Quality and Advertising in a Vertically Differentiated Market

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Abstract

We examine firms' quality positions when consumers can only consider purchasing products that they are informed about through advertising. Consumers compare the alternatives in their consideration set and choose the product that maximizes their utility net of price. Firms choose product quality in a first stage, advertising strategy in a second stage, and prices in the last stage. We study two forms of advertising– *blanket* and *targeted*. Under blanket advertising, firms communicate indiscriminately and a consumer's probability of seeing an ad depends on the level of ad expenditure. We find that when blanket advertising is relatively ineffective, i.e., it is costly to ensure that all consumers are informed, both firms choose a light ad spending. This allows the firms to be relatively undifferentiated in qualities, without concern of intense price competition. When blanket advertising is very effective, the high quality firm expends heavily on advertising, while its rival differentiates with a low quality product and expends lightly on advertising. Interestingly, in a mid range of advertising effectiveness one firm chooses a high quality product and its rival positions close by. In this way, the lower quality firm induces its rival to advertise only lightly to avoid fierce price competition. Under targeted advertising, firms choose the specific segment(s) they wish to inform. We identify conditions such that both firms choose equally high quality products, but advertise to different segments. This can result in a middle pocket of unserved consumers, even though consumers with lower willingness to pay are served.

(*Product Quality, Advertising Strategy, Differentiation, Competition*)

1 Introduction

In order for consumers to consider the purchase of a product, they must first be aware of its existence and informed about its characteristics.[1] Although there are a number of ways for consumers to become informed about the products available in the marketplace, firm initiated communications are a primary vehicle. In some cases, firms can be selective and send messages only to those consumers who are part of their target market. For example, by purchasing or compiling a list of consumers that meet certain criteria, firms can send targeted e-mails or direct mail; in B2B settings firms can send their sales force to a subset of potential customers with common characteristics. In other cases, firms cast a much wider net and use media outlets that preclude direct control over who sees their ads. For example, by running a commercial on national TV or placing an ad in a general interest magazine, a firm will potentially reach a broad set of consumers. It is noteworthy that in the U.S. alone companies spent over $155 billion in 2006 to advertise their offerings to consumers.[2]

Consumers consider the various products they are informed about and choose the one that delivers them maximum utility. Consequently, the return on advertising for a firm will critically depend on how its offering compares to the other offerings in the marketplace *and* how aggressively those products are advertised, i.e., which products constitute consumers' consideration set. In this context, the decision of what quality product to offer and then how heavily to promote it through advertising become intertwined, and further depend on the rival's product positioning and advertising strategy.

For example, in any given automobile class (such as sedans, sports utility, roadsters) manufacturers need to make decisions on the quality of their models prior to development. Upon launch, these manufacturers (e.g., Mazda, BMW, Porsche) need to advertise their cars to engender demand. High-tech firms, such as manufacturers of storage hardware, decide whether to position their devices as suitable for the high- or low-end spectrum of computing needs, and their sales force must then communicate these positions to relevant customers. In several beverage categories the same is true. For instance, firms that market ground coffee

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[1]Behavioral models of the consumer's decision making process (DMP) typically include the stages of awareness, knowledge, consideration/evaluation, preference and ultimately purchase (Dolan 1999, Keller and Kotler 2006).

[2]Advertising Age, June 25, 2007.

(e.g., Maxwell House, Foldger's, Cafe de Columbia) select the quality of beans to use and determine how finely they will be roasted prior to expending resources on promotion; firms that produce vodka (e.g., Smirnoff, Absolut, Russian Standard) make decisions on the type of grains to be used and on the consistency of the distilling process, and subsequently carry out advertising campaigns to inform consumers.

In this paper, we study how the need to advertise in order to be included in the consumer's consideration set affects a firm's decision of where to position its product vis-à-vis the competition. We develop a duopoly model in which firms choose product quality in a first stage, their advertising strategy in a second stage, and set prices in the last stage. Consumers are heterogeneous with respect to their valuation for quality, allowing for vertical differentiation.

We study two forms of advertising that firms can use to inform consumers about the existence (and characteristics) of their products. The first type– *blanket advertising*– captures situations where firms communicate to all consumers indiscriminately (e.g., during widely popular shows on broadcast television or radio). The probability that a firm's product enters a consumer's consideration set is a function of the advertising expenditure. Specifically, a heavy ad spend guarantees that all consumers will receive the firm's message and consider its product, while a light ad spend only results in a likelihood that each consumer receives the message. Three main equilibria can emerge depending on the effectiveness of blanket advertising, which is a function of the cost differential between heavy and light advertising expenditures (i.e., the more it costs to advertise heavily the less effective advertising is). When advertising is relatively ineffective, the return on advertising heavily is small and both firms advertise lightly. Price competition is softened because the likelihood that a given consumer receives ads from both firms is low; hence one firm selects maximal quality and its rival can also choose a relatively high quality, leading to a moderate level of quality differentiation. When, at the other extreme, blanket advertising is very effective, one firm chooses maximal quality and expends heavily on advertising. The rival now prefers to differentiate with a much lower quality product and to expend lightly on advertising to avoid harsh price competition. In a mid range of advertising effectiveness, we get the intriguing result that by choosing quality appropriately, the lower-quality firm induces the high-quality firm to expend

only lightly on advertising. Specifically, because the products are minimally differentiated, the high-quality firm seeks to avoid fierce price competition by refraining from advertising heavily. Somewhat counterintuitively, the more effective advertising is in this range the less quality differentiation is observed.

The second type of advertising we examine–*targeted advertising* –captures situations where firms can communicate to specific segment(s) they wish to inform about their product. We characterize equilibria where each firm advertises to a distinct segment. Because each consumer only considers one product, the firms do not compete in the pricing stage and hence both choose equally high quality– that is, their products are entirely undifferentiated. Interestingly, due to the relatively high prices firms charge, there exist conditions such that a set of consumers with moderate valuation for quality are unserved, even though consumers in a different segment with lower willingness to pay are served. We also show that the total number of consumers served follows an inverted-U shape as a function of the size of the low willingness-to-pay segment.

In an extension, we study the role of persuasive advertising. While the two types of informative advertising impact awareness and consideration, persuasive advertising changes the way consumers perceive product quality; thereby affecting their willingness to pay. Our main result here is that firms may differentiate less in objective qualities than under no advertising. However, the difference in their persuasive advertising levels counterbalances this effect, leading to the same degree of differentiation in perceived qualities as when consumers are fully aware of objective product qualities.

The rest of the paper is organized as follows. The next section relates our work to the relevant literature and summarizes our contribution. Section 3 sets up the basics of the model in terms of demand and firm behavior. Section 4 solves the case of blanket advertising and Section 5 solves the case of targeted advertising. We consider the persuasive advertising extension in Section 6. Finally, Section 7 concludes, discusses limitations and outlines opportunities for future research. To enhance readability, we postpone all proofs to the Appendix.

2 Related Literature

Our work is primarily related to the vertical differentiation literature, beginning with the widely known model of Shaked and Sutton (1982). They examine price competition between firms that first choose the quality of the products they sell. Since consumers, who are fully informed about product qualities, are heterogeneous with respect to their valuation of quality, in equilibrium firms choose different qualities in order to reduce price competition in the last stage. Moorthy (1988) relaxes Shaked and Sutton's zero cost of production assumption by introducing a quadratic cost function for quality, resulting in an equilibrium where the firm choosing the lower quality may be better off. Moorthy (1988) further shows that when firms choose qualities (simultaneously or sequentially) the equilibrium strategies are to differentiate in qualities. Assuming that firms do not cover the market, Choi and Shin (1992) show that the low quality firm will choose a quality level which is a fixed proportion of the high quality firm's choice. Wauthy (1996) on the other hand, gives a full characterization of quality choices, allowing the coverage of the market to be endogenous. More recently, Choudhary et al. (2005) and Jing (2006) both find that the higher quality firm can be worse off in equilibrium. Choudhary et al. (2005) examine a vertical differentiation model where personalized pricing is allowed. Personalized pricing results in greater market coverage, but also intensifies competition, which can hurt the high quality firm. Jing (2006) identifies the conditions on the cost structure under which producing the low-quality good is more profitable.

The previous studies show that quality differentiation is a robust equilibrium outcome. However, in reality similar quality products are often observed in the marketplace. Rhee (1996) cites evidence for this and offers an explanation that incorporates consumer heterogeneity along unobservable attributes into the vertical differentiation model. As a result, if consumers are sufficiently heterogeneous on the extra dimensions, in equilibrium firms offer products that are identical on the observed quality dimensions (but differentiated on the unobserved dimensions). Banker et al. (1998) investigate the relationship between equilibrium quality levels and the intensity of competition between firms. They find that the degree of differentiation depends on the market structure and production cost differences.

Notably, the papers in this stream analyze firms' quality positions under various price

and cost assumptions, but assume that all consumers are fully informed about these qualities and ignore the role or need for advertising.

Another stream of literature related to our work is the vast amount of studies on advertising. We focus on prior research that examines the relationship between advertising and product quality. It is theoretically well-established in economics and marketing that advertising level can be a signal of quality (Nelson 1974, Milgrom and Roberts 1986). However, this work ignores the informativeness of advertising and thus its effect on market size. In contrast, Zhao (2000) shows that when advertising raises awareness, spending less can be the optimal signaling approach of the high-quality firm. Iyer et al. (2005), on the other hand, investigate how firms should target their advertising when consumers have horizontal tastes. They find that firms advertise more to consumers who have a strong preference for their product, and argue that this is a way to soften price competition. For the most part, the literature in this stream takes product qualities as exogenous.

Our contribution lies in extending the first stream of literature by exploring the strategic interaction between advertising spending and quality choice and by showing how advertising can lead to less or even no differentiation. Relative to the second stream, we endogenize product qualities and analyze how this decision is impacted by foreseeing the need to advertise. In this context, we employ models where advertising impacts consumer consideration of products either at the general (blanket) or specific (targeted) reach level.

3   Model Setup

We assume that there are two competing firms that seek to sell their products in a given market. We index the firms by the numbers 1 and 2 or the letters i and j, always assuming that $i \neq j$. If firms offer different quality products, we denote the firm offering the lower quality product by 1 and the higher quality product by 2. We assume that every consumer purchases at most one unit of the product. We further assume that consumers are heterogeneous with respect to their valuation of quality, denoted by ϑ. The parameter ϑ is uniformly distributed in the interval $I = [0, 1]$. A consumer with parameter ϑ gains utility $\vartheta s - p$ from a product with quality s priced at p, and purchases the product for which his/her utility is greater. However, consumers can only consider purchasing products they are informed about

(Keller and Kotler 2006). By advertising, a firm communicates the existence and characteristics of its product (including price), thereby affecting the likelihood that its product enters a consumer's consideration set. Our characterization is thus consistent with the informative view of advertising (see, e.g., Tirole 1988, Iyer et al. 2005, and references therein).[3] In Section 7.1 we discuss our assumptions on the impact of advertising and possible extensions to consumers' knowledge about products (e.g., through search).

We study two different types of informative advertising mechanisms. In the first type, which we call *blanket advertising*, the probability that a consumer considers firm i's product is a_i, independently for each consumer. This probability depends on how heavily firm i advertises and is independent of firm j's advertising level. In the second type, which we call *targeted advertising*, firms can communicate to subsets of consumers that belong to distinct segments. If a consumer is in a firm's targeted segment, s/he considers that firm's product with probability 1.

Timing

The timing of the game is as follows. First, firms choose their qualities s_i ($s_1 \leq s_2$). Qualities are positive and have a maximum value that is normalized to 1. Second, firms make their advertising decisions, and incur the associated promotion costs. In the blanket case they set their advertising level and in the targeted advertising case they choose the segment(s) they wish to advertise to. Third, firms set prices. Finally, consumers make their purchases. This timing reflects the notion that quality choice tends to be a long-term decision, whereas prices can be easily changed. The time-scope of advertising decisions is somewhere in between; consistent with this timing, in practice advertising budget is typically set only after the product characteristics have been determined.

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[3]Consistent with consumer behavior research (e.g., Mitra and Lynch 1995), the role of advertising in our model can also be regarded as creating awareness or a trigger for the consumer to include a given product in their consideration set. Trivially, an individual cannot consider buying a product that s/he is not aware of. Yet the consideration set can be narrower than the set of products the consumer is aware of, since some of these products might be excluded before carefully comparing the utilities derived from them (Hauser and Wernerfelt 1990). For example, because advertising impacts the salience of products and their characteristics in consumers' memory, it affects which products are "top of mind" at the time of purchase (Iyer et al. 2005, provide an excellent discussion of this issue). Hence, our model applies even if we assume that consumers are aware of all products; all we require for our results to go through is that advertising sufficiently increases the likelihood of a product being part of the consumer's consideration set.

Costs and Profits

We assume no fixed entry costs, hence both firms participate in the market. Furthermore, we assume that variable costs are constant and normalize them to zero. Our model is thus consistent with Shaked and Sutton (1982).[4] However, advertising costs, denoted $c(\cdot)$, depend on the advertising level chosen in the following way. In the blanket advertising case, we assume that $c(a_i)$ is an increasing function of a_i (i.e., to achieve a higher likelihood of being considered, a firm has to spend more on advertising). In the targeted advertising case, we assume that the cost is a linear function of the segment size. Firms' profits are therefore simply their revenues (price × quantity sold) minus advertising costs:

$$\Pi_i = p_i D_i - c_i^{adv}.$$

We solve for the pure-strategy, sub-game perfect equilibria of the game. Next, we characterize the equilibria in the two different cases: blanket advertising and targeted advertising (Sections 4 and 5, respectively).

4 Blanket Advertising

In this setup, both firms advertise to the entire mass of consumers but the probability of being included in a consumer's consideration set depends on the effort each firm selects. For the sake of simplicity, let us assume that there are only two levels of advertising: heavy (H) and light (L), with costs $c_H > c_L$, respectively. The probabilities of being considered by consumers are then $0 < a_L \leq a_H = 1$. That is, a firm can either choose to heavily advertise at a high cost, thereby ensuring every consumer is informed about its product, or to advertise lightly at a low cost, thereby expecting only a portion of consumers to be informed. We further assume that the impact of light advertising is not too low. Formally, this means $a_L > a^*$, where a^* is defined in the Appendix (proof of Claim 2). This assumption ensures

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[4]Having a fixed cost of developing a product would not qualitatively affect our results. Our normalizing both variable and fixed costs of production to zero might seem as an oversimplifying assumption but is done for two reasons. First, our assumptions in this respect correspond to Shaked and Sutton (1982), allowing us to directly compare our results to theirs. Second, it allows us to focus on the strategic incentives to differentiate in qualities when there are advertising costs involved; including production costs would complicate this analysis without providing much added insights into the questions of interest, as our results would hold over a certain range of cost parameters.

that firms don't trivially always advertise heavily; though we will discuss (after Proposition 1) the case of very low a_L.

As a benchmark, let us solve the case of a monopolist. A consumer who considers the monopolist's product buys it if and only if $\vartheta s_m - p_m \geq 0$. That is, the monopolist's demand consists of consumers that are informed about the product and for which $\vartheta \geq p_m/s_m$. Therefore,

$$D_m(p_m, a_m) = (1 - p_m/s_m)a_m,$$

where $a_m \in \{a_L, a_H\}$. The monopolist chooses a price to maximize its revenue, $R(p_m, a_m, s_m) = p_m(1 - p_m/s_m)a_m$. Given a_m and s_m, $p_m^* = s_m/2$, thus its profit can be written as

$$\Pi_m(a_m, s_m) = s_m a_m/4 - c(a_m),$$

which is increasing in s_m no matter what the advertising level is. Therefore, the monopolist sets $s_m^* = 1$ and chooses a_H over a_L if and only if $\frac{a_H - a_L}{4} > c$, where $c = c_H - c_L$. Thus, the monopolist advertises heavily if the extra revenue that results from this level is greater than the additional cost needed to advertise heavily rather than lightly. Said differently, if the effectiveness of advertising heavily is above a certain threshold then the monopolist chooses this level. We define the *effectiveness* of advertising as $e \equiv \frac{a_H - a_L}{c_H - c_L} = \frac{1 - a_L}{c}$. The monopolist chooses to advertise heavily iff $e > 4$.

4.1   Duopoly

Let us now turn to the duopoly case. From this point, we will assume that $e > e_M = 4$, which is the critical effectiveness level for a monopolist to advertise heavily. The rationale behind focusing on this region is that if the effectiveness of advertising heavily is so meager that even a monopolist would choose to advertise lightly, trivially, both duopolists would also advertise lightly. Furthermore, facing a competitor, if a firm does choose to advertise lightly in this region then we are assured that this is a result of the strategic interaction between the firms.

We solve the game using backward induction and begin with the last stage pricing game, treating the qualities and advertising levels as given. Fixing the qualities chosen by firms 1 and 2 at s_1, s_2 (recall that we use the convention whereby s_1 denotes the lower-quality firm)

and the advertising probabilities at a_1, a_2, we can characterize the pricing equilibria. The existence of a pure-strategy equilibrium will depend on how close s_1 is to s_2 (or how close the ratio s_1/s_2 is to 1). The critical ratio of qualities necessary for the equilibrium to exist also depends on the proportion of consumers impacted by the advertising of each firm, which we capture through a function $f(a_1, a_2)$.

Claim 1 *There exists a function $f(a_1, a_2)$: $0 \leq f(a_1, a_2) \leq 1$, $f(a_1, 1) = 1$ for any a_1, and $f(a, a)$ is increasing for $a \geq 1/2$, such that*

1. If $s_1 < s_2$ and $s_1/s_2 \leq f(a_1, a_2)$, then the equilibrium prices are

$$p_1^* = \frac{s_1(s_2 - s_1)(2(s_2 - s_1) - a_2 s_2 + 2 s_1(a_1 + a_2 - a_1 a_2))}{4 s_1^2 (1 + a_1 a_2 - a_1 - a_2) + s_1 s_2 (4 a_1 + 4 a_2 - a_1 a_2 - 8) + 4 s_2^2}, \tag{1}$$

$$p_2^* = \frac{s_2(s_2 - s_1)(2(s_2 - s_1) + s_1(a_1 + 2 a_2 - a_1 a_2))}{4 s_1^2 (1 + a_1 a_2 - a_1 - a_2) + s_1 s_2 (4 a_1 + 4 a_2 - a_1 a_2 - 8) + 4 s_2^2}. \tag{2}$$

2. If $s_1 < s_2$ and $s_1/s_2 > f(a_1, a_2)$, then there is no pure-strategy equilibrium.

3. If $s_1 = s_2$, then the equilibrium prices are always zero.

As in other models with vertical consumer heterogeneity, when the firms' qualities are exactly the same (case 3 of the claim) the only sustainable equilibrium is that of zero pricing. Furthermore, if the quality levels are sufficiently different (case 1 of the claim), we can sustain a pure strategy equilibrium with positive prices. Note that here the exact prices firms charge will depend not only on the difference between the qualities $(s_2 - s_1)$ but also on the advertising levels. The equilibrium prices in (1) and (2) directly generalize the no-advertising model with fully informed consumers (per Shaked and Sutton 1982). The lower-quality firm charges a lower price than its higher-quality rival $(p_1^* < p_2^*)$. Further note that prices generally decrease with greater advertising levels due to the fact that more consumers can compare both offerings and hence price competition intensifies (mathematically, $\frac{\partial p_i^*}{\partial a_i} < 0, \frac{\partial p_i^*}{\partial a_j} < 0$). However, when the qualities are very similar (yet not identical) a pure-strategy price equilibrium does not exist (case 2 of the claim). This is because the lower-quality firm may have an incentive to deviate from the pricing schedule in (1) and set a higher price to serve those consumers who only consider its product and not the competitor's. For

the deviation to be profitable though, it must be the case that a sufficient proportion of consumers do not consider the high-quality firm's product (because with both high- and low-quality goods offered at high prices, consumers will surely prefer the former if they are informed about it). Moreover, if the high-quality firm advertises heavily ($a_2 = 1$ so that $f(a_1, 1) = 1$) a pure-strategy equilibrium always exists.

Now let us turn our attention to the second stage of the game where firms decide whether to advertise heavily or lightly. At this stage, quality levels are already set, thus we treat them as parameters. Since payoffs only depend on the ratio of s_1 and s_2 (see Appendix), without loss of generality we may normalize $s_2 = 1$ and fix $0 < s_1 \leq 1$. Let $R^i_{l_i l_j}(s_1)$ denote firm i's revenues in the pricing-stage equilibrium, if it exists, given the advertising levels chosen $l_i, l_j \in \{H, L\}$ (these levels result in probabilities a_i and a_j of informing a consumer). Also, let $\Delta_1 = R^1_{HH}(s_1) - R^1_{LH}(s_1)$ denote the gains to firm 1 of advertising heavily instead of lightly given its rival advertises heavily, and let $\Delta_2 = R^2_{HL}(s_1) - R^2_{LL}(s_1)$ denote the gains to firm 2 of advertising heavily instead of lightly given its rival advertises lightly. Recall that the extra cost the firm has to incur when shifting from light to heavy advertising is denoted by $c = c_H - c_L$. Clearly, the greater Δ_i relative to c, the more incentive a firm has to advertise heavily. The following claim describes the possible equilibria at the advertising stage, denoted by (L, L), (L, H) or (H, H), where the first argument is the advertising level chosen by the lower-quality firm and the second is the level chosen by the higher-quality firm.

Claim 2

1. If $c \geq \Delta_2$ and $s_1 \leq f(a_L, a_L)$ then the advertising equilibrium is (L, L).

2. If $\Delta_2 \geq c \geq \Delta_1$ then the advertising equilibrium is (L, H).

3. If $\Delta_1 \geq c$ then the advertising equilibrium is (H, H).

Claim 2 reveals several interesting properties of the possible second-stage advertising equilibria. First, if the cost of advertising heavily is high relative to the impact it will have on generating extra revenue ($c > \Delta_2$), then both firms will advertise lightly (of course, we require sufficient difference in qualities per Claim 1 for the equilibrium to exist; hence
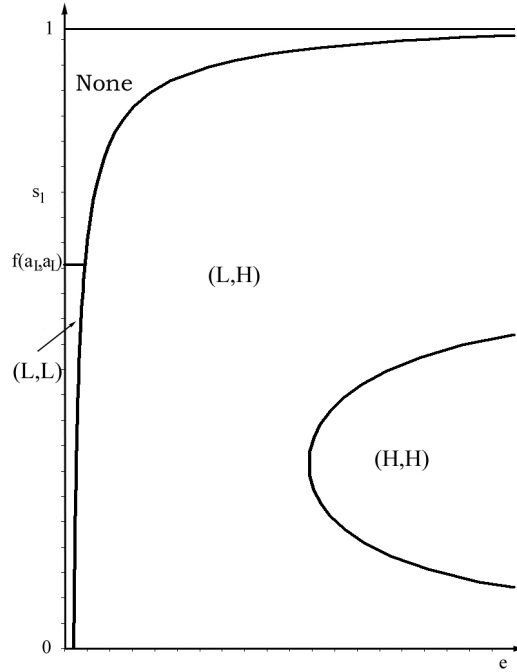
Figure 1: The Different Equilibria in the Advertising Stage as a Function of the Lower-Quality Firm's Position (s_1) and Advertising Effectiveness (e)

the condition on s_1).[5] Conversely, per part 3 of Claim 2, if the cost is low relative to the impact ($c \leq \Delta_1$), then both firms will advertise heavily. However, for a mid-range of cost c only the high-quality firm will advertise heavily, while the lower-quality firm will advertise lightly. To understand the intuition for this asymmetry in advertising strategies, we note that the return on shifting to advertising heavily is greater for the high-quality firm than it is for the lower-quality firm (i.e., $\Delta_2 \geq \Delta_1$). When consumers are informed about the high-quality product, the supplying firm is at an advantage because of the higher willingness to pay for its product. If the high-quality firm advertises heavily and informs all consumers, the lower quality firm faces a tradeoff. On the one hand, advertising heavily means that more consumers will consider its product, thus increasing its market potential. But, on the other hand, because all consumers consider the high-quality product, the surplus that can be extracted from these additional consumers is small as they will only purchase the lower-quality product if it is cheap enough. Furthermore, advertising lightly softens price competition with the high-quality firm that can then exploit the fact that some consumers

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[5]Specifically, if $c \geq \Delta_2$ and $s_1 > f(a_L, a_L)$ then there is no pure-strategy advertising equilibrium.

are only aware of its product. Consequently, there exists a range of costs such that only the high-quality firm will find it beneficial to advertise heavily.

Recall that we have defined advertising effectiveness as $e = (a_H - a_L)/c = (1 - a_L)/c$. One can therefore also express the advertising equilibria in Claim 2 in terms of e. When advertising is relatively ineffective ($e < (1 - a_L)/\Delta_2$) neither firm advertises heavily, when advertising is very effective ($e > (1 - a_L)/\Delta_1$) both firms advertise heavily, and for mid effectiveness values only the high-quality firm advertises heavily. Figure 1 depicts the various equilibria in the (s_1, e) space.

Having characterized the possible equilibria in the pricing and advertising sub-games, we can now solve for the qualities chosen in the first stage.

Proposition 1 *There exist values $\underline{e}$ and $\bar{e}$ ($e_M < \underline{e} < \bar{e}$) such that*

1. *For $e_M < e \leq \underline{e}$: $s_1 = \bar{s}$ and the advertising levels are (L, L).*

2. *For $\underline{e} < e < \bar{e}$: s_1 is an increasing function of advertising effectiveness, $s_1 > \bar{s}$, and the advertising levels are (L, L).*

3. *For $\bar{e} < e$: $s_1 = \underline{s}$ and the advertising levels are (L, H).*

We have $s_2 = 1$ and $\bar{s} > \underline{s}$. Equilibrium prices are given in Claim 1.

Figure 2 depicts the various equilibria in terms of the relative quality chosen by the lower-quality firm (s_1) as a function of advertising effectiveness (e). As can be gleaned, when advertising is relatively ineffective, and consistent with Claim 2, both firms will advertise lightly. In this case, the firm choosing a lower quality realizes that not all consumers will be aware of the high-quality firm's product. Thus, it can position relatively close to the maximal quality of 1 without the fear of triggering intense price competition; resulting in moderate quality differentiation between the firms.

At the other extreme, when advertising is effective, the high-quality firm (which always selects a maximal quality of 1) will surely want to advertise heavily to let all consumers know about its product. For the rival firm, advertising heavily can only be beneficial if the two products are sufficiently differentiated. Hence, the rival firm faces a dilemma: either produce

a very low quality product, advertise heavily and charge a very low price, or, alternatively, produce a moderate quality product, advertise lightly and price moderately. The latter option is more attractive in this range given the softened price competition (the high-quality firm need not be so aggressive as some consumers will only consider its product), and because of the savings on advertising costs.

In the mid-region of advertising effectiveness, we get the intriguing result whereby the best response of the lower-quality firm is to select a quality level that is increasing in advertising effectiveness. The intuition is as follows. Since advertising is somewhat effective in this range, the high-quality firm would like to advertise heavily as its return on such advertising is quite appealing. But this is detrimental to the rival firm that would prefer some consumers remain unaware of the high-quality option so that it can charge a high price. By minimizing the degree of differentiation, i.e., choosing a quality level closer to 1, the rival firm makes advertising heavily less beneficial for the high-quality firm because of the intense price competition that would ensue (as some consumers would be informed about both products). Said differently, by choosing its quality appropriately, the lower-quality firm reduces the return on advertising for the high-quality firm, and thus keeps advertising at light levels for both. As the advertising effectiveness (e) increases, an even higher quality position needs to be chosen in order to prevent the high-quality firm from switching to the heavy level of advertising, which explains why $\frac{\partial s_1^*}{\partial e} > 0$ in this region. There is clearly an upper limit to such behavior, since we know from Claim 1 that if the firms are completely undifferentiated then prices (and profits) will be zero. Indeed, at $e = \bar{e}$ the rival firm prefers to switch to a much lower quality level and focus on the low willingness to pay consumers and allow the high-quality firm to advertise heavily.[6]

To make the above findings more concrete, we provide the following numerical example.

Example 1 *If $a_L = 3/4$ and $a_H = 1$, Proposition 1 describes the equilibria with $\underline{e} \approx 10.5$, $\bar{e} \approx 12.2$ and $\underline{s} = 0.64$, $\bar{s} = 0.65$. The minimal level of differentiation occurs when $e = \bar{e}$, and $s_1 \approx 0.71$.*

We would like to relate our findings here on product differentiation to those of the Shaked

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[6]Consistent with Claim 1, the quality level s_1 satisfies $s_1 < f(a_L, 1)$ so the equilibrium exists.
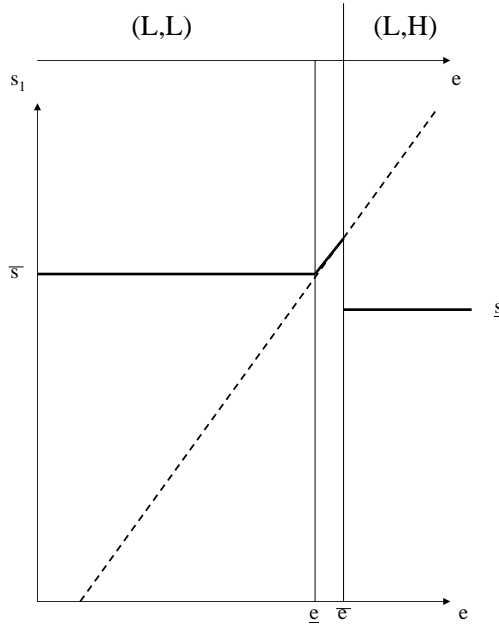
Figure 2: The Different Equilibria in Quality Positions and Advertising Levels
(The bold line shows the different s_1^* values as a function of e. The top part summarizes the advertising strategies in each region)

and Sutton (1982) model. Recall that in their model all consumers are assumed to be informed about firms' quality positions (and consider both products). This would correspond in our model to the case of both firms advertising heavily and choosing qualities in anticipation of this. However, as is evident from Proposition 1, there is no case under which both firms advertise heavily in equilibrium. In particular, the lower-quality firm always prefers to advertise lightly. This strategy works in concert with a smaller degree of product differentiation as explained above. In Shaked and Sutton (1982) the ratio of qualities is $s_1/s_2 = 4/7 \approx 0.57$, whereas is our numerical example this ratio is at least 0.64 and the minimal level of differentiation is when e=$\bar{e} \approx 12.2$ and the lower-quality firm selects $s_1 \approx 0.71$.

So far we have assumed that a_L is not too low. It is natural to ask what happens if the light advertising level results in only a very small probability of the product entering a consumer's consideration set; note that holding constant all other parameters, lowering a_L increases the effectiveness of advertising (since $e = \frac{1-a_L}{c}$). Although this complicates the analysis, the nature of the equilibria would be similar to those in Proposition 1, with the

exception that for very high advertising effectiveness a new region emerges where both firms advertise heavily (and differentiation is therefore as in Shaked and Sutton 1982). In the extreme case of $a_L = 0$, firms that do not set $a_i = a_H$ cannot earn revenue for any level of advertising effectiveness, and both firms advertise heavily (assuming c_H is not too high).

Our findings here bear on several phenomena pertaining to vertical product differentiation. First, they offer a new explanation for why in some categories we observe firms "clustering together" in terms of quality rather than maximally differentiating to cope with competition. Notably, the industries cited in Rhee (1996) as exhibiting this pattern are also ones that utilize advertising. Hence, our explanation involves empirically measurable variables directly under the firm's control (rather than explanations that rely on unobserved factors/attributes). Second, our findings here can shed additional light on why we observe variance in the degree of vertical differentiation across industries and even within industry in different product classes. The automobile industry is a case in point. In certain car categories we observe very little quality differentiation and also limited advertising support, while in other car categories we witness more extensive advertising spend by at least some of the models in the class. Given that, in general, advertising is an important part of firm strategy in automobile markets, our theory would suggest that the interaction between competitive concerns and advertising effectiveness (impact relative to cost) can explain at least some of this variance.[7]

In closing the analysis of the blanket advertising setup, the following corollary provides the equilibrium profit levels.

Corollary 1 *The profits of the high-quality firm, π_2, are a constant function of e for $e \in [e_M, \underline{e}]$, are decreasing for $e \in [\underline{e}, \overline{e}]$ and increasing for $e > \overline{e}$. Profits of the lower-quality firm, π_1, are constant for $e \in [e_M, \underline{e}]$ and $e > \overline{e}$, and are decreasing for $e \in [\underline{e}, \overline{e}]$.*

Figure 3 shows firms' profits as a function of advertising effectiveness. Although the firm producing the lower-quality product always makes less profits than its higher-quality rival, the Corollary reveals a surprising phenomenon in the $[\underline{e}, \overline{e}]$ interval: both firms' profits

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[7]For example, in the SUV category Lexus models are of higher quality than Honda models (consumer reports); the former are also more heavily advertised. In the roadster category, several models are of roughly similar quality (e.g., in the late 90s the BMW Z3 was regarded as having slightly lower quality than the Porsche Boxter) and were supported with less than average advertising budgets (Fournier and Dolan 1997).
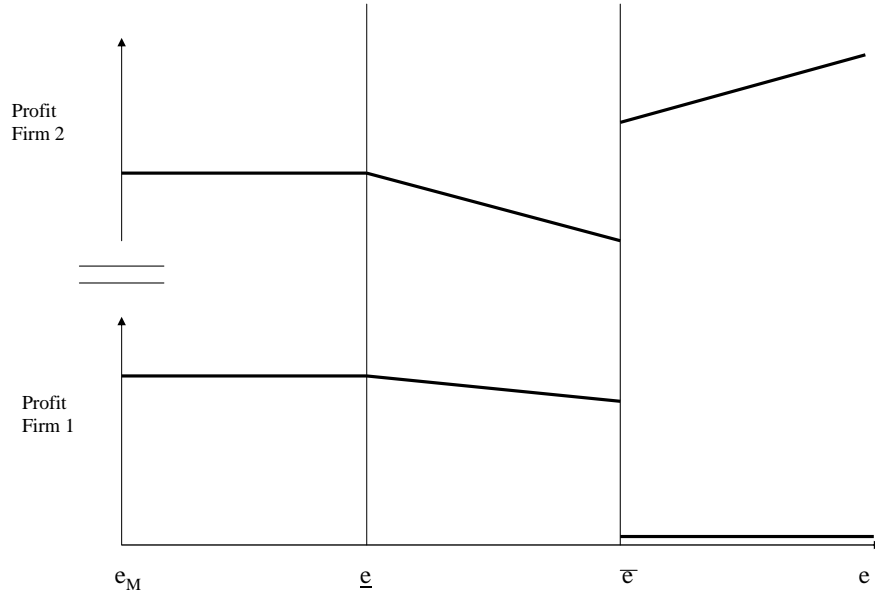
Figure 3: Profits of the Two Firms as a Function of Advertising Effectiveness (e)

are decreasing functions of advertising effectiveness (e). The intuition is as follows. Recall from Proposition 1 that in this region neither firm advertises heavily and that the degree of quality differentiation decreases with e. Due to the firms getting closer and closer to each other in terms of their product qualities, clearly the high-quality firm is worse off (from the price schedules in Claim 1: as $s_1 \rightarrow s_2$, p_2^* declines). The decreased differentiation is also detrimental to the lower-quality firm, but not as much as the dramatic drop in profits it experiences when the high-quality firm shifts to advertising heavily (see Figure 3).

5 Targeted Advertising

In this advertising setup, firms can communicate to specific segments of consumers, i.e. *target* them. This gives advertisers the possibility to focus their efforts on groups of consumers. Firms need to decide which subsets of consumers to target, and a consumer can only consider buying a firm's product if s/he is in a segment the firm has communicated to. In order to focus on the targeting aspect, we assume that advertising is perfectly efficient, in the sense that every consumer in the targeted segment is informed about the firm's product. For the sake of simplicity, we consider a two-segment model. Specifically, let us divide the continuum

of consumers into two disjunct intervals. Let the parameter $0 < t < 1$ determine the two intervals, such that consumers who are located at $\vartheta \geq t$ belong to the high-valuation segment and consumers who are located at $\vartheta < t$ belong to the low-valuation segment. Therefore, the size of the low-valuation segment (denoted $\mathcal{L}$) is t, whereas the size of the high-valuation segment (denoted $\mathcal{H}$) is $1 - t$. A firm can target neither, both, or just one of these segments. Formally, the targeting strategy of firm i (denoted by S_i) can be $\emptyset$, $\mathcal{L}$, $\mathcal{H}$ or $\mathcal{L} \cup \mathcal{H}$. The costs associated with the four possible advertising strategies are $0, c_{\mathcal{L}}, c_{\mathcal{H}}, c_U$, respectively. We assume that advertising costs are linear in the absolute size of the segment(s) targeted. That is, $c_{\mathcal{L} \cup \mathcal{H}} = c_U$, $c_{\mathcal{L}} = c_U t$, and $c_{\mathcal{H}} = c_U(1 - t)$. We assume that $c_U < t/4$ to ensure that both firms make positive profits in equilibrium. The general setup and timing of the game are the same as those described in Section 3.

A good way to interpret this setup is that the parameter t reflects the "targeting criterion" that the firms can use to identify consumers as having a high vs. low willingness to pay for quality in this market.[8] For example, certain demographic variables (income, education level, etc.) can indicate which individuals are willing to pay more for certain products; those consumers that patron certain venues (say amusement parks, or sports events) typically have greater willingness to pay for related items.

It should be obvious that if t approaches 0 or 1, i.e., one of the segments is negligible in size, the problem reduces to the basic vertical differentiation model of Shaked and Sutton (1982). The following proposition shows that as long as t is not too close to either of these extremes, then in equilibrium both firms choose maximal quality and do not differentiate.

Proposition 2 *If $1/3 \leq t \leq 4/5$ then the only equilibria of the game are represented by those sets of strategies where $s_i = s_j = 1$, $S_i = \mathcal{L}$, $S_j = \mathcal{H}$, $p_i = t/2$ and $p_j = \max(t, 1/2)$. Firms' profits are $\Pi_i = t^2/4$ and $\Pi_j = 1/4$ for $t < 1/2$ and $\Pi_j = t(1 - t)$ for $t \geq 1/2$.*

The intuition behind this result is that in the advertising stage firms prefer to choose disjunct segments, thereby "dividing" the market. Doing so allows them to act as de facto monopolists in their chosen segments and avoid price competition in the final stage. Fore-

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[8]Thus, we examine the realistic case where firms cannot identify each and every consumer and know their exact willingness to pay for quality. Through market research and experience, though, the firms can use proxy variables that provide a relatively accurate split of consumers as those having a greater vs. lower valuation for product quality in the category.
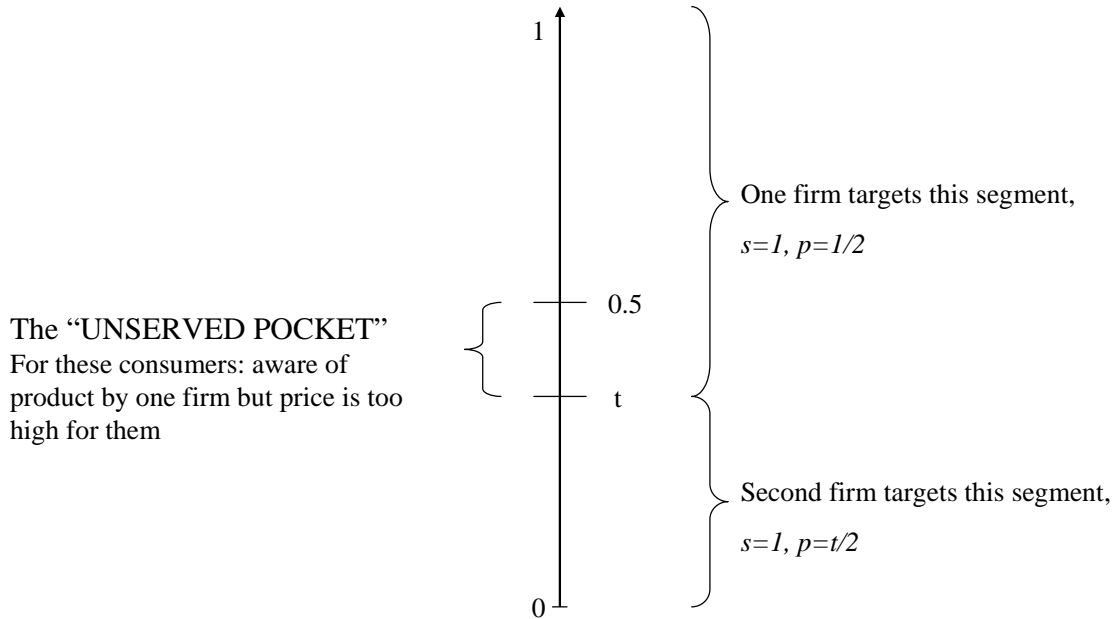
Figure 4: Consumers with a Valuation in the Middle Region are Unserved

seeing this possibility, when the firms select quality positions in the first stage they both choose maximal quality.

In the equilibrium characterized in Proposition 2, both firms select $s = 1$. Let us denote the firm advertising to the low-valuation segment by 1 and the other firm by 2. In the next two corollaries we highlight important features of the equilibrium.

Corollary 2 *If* $1/3 < t < 1/2$*, then equilibrium prices are* $p_1^* = t/2$ *and* $p_2^* = 1/2$*. Consumers with valuation* $\vartheta \in [t, 1/2]$ *are unserved, in the sense that they do not buy either product.*

It is common in vertical differentiation models that consumers with the lowest valuation, here below $t/2$, are unserved because prices are too high for them to purchase even the low-quality product. However, with targeted advertising, if $1/3 < t < 1/2$, then consumers in a mid-range, with valuations between t and $1/2$, are also unserved (see Figure 4). The intuition for why a "pocket" of unserved consumers exists is as follows. Based on the equilibrium in Proposition 2, these consumers are part of the $\mathcal{H}$ segment and are targeted by firm 2; hence they only consider buying the product of the firm advertising to the high-valuation segment,

and this firm charges a price of $p_2^* = 1/2$. But since for them $\vartheta - 1/2 < 0$, i.e., the price is above their maximum willingness to pay, they do not purchase any product.

The next corollary summarizes how the demand changes as a function of t, the size of the low-valuation segment.

Corollary 3 *Firm 1's demand is an increasing function of t, and firm 2's demand is a non-increasing function of t. The total demand (consumers served) is first increasing ($t \in (1/3, 1/2]$) and then decreasing ($t \in [1/2, 4/5]$) as a function of t, attaining its maximum at $t = 1/2$.*

Firm 1 serves a demand of size $t/2$ (specifically, those with a ϑ between $t/2$ and t). Hence, as t increases, firm 1 sells to more and more consumers. However, from Corollary 2 we know that firm 2 does not sell to those consumers with valuations between t and $1/2$; hence its demand is initially constant (and equals $1/2$) and then declines as t increases beyond $1/2$. The combination of the two demand schedules yields a total number of consumers served that is an inverted-U shaped function, with a maximum at $1/2$. This is an interesting pattern if we bear in mind that both firms are selling equally high quality products. One could have expected that as long as the equilibrium is for firms to target different segments and for both to set qualities at 1, then how the market is split between them should not matter for total demand. However, because optimal pricing for the high-end consumers is not symmetric to pricing for the low-end consumers, and because of the existence of the unserved pocket, the location of t does matter for total demand. Note that the firm serving the low-valuation segment sees its profits increase with t, whereas the firm serving the high-valuation segment sees constant and then decreasing profits. The firm serving the high-valuation segment makes greater profits as long as $t < 4/5$, that is, if the low valuation segment is less then 4 times larger than the high-valuation segment it is more lucrative to serve consumers at the "top of the pyramid". We note that the total profits that the firms make are maximal at $t = 2/3$.

The findings in this section might help explain why certain product categories exhibit "market polarization" (Knudsen et al. 2005), whereby two primary segments exist with respect to their valuation for quality– the premium-end and the value-end. Examples from a range of industries include: home appliances (Knudsen 2006), beverages such as coffee and

vodka (Deshpande 2001), and network storage hardware (Ofek and Hamid 2005). In these cases, we typically find that each firm focuses on a distinct segment and targets its communications and distribution to that segment. Moreover, and consistent with our findings, even though in reality quality and production costs can be similar across offerings, the prices of products targeted at value-oriented customers are typically much lower than those targeted at premium-oriented customers.[9] Mounting evidence also suggests that serving consumers with intermediate willingness to pay values is not beneficial. Indeed, recent management consulting practice calls on firms to "escape the middle-market trap" (Knudsen 2006).

We have focused in this section on the case where the low willingness to pay segment comprises at least 33% of the market and the high willingness to pay segment at least 20%. This ensures that these segments are advertised to in equilibrium.[10] We believe this is the relevant case to analyze because for many real world settings, targeting criteria are only meaningful if they result in segments with a sufficient proportion of consumers. That said, we now explain what happens when t does not satisfy the conditions in Proposition 2. If $t < 1/3$, then the low willingness to pay interval ($\mathcal{L}$) might be too small for either firm to want to advertise to it, and both may end up advertising only to consumers in the high willingness to pay interval ($\mathcal{H}$). In this case, if one firm sets $s_2 = 1$, then the rival will set a quality level higher than $s_1 = 4/7$ because its market size is limited by the lower bound of the interval. On the other hand, if t is close to 1 neither firm can afford to forgo the low-valuation segment, and both firms will include it in their target market (and advertise to it). In this region, it is complex to characterize the full set of possible equilibria, but it should be clear that as t approaches 1 the model effectively reduces to Shaked and Sutton (1982); one firm selects the maximal quality of $s = 1$ and its rival sets $s = 4/7$.

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[9]In the case of home appliances, outsourcing parts or complete production has resulted in similar variable costs, yet products aimed at the premium end are 2-3 times more expensive. In the vodka market, blind taste tests consistently reveal the top brands as roughly equally rated and their ingredients and processes are similar. However, the average price of a 750ml bottle of Smirnoff is about $13 while that of Grey Goose is about $32. The former vodka is primarily targeted at casual consumption (low willingness to pay consumers), while the latter for heavy vodka drinkers. In the market for SAN storage devices, firms were able to target separate segments despite selling equally advanced products (the technology used was similar, the only difference was the configuration of the device; McData sold to high-end data centers and Brocade sold to customers with less data-intensive applications).

[10]It is not surprising that the lower bound (0.33) is larger than the upper bound (0.20). The margin on the low willingness to pay consumers is small hence this segment ($\mathcal{L}$) has to be big enough in size to be targeted, while the margin on the highest willingness to pay consumers can compensate for a smaller segment size ($\mathcal{H}$).

6 Persuasive Advertising

In this section, instead of focusing on the informative features of advertising as before, we examine the case of persuasive advertising. We assume that all consumers are aware of the products available but the quality they perceive each of the offerings to possess is affected by the advertising they are exposed to. Thus, in determining which product to buy, what matters for the consumer is the net perceived utility. There is evidence that in certain cases advertising affects buying decisions in this way. For example, in an empirical study Moorthy and Zhao (2000) find (across ten product categories) that advertising effort (spending) has a positive effect on perceived quality. Based on this phenomenon, we modify the model in the following way.[11]

Let s_i denote the real (or objective) quality of the product offered by firm i. As before, let a_i denote firm i's advertising impact. We assume that $0 \le a_i \le 1$ and that the perceived quality of product i is $a_i s_i$. This simple formulation captures the fact that by advertising more the firm can raise its product's perceived quality. However, the real quality forms an upper limit to the perceived quality.[12] Aside from the way advertising impacts demand, the setup of the game is equivalent to that of the blanket advertising model presented in Section 4. Firms first choose qualities, then advertising efforts and, finally, prices (decisions in each stage are simultaneous).

As a first step, let us examine a very simple case in which advertising is costless. Despite the oversimplifying assumption, the analysis sheds light on how firms differentiate in qualities when they can use advertising to change product perceptions.

Claim 3 *The game has infinitely many sub-game perfect equilibria, where $s_j = 1$ and s_i can take any value between 4/7 and 1. Furthermore, in every equilibrium $a_i s_i = (4/7) a_j s_j$.*

Interestingly, the degree of objective quality differentiation is less than or equal to the ratio $s_1 / s_2 = 4/7$, which is the degree of differentiation in the basic model (with no persuasive

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[11]Colombo and Lambertini (2003) study persuasive advertising in a dynamic setting with endogenous quality levels. An important difference is that we examine how advertising changes consumers' valuations of the product, whereas they assume a reduced functional form for the effect of advertising on demand. Furthermore, these authors do not consider informative advertising.

[12]This formulation produces equivalent results as a formulation in which the upper limit is any linear function of the real quality.

advertising and full awareness). However, note that firms will use persuasive advertising to increase the perceived differentiation so that, from consumers' perspective, the ratio of perceived qualities is always $4/7$.

In order to relax the assumption of costless advertising, let us assume that advertising effort is a discrete variable (as in Section 4). Firms can choose between $a_i = a_L$ and $a_i = a_H$, where $0 \leq a_L < a_H \leq 1$. Let c_L and c_H denote the costs associated with the two advertising levels, with c_L sufficiently low so that both firms advertise. Again, let $c = c_H - c_L$.

Proposition 3 *There exist values $\underline{c}$ and $\overline{c}$ ($0 < \underline{c} < \overline{c}$), such that we get the following sub-game perfect equilibria*

1. *If $\overline{c} \leq c$, then $s_i = 4/7$, $s_j = 1$ and the advertising equilibrium is (L, L).*

2. *If $\underline{c} \leq c < \overline{c}$, then $s_i = (4/7)(a_H/a_L)$, $s_j = 1$ and the advertising equilibrium is (L, H).*

3. *If $0 < c < \underline{c}$, then $s_i = 4/7$, $s_j = 1$ and the advertising equilibrium is (H, H),*

Intuitively, if the extra cost of advertising heavily is sufficiently high, then both firms choose the light level. If the cost difference is in the middle range, one firm chooses to advertise heavily and the other lightly. Lastly, if the cost differential is low, both firms find it beneficial to advertise heavily.

Turning now to explaining the equilibrium quality decisions, note that in the two extreme advertising cost regions the objective qualities are as in the original Shaked and Sutton (1982) model ($s_1/s_2 = 4/7$). Given that both firms either advertise heavily or lightly, the degree of perceived differentiation is the same as without advertising. However, in the middle range we find the same phenomenon as in the simple case of Claim 3. The objective quality ratio s_1/s_2 is higher than $4/7$, corresponding to a smaller degree of objective differentiation, which is then counterbalanced by the difference in advertising efforts; leading to a perceived quality ratio of exactly $4/7 = (s_1 a_1)/(s_2 a_2)$.

To summarize, with persuasive advertising the degree of perceived quality differentiation is always the same and corresponds to the case where advertising has no persuasive effect (and with all consumers informed). That said, in choosing its objective quality a firm has to be mindful of the advertising levels that will be chosen. In particular, if a firm faces a

rival that selects maximal quality and the equilibrium advertising levels are expected to be asymmetric, then the firm should select a relatively high objective quality to overcome the fact that it will only advertise lightly.

7 Conclusion

In this paper, our goal has been to study the interaction between endogenous quality choice and the desire to advertise products to consumers. We developed a model in which firms choose product quality in a first stage, their advertising strategy in a second stage, and prices in the last stage. We explored two forms of informative advertising – blanket and targeted. In an extension, we also examined the implications of persuasive advertising, which serves to affect consumers' perception of product quality.

In the case of blanket advertising, we showed that firms can be better off under light levels of advertising because this allows them to choose higher product quality positions yet avoid fierce price competition. Moreover, the equilibria of the game depend on the cost-efficiency of advertising. When advertising is relatively ineffective, differentiation is relatively modest and both firms advertise at a light level. When, at the other extreme, advertising is very effective, one firm chooses maximal quality and expends heavily on advertising. The rival now prefers to differentiate with a much lower quality product and expend lightly on advertising. The smallest degree of quality differentiation occurs in a mid range of advertising effectiveness. By positioning its product close by, a firm prevents its high-quality rival from switching to a heavy level of advertising expenditure. Moreover, in this range as ad effectiveness increases the firm will select an even higher quality position. This results in products that are minimally differentiated and, in turn, prices decline.

Under targeted advertising, our main result was that each firm advertises to a different segment. Thus, firms do not directly compete in the pricing stage and can both choose equally high quality products– that is, their offerings are entirely undifferentiated. Interestingly, due to the relatively high prices firms charge, there exist conditions such that a set of consumers with moderate valuation for quality are unserved by the firm that advertises to them– even though consumers in a different segment with lower willingness to pay are served by the rival firm. We also showed that the total number of consumers served is an

inverted-U function of the low-valuation segment size.

In an extension, we studied the role of persuasive advertising. Our main result there is that firms may differentiate less in objective qualities, but the asymmetry in advertising levels counterbalances this effect; leading to the same degree of perceived quality differentiation as when no persuasive advertising is possible (and all consumers are fully informed about the products available).

These findings have several important managerial implications. First, they suggest that the conventional wisdom whereby a firm should seek a high degree of product differentiation from its rival to avoid head-to-head price competition is qualified, and depends on whether or not there is a role for informative advertising that generates awareness or impacts consumer consideration of the products offered. It further matters what kind of communication vehicles the firms can use to reach consumers. If firms cannot identify consumers through any targeting criterion and use blanket advertising (also referred to as national advertising), the degree of quality differentiation depends on how effective advertising is. Most managerial treatments of advertising classify media based on their impact on consumers and their cost (see, e.g., Thomas et al. 2000, exhibit 14.17). Our results thus tell managers how the specific characteristics of advertising vehicles in their market should affect the quality of products they choose to offer. By contrast, when firms can target distinct segments our findings suggest that no differentiation in quality may actually work best, provided the firms can "stay out of each other's turf". Importantly, our analysis reveals that a firm need not feel forced to lower price so as to serve all the consumers who have been targeted; the firm may wish to ignore consumers that have moderate willingness to pay, even though low-end consumers are being served by the rival firm. In other words, "avoiding the middle segment" may be the best course of action.

Second, it is important to bear in mind that while industry funded R&D is approximately $240 billion per year in the U.S. (NSF 2008), companies spend comparable amounts on advertising (as noted in the Introduction, about $155 billion in 2006); and this holds for many high-tech firms as well.[13] Hence, when setting new product strategy, which is often aimed at improving the quality of existing products (e.g., higher levels of key attributes),

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[13]For instance, in 2006 Intel Corp. spent $5.8 billion on R&D compared to $6.1 billion on marketing (source: Intel 2006 10k report).

to ensure the best possible commercial outcome it is crucial to take into account how the new products will be advertised to potential customers. In their seminal work surveying managers on how firms appropriate returns from R&D investments given competitors, Levin et al. (1987) found that marketing and sales effort were cited as the primary vehicle (over such means as patents and trade secrets). This clearly suggests that managers need to understand the strategic implications of communication strategy on the products they choose to develop vis-à-vis the products their rivals are expected to develop.

7.1 Limitations and Future Research

Although our study encompasses three different types of advertising and we have justified many of our assumptions, we acknowledge several limitations. First, we analyzed the advertising models separately. One could imagine a general a model where advertising is informative and persuasive at the same time (or that firms need to choose the levels of both types of advertising). Also, by combining the first two models presented (in Sections 4 and 5), we would get a general setting in which advertising can be targeted but its impact within the segment may not be perfect, resulting in two decision variables: which segment(s) to target and how aggressively to try and reach consumers in the chosen segment(s).

Second, in Sections 4 and 5 we assumed that the only way for consumers to become informed was through advertising. This allowed us to focus on advertising's role in affecting whether consumers consider a given product and, in turn, firms' strategic incentives to expend on advertising and their quality choices. In reality, of course, consumers have other ways to become informed about the products available, e.g., through search (prior to visiting the store or at the store).[14] Assuming that a given consumer has a positive probability of becoming informed about the firm's product even without receiving an ad for it would not change our model findings, as long as this probability is not too high. In this case, we would interpret advertising in our model as increasing this probability by a_i. Our results would also hold if consumer search costs are sufficiently high so that not all consumers have an incentive to become fully informed. However, if consumer search costs are only moderate, uninformed consumers may have a greater incentive to search than informed

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[14]Note that sales assistance at the store can be regarded as part of a firm's promotion strategy (as a firm can offer more or less retail incentives; choose the retailers it provides the product to, etc.).

consumers, which could impact our findings. We leave for future research the investigation of how combining informative advertising and consumer search affects endogenous quality positions.

Third, we have employed very stylized assumptions on costs. In order to simplify the presentation and analysis, we assumed that products are produced at a zero variable cost (that is not a function of quality). In this way we were able to focus on the strategic forces that drive the degree of quality differentiation and the interaction with advertising. It is also a reasonable assumption in many product categories where variable costs are either negligible, e.g., digital goods or drugs, or only weakly related to quality positions. Future research could examine the implications of including variable costs that are a function of product quality. It is conceivable, for example, that if variable costs are a convex function of quality that the high-quality firm earns a smaller margin on its product than the lower-quality rival does, and hence has less incentive to advertise heavily than in the model presented here; this could affect the nature of the results.

Fourth, in this paper we have focused on consumer heterogeneity in willingness to pay for quality. This allowed us to examine how vertical differentiation is impacted by the need to inform consumers about product offerings. In reality, of course, consumers may also exhibit heterogeneity in horizontal tastes (e.g., some consumers preferring contemporary furniture designs while others preferring old-fashioned designs). It is conceivable that firms' choice of product positioning along a horizontal continuum may be impacted by the need to advertise to make consumers aware of their offerings, and there may be an even greater role for persuasive advertising in this case. The interaction between product location and advertising with horizontal heterogeneity can be fruitful ground for future research.

Appendix: Proofs

Proof of Proposition 1

PROOF OF CLAIM 1

The demand structure is very similar to the original model for those consumers who consider both firms' products. However, we have to take into account those who only consider one of the two products. Let us introduce notations for three types of indifferent consumers. The consumer whose ϑ parameter is $t_2 = \frac{p_2 - p_1}{s_2 - s_1}$ obtains the same utility from buying firm 1's and firm 2's product. The consumer with $\vartheta = t_2' = \frac{p_2}{s_2}$ obtains 0 utility from buying firm 2's product, that is, s/he is indifferent between purchasing from firm 2 and not buying anything. Finally, $t_1 = p_1 s_1$ is the critical point of buying firm 1's product versus not buying anything.

If firm 1's relative price is lower ($\frac{p_1}{s_1} < \frac{p_2}{s_2}$), then $t_1 < t_2' < t_2$. In this case, the demands are the following.

$$D_1(p_1, p_2) = a_1(t_2 - t_1) + a_1(1 - a_2)(1 - t_2).$$

$$D_2(p_1, p_2) = a_2(1 - t_2) + a_2(1 - a_1)(t_2 - t_2').$$

If, on the other hand $\frac{p_1}{s_1} \geq \frac{p_2}{s_2}$, then firm 1 only sells its product to the consumers who are not aware of firm 2. In this case

$$D_1'(p_1, p_2) = a_1(1 - a_2)(1 - t_1), \quad D_2'(p_1, p_2) = a_2(1 - t_2').$$

Since the variable cost is zero, the revenue is simply the price multiplied by the demand.

$$R_1(p_1, p_2) = \begin{cases} p_1 D_1(p_1, p_2), & \text{if } p_1 < \frac{p_2 s_1}{s_2}, \\ p_1 D_1'(p_1, p_2), & \text{if } p_1 \geq \frac{p_2 s_1}{s_2}. \end{cases}$$

$$R_2(p_1, p_2) = \begin{cases} p_2 D_2(p_1, p_2), & \text{if } p_2 > \frac{p_1 s_2}{s_1}, \\ p_2 D_2'(p_1, p_2), & \text{if } p_2 \leq \frac{p_1 s_2}{s_1}. \end{cases}$$

First, we calculate the best response function of firm 2. Its revenue function consists of two quadratic functions, with maximums at $\frac{s_2}{2}$ and $\frac{s_2 - s_1 + p_1 a_1)s_2}{2(s_2 - s_1 + s_1 a_1)}$, respectively. Depending on the value of p_1, the maximum is either attained in the first interval, the second interval, or the intersection. If $p_1 < \frac{s_1(s_2 - s_1)}{2(s_2 - s_1) + s_1 a_1}$, then the best response is $b_2(p_1) = \frac{s_2 - s_1 + p_1 a_1)s_2}{2(s_2 - s_1 + s_1 a_1)}$, if $\frac{s_1(s_2 - s_1)}{2(s_2 - s_1) + s_1 a_1} \leq p_1 < \frac{s_1}{2}$, then the best response is $b_2(p_1) = \frac{p_1 s_2}{s_1}$, finally, if $\frac{s_1}{2} \leq p_1$, then the best response is $b_2(p_1) = \frac{s_2}{2}$.

For firm 1, the best response is either $b_1(p_2) = \frac{(s_2 - s_1 - a_2 s_2 + a_2 s_1 + a_2 p_2)s_1}{2(s_2 - s_1 + s_1 a_1)}$ or $b_1(p_2) = \frac{s_1}{2}$, depending on where the revenue is higher. Since $b_1(s_2/2) < s_1/2$, the only possible equilibrium is

$$p_1^* = \frac{s_1(s_2 - s_1)(2(s_2 - s_1) - a_2 s_2 + 2s_1(a_1 + a_2 - a_1 a_2)}{4s_1^2(1 + a_1 a_2 - a_1 - a_2) + s_1 s_2(4a_1 + 4a_2 - a_1 a_2 - 8) + 4s_2^2},$$

$$p_2^* = \frac{s_2(s_2 - s_1)(2(s_2 - s_1) + s_1(a_1 + 2a_2 - a_1 a_2))}{4s_1^2(1 + a_1 a_2 - a_1 - a_2) + s_1 s_2(4a_1 + 4a_2 - a_1 a_2 - 8) + 4s_2^2}.$$

This is an equilibrium if and only if firm 1 has no incentive to deviate, that is, if

$$p_1^* D_1(p_1^*, p_2^*) \leq \frac{s_1}{2} D_1'\left(\frac{s_1}{2}, p_2^*\right) = \frac{s_1 a_1(1 - a_2)}{4}. \tag{3}$$

In order to define f, we need the following the lemma.

Lemma 1 $R(s_1) = p_1^* D_1(p_1^*, p_2^*)$ *is a concave function of s_1 for $0 \le s_1 \le s_2$, $R(0) = R(s_2) = 0$ and $R'(0) > a_1(1 - a_2)/4$ for $a_1, a_2 > 0$.*

PROOF: We can assume without loss of generality that $s_2 = 1$. Then

$$R(s_1) = \frac{(1 - s_1)(1 - s_1 + a_2 s_1)(-2a_2 s_1 + 2s_1 - 2s_1 a_1 + 2a_1 s_1 a_2 - 2 + a_2)^2 a_1 s_1}{(-a_1 s_1 a_2 + 4 - 8s_1 + 4s_1 a_1 + 4s_1^2 - 4s_1^2 a_1 + 4a_2 s_1 - 4a_2 s_1^2 + 4a_2 s_1^2 a_1)^2},$$

which is concave for $0 \le s_1 \le 1$, and

$$R'(0) = \frac{(2 - a_2)^2 a_1}{16} > \frac{a_1(1 - a_2)}{4}$$

for $a_1, a_2 > 0$ ☐

Now, let $f(a_1, a_2)$ be the solution of the equation

$$p_1^* D_1(p_1^*, p_2^*) = \frac{s_1 a_1(1 - a_2)}{4},$$

with respect to s_1, setting s_2 to 1. Since the left hand side is a concave and the right hand size is linear function of s_1, the solution is unique in the interval $0 \le s_1 \le 1$, and according to the lemma $0 < f < 1$. Since s_1 only satisfies (3) if and only if $s_1 \le f(a_1, a_2)$, we have proved parts *1* and *2*.

Then $f(a_1, 1) = 1$ for any $0 < a_1 \le 1$ also obviously follows from the lemma. In order to prove that $f(a, a)$ is increasing in a, we have to examine the function $R(s_1)$ more carefully. One can check that

$$R(s_1) - \frac{s_1 a(1 - a)}{4}$$

is increasing in a if $1/2 \le a = a_1 = a_2$ for any $0 \le s_1 \le 1$. Since $R(s_1)$ is concave and $R(0) = 0$ this proves that $f(a, a)$ is increasing for $a \ge 1/2$.

If $s_1 = s_2$ and both advertising probabilities are positive, then there is always a positive mass of consumers, who are aware of both products. Thus, in a symmetric equilibrium, both prices have to be zero as a consequence of the Bertrand-type competition. On the other hand, asymmetric equilibria do not exist, since any of the firms would be better off by setting a price of $s_1/2 = s_2/2$.
☐

PROOF OF CLAIM 2

Let $G_L^1(s_1) = R_{HL}^1(s_1) - R_{LL}^1(s_1)$, $G_H^1(s_1) = R_{HH}^1(s_1) - R_{LH}^1(s_1)$ $G_L^2(s_1) = R_{LH}^2(s_1) - R_{LL}^2(s_1)$, $G_H^2(s_1) = R_{HH}^1(s_1) - R_{HL}^2(s_1)$ denote the gains of setting advertising to heavy instead of light. In order to determine a^*, we need the following observations, that can be proved by basic algebraic calculations.

Observation 1 *Let $G(q)$ denote the derivative $(G_H^1(1) - G_H^2(1))'$ as a function of q for $0 \le q \le 1$. Then $G(q)$ is an increasing function, and $G(q) = 0$ has a solution in the interval $0 \le q \le 1$.*

Let a^* be the solution of $G(q) = 0$, which is $a^* \approx 0.7032$. Note that since $a^* > 1/2$, our assumption that $a_L > a^*$ is consistent with numerous empirical studies showing that the advertising response function is concave (Lilien and Rangaswamy 2004).

Observation 2 *If $a_L > a^*$, then $G_L^1(s_1) < G_L^2(s_1)$ and $G_H^1(s_1) < G_H^2(s_1)$ for $0 < s_1 < 1$.*

Now let us examine when the different types of equilibria are possible.

- (L,L)

  If firm 1 chooses L, then firm 2's best response to this is L if and only if $c \geq G_L^2(s_1)$. Firm 1's best response to this is L if and only if $c \geq G_L^1(s_1)$. Since $G_L^1(s_1) < G_L^2(s_1)$, this type of equilibrium emerges only if $c \geq G_L^2(s_1)$. For its existence we also need that an equilibrium in the last stage exist, that is, $s_1 \leq f(a_L, a_L)$, otherwise no equilibrium exists.

- (L,H)

  If firm 1 chooses L, then firm 2's best response to this is H if and only if $c \leq G_L^2(s_1)$. Firm 1's best response to this is L if and only if $c \geq G_H^1(s_1)$. Since in this case the pricing equilibrium always exists (f(q,1)=1), this type of equilibrium emerges if and only if $G_H^1(s_1) \leq c \leq G_L^2(s_1)$.

- (H,H)

  If firm 1 chooses H, then firm 2's best response to this is H if and only if $c \leq G_H^2(s_1)$. Firm 1's best response to this is H if and only if $c \leq G_H^1(s_1)$. Since $G_H^1(s_1) < G_H^2(s_1)$ and the pricing equilibrium always exists, this type of equilibrium emerges if and only if $c \geq G_H^1(s_1)$. This completes the proof of the claim

In order to complete the proof we need the following observations, that can be proved by basic algebraic calculations.

Observation 3 $G_L^2(s_1)$ *is decreasing for* $0 \leq s_1 \leq 1$.

Observation 4 *We have* $R_{LH}^2(s_1) < R_{LL}^2$ *for* $0 < s_1 < 1$.

Observation 5 *We have* $\max_{\{0 \leq s_1 \leq 1\}} R_{LH}^1(s_1) = \max_{\{0 \leq s_1 \leq 1\}} R_{HH}^1(s_1)$.

Let $\bar{s} = \arg\max(R_{LL}^1(s_1))$ and $\underline{s} = \arg\max(R_{LH}^1(s_1))$ and let $\bar{c} = G_L^2(\bar{s})$ and $\underline{c} = G_L^2(f(a_L, a_L))$. Consequently, $\bar{e} = \frac{1-a_L}{G_L^2(\bar{s})}$ and $\underline{e} = \frac{1-a_L}{G_L^2(f(a_L,a_L))}$.

We now start analyzing the equilibria at the quality choice stage. Let us first examine firm 1's best response quality choice to $s_2 = 1$.

- If $c_M > c > \bar{c}$, that is, if $e_M < e < \bar{e}$, then depending on s_1 the advertising equilibrium is either (L, H) or (L, L). Let $s' = (G_L^2)^{-1}(c)$ denote the critical value for (L, L) to realize. Therefore, firm 1 maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$ and R_{LL}^2 in the interval $s' \leq s_1 \leq 1$. According to Observation 4, the maximum of R_{LL}^2 is greater than the maximum of R_{LH}^1. On the other hand, it follows from the definition of $\bar{c}$ that R_{LL}^2 attains its maximum in the interval $s' \leq s_1 \leq 1$, hence firm 1's best response is $\bar{s}$ and the advertising strategies are (L, L). Note that according to the definition of $\underline{c}$, the pricing equilibrium exist in this case.

- If $\bar{c} > c > \underline{c}$, that is, if $\bar{e} < e < \underline{e}$, then firm 1 still maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$ and R_{LL}^2 in the interval $s' \leq s_1 \leq 1$. However, in this case, $R_{LL}^2()$ does not attain its maximum in the interval $s' \leq s_1 \leq 1$. On the other hand, $R_{LL}^2(s') \geq \max_{\{0 \leq s_1 \leq 1\}} R^2 LH(s_1)$, therefore, the best response of firm 1 is $s' = (G_L^2)^{-1}(c)$ and the advertising strategies are (L, L). Note that according to the definition of $\underline{c}$, the advertising equilibrium still exist in this case.

- If $\underline{c} > c \geq 0$, that is, if $e > \underline{e}$, let s'' and s''' denote the two solutions of $G_H^1(s_1) = c$, if they exist. Then firm 1 maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$, except for the interval $s'' \leq s_1 \leq s'''$, where it maximizes $R_{HH}^1 - c$, if s'' and s''' exist, that is, if $c \leq \max G_H^1(s_1)$. Since $\max_{\{0 \leq s_1 \leq 1\}} R_{LH}^1(s_1) = \max_{\{0 \leq s_1 \leq 1\}} R_{HH}^1(s_1)$, firm 1's best response is always $\underline{s}$ and the advertising strategies are (L, H).

In order to show that the above strategies consist equilibria, we have to make sure that firm 2's response to firm 1's actual action is $s_2 = 1$. Note that firm 2's profit functions $R_{LL}^2(s_2)$, $R_{LH}^2(s_2)$ and $R_{HH}^2(s_2)$ are all increasing if we fix s_1 at a certain level. Thus, the only incentive for firm 2 to not to choose $s_2 = 1$ would be to change the advertising equilibrium and increase its payoff trough that. This is, however, not possible as the only change that it could attain by decreasing s_2 from 1 is to change an (L, H) equilibrium to an (L, L) which is obviously not profitable. Thus, we have confirmed that all the above described strategies are equilibria.

We have to show that no other equilibrium exists, that is, that s_2 is always 1 in equilibrium. Let us assume that an equilibrium exists with $s_2^* < 1$. The advertising equilibria and the best response of firm 1 can be calculated the same way as for $s_2 = 1$, except that the s_1 values have to multiplied by s_2^*. As mentioned before, for a fixed advertising equilibrium and a fixed s_1, the profit of firm 2 is strictly increasing in s_2. Thus, the only way firm 2 has no incentive to increase s_2 is if that would change the advertising equilibrium and decrease profits. However, the above cases show that the only possibility to such a change would be from (L, L) to (L, H), but that does not decrease firm 2's profit. This completes the proof of the proposition. $\square$

Proof of Proposition 2

First, we examine the pricing stage of the game given the quality levels and the advertising segments.

1. If the two segments are distinct, that is, if $S_1 \cap S_2 = \emptyset$, then there is no price competition between the firms. They both maximize their income in their own segment. For the firm choosing the high interval ($S_i = \mathcal{H}$), the optimal price is $p_i^* = \max(s_i/2, s_i t)$. For the firm choosing the low interval ($S_j = \mathcal{L}$), the optimal price is $p_i^* = s_j t/2$.

2. If both firms choose both intervals, that is , if $S_1 = S_2 = \mathcal{L} \cup \mathcal{H}$, then there is full price competition and the pricing equilibria are the same as in the Shaked and Sutton model. We have also covered this case in Claim 1 (substituting $a_1 = a_2 = 1$): if $s_1 = s_2$, then prices go down to zero, whereas if $s_1 < s_2$, then

$$p_1^* = \frac{s_1(s_2 - s_1)}{4s_2 - s_1}, \quad p_2^* = \frac{2s_2(s_2 - s_1)}{4s_2 - s_1},$$

where the indifferent consumer has a ϑ of $t_2^* = \frac{p_2^* - p_1^*}{s_2 - s_1} = \frac{2s_2 - s_1}{4s_2 - s_1}$.

3. If the firms choose the same intervals, for example $S_1 = S_2 = \mathcal{H}$, then in case of $s_1 = s_2$, equilibrium prices are zero. If $s_1 < s_2$, then depending on t two equilibria are possible. If $\frac{s_2 - s_1}{4s_2 - s_1} \leq t \leq \frac{s_2 - s_1}{2s_2 + s_1}$, then

$$p_1^* = ts_1, \quad p_2^* = \frac{s_2 - s_1 + ts_1}{2}.$$

If $t \geq \frac{s_2 - s_1}{2s_2 + s_1}$, then

$$p_1^* = \frac{(s_2 - s_1)(1 - 2t)}{3},$$

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$$p_2^* = \frac{(s_2 - s_1)(2 - t)}{3}.$$

4. If $S_1 = S_2 = \mathcal{L}$, then in case of $s_1 = s_2$, equilibrium prices are zero. If $s_1 < s_2$, then the game is equivalent to Case 2, but we have to normalize the top of the interval to 1, that is,

$$p_1^* = t\frac{s_1(s_2 - s_1)}{4s_2 - s_1}, \quad p_2^* = t\frac{2s_2(s_2 - s_1)}{4s_2 - s_1},$$

5. If $S_1 = \mathcal{L}$ and $S_2 = \mathcal{L} \cup \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then the price competition is limited to consumers below t. That is, firm 1 only has an incentive to decrease prices until the position of the indifferent consumer ($t_2 = \frac{p_2 - p_1}{s_2 - s_1}$) reaches either t or t_2^*. That is, in case of $t > t_2^*$, the equilibrium prices are the same as in Case 2. On the other hand, if $t \leq t_2^*$ then $p_1^* = s_1 t/2$ and $p_2^* = \max(s_2/2, s_2 t)$.

6. If $S_1 = \mathcal{L} \cup \mathcal{H}$ and $S_2 = \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, the price competition is limited to consumers above t. That is, firm 2 only has an incentive to decrease prices until the position of the indifferent consumer ($t_2 = \frac{p_2 - p_1}{s_2 - s_1}$) reaches either t or t_2^*. That is, in case of $t < t_2^*$, the equilibrium prices are the same as in Case 2. On the other hand, if $t \geq t_2^*$ then $p_1^* = s_1 t/2$ and $p_2^* = \max(s_2/2, s_2 t)$.

7. If $S_1 = \mathcal{H}$ and $S_2 = \mathcal{L} \cup \mathcal{H}$, then in case of $s_1 = s_2$, equilibrium prices are zero. If $s_1 < s_2$, then

$$p_1^* = \frac{(s_2 - s_1)(2ts_1 + s_2 - 3ts_2)}{7s_2 - 4s_1}, \quad p_2^* = \frac{(s_2 - s_1)(t + 2)s_2}{7s_2 - 4s_1}$$

form an equilibrium, since $t > 1/3$ is assumed.

8. If $S_1 = \mathcal{L} \cup \mathcal{H}$ and $S_2 = \mathcal{L}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then

$$p_1^* = \frac{s_1(s_2 - s_1)(2 - t)}{4s_2 - s_1}, \quad p_2^* = \frac{(s_2 - s_1)(2ts_2 - ts_1 + s_1)}{4s_2 - s_1}.$$

9. If either firm chooses not to advertise then the other firm is a monopolist in its segment, setting the price to $\max(s_i x, s_i y/2)$ if the segment is the $[x, y]$ interval.

Let us now turn our attention to the advertising stage of the game and determine which segments firms choose to advertise to. Let us examine the different types of possible equilibria. An equilibrium is described by a pair (S_1, S_2), where the first set denotes the segment chosen by firm 1 and the second set denotes the segment chosen by firm 2. First we fix $s_1 < s_2$ and determine which types of equilibria are possible if $4/5 > t \geq 1/3$. Note that we do not consider cases where a firm does not advertise.

- $(\mathcal{L}, \mathcal{H})$: In this case, $R_1 = s_1 t^2/4$, $R_2 = s_2/4$ if $t < 1/2$ and $R_1 = s_1 t^2/4$, $R_2 = s_2 t(1 - t)$ if $t \geq 1/2$. In order to check when this constitutes and equilibrium we have to check three cases. First, if $t \geq 1/2$, then firm 1 has no incentive to deviate since it cannot reach consumers above $t_2^* = \frac{2s_2 - s - 1}{4s_2 - s_1} \leq 1/2$. Firm 2 is better off by choosing $S_2 = \mathcal{L} \cup \mathcal{H}$ if and only if

$$\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2} - s_2 t(1 - t) > c_L = tc_U. \tag{4}$$

If $1/2 > t > t_2^*$, then the condition becomes

$$\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2} - \frac{s_2}{4} > c_L = tc_U. \tag{5}$$

Finally, if $t_2^* \geq t$, then firm 2 has no incentive to deviate, however firm 1 does if and only if

$$\frac{s_1 s_2 (s_2 - s_1)}{(4s_2 - s_1)^2} - s_1 t^2/4 > c_H = (1 - t)c_U. \tag{6}$$

- $(\mathcal{L} \cup \mathcal{H}, \mathcal{H})$: In this case, firm 2 has no incentive to deviate. However, firm 1 is better off setting $S_1 = \mathcal{L}$ if $t \geq t_2^*$. If $t < t_2^*$, then firm 1 deviates if and only if (6) does not hold.

- $(\mathcal{L}, \mathcal{L} \cup \mathcal{H})$: In this case, firm 1 has no incentive to deviate. However, firm 2 is better off setting $S_1 = \mathcal{H}$, if $t \leq t_2^*$. If $t > t_2^*$, then firm 2 will deviate if and only if (4) and (5) do not hold in the cases of $t \geq 1/2$ and $1/2 > t > t_2^*$, respectively.

In the following cases one of the firms always has an incentive to deviate, thus, they do not constitute equilibria. We assume that firms make positive profits, otherwise it would be profitable for them to not advertise at all.

- $(\mathcal{L} \cup \mathcal{H}, \mathcal{L} \cup \mathcal{H})$: In this case, either firm 1 or firm 2 has an incentive to deviate. If $t \leq t_2^* = \frac{2s_2 - s_1}{4s_2 - s_1}$, then consumers below t_2^* will not buy from firm 2, therefore firm 2 is better off setting $S_2 = \mathcal{H}$. On the other hand, if $t \geq t_2^*$, then firm 1 is better off setting $S_1 = \mathcal{L}$.

- $(\mathcal{L}, \mathcal{L})$: In this case, firm 2's revenues are $t^2 \frac{4s_2^2(s_2 - s_1)}{4s_2 - s_1}$. If firm 2 chooses $S_2 = \mathcal{L} \cup \mathcal{H}$ instead, then its revenues become at least $\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2}$ which is at least $1/t^2$ times more. Since costs are only $1/t$ times more, firm 2 has incentive to deviate.

- $(\mathcal{H}, \mathcal{H})$: If $t \geq \frac{s_2 - s_1}{2s_2 + s_1}$, then firm 1 has an incentive to choose $S_1 = \mathcal{L}$, because then it makes $s_1 t^2/4 - tc_U$ instead of $(2t - 1)^2(s_2 - s_1)/9 - (1 - t)c_U$ and $\frac{s_1}{s_2 - s_1} \geq \frac{1 - 2t}{3t}$ in this region. Then one can check that if $1/5 \leq t \leq 1/2$, then

$$(1 - t)\frac{1 - 2t}{3t}\frac{t^2}{4} - t\frac{(2t - 1)^2}{9} \geq 0.$$

If $1/3 \leq t \leq \frac{s_2 - s_1}{2s_2 + s_1}$, then firm 1 has an incentive to choose $S_1 = \mathcal{L}$, because then it makes $s_1 t^2/4 - tc_U$ instead of $\frac{ts_1(s_2 - s_1 + ts_1 - 2ts_2)}{2(s_2 - s_1)} - (1 - t)c_U$. If $t \leq 1/3$, one can check that

$$(1 - t)\frac{s_1 t^2}{4} - t\frac{ts_1(s_2 - s_1 + ts_1 - 2ts_2)}{2(s_2 - s_1)} \geq 0.$$

- $(\mathcal{H}, \mathcal{L})$: In this case, firm 2's revenues in the pricing stage are $R_2 = s_2 t^2/4$, thus it has an incentive to deviate to $S_2 = \mathcal{H}$ or $S_2 = \mathcal{L} \cup \mathcal{H}$.

- $(\mathcal{L} \cup \mathcal{H}, \mathcal{L})$: In this case firm 2 wants to choose $S_2 = \mathcal{H}$ or $S_2 = \mathcal{L} \cup \mathcal{H}$ depending on whether $t_2^* < t$ or the opposite.

- $(\mathcal{H}, \mathcal{L} \cup \mathcal{H})$: In this case, firm 1 has an incentive to choose $S_1 = \mathcal{L}$ or $S_1 = \mathcal{L} \cup \mathcal{H}$.

If now we fix $s_1 = s_2$, then in the cases where $|S_1 \cap S_2| > 0$, equilibrium prices are zero, that is, firms always have an incentive to deviate. If $s_1 = s_2$, then the only possible advertising equilibria are $S_i = \mathcal{L}$ and $S_j = \mathcal{H}$. In this case, firms do not have an incentive to deviate if they make positive profits.

Now we can examine the first (quality choice) stage of the game. We start with the case of $4/5 > t \geq 1/2$. Note that firm 2's revenue is increasing in s_2 in any case, that is, in equilibrium $s_2 = 1$. Firm 1's profit is $\frac{s_1 s_2 (s_2 - s_1)}{(4s_2 - s_1)^2}$ if (4) holds and $s_1 t^2/4$ if not. Therefore, firm 1 is better off if (4) does not hold, that is, in equilibrium $s_1 = 1$ must hold. It is easy to check that there are equilibria with $s_1 = s_2 = 1$. Firms choose disjunct segments: $S_i = \mathcal{L}, S_j = \mathcal{H}$ and the corresponding prices $p_i = t/2$, $p_j = t$.

If $1/3 < t < 1/2$, firm 2 again sets $s_2 = 1$ in equilibrium. Firm 1 is obviously better off if $t_2^* > t$, since t_2^* is decreasing in s_1 and firm 1's profit is at least $s_1 t^2/4 - c_{\mathcal{L}}$ in this case. As s_1 approaches s_2, t_2^* goes to $1/2$, hence firm 1 can reach $t_2^* > t$. However, if $t_2^* > t$, then firm 1's profit is increasing in s_1 thus, in equilibrium, $s_1 = 1$ must hold. As in the previous case, it is easy to check that there are equilibria with $s_1 = s_2$. Firms choose disjunct segments: $S_i = \mathcal{L}, S_j = \mathcal{H}$ and the corresponding prices $p_i = t/2$, $p_j = 1/2$. This completes the proof of the proposition.

Proof of Claim 3

The pricing stage of the game is equivalent to that of the original vertical differentiation model with $s_1' = a_1 s_1$ and $s_2' = a_2 s_2$. Therefore, the revenues, as functions of s_1' and s_2', are

$$R_1 = \frac{s_1' s_2' (s_2' - s_1')}{(4s_2' - s_1')^2} \quad R_2 = \frac{4s_2'}{s_1'} R_1. \tag{7}$$

At the advertising stage, differentiating the two functions with respect to a_1 and a_2 gives the best response functions

$$a_1 = \min\left(\frac{4s_2 a_2}{7s_1}, 1\right)$$

for the low-quality firm and $a_2 = 1$ for the high quality firm. Plugging these in the expressions in (7), we get $R_2 = 7/48s_2$. Thus, one firm always chooses $s_j = 1$. The other firm can choose anything above or equal to $4/7$, that is $1 \geq s_i \geq 4/7$. A lower choice for s_i would not allow the low-quality firm to reach the optimal advertising level in the next stage. All the described strategies constitute sub-game perfect equilibria, completing the proof.

Proof of Proposition 3

The proof goes on similar lines as the proof of Proposition 1. However, the pricing stage of the game is much simpler here. In fact, it is equivalent to that of Claim 3. Thus, we can use the resulting revenue functions from (7) and turn our attention to the stage where firms decide whether they want to advertise at high or low levels. Let us normalize s_2 to 1 and fix $0 < s_1 < 1$. Let $R_{LL}^i(s_1)$ denote the revenue of firm i in the equilibrium of the pricing stage when both firm's choose $a_1 = a_2 = L$. In the other three cases let $R_{LH}^i(s_1)$, $R_{HL}^i(s_1)$, and $R_{HH}^i(s_1)$ denote the same revenue functions of firm i where the indices show the advertising level chosen by firm 1 and 2, respectively. Also, let $G_L^1(s_1) = R_{HL}^1(s_1) - R_{LL}^1(s_1)$, $G_H^1(s_1) = R_{HH}^1(s_1) - R_{LH}^1(s_1)$ $G_L^2(s_1) = R_{LH}^2(s_1) - R_{LL}^2(s_1)$, $G_H^2(s_1) = R_{HH}^1(s_1) - R_{HL}^2(s_1)$ denote the gains of setting advertising to high instead of low. One

can check that $G^1_L(s_1) < G^1_H(s_1) < G^2_L(s_1) < G^2_H(s_1)$ if $1 \geq s_1 > 0$. In the following cases, we determine the possible advertising equilibria given different c and s_1 values.

- (L,L)
  If firm 1 chooses $a_1 = L$, then firm 2 chooses $a_2 = L$ iff $c \geq G^2_L(s_1)$. However, firm 1's best response to this is L iff $c \geq G^1_L(s_1)$. That is, an (L, L) advertising equilibrium exist iff $G^2_L(s_1) \leq c$.

- (L,H)
  If firm 1 chooses $a_1 = L$, then firm 2 chooses $a_2 = H$ iff $c < G^2_L(s_1)$. However, firm 1's best response to this is L iff $c \geq G^1_H(s_1)$. That is, an (L, H) advertising equilibrium exist iff $G^1_H(s_1) \leq c < G^2_L(s_1)$.

- (H,L)
  If firm 1 chooses $a_1 = H$, then firm 2 chooses $a_2 = L$ iff $c \geq G^2_H(s_1)$. However, firm 1's best response to this is H iff $c < G^1_L(s_1)$. That is, an (H, L) advertising equilibrium never exists.

- (H,H)
  If firm 1 chooses $a_1 = H$, then firm 2 chooses $a_2 = H$ iff $c < G^2_H(s_1)$. However, firm 1's best response to this is H iff $c < G^1_H(s_1)$. That is, an (H, H) advertising equilibrium exist iff $0 < c < G^1_H(s_1)$.

We now determine the equilibria at the quality choice stage. Let us first examine firm 1's best response quality choice to firm 2 choosing $s_2 = 1$. Given that the advertising equilibrium is (L, L) or (H, H), firm 1's best response is to choose $s_1 = 4/7$ as in the original model, since advertising has the same effect on both firms' perceived qualities. However, if the advertising equilibrium is (L, H), then firm 1 maximizes $R^1_{LH}(s_1)$ yielding $s_1 = 4/7(a_H/a_L)$. Note that $\max_{s_1} R^1_{LH}(s_1) > \max_{s_1} R^1_{LL}(s_1)$ and $G^2_L(s_1)$ is increasing, thus firm 1 always chooses an s_1 that leads to an (L, H) equilibrium over an (L, L) when it is possible, yielding $\bar{c} = G^2_L(4/7(a_h/a_L))$. Furthermore, firm 1 chooses an (H, H) equilibrium over (L, H) if and only if $c < \underline{c} = R^1_{LL}(4/7) - R^1_{LH}(4/7(a_h/a_L)) > 0$. In order to show that firm 2 chooses $s_2 = 1$ in equilibrium, one can check that given any advertising equilibrium, firm 2's profit is an increasing function of s_2, thus it chooses the maximum quality of 1. □

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