A to D.

In other words, if there exists an equilibrium satisfying characteristics A to D, $\forall i \in C \setminus L\ \delta_i > \delta_L$. This completes the proof. Note that the inequality sign in "$\forall i \in C \setminus L\ \delta_i > \delta_L$" in the proposition is always strict because of Tie-Breaking Rule 2.

PROOF OF THE EFFICIENCY INEQUALITY IN SECTION 6.1. The inequality to be proved is:

$$\delta_c / \delta^*_{M\text{-}1} > (\delta_v / \delta^*_1)(M\text{-}1) - (M\text{-}2),$$

the fulfillment of which indicates that it is less efficient for the consumers to have the central consumer as the sole opinion leader in equilibrium than to have the central consumer as the sole follower in equilibrium. To deduce this, first note that the total ex ante consumer utility when the central consumer is the sole opinion leader is (using the results from previous proofs):

$$U(\overline{q}) + (M-1)\delta_v (U(\overline{q}) / \delta^*_1).$$

On the other hand, the total ex ante consumer utility when the central consumer is the sole follower is:

$$\delta_c (U(\overline{q}) / \delta^*_{M-1}) + (M-1)U(\overline{q}).$$

Comparison of the two yields the inequality. Note that the inequality result is applicable only when $\delta_c > \delta^*_{M\text{-}1}$ and $\delta_v > \delta^*_1$ so that both equilibria are feasible (and, in which case, the new product is a priori preferred over the outside option).

PROOF OF THE RESULTS AT THE END OF SECTION 6.2. What we first need to establish is that:

$$p'\left[1 + \delta_F q(M-1) - \left(M - 1 + \delta_F \sum_{m'=m}^{M-1} C_{m'}^{M-1} q^{m'} (1-q)^{M-1-m'}\right)\right] \le 0.$$

To prove the above, observe that:

$$1 + \delta_F q(M-1) - \left(M - 1 + \delta_F \sum_{m'=m}^{M-1} C_{m'}^{M-1} q^{m'} (1-q)^{M-1-m'}\right) \le 1 + \delta_F q(M-1) - \left(M - 1 + \delta_F q^{M-1}\right).$$

The first-order derivative of the right-hand side with respect to q is:

$$\delta_F (M-1)(1 - q^{M-2}) \ge 0,$$

within the range of parameters under consideration. That is, the right-hand side is maximized at $q = 1$ within the range of q under consideration. But its maximized value is simply $(M-2)(\delta_F - 1) \le 0$, meaning that the left-hand side must be non-positive. In fact,

it is easy to see that the left-hand side is strictly negative except when either of the following is satisfied: (1) $M = 2$ and $q = 0$; (2) $M = 2$ and $m = 1$; (3) $\delta_F = 1$ and $m = M\text{-}1$. In all three cases (which are not mutually exclusive), the left-hand side is zero.