Information Acquisition and Sharing in a Vertical Relationship*

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ABSTRACT

Suppliers are increasingly acquiring consumer information in a sequential fashion with negligible marginal costs, and influencing downstream retailer actions by sharing the acquired information using either ex-ante mandatory or ex-post voluntary sharing formats. This paper examines the interaction between a manufacturer's optimal strategies for sequential information acquisition on product fit/quality and for information sharing in a vertical relationship. We examine how the flexibility to sequentially control information collection, and how the flexibility in ex-post voluntary information sharing, may influence the manufacturer's equilibrium amount of information acquired.

We show that, when information acquisition is sequential, the manufacturer may not acquire perfect information even if it is costless to do so. This self-restriction in information acquisition follows from the manufacturer's motivation to strategically influence retail behavior. When information acquisition is inflexible and constrained to be either zero or perfect information, the manufacturer will acquire more (less) information under voluntary (mandatory) sharing. Nevertheless, voluntary information sharing will unambiguously lead to more information being generated, because the manufacturer has the option to strategically withhold the disclosure if the acquired information turns out to be unfavorable. Finally, the conditions under which the manufacture prefers a particular sharing format are examined.

Key words: information acquisition; information disclosure; information sharing; ex-ante and ex-post sharing; sequential acquisition; vertical relationship

1. Introduction

The increasing sophistication of marketing research instruments and the availability of consumer databases has led to an increase in the scope and volume of consumer information that can be acquired by firms. For example, suppliers in a variety of markets can improve their knowledge about consumers (e.g., preferences, product evaluations) through subscribing to third-party syndicated databases. There is also a growing trend for vendors to develop web-based information collection and consumer tracking technologies that help them test new products, monitor reactions to product modifications, and detect changes in consumer preferences. An important feature of these information acquisition activities is the firms' increasing flexibility to sequentially control the generation of information. For example, with syndicated databases or online surveys, signals that are potentially useful for decision making (i.e., data) may flow in continuously and sequentially, and a firm can decide, at each point in time after observing the signals that have been collected, whether or not to acquire additional signals. The optimal information acquisition strategy in these scenarios is therefore characterized by a sequence of sequentially-related binary decisions about whether to continue or to terminate the data generation process (i.e., a stopping rule).

In a vertical channel, upstream firms can potentially share their acquired information about consumers with their downstream uninformed retailers. There are two distinct types of information sharing arrangements. For example, some manufacturers (e.g., Procter and Gamble, Warner-Lambert) streamline the sharing process by setting up data pooling systems, such as Collaborative, Planning, Forecasting, and Replenishment (CPFR), that automatically transmit the acquired data to the collaborating downstream firms (Gal-Or et al. 2007). Similarly, contractual arrangements with industry associations/authorities that ensure truthful sharing of information are seen in some markets. Nevertheless, anecdotal evidence suggests that many firms share insights with their downstream retailers from time to time, but do not contractually commit to sharing information on a long-term basis. According to the extensive surveys conducted by BearingPoint in recent years, firms in the US rely primarily on traditional devices (e.g., E-mail, phone, fax, meeting) to communicate with their channel partners (Chain Store Age 2003). Note that the major difference between these alternative sharing formats hinges on whether the sharing decision is committed/made prior to the acquisition of information or can be flexibly made after having observed the acquired information.

These observations raise some important questions: How much consumer information should an upstream manufacturer acquire? When should the acquired information be shared with downstream retailers? How do the information acquisition and the information sharing decisions interact with each other? Moreover, what type of sharing format should be implemented by manufacturers?

To address these questions, we consider a model where a manufacturer sells to end consumers through a retailer. The firms have uncertainty about how well the product fits consumer preferences, which we label as (perceived) product quality. The manufacturer can potentially resolve the uncertainty through acquiring signals, each of which can provide imperfect information about product quality. There is uncertainty about whether useful information will be available, which is represented by two possible states of information acquisition. The manufacturer can acquire informative quality signals only in the state when information useful for decision making is available. The retailer is uninformed of the realization of the information acquisition states. We focus on the manufacturer's strategic incentives for information acquisition, where the motivation for the manufacturer to acquire information is to manipulate the downstream retailer's belief on product quality (through information sharing) and thus to influence the retailer's actions (i.e., ordering quantity, retail price) to the manufacturer's own advantage.

We analyze and compare several scenarios depending upon whether or not information acquisition/sharing is flexible. First, we examine the effect of the flexibility in sequential information acquisition. When information acquisition is "inflexible," the manufacturer can choose to acquire only either none or an unlimited number of signals (which almost perfectly reveal true quality). Alternatively, and as exemplified in the above-mentioned examples, the manufacture may sequentially decide at each point in time whether or not to generate an additional signal. We also investigate two alternative information sharing formats. In the "ex-ante mandatory sharing" case, the manufacturer commits in a contract to share all the acquired information with the retailer before the information is actually acquired. This can represent long-term information sharing agreements in vertical channels. In contrast, under the "ex-post voluntary sharing" format, the manufacturer can decide after the information acquisition process whether or not to share the acquired information. Thus information sharing is voluntary as the manufacturer is not bound by an ex-ante contract. For both sharing formats, we assume that information sharing is truthful (e.g., Gal-Or 1985, Li 1985), but the manufacturer can choose not to share at all unless contractually committed. Comparing these two formats therefore allows us to examine the impact of the flexibility in information sharing.

1.1. Summary of Results

The first result of our analysis is that, when information acquisition is sequential, the manufacturer may not want to acquire an unlimited number of signals that perfectly reveal quality, *even if* information acquisition is costless. The manufacturer in equilibrium exercises self-restriction in information acquisition, and continues to generate signals if and only if the posterior belief on quality is between some upper and lower bound. This stems from the manufacturer's motivation to strategically influence retailer behavior. If the information acquisition process reaches a stage where the updated posterior belief is at a high enough level, the manufacturer is induced to stop the acquisition simply because no better outcome can be achieved. However, the manufacturer may also want to terminate information acquisition when a sufficiently low posterior belief is reached, because of the risk that further collection of additional signals might lead to overly adverse results.

The equilibrium amount of information generated is influenced by the interaction between the flexibility in information acquisition (sequential versus inflexible) and the flexibility in information sharing (ex-post voluntary versus ex-ante mandatory). First, the flexibility to sequentially control the information acquisition process may or may not lead to an increasing incentive for the manufacturer to acquire more information. In particular, when the manufacturer has committed ex-ante to mandatorily share the acquired information, it will choose not to acquire any information if the acquisition of information is inflexible in that the manufacturer is exogenously constrained to acquire either zero or perfect information. Thus under ex-ante mandatory sharing, a positive amount of information is acquired only when the manufacturer has the flexibility to sequentially decide on the amount of information acquired. In contrast, under ex-post voluntary sharing, the manufacturer's flexibility in sequentially controlling the number of signals does not lead to an increase in the equilibrium amount of information acquisition. Indeed in the inflexible information acquisition case, the manufacturer in equilibrium always acquires information. As a result, the manufacturer's flexibility in information acquisition actually leads to reduced information generation. Overall, this suggests that the impact of the flexibility in information acquisition on the equilibrium amount of information acquired is moderated by the flexibility in information sharing.

Interestingly, we find that an increasing flexibility in information sharing can unambiguously induce the manufacturer to generate a larger amount of information. This arises from the manufacturer's ability to selectively decide, under ex-post voluntary sharing, whether or not to disclose the acquired information: Unlike that under ex-ante mandatory sharing, suppose that the manufacturer were to continue to acquire additional information and if the acquired information turns out to be unfavorable, the manufacturer has the option to withhold the disclosure of information and hence may not necessarily induce unfavorable responses from the retailer. Such information concealment is credible: The retailer cannot distinguish between the cases when the manufacturer's acquired information is unfavorable and when useful information is indeed unavailable, since under both cases no information will be received by the retailer.

Finally, we examine the manufacturer's preference for the different information sharing formats by comparing the manufacturer's equilibrium ex-ante payoffs between voluntary and mandatory information sharing. We find that, counter-intuitively, when the prior belief about product fit is sufficiently low, the manufacturer prefers to ex-ante commit to mandatorily share any information that will be acquired and thereby gives up the ex-post flexibility to voluntarily disclose information. This is because more information acquisition may induce on average lower retail ordering, which can be alleviated by mandatory sharing contracts that serve as the manufacturer's self-discipline in information generation. This result seems consistent with the high incidence of formal ex-ante information sharing arrangements that are documented in fashion markets where failure rates of new designs are likely to be high.

1.2. Related Research

There is a substantial literature starting from Novshek and Sonnenschein (1982) and Vives (1984) on information sharing between oligopolistic firms involved in market competition. The general theme in this literature is that the equilibrium impact of information sharing depends upon the interplay of two effects: First, there is an efficiency effect because each firm has better information about supply or demand uncertainty. Second, the sharing of information leads to greater correlation in the strategies of the competing firms. The net impact of these effects differs across the various contexts considered. For example, Gal-Or (1985) and Li (1985) investigate a Cournot oligopoly with uncertainty about a common demand parameter and show that firms will not share information, while Gal-Or (1986) and Shapiro (1986) derive the opposite result with uncertainty about private cost values. Vives (1984) shows that the incentives to share information change depending upon whether the products are substitutes or complements and whether the competition is Cournot or Bertrand.[1] Villas-Boas (1994) studies a related question of whether or not competitors should share the same advertising agency and the effects of information transmission that might result from the sharing of the same agency. In contrast to the above literature, this paper investigates the acquisition and the transmission of information in a vertical relationship from an upstream manufacturer to a retailer. The economic incentives at play in our paper involve the effects of information on retailer ordering and pricing.

Another stream of research examines the acquisition of information by oligopolistic firms. For example, Li et al. (1987) and Vives (1988) analyze optimal information acquisition about demand uncertainty prior to firms' production/market decisions and show that the equilibrium level of information acquisition decreases with the cost of information and the slope of the demand function. Hwang (1993) extends these models to the case of non-identical and increasing marginal costs to show that the firm with a smaller slope of marginal costs acquires more information in equilibrium. In these studies, firms make an one-time (static) decision on how much information should be acquired, and perfect information acquisition would arise in equilibrium when the acquisition cost is zero. In contrast, we examine the sequential acquisition of information, which implies that after each signal there is an optimal stopping decision on whether or not to acquire additional signals. As a result, in our analysis the manufacturer may prefer not to acquire full information even if it is costless because of the motivation to control retail behavior. Moreover, we examine how information acquisition interacts with the subsequent sharing decision in a vertical channel, and show that the equilibrium amount of information generation is influenced by the flexibility in information acquisition and in the sharing format.

There are also some papers that solely focus on information sharing issues in vertical relationships (e.g., Niraj and Narasimhan 2004, Gu and Chen 2005, He et al. 2008).[2] Li (2002) is perhaps

[1]In this vein, Raith (1996) presents a general model of information sharing which accommodates and clarifies the effects of information sharing in many of the major models in the literature, including cost versus demand uncertainty, and Cournot versus Bertrand competition.

[2]There is a substantial literature in supply chain management, originating from Lee et al. (1997), that examines the efficiency-improving role of sharing information in reducing production, logistical, and inventory-related costs (e.g., Cachon and Fisher 2000, Kulp et al. 2004).

the first analysis of the incentives of competing retailers to share their private information about retail demand or costs with an upstream manufacturer.[3] It identifies a direct effect of information sharing due to changes in the actions of the parties involved in the sharing, and an indirect effect due to the changes in the actions of the competing retailer. Recently, Gal-Or et al. (2007) examine information sharing between a manufacturer and two competing retailers, each of whom has imperfect private signals of the true demand. Information sharing between the upstream and downstream parties can alleviate the distortion in the wholesale prices caused by demand uncertainty. Our focus diverges significantly from these papers in that we analyze the sequential acquisition of information by an upstream manufacturer and its subsequent sharing with the retailer. Moreover, only ex-ante mandatory sharing is considered in these studies,[4] whereas we examine how the format of sharing (ex-post voluntary versus ex-ante mandatory) affects the amount of information that the manufacturer will acquire.

The rest of the paper is organized as follows. The next section describes the model. Section 3 presents some preliminary results. This is followed by the analysis of ex-ante mandatory sharing in Section 4. Next, Section 5 addresses the case of ex-post voluntary sharing, where the manufacturer's equilibrium ex-ante payoffs across the alternative sharing formats are also compared. The final section concludes the paper and discusses potential directions for future research.

2. The Model

An upstream manufacturer produces a good/service and sells to end consumers through a retailer. The manufacturer has a constant marginal cost of production, which is normalized to zero without any loss of generality. There is a competitive supply of the product in the upstream market, over which the manufacturer enjoys a margin $d > 0$. The retailer would order the product from the manufacturer if and only if the per-unit wholesale price ω is not higher than d. As a result, the parameter d represents a measure of the competitiveness in the upstream market. For example, d can be seen as the manufacturer's cost advantage over other suppliers. The firms are risk neutral and maximize expected payoffs.

There is a unit mass of two-segment consumers in the market, whose product valuation is:

$$V_i = \theta_i Q, \tag{1}$$

[3] Gal-Or et al. (2006) study whether buyers should share supplier-specific fit information with prospective suppliers in order to extract more surplus in input procurement. Creane (2007) looks at an analogous problem of downstream firms sharing their productivity information with an input supplier.

[4] One exception is Guo (2008) which examines the payoff implications of a downstream retailer sharing privately acquired information with an upstream manufacturer on an ex-post voluntary basis.

where $i \in \{h, l\}$ denotes consumer segments; Q represents the fit of the product with consumer preferences, which can also be interpreted as quality in that higher quality products fit consumer preferences better; and θ_i captures consumer segment i's valuation or willingness to pay for quality, where $0 < \theta_l < \theta_h$. The relative size of the high and the low consumer segments are α and $1 - \alpha$, respectively, where $\alpha \in (0, 1)$. To capture the firms' uncertainty about consumer valuation, we assume that whether the product is a good fit or not is known to consumers at the time of purchase but initially unknown to the firms.[5] This firm uncertainty establishes the role for the acquisition and sharing of consumer information. Suppose without loss of generality that there are two possible states of nature, $S \in \{G, B\}$, such that the product's quality is $Q = 1$ if the product is "good" (i.e., $S = G$) and $Q = 0$ if it is "bad" instead (i.e., $S = B$). The firms have common prior belief about the true quality state, i.e., $\Pr(S = G) = \beta$ and $\Pr(S = B) = 1 - \beta$, where $\beta \in (0, 1)$.

This paper deals with quality information which the manufacturer can acquire and transmit to the retailer. This information may be particularly relevant for new products or changes to products, because it is in these cases that there would be significant firm uncertainty about product fit with consumer preferences. Manufacturers typically have better access to testing objective product quality than retailers do.[6] For example, a manufacturer may collect consumer information through focus groups, online surveys, feedback from sales forces, or subscribing to syndicated information databases.

Nevertheless, useful information may not always be available. For example, the data collection technology may only yield raw signals that are too complex or irrelevant for decision making, or human resources may be absent or incapable to process the collected raw data. Specifically, the states of the world about the availability of useful information are denoted as $I \in \{y, \bar{y}\}$, where y is realized with a probability half and represents the state in which useful information can be acquired, while with the remaining probability of half the information state $\bar{y}$ is realized in which no useful information is available. As we will see in the analysis, this uncertainty has implications for the information sharing strategy of the manufacturer.

If the information state is $I = y$, the manufacturer can acquire (imperfect) signals about the true product quality at zero marginal cost per signal.[7] We deliberately make this assumption, precisely

[5] One can then interpret Q as the consumers' "perceived fit/quality." Nevertheless, to simplify exposition, we will refer to Q as product fit/quality where no confusion arises.

[6] Note that the insights of the paper can be readily applied to cases where the manufacturer can test the objective product quality which is positively correlated with consumers' perceived quality.

[7] This may represent the cost of converting the data obtained from a vendor or an online survey, if informative (i.e., $I = y$), into useful insights for decision making. For example, many information vendors typically charge a fixed fee for unlimited access to the database. In online surveys, once the fixed cost of setting up the survey is incurred, the cost of eliciting an additional response is negligible. Generally, one may interpret the setup as one in which masses of data can be generated by investing a fixed cost on a data-collection technology, and the subsequent cost

because our intention is to highlight the point that *even* if the marginal cost of acquiring additional signals is zero the manufacturer might not want to collect an infinite number of signals. Each imperfect signal $s \in \{g, b\}$ is assumed to be generated from the true quality state S with probability $\gamma \in [1/2, 1]$: $\Pr(g|G) = \Pr(b|B) = \gamma$. Conditional on a signal $s \in \{g, b\}$, the probabilities of the states of product quality are given by $\Pr(G|g) = \frac{\beta\gamma}{\beta\gamma+(1-\beta)(1-\gamma)}$ and $\Pr(B|b) = \frac{(1-\beta)\gamma}{\beta(1-\gamma)+(1-\beta)\gamma}$. The parameter γ captures the informativeness of the quality signals. When $\gamma \to 1/2$, a signal provides no additional information beyond the prior: $\Pr(G|g) = \beta$ and $\Pr(B|b) = 1 - \beta$. When $\gamma \to 1$, the signals reveal the true quality with full certainty: $\Pr(G|g) = \Pr(B|b) = 1$. Conditional on a true quality state $S \in \{G, B\}$, the signals are assumed to be independently generated.

The timing of the game is shown in Figure 1. In the first stage, the firms sign and commit to a contract which includes the specification of a per-unit wholesale price ω under which the product is transferred to the retailer, and may or may not include the specification of information sharing. Two alternative information sharing arrangements therefore arise: The first one is "ex-ante" information sharing in which the manufacturer commits in the contract to share information with the retailer. This ex-ante commitment to share information can also be labeled as "mandatory" information sharing and has been analyzed in the literature (e.g., Li 2002). For example, manufacturers such as Procter and Gamble collaborate with retailers to develop shared databases. In contrast, the manufacturer can choose to share information "ex-post" in which case the manufacturer does not commit in the contract to transfer the to-be-acquired information to the retailer. Rather, after acquiring the information in stage 2, the manufacturer ex-post selects whether or not to disclose the acquired information to the retailer. Thus information sharing in this case is "voluntary" and is not mandated by the contract before its acquisition. The essential difference between these two sharing formats hinges on whether or not the manufacturer, prior to knowing the content of the acquired information, ex-ante commits to share the information that will be acquired. Note that if information sharing cannot be credibly contracted, the manufacturer can always share its private information voluntarily on an ex-post basis. We will investigate and compare these two alternative sharing cases, which allows us to capture the impact of the flexibility in information sharing.

After the contract is written, the information state $I \in \{y, \bar{y}\}$ about the availability of useful information is realized and revealed to the manufacturer in the second stage of the game. The realization of I is private to the manufacturer and remains unrevealed to the retailer. As a result, the retailer is uncertain about whether or not the manufacturer has collected useful information, unless the acquired information is shared (either mandatorily or voluntarily). Subsequently, if $I = y$, the manufacturer decides on the amount of information to be acquired, in anticipation of how the

of converting each unit of the collected data into managerially-relevant information is negligible.

Figure 1: Timing of the Model

acquired information may be strategically used to influence the retailer's behavior. We will consider two alternative scenarios, depending upon whether or not the manufacturer can acquire the quality signals in a sequential fashion. In the first scenario, information is "inflexible" in the sense that the manufacture can choose to acquire only either none or an infinite number of quality signals. In contrast, if information acquisition is sequential, at each point in time when a total of $n \geq 0$ pieces of signals have been accumulated and processed, the manufacturer decides whether or not to generate an additional signal.[8] The comparison between these two scenarios allows us to capture the impact of the flexibility in information acquisition. To facilitate notation, we define the manufacturer's and the retailer's expected payoffs in stage 2 as Π' and π' (or Π'' and π''), respectively, conditional on the wholesale price ω and the realized information state being $I = y$ (or $I = \bar{y}$), and taking into account the manufacturer's optimal information acquisition and sharing strategies. Moreover, the firms' conditional expected profits, prior to the realization of the information state I, are defined as $\Pi(w) = (\Pi' + \Pi'')/2$ and $\pi(w) = (\pi' + \pi'')/2$, respectively.

In stage 3 the manufacturer can share the acquired information with the retailer, who may then update its belief on the true product quality. In the mandatory sharing case, all the signals obtained by the manufacturer in stage 2 on the product's quality are truthfully and completely transmitted to the retailer, and the manufacturer's sharing decision is hence immaterial. Therefore this case is equivalent to the manufacturer and the retailer having a common posterior belief about the product's quality, and is exemplified by shared information systems established by manufacturers and retailers in many industries. When information sharing is ex-post and voluntary, the manufacturer decides whether or not to transfer the acquired signals to the retailer. The manufacturer has to truthfully disclose all the acquired signals if it decides to share, with the alternative option to remain quiet and disclose none of the signals.[9] The firms' posterior beliefs on product quality will then be the same if sharing occurs, but may diverge from each other if otherwise.

[8]An equivalent interpretation of this information acquisition problem is that the signals flow in sequentially (e.g., online surveys) and the manufacturer chooses when to stop the information acquisition process.

[9]The truth-telling assumption in the voluntary case would hold if the firms take a long-term perspective and care about reputation in the vertical relationship, or if verification costs are negligible.

The fourth and final stage involves two retail decisions. The retailer decides on the unit of products, $x \in [0, 1]$, to be ordered from the manufacturer and sets the retail price p. The retailer has to carry the ordered stock to be able to sell to the end consumers. When making the ordering and the pricing decisions, we assume that the retailer remains uncertain about the true product quality unless an infinite number of signals (i.e., almost perfect information) have been acquired and disclosed by the manufacturer. It is only with this assumption that we can investigate the strategic role of information acquisition/disclosure in influencing the retailer's decisions.[10]

Some discussions on model assumptions are warranted. We intentionally assume that the determination of the wholesale price precedes the manufacturer's information acquisition and disclosure, because this enables us to isolate the strategic effect of upstream information acquisition/disclosure on the retailer's decisions from its efficiency effect of improving the manufacturer's own decision making. Note also that under the assumption of zero marginal cost of information acquisition, the manufacturer can choose to acquire an infinite amount of signals and become (almost) fully informed of the true quality without incurring any additional cost. But the interesting point we want to investigate is the possibility that the manufacturer may choose not to know the true quality for sure even when it is completely costless to do so, potentially because of the incentive to strategically manipulate the downstream retailer's belief. It is this role of information sharing in influencing the manufacturer's information acquisition strategy that we highlight in the analysis.

We will focus on the market conditions where the relative size of the consumer segments satisfies $\alpha\theta_h < \theta_l < \alpha(2 - \alpha)\theta_h$. The first inequality rules out the trivial scenario when the low type consumers are never served, and ensures that a vertically integrated manufacturer would serve the whole market. On the other hand, the second inequality represents the case of interest in which the low type consumers will not buy in equilibrium in a decentralized channel without quality information. This then captures the relevant range of market conditions which helps us examine the relationship between information acquisition/sharing and the trade-off between greater market coverage and surplus extraction through higher wholesale prices. To solve the game, we will use backward induction to insure sub-game perfection.

3. Preliminaries

We start by presenting some of the model preliminaries. This involves three parts: 1) deriving the retailer's optimal ordering and pricing decisions, conditional on the wholesale price ω and on the

[10]Nevertheless, the results are unchanged in an alternative setup where the retail price is determined after the true quality is revealed to the retailer.

retailer's updated belief $\hat{\beta}$; 2) characterizing how a firm's belief about the true product quality is updated when a series of some n quality signals are sequentially obtained; and finally 3) presenting the benchmark case without information sharing/acquisition.

3.1. Retailer Decisions

Let the retailer's (updated) belief about the quality be $\hat{\beta}$ when ordering and choosing the retail price. With probability $\hat{\beta}$, the product valuation of the consumers are $V_h = \theta_h$ and $V_l = \theta_l$, respectively; and with probability $1 - \hat{\beta}$, we have $V_h = V_l = 0$. As a result, the retailer would consider only three possible levels of ordering quantity, i.e., $x \in \{0, \alpha, 1\}$, serving none, the high type, or all of the consumer types, respectively. In particular, if the retailer charges $p = \theta_h$, then the optimal ordering is either α or 0, with an expected payoff $\pi = \max\{\alpha\hat{\beta}\theta_h - \alpha\omega, 0\}$. Similarly, the retailer would optimally order a size of either one or zero when it charges $p = \theta_l$, leading to an expected payoff $\pi = \max\{\hat{\beta}\theta_l - \omega, 0\}$. Let us denote $\beta_l \equiv \frac{\omega}{\theta_h}$, and $\beta_h \equiv \frac{(1-\alpha)\omega}{\theta_l - \alpha\theta_h}$. We can then characterize the retailer's optimal ordering strategy:[11]

$$x = \begin{cases} 0, & \text{if } \hat{\beta} < \beta_l; \\ \alpha, & \text{if } \beta_l \le \hat{\beta} < \beta_h; \\ 1, & \text{if otherwise.} \end{cases}$$

The retailer's optimal ordering quantity increases as its updated belief $\hat{\beta}$ on product quality becomes higher, since the retailer makes a trade-off between serving more consumer types when the product turns out to be good and saving on over-ordering when the product is bad. If the retailer believes that the product is unlikely to be a good ($\hat{\beta} < \beta_l$), the saving incentive dominates and the retailer would order nothing from the manufacturer. When the perceived likelihood that the product will be a good is sufficiently high ($\hat{\beta} \ge \beta_h$), the retailer would make an order of size one for the whole market. In the same vein, an intermediate belief ($\beta_l \le \hat{\beta} < \beta_h$) would lead the retailer to stock only an intermediate amount such that only the high type consumers are expected to be served. Note also that the thresholds β_l and β_h are both increasing in ω, and we have $\beta_h > 1$ when $\omega > \frac{\theta_l - \alpha\theta_h}{1-\alpha}$. This implies that the low type consumers may not be served for all $\hat{\beta} \in [0, 1]$, even when the wholesale price is at an intermediate level (i.e., $\frac{\theta_l - \alpha\theta_h}{1-\alpha} < \omega \le \theta_l$) under which all consumers will buy in the absence of double marginalization. We will show in Section 3.3 that this can indeed arise in equilibrium in a benchmark model without information sharing/acquisition.

[11] Given the retailer's optimal ordering strategy, ex-post over-ordering never occurs unless the product turns out to be a bad in which case the salvage value of the product is zero anyway, making product return an irrelevant issue.

3.2. Sequential Quality Signals and Posterior Beliefs

Let us then investigate how sequential information acquisition influences the updating of posterior beliefs on true quality, starting with an arbitrary initial belief $\dot{\beta}$. Denote the number of "good" and "bad" signals that are generated as n_g and n_b, respectively. Conditional on a series of $n = n_g + n_b$ signals having been acquired, the posterior belief is updated as follows:

$$\Pr(G|n_g, n_b) = \frac{\Pr(G)\Pr(n_g, n_b|G)}{\Pr(G)\Pr(n_g, n_b|G) + \Pr(B)\Pr(n_g, n_b|B)} = \frac{\dot{\beta}}{\dot{\beta} + (1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{(n_g - n_b)}}.$$

Note that the updated posterior belief is dependent only upon the difference between the number of "good" and "bad" signals. We can then define the posterior belief, updated from an initial belief $\dot{\beta}$, as a function of the number of accumulated "net good" signals $N \equiv n_g - n_b$:

$$\hat{\beta}(N) \equiv \frac{\dot{\beta}}{\dot{\beta} + (1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{N}}. \tag{2}$$

To facilitate the analysis, suppose that N is a real number, which is without loss of generality when the number of signals becomes sufficiently large. Given that $\gamma \in [1/2, 1]$, it is then obvious that $\frac{\partial \hat{\beta}(N)}{\partial N} > 0$. Intuitively, this suggests that the larger the number of "good" signals relative to that of "bad" ones, it is believed that the true state is more likely to be the former. Moreover, we have $\lim_{N\to-\infty} \hat{\beta}(N) = 0$ and $\lim_{N\to+\infty} \hat{\beta}(N) = 1$, which implies that the true quality can become almost certainly known if an infinite number of signals are acquired.

Let us suppose that, starting from an initial belief $\dot{\beta}$, information acquisition will not stop until either a number of $N_H > 0$ or $N_L < 0$ signals are generated. Then conditional on the true state being $S \in \{G, B\}$, what is the probability, formally defined as $\Phi^S(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$, of reaching the posterior belief $\hat{\beta}_H \equiv \hat{\beta}(N_H) > \dot{\beta}$ before reaching $\hat{\beta}_L \equiv \hat{\beta}(N_L) < \dot{\beta}$? Similarly, we can define the unconditional probability as $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) \equiv \dot{\beta}\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) + (1-\dot{\beta})\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$.

To derive these probabilities, let us define $\Psi^S(N) \equiv \Phi^S(\hat{\beta}(N)|\hat{\beta}_L, \hat{\beta}_H)$ as the conditional transition probability that the upper bound N_H is hit before the lower bound N_L, as a function of the current "net good" signals $N \in [N_L, N_H]$. Consider a situation when N signals have been accumulated and the current updated belief is given by $\hat{\beta}(N)$. Suppose an additional signal is to be generated. Then conditional on $S = G$, the posterior belief would be updated upward to $\hat{\beta}(N+1)$ or downward to $\hat{\beta}(N-1)$, with a probability γ or $1-\gamma$, respectively. This implies that the conditional transition function $\Psi^G(N)$ is also updated to $\Psi^G(N+1)$ or $\Psi^G(N-1)$ with probability γ or $1-\gamma$, respectively. If the true state is $S = B$ instead, then an additional signal would move the updated posterior belief toward $\hat{\beta}(N+1)$ or $\hat{\beta}(N-1)$, leading to the updated conditional transition function $\Psi^B(N+1)$ or $\Psi^B(N-1)$, with probability $1-\gamma$ or γ, respectively.

The above discussion suggests that we can derive second-order difference equations for $\Psi^G(N)$ and $\Psi^B(N)$, respectively. These difference equations can be solved using the boundary conditions $\Psi^S(N_L) = 0$ and $\Psi^S(N_H) = 1$, where $S \in \{G, B\}$. Noticing that by definition $\Phi^S(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^S(0)$, we can obtain:

LEMMA 1: $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\hat{\beta}_H(\dot{\beta}-\hat{\beta}_L)}{\dot{\beta}(\hat{\beta}_H-\hat{\beta}_L)}$, $\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{(1-\hat{\beta}_H)(\dot{\beta}-\hat{\beta}_L)}{(1-\dot{\beta})(\hat{\beta}_H-\hat{\beta}_L)}$, *and* $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\dot{\beta}-\hat{\beta}_L}{\hat{\beta}_H-\hat{\beta}_L}$.

This lemma captures several interesting features of the probabilities of arriving at a posterior upper bound $\hat{\beta}_H$ before a lower bound $\hat{\beta}_L$, starting from an initial belief $\dot{\beta}$. First, it can be seen that $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) > \Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) > \Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H)$, for all $\hat{\beta}_L$, $\hat{\beta}_H$, and $\dot{\beta} \in [\hat{\beta}_L, \hat{\beta}_H]$. Thus the accumulated quality signals are more likely to lead to the posterior belief being updated toward the upper bound before the lower bound, when the true state is G than when it is B. Intuitively, the g signals move the posterior belief upward while the b ones move it downward. Therefore, the conditional likelihood of reaching the upper bound first is higher when $S = G$ than when $S = B$.

Note also that these probabilities are proportional to the distance between the initial belief and the posterior lower bound (i.e., $\dot{\beta} - \hat{\beta}_L$) relative to the difference between the posterior upper bound and the lower bound (i.e., $\hat{\beta}_H - \hat{\beta}_L$). All else being equal, the closer the initial belief is to the upper bound (or the farther from the lower bound), the more likely the upper bound is reached before the lower bound. Moreover, one can readily verify that $\Phi^S(\hat{\beta}_L|\hat{\beta}_L, \hat{\beta}_H) = \Phi(\hat{\beta}_L|\hat{\beta}_L, \hat{\beta}_H) = 0$, and $\Phi^S(\hat{\beta}_H|\hat{\beta}_L, \hat{\beta}_H) = \Phi(\hat{\beta}_H|\hat{\beta}_L, \hat{\beta}_H) = 1$, $S \in \{G, B\}$. This suggests that, irrespective of the true quality state, a posterior bound would be almost surely approached if it is sufficiently close to the initial belief, before reaching the opposite posterior bound.

3.3. The Benchmark without Information Sharing/Acquisition

Before we begin the main analysis, let us consider the benchmark case where it is infeasible for the manufacturer to share any of the acquired information with the retailer. This would be the case, for example, if the message sent by the manufacturer is completely unverifiable and costless (i.e., cheap talk). As a result, no information would be acquired by the manufacturer, since information acquisition has no role under the current setup if it is not possible for the manufacturer to influence the retailer's behavior through information sharing. The retailer will then maintain the prior belief β. Given the assumption $\alpha\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$, one can then readily obtain that in equilibrium the manufacturer's and the retailer's ex-ante profits are, respectively, given by:

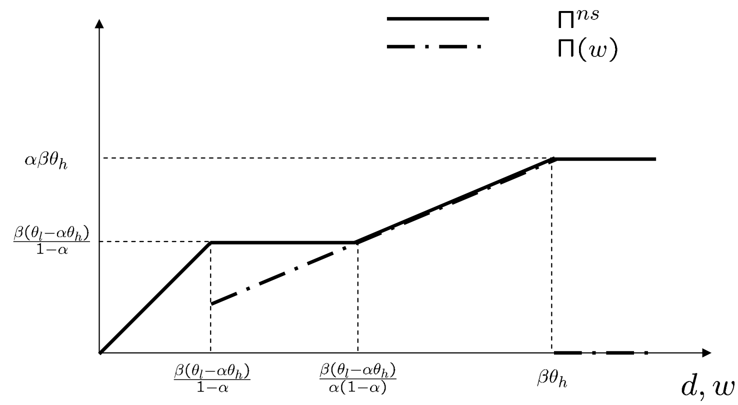

Figure 2: The Manufacturer's Expected Payoffs without Information Sharing

$$\Pi^{ns} = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}; \\ \alpha \min\{d, \beta\theta_h\}, & \text{if otherwise.} \end{cases}$$

$$\pi^{ns} = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}; \\ \alpha\beta\theta_h - \alpha \min\{d, \beta\theta_h\}, & \text{if otherwise.} \end{cases}$$

The manufacturer's equilibrium ex-ante payoff without information sharing, Π^{ns}, is shown in Figure 2. When the competitive margin is sufficiently small, i.e., $d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, the manufacturer is induced to charge low wholesale prices such that the whole market is served in equilibrium. However, when the manufacturer's competitive margin is sufficiently large, i.e., $d > \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, the low type consumers are not served in equilibrium, resulting in a loss of market coverage.

4. Ex-ante Mandatory Information Sharing

In this section we analyze the case of mandatory information sharing in which the manufacturer ex-ante commits in an enforceable contract to share with the retailer all the information that is to be acquired. The manufacturer's decision in the third stage of the game hence becomes immaterial, and we can focus on the manufacturer's second-stage decision about how much information should be acquired conditional on useful information being available (i.e., $I = y$). Note also that the firms necessarily share the same posterior belief, irrespective of the manufacturer's information acquisition strategy, which can be denoted as $\hat{\beta}$. We will start with the scenario when information acquisition is inflexible such that only either none or an infinite number of quality signals can be generated. We will then investigate the scenario when the manufacturer can sequentially decide on the number of quality signals to accumulate, i.e., when to stop the information acquisition process. In characterizing the manufacturer's optimal information acquisition strategy, we highlight the strategic influence exerted on the extent to which the retailer's posterior belief, and therefore its ordering and pricing decisions, can be influenced in the direction beneficial to the manufacturer.

4.1. Inflexible Information Acquisition

In this scenario, the manufacturer's information acquisition decision when $I = y$ amounts to either maintaining the prior belief β or becoming (almost) fully informed of the true quality state. Should no information be acquired at all, the retailer's belief will not be updated from the prior. If the manufacturer decides to acquire information, then the retailer' posterior belief is updated to either $\hat{\beta} = 1$ or $\hat{\beta} = 0$, with ex-ante probability β or $1 - \beta$, respectively. Given that the belief updating can subsequently influence the retailer's optimal ordering and pricing decisions, which of these two options is more beneficial for the manufacturer?

Proposition 1: *Under ex-ante mandatory information sharing and when information acquisition is inflexible, in equilibrium the manufacturer decides not to acquire information even when the information state is $I = y$. The equilibrium wholesale price and the firms' ex-ante payoffs are the same as those in the benchmark without information sharing (i.e., Π^{ns} and π^{ns}).*

Interestingly, when the acquired information is mandatorily shared with the retailer and the acquisition process is inflexible, the manufacturer in equilibrium would not charge wholesale prices that lead to information acquisition being desirable. Under the charged equilibrium wholesale prices, the manufacturer is strictly better off acquiring no information at all than making the retailer (almost) perfectly informed of the true quality. Intuitively, here providing information to the retailer only leads to more stochastic retail ordering, intensifying the double marginalization and market recession problem when the acquired information indicates bad quality (i.e., $\hat{\beta} = 0$). This implies that, even when the cost of information acquisition is negligible, the manufacturer may not necessarily acquire information if it is constrained to choose between zero and perfect information.

4.2. Sequential Information Acquisition

Let us now investigate the scenario when the manufacturer can flexibly decide on the number of quality signals to acquire in a sequential manner. We start by investigating the manufacturer's optimal stopping rule for information acquisition when $I = y$, and then examine the payoff implications of manipulating the information acquisition process strategically.

4.2.1. Optimal Information Acquisition Strategy

The manufacturer's optimal information acquisition decision pertains to whether or not to stop the acquisition process at each updated posterior belief $\hat{\beta} \in (0, 1)$ that is commonly shared by both firms.[12] Recall that the manufacturer's expected payoff is given by $\Pi = \omega x$, where x is the retailer's optimal ordering quantity which increases with the posterior belief $\hat{\beta}$. It is obvious that the manufacturer would not continue the information acquisition process when $\hat{\beta} \geq \beta_h$, because the retailer would already order the maximum amount of one unit. Conversely, the manufacturer would continue to accumulate more information whenever $\hat{\beta} < \beta_l$. This is because, the manufacturer's payoff would be zero if the retailer's belief remains below β_l, while sampling additional signals may induce the retailer to order $x = \alpha$ if β_l is reached. In the case when $\hat{\beta} \in (\beta_l, \beta_h)$, the manufacturer's optimal information acquisition strategy is dependent upon the possible levels of the retailer's optimal ordering quantity that is determined by the charged wholesale price. In particular, information acquisition will never be terminated, if the wholesale price is low (i.e., $\omega \leq \frac{\theta_l - \alpha\theta_h}{1-\alpha}$) such that $\beta_h \leq 1$ and the retailer may order at all the three possible levels: $x \in \{0, \alpha, 1\}$. This is because sampling an additional signal would not decrease the amount ordered by the retailer, but may induce more ordering if the posterior belief $\hat{\beta}$ is updated upward to reach β_h. In contrast, if the wholesale price is high, i.e., $\omega > \frac{\theta_l - \alpha\theta_h}{1-\alpha}$, which implies that $\beta_h > 1$, the optimal ordering can take only two possible values: $x \in \{0, \alpha\}$; and the manufacturer would then not have the incentive to continue the information acquisition process.

Finally, what remains to be determined is the information acquisition strategy when $\hat{\beta} = \beta_l$. If no additional information is collected, then the manufacturer can be guaranteed a payoff of $\Pi = \alpha\omega$. When $\beta_h > 1$, this is the best possible outcome, and therefore no additional information would be acquired. However, when $\beta_h \leq 1$, collecting an additional piece of information would lead to either $\hat{\beta} > \beta_l$ or $\hat{\beta} < \beta_l$, and from the discussion above we know that the information acquisition process will continue from then on until the posterior belief reaches either β_h or 0, respectively. In other words, continuing the information acquisition process may move the manufacturer's ultimate payoff to either ω or 0, with a probability $\Phi(\beta_l|0, \beta_h)$ or $1 - \Phi(\beta_l|0, \beta_h)$, respectively. Using Lemma 1, we have $\Phi(\beta_l|0, \beta_h) = \frac{\beta_l}{\beta_h} = \frac{\theta_l - \alpha\theta_h}{(1-\alpha)\theta_h}$. It follows that $\alpha\omega > \Phi(\beta_l|0, \beta_h)\omega$. As a result, the manufacturer will stop collecting more information when $\hat{\beta} = \beta_l$. Collecting an extra piece of information is

[12] Note that it is assumed that when $\hat{\beta}$ is arbitrarily close to either 0 or 1, perfect information is (almost) surely obtained and the information acquisition process is hence terminated naturally.

a "double-edged sword" here, moving the posterior belief either upward toward β_h or downward toward 0. It turns out that the expected improvement in the manufacturer's payoff when the former is reached cannot compensate for the potential loss when the latter is reached.[13]

Proposition 2: *Under ex-ante mandatory information sharing and when information acquisition is sequential, the optimal stopping rule of information acquisition when $I = y$ is characterized by two boundary points, $\underline{\beta} \in [0, \beta]$ and $\overline{\beta} \in [\beta, 1]$, such that the manufacturer continues to collect information unless the updated posterior belief $\hat{\beta}$ reaches either $\underline{\beta}$ or $\overline{\beta}$. In particular:*

i. If $\omega \leq \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$, then $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$;

ii. If $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l - \alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = \beta_l$, $\overline{\beta} = \beta_h$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l - \alpha\theta_h)}{\theta_h - \theta_l} + \frac{[\alpha(2-\alpha)\theta_h - \theta_l]\omega}{\theta_h - \theta_l}$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iii. If $\frac{\theta_l - \alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, then $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iv. If $\omega > \beta\theta_h$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_l$, $\Pi' = \alpha\beta\theta_h$, and $\pi' = 0$.

This proposition establishes an important result of the paper: The manufacturer will not want to collect an unlimited number of signals to fully resolve the quality uncertainty, even if the costs of information acquisition (both fixed and marginal) are zero. In other words, the manufacturer in equilibrium exercises self-restriction in information acquisition. The optimal information acquisition is bounded by two posterior beliefs $\underline{\beta}$ and $\overline{\beta}$, where $|\overline{\beta} - \underline{\beta}| < 1$. The incentive underlying this self-restriction in information acquisition is the strategic influence exerted on the retailer's behavior. Under ex-ante mandatory sharing, the only way the manufacturer can manipulate the retailer's belief is through controlling the acquisition of information. To control the generation of information to its own advantage, all else being equal, the manufacturer would like to stop the information acquisition process when the updated posterior belief is sufficiently high.[14] In contrast, the manufacturer would like to continue the acquisition process when the converse is true about the updated belief.

[13]This is because the market condition of interest which our analysis concentrates on is the one in which a decentralized vertical structure will not find it sufficiently profitable to serve the low type consumers without any information, i.e., $\theta_l < \alpha(2-\alpha)\theta_h$. Note that if instead $\theta_l \geq \alpha(2-\alpha)\theta_h$, then the manufacturer would continue the information acquisition process when $\hat{\beta} = \beta_l$. The manufacturer's optimal information acquisition strategy in this case can be characterized in a similar manner.

[14]Note that this does not necessarily suggest that the quality signals ultimately generated include on average more "good" ones than "bad" ones—or more generally that the empirical sample is biased upward. For example, when β is sufficiently close to β_l from above, it is almost the case that only "bad" signals would be observed in the end when the manufacturer stops collecting further information. In this case, as we have already shown, it is optimal for the manufacturer to stop when the updated posterior belief hits β_l—rather than to continue until 0 is reached.

Two points are warranted in interpreting the manufacturer's equilibrium self-restriction in generating information. First, under ex-ante sharing that is mandated in the contract, the manufacturer does not enjoy any informational advantage over the retailer, i.e., any incompleteness in information is symmetric across the firms at every point in the game. As a result, the discontinuation of information acquisition and transmission to the retailer before the true quality state is (almost) fully revealed, does not have any signaling role. Second, because the information transmitted by the manufacturer is truthful, the retailer would always believe in the received information, even though it is known that the acquisition of information has been strategically controlled by the manufacturer. Therefore, the retailer would not, even if it could, ex-ante commit not to accept and act on the ex-post shared information.

Nevertheless, in comparison to the scenario with inflexible information acquisition, the manufacturer in equilibrium acquires more information when information acquisition can be sequentially controlled. Recall that the manufacturer in equilibrium prefers not to acquire any information at all if it is constrained to generate either none or unlimited signals. Thus, paradoxically, it is only when the manufacturer can sequentially control the amount of information acquisition that there will be a positive amount of information acquired. In other words, the flexibility in the manufacturer's information acquisition leads to a larger amount of equilibrium information acquisition.

One may expect that the manufacturer can derive economic rents from sequentially controlling the information acquisition process. From Proposition 1(*ii*), for instance, it is obvious that when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, the manufacturer's expected payoff under sequential information acquisition when the information state is $I = y$ (i.e., Π') is indeed higher than that when the state is $I = \bar{y}$ (i.e., $\Pi'' = \alpha\omega$). This is demonstrated in Figure 3. Note that when the wholesale price is within this range, the retailer would order only $x = \alpha$ if it maintains its prior belief β. However, with the strategic control of information flow, the manufacturer may induce the retailer to order $x = 1$ when the updated posterior belief reaches the upper bound β_h, and never order below $x = \alpha$ even when the lower bound β_l is hit.

Interestingly, this strategic effect of sequential information acquisition works to the advantage of the manufacturer without necessarily hurting the retailer. Indeed, conditional on any wholesale price ω, the retailer's expected payoff remains the same no matter whether the manufacturer acquires information or not at all. Again, this is because the information transmitted to the retailer, although strategically manipulated, is truthful. When the retailer's updated posterior belief is manipulated upward, both its expected revenue and expected payment are increased, making the overall expected payoff remain unchanged. The overall message here is that, with the manufacturer's sequential control of information acquisition, there can be a Pareto improvement in the vertical system's

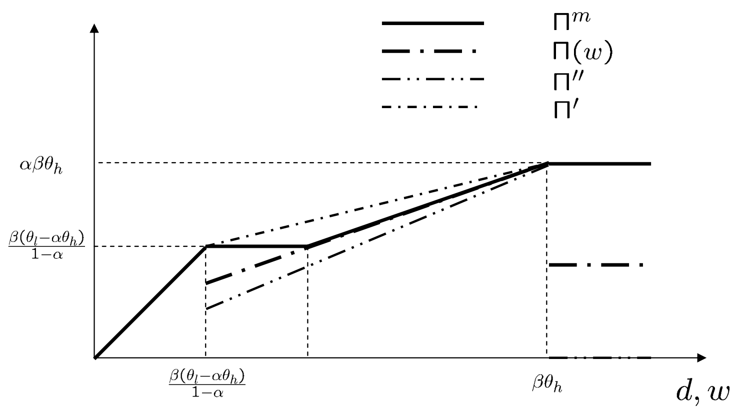

Figure 3: The Manufacturer's Expected Payoffs under Mandatory Sharing ($\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$)

expected payoffs. Essentially, the demand recession problem of being unable to fully cover the market is mitigated and the retailer's expected ordering quantity is higher, when more information is acquired and provided to the retailer to boost its updated posterior belief. Nevertheless, these payoff implications are conditional on the wholesale price remaining unchanged, which may not necessarily hold from an ex-ante perspective when the wholesale price is optimally set. We will tackle this issue in the following section.

4.2.2. Equilibrium Ex-ante Profits

We can then characterize the manufacturer's optimal wholesale price, and derive the firms' equilibrium ex-ante payoffs.

PROPOSITION 3: *Under ex-ante mandatory information sharing and when information acquisition is sequential, in equilibrium:*

(i) If β is sufficiently large (i.e., $\beta > \tilde{\beta}$), the manufacturer does not acquire any information even when $I = y$, where $\tilde{\beta} \in \left(\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}, 1\right)$ is given in the Appendix. The manufacturer's equilibrium ex-ante payoff is higher than that without information sharing (i.e., Π^{ns}), if and only if β is sufficiently small and d is intermediate.

(ii) The retailer's equilibrium ex-ante payoff can be higher than that without information sharing (i.e., π^{ns}), if both β and d are intermediate.

The first point of this proposition is that sequential information acquisition may not strictly improve the manufacturer's ex-ante payoff when the prior belief β is sufficiently high. This has to do with the uncertainty about the availability of useful information, at the time when the wholesale price is chosen. Recall that the manufacturer can benefit from sequential information acquisition (i.e., $\Pi' > \Pi''$) when the charged wholesale price is such that the retailer would order $x = \alpha$ in the absence of information sharing (i.e., $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega < \beta\theta_h$). When the prior is already high, the extent to which the retailer's belief can be manipulated upward is limited, and this constrains the potential benefit of sequential information acquisition. However, with one half probability, no information can be acquired (i.e., $I = \bar{y}$), in which case the manufacturer would have been better off had it charged a lower wholesale price (i.e., $\omega = \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$) under which the whole market is served. As a result, when β is sufficiently large, the equilibrium wholesale price is such that it is optimal for the manufacturer to acquire no information even when $I = y$.

It is only when the prior belief β is sufficiently small that the manufacturer would in equilibrium acquire information and thus benefit from sequentially controlling the information acquisition process. In this case, the difference between the manufacturer's equilibrium ex-ante profits under sequential and inflexible information acquisition, has an inverted-U relationship with the manufacturer's competitive margin d. In other words, the equilibrium ex-ante benefit from manipulating the retailer's belief through sequential information acquisition reaches its peak when d is intermediate. On the one hand, the manufacturer can charge a higher wholesale price as d increases, magnifying the benefit of sequential information acquisition in that on expectation there is greater coverage of consumer segments. While on the other hand, both β_l and β_h increase with a higher wholesale price, lowering the probability that the retailer's posterior belief reaches β_h before β_l (i.e., lowering the likelihood that both consumer segments are covered in equilibrium). Consequently the effect of d on the difference between the manufacturer's equilibrium ex-ante profits is non-monotonic.

Interestingly, from an ex-ante perspective, it is possible that the retailer can benefit from the manufacturer's sequential information acquisition. This occurs when β is neither too large such that information acquisition can arise in equilibrium, nor too small such that there exists a set of wholesale prices (i.e., $\frac{\theta_l - \alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$) under which the retailer orders at least $x = \alpha$ but never $x = 1$.[15] In this case, the manufacturer would necessarily acquire information when $\omega = \frac{\theta_l - \alpha\theta_h}{1-\alpha}$ is charged (and when $I = y$), leading to a discontinuous increase in the manufacturer's expected payoff from that when $\omega = \frac{\theta_l - \alpha\theta_h}{1-\alpha} + \epsilon$ (i.e., a sufficiently small increase in the wholesale price). Therefore, if the competitive margin d is sufficiently close to the cut-off point $\frac{\theta_l - \alpha\theta_h}{(1-\alpha)}$ from above, the equilibrium wholesale price will be $\omega = \frac{\theta_l - \alpha\theta_h}{(1-\alpha)} < d$. In contrast, in the absence of information sharing/acquisition, the optimal wholesale price would be $\omega = d$. What this means is that the prospect of sequentially manipulating information may induce the manufacturer to charge a lower

[15] Note that $\frac{\theta_l - \alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$ implies that $\beta_l \leq \beta < 1 < \beta_h$.

equilibrium wholesale price compared to the case when information acquisition is inflexible. As a result, both the manufacturer and the retailer can be strictly better off when the information flow is strategically controlled by the manufacturer, i.e., a "win-win" situation.

5. Ex-Post Voluntary Information Sharing

In this section we look at voluntary information sharing where the manufacturer can decide whether or not to share information on an ex-post basis after the information has been acquired. Thus, in contrast to the ex-ante mandatory sharing case, the retailer's updated posterior belief, $\hat{\beta}_r$, may not coincide with that of the manufacturer, $\hat{\beta}_m$, because the manufacturer can choose not to disclose the acquired signals. Formally, we can define the manufacturer's information disclosure strategy at stage 3 as $m(\hat{\beta}_m) : \hat{\beta}_m \rightarrow M \equiv \{\hat{\beta}_m, \otimes\}$, where M is the manufacturer's feasible set of messages conditional on $\hat{\beta}_m$, and $\otimes$ represents "no disclosure." Note that the manufacturer's updated posterior belief $\hat{\beta}_m$ at the time of making the disclosure decision is determined by its information acquisition strategy in the second stage of the game.

Moreover, to derive the manufacturer's optimal information acquisition and disclosure strategies, we also need to characterize the updating of the retailer's posterior belief, $\hat{\beta}_r(m)$, in response to the message received $m \in M$. Note first that under truthful disclosure, we have $\hat{\beta}_r(\hat{\beta}_m) = \hat{\beta}_m$. We then have to derive the retailer's updated posterior belief $\hat{\beta}_r(\otimes)$ in the event of a "no disclosure" message. This message may be sent by the manufacturer, for example, either when no useful information is available (i.e., $I = \bar{y}$), or when the realized information state is $I = y$ and the manufacturer withholds its acquired information. This implies that the retailer's updated posterior belief $\hat{\beta}_r(\otimes)$ is influenced by the manufacturer's optimal information acquisition and disclosure strategies, which need to be characterized simultaneously with the derivation of the retailer's belief updating.

5.1. Inflexible Information Acquisition

Let us start with deriving the manufacturer's optimal information acquisition and sharing strategies. Recall that in this scenario the manufacturer can acquire either none or an infinite number of quality signals. Suppose that the manufacturer choose to acquire information when $I = y$, and thus with probability β learn that the true quality state is $S = G$, or $S = B$ with probability $1 - \beta$. Recall also that the retailer's optimal ordering quantity, and thus the manufacturer's expected payoff, conditional on a given wholesale price ω, increases with the retailer's updated posterior belief $\hat{\beta}_r$. As a result, when it is revealed to the manufacturer that the true quality sate is G (i.e., $\hat{\beta}_m = 1$), it is in the best interest of the manufacturer to disclose the information and move the retailer's updated posterior belief toward $\hat{\beta}_r = \hat{\beta}_m = 1$. Conversely, when the manufacturer learns that $\hat{\beta}_m = 0$, it would keep silent and choose not to disclose the information to the retailer. This implies that the manufacturer's optimal information sharing strategy is: $m(1) = 1$, and $m(0) = \otimes$. Therefore, the "no disclosure" message $m = \otimes$ would be sent by the manufacturer either when $I = \bar{y}$, or when $I = y$ and the manufacturer acquires and conceals its updated posterior belief (i.e., $m(0) = \otimes$). The ex-ante probabilities for these two scenarios are given by $1/2$ and $(1-\beta)/2$, respectively. Using the Bayes theorem, we therefore have $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$.

We can then determine whether or not the manufacturer would acquire information when useful information is available (i.e., $I = y$), and derive the manufacturer's equilibrium ex-ante payoff.

PROPOSITION 4: *Under ex-post voluntary information sharing and when information acquisition is inflexible, in equilibrium the manufacturer decides to acquire information when the information state is $I = y$. The manufacturer's equilibrium ex-ante payoff is higher than that in the benchmark without information sharing (i.e., Π^{ns}) if and only if* $\frac{4\theta_l - 2\alpha(3-\alpha)\theta_h}{2\theta_l + [1-\alpha(4-\alpha)]\theta_h} < \beta < \frac{2\alpha}{1+\alpha}$ *and* $\frac{2\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \min\left\{\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}, \frac{[\alpha(2-\beta)+\beta](\theta_l - \alpha\theta_h)}{2\alpha(1-\alpha)}\right\}$, *and (weakly) lower if otherwise..*

Several interesting results emerge from this proposition. These results pertain to the impacts of the manufacturer's flexibility in information sharing, which can be obtained by comparing the voluntary sharing case in this section with the mandatory sharing case in Section 4.1—recall that the latter case is in equilibrium equivalent to the benchmark without information sharing. First, this proposition indicates that, in contrast to the case of mandatory information sharing, acquiring information is an equilibrium strategy for the manufacturer under voluntary sharing. This implies that the manufacturer would acquire information when it is not bound to disclose all the acquired information. Intuitively, when information sharing is mandatory, the manufacturer is forced to disclose information that is ex post unfavorable compared to the prior belief, which may not be desirable prior to the acquisition of information. On the other hand, when information sharing is voluntary, the manufacturer can withhold the ex post unfavorable information but disclose the favorable one. The retailer hence maintains the same updated posterior beliefs either when unfavorable quality signals are generated or when no information is acquired, but moves its posterior belief upward when favorable information is received. Information acquisition is therefore a dominant strategy for the manufacturer when useful information is available.

Second, this proposition suggests that, when information acquisition is inflexible, the manufacturer may or may not benefit from the flexibility to selectively disclose favorable information while concealing unfavorable information. To understand this, let us investigate the two effects exerted by voluntary information disclosure that, in contrast to the mandatory sharing case, induces the manufacturer to collect information in equilibrium. As discussed previously, when the acquired information conveys negative news, the low segment may be unserved, which may be otherwise served without information acquisition when the prior belief is maintained by the retailer, i.e., $\hat{\beta}_r(\otimes) < \beta$. As a result, there may be a market recession effect of information acquisition. Interestingly, this market recession effect may arise even in the scenario when no useful information is available (i.e., $I = \bar{y}$). This is because, the retailer cannot distinguish between the scenarios when the manufacturer withholds unfavorable information and when no useful information is available. While on the other hand, the manufacturer can benefit from acquiring information, if the consumer segments are more likely to be served when the acquired information is favorable than when no information is acquired, i.e., $\beta < \hat{\beta}_r(1) = 1$.

The above discussion suggests that the net effect of voluntary information sharing hinges on whether or not a higher equilibrium market coverage is induced on average across the two ex-post sharing scenarios (i.e., $\hat{\beta}_r(\otimes)$ and $\hat{\beta}_r(1)$). It turns out that, as the proof demonstrates, whether or not the overall impact of voluntary information sharing is beneficial depends upon the comparison between the manufacturer's equilibrium payoffs under the following two scenarios: 1) in the case under voluntary information sharing, only the high segment consumers are served in equilibrium when the retailer's updated posterior belief is given by $\hat{\beta}_r(\otimes)$, and both consumer segments are covered when $\hat{\beta}_r = 1$, i.e., $\Pi^{iv} = [\alpha(2-\beta)+\beta]d/2$; and 2) in the case without information sharing/acquisition when the retailer maintains the prior belief β, only the high segment consumers are served in equilibrium, i.e., $\Pi^{ns} = \alpha d$. In particular, the manufacturer's equilibrium payoff is higher under voluntary information sharing, if and only if the equilibrium payoff at the upper boundary under scenario 1) is higher than that at the lower boundary under scenario 2). Intuitively, it is only then that the equilibrium market coverage can be enhanced when the acquired information is favorable, but not lowered when the information is unfavorable.

The necessary and sufficient condition for voluntary information sharing to be beneficial requires that the manufacturer's competitive margin d is intermediate. When d is small, the manufacturer is constrained to set low wholesale prices such that both consumers segments are served in equilibrium in the case without information sharing/acquiaition. Conversely, when d is sufficiently large, excessively high wholesale prices would be charged, such that no consumer would be served when the retailer's updated posterior belief is given by $\hat{\beta}_r(\otimes)$, and/or only the high segment would be covered when $\hat{\beta}_r = 1$. In the same vein, the manufacturer is better off with voluntary information sharing only if it is satisfied that $\frac{4\theta_l - 2\alpha(3-\alpha)\theta_h}{2\theta_l + [1-\alpha(4-\alpha)]\theta_h} < \beta < \frac{2\alpha}{1+\alpha}$. Note that the upper boundary for the above scenario 1) is given by $\min\{\hat{\beta}_r(\otimes)\theta_h, \frac{\hat{\beta}_r(1)(\theta_l - \alpha\theta_h)}{1-\alpha}\}$, and the lower boundary for scenario 2) is $\frac{\beta(\theta_l - \alpha\theta_h)}{\alpha(1-\alpha)}$. As a result, the prior belief β cannot be too small, because the extent to which the market is recessed under unfavorable information (and/or when no useful information is available), i.e., $\frac{\beta-\hat{\beta}_r(\otimes)}{\beta}$, decreases with β. Nevertheless, when the low segment becomes unimportant (i.e, $4\theta_l - 2\alpha(3-\alpha)\theta_h < 0$) such that market recession is less of a concern, voluntary information sharing can be beneficial even when β goes to zero. On the other hand, the prior belief cannot be too large either, because otherwise the potential benefit from inducing higher market coverage through information acquisition would become less important as β approaches $\hat{\beta}_r = 1$.

5.2. Sequential Information Acquisition

5.2.1. Optimal Information Acquisition and Sharing Strategies

Let us first derive the manufacturer's optimal information acquisition and sharing strategies when $I = y$. The manufacturer sequentially decides on the number of quality signals to be collected and whether or not to terminate the information acquisition process at each updated posterior belief. Next, the manufacturer determines whether or not to disclose the acquired information when it terminates information acquisition (i.e., $\hat{\beta}_m$) to the retailer. If the manufacturer's optimal information acquisition strategy is similar to that in Section 4.2.1—that is, continuing to collect information unless the updated posterior belief reaches either $\underline{\beta} \in [0, \beta]$ or $\overline{\beta} \in [\beta, 1]$—then in equilibrium we have $\hat{\beta}_m \in \{\underline{\beta}, \overline{\beta}\}$. This implies that the manufacturer's optimal disclosure strategy is influenced by its stopping rule for information acquisition, which in turn is affected by whether or not the manufacturer would choose to disclose the collected information.

To proceed, we will first take as given the retailer's updated belief $\hat{\beta}_r(\otimes)$ when it receives no information from the manufacturer, in deriving the manufacturer's optimal information acquisition and disclosure strategies. We then investigate the equilibrium conditions under which the retailer's updated belief $\hat{\beta}_r(\otimes)$ can be sustained, taking into account the derived information acquisition and disclosure strategies. Suppose first that $\hat{\beta}_r(\otimes) \geq \beta_h$ (or $\beta_l \leq \hat{\beta}_r(\otimes) < 1 < \beta_h$). That is, sending the "no disclosure" message leads the retailer to order the maximum possible level 1 (or α). It follows that, starting from the prior belief β, the manufacturer would not collect any information at all. The retailer's rational expectation hence requires that $\hat{\beta}_r(\otimes) = \beta$. As a result, such a retailer's belief can be sustained if and only if $\beta \geq \beta_h$ (or $\beta_l \leq \beta < 1 < \beta_h$).

Suppose then $\beta_l \leq \hat{\beta}_r(\otimes) < \beta_h \leq 1$. Following the "no disclosure" message, the retailer would order an amount equal to $x = \alpha$. Note that the manufacturer would strictly prefer to stop and disclose when $\hat{\beta}_m \geq \beta_h$. When $\hat{\beta}_m < \beta_h \leq 1$, it is strictly dominant for the manufacturer to accumulate more information, and disclose the acquired information if $\hat{\beta}_m = \beta_h$ is reached or withhold the information if $\hat{\beta}_m = 0$ is reached. As a result, such a retailer's updated belief $\hat{\beta}_r(\otimes)$ would not be sustained if $\beta \geq \beta_h$. This is because the "no disclosure" message would then be sent only when $I = \bar{y}$, leading to $\hat{\beta}_r(\otimes) = \beta$, which is a contradiction to $\hat{\beta}_r(\otimes) < \beta_h$. If instead $\beta < \beta_h \leq 1$, given the manufacturer's optimal information acquisition and disclosure strategies, the retailer is expected to receive no quality signals from the manufacturer when: 1) the information state is $I = \bar{y}$; or 2) the information state is $I = y$ and the manufacturer's updated belief reaches $\hat{\beta}_m = 0$. Noticing that the ex-ante probabilities for these scenarios are 1/2 and $(1 - \Phi(\beta|0, \beta_h))/2 = (1 - \beta/\beta_h)/2$, respectively, one can obtain:

$$\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1 - \beta/\beta_h)/2} = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega - \beta(\theta_l - \alpha\theta_h)}, \tag{3}$$

which sustains $\beta_l \leq \hat{\beta}_r(\otimes) < \beta_h \leq 1$ if and only if $\frac{2(1-\alpha)\omega}{\theta_l + (1-2\alpha)\theta_h} \leq \beta < \beta_h \leq 1$.

Next, suppose that $\hat{\beta}_r(\otimes) < \beta_l$. In the case, the retailer would order zero if no information is disclosed by the manufacturer. It is then strictly dominant for the manufacturer to disclose its acquired information, as long as the updated belief $\hat{\beta}_m$ when the manufacturer stops information accumulation is not less than β_l. When $\hat{\beta}_m < \beta_l$, the manufacturer would continue to collect information until either β_l or 0 is hit. One may therefore obtain that the manufacturer's optimal information acquisition strategy is the same as that under mandatory sharing in Section 4.2.1. That is, the manufacturer would continue the information acquisition process if and only if $\hat{\beta}_m < \beta_l$ or $\beta_l < \hat{\beta}_m < \beta_h \leq 1$. Given this, we can determine how the retailer's posterior belief should be updated. Note first that when $\beta \geq \beta_h$ or $\beta_l \leq \beta < 1 < \beta_h$, the manufacturer would not acquire any information, leading to the contradicting result $\hat{\beta}_r(\otimes) = \beta$. When $\beta_l \leq \beta < \beta_h \leq 1$, the manufacturer continues to collect information until either β_h or β_l is reached, which will be disclosed to the retailer; so the only scenario under which the retailer may receive the "no disclosure" message is when $I = \bar{y}$, which again leads to the contradicting result $\hat{\beta}_r(\otimes) = \beta$. When $\beta < \beta_l$, the information acquisition process continues until β_l or 0 is reached, and the manufacturer would disclose the former situation but withhold the latter; so we have $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2 + (1-\beta/\beta_l)/2} = \frac{\beta\omega}{2\omega - \beta\theta_h} < \beta_l$. Overall, there exists an equilibrium where $\hat{\beta}_r(\otimes) < \beta_l$ is satisfied, if and only if $\beta < \beta_l$.

It follows that there does not exist a pure-strategy equilibrium when $\beta_l \leq \beta < \frac{2(1-\alpha)\omega}{\theta_l + (1-2\alpha)\theta_h}$ (and $\beta_h \leq 1$). In the mixed-strategy equilibrium, it must be that: 1) the manufacturer is indifferent and randomizes between continuing and stopping information acquisition when its updated belief reaches β_l; and 2) the retailer is indifferent and randomizes between ordering α and 0 when it receives no information from the manufacturer. Let us define the probability as λ that the manufacturer continues to accumulate signals when $\hat{\beta}_m = \beta_l$, and the probability as ρ (or $1 - \rho$) that the retailer orders $x = \alpha$ (or $x = 0$) when $m = \otimes$. The mixed-strategy equilibrium requires:

$$\alpha\omega = \Phi(\beta_l|0,\beta_h)\omega + [1-\Phi(\beta_l|0,\beta_h)]\rho\alpha\omega, \tag{4}$$

$$\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+\lambda(1-\beta/\beta_h)/2} = \beta_l, \tag{5}$$

which lead to $\rho = \frac{\alpha\beta_h-\beta_l}{\alpha(\beta_h-\beta_l)} = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)} \in (0,1)$, and $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)} = \frac{(1-\alpha)(\beta\theta_h-\omega)}{(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)} \in [0,1)$, respectively. Summarizing the above discussions, we have the following proposition:

Proposition 5: *Under ex-post voluntary information sharing and when information acquisition is sequential, the optimal stopping rule of information acquisition when $I=y$ is characterized by two boundary points, $\underline{\beta} \in [0,\beta]$ and $\overline{\beta} \in [\beta,1]$, such that the manufacturer continues to collect information unless the updated posterior belief $\hat{\beta}_m$ reaches either $\underline{\beta}$ or $\overline{\beta}$; the optimal disclosure strategy is characterized by $m(\underline{\beta}) \in \{\underline{\beta},\otimes\}$ and $m(\overline{\beta}) = \overline{\beta}$, when the manufacturer stops information acquisition at $\underline{\beta}$ or $\overline{\beta}$, respectively; and the retailer's updated belief is given by $\hat{\beta}_r(m) = m$ if $m \neq \otimes$, and by $\hat{\beta}_r(\otimes)$ if $m = \otimes$. In particular:*

i. If $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, then $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$;

ii. If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_h$, $m(\underline{\beta}) = \otimes$, $\hat{\beta}_r(\otimes) = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)}$, $\Pi' = \beta(\theta_l-\alpha\theta_h)+\alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

iii. If $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, then $\underline{\beta} = 0$ or $\underline{\beta} = \beta_l$ with probability $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)}$ or $1-\lambda$ respectively, $\overline{\beta} = \beta_h$, $m(0) = \otimes$, $m(\beta_l) = \hat{\beta}_r(\otimes) = \beta_l$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and $\pi' = \frac{[\alpha^2\theta_h+(1-2\alpha)\theta_l](\beta\theta_h-\omega)}{\theta_h-\theta_l}$;

iv. If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, then $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$;

v. If $\omega > \beta\theta_h$, then $\underline{\beta} = 0$, $\overline{\beta} = \beta_l$, $m(\underline{\beta}) = \otimes$, $\hat{\beta}_r(\otimes) = \frac{\beta\omega}{2\omega-\beta\theta_h}$, $\Pi' = \alpha\beta\theta_h$, and $\pi' = 0$.

In comparison to the case under mandatory sharing (i.e., Proposition 2), voluntary information sharing leads the manufacturer to generate a larger amount of information.[16] This implies that, similar to the cases when information acquisition is inflexible (i.e., Sections 4.1 and 5.1), the manufacturer's flexibility to selectively disclose its acquired information can promote more information acquisition. When $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, for example, the lower bound at which the

[16] To see this, note that parts *i*, *iv*, and *v* of Proposition 5 correspond to the ranges of ω which are the same as parts *i*, *iii*, and *iv*, respectively in Proposition 2, and the amount of information acquisition is identical in both propositions. Then by comparing the amount of information acquisition under ex-post voluntary sharing in parts *ii* and *iii* of Proposition 5 to that under ex-ante mandatory sharing in part *ii* of Proposition 2, we can see that ex-post voluntary sharing leads to greater information acquisition.

manufacturer stops collecting information is $\underline{\beta} = \beta_l$ under mandatory information sharing, but can be extended to $\underline{\beta} = 0$ if information sharing is ex-post voluntary. This is because, when the updated posterior belief is at $\hat{\beta}_m = \beta_l$, if the information acquisition process continues and the acquired information turns out to be unfavorable (i.e., $\hat{\beta}_m$=0), the manufacturer can strategically withhold the unfavorable information and therefore does not necessarily induce less retail ordering.

Interestingly, there may exist a mixed-strategy equilibrium (i.e., part *iii* of Proposition 5) where the manufacturer randomizes between stopping information acquisition at $\underline{\beta} = 0$ and at $\underline{\beta} = \beta_l$. In contrast to the mandatory sharing case, here voluntary information disclosure leads the manufacturer to randomize the lower bound of information acquisition. Accordingly, the retailer's ordering amount, when no information is disclosed by the manufacturer, is also a randomized one.

However, the flexibility for the manufacturer to sequentially determine how much information to acquire does not necessarily promote the amount of information generated in equilibrium. Whether or not it does so depends upon the nature of the information sharing format. Consider first voluntary information sharing. Recall from Proposition 4 that, when information acquisition is inflexible and the manufacturer is constrained to acquire either none or perfect information, acquiring information is a dominant strategy for the manufacturer when $I = y$. However, as can be seen from Proposition 5, the manufacturer's ability to sequentially control the amount of acquired information actually leads to less information generation. In this case the firms may in equilibrium find out with certainty whether or not the product is "bad" (when the lower bound of information acquisition is at $\underline{\beta} = 0$), but cannot find out with certainty whether or not the product is "good" (i.e., $\overline{\beta} < 1$). Thus, under ex-post voluntary information sharing, the equilibrium amount of information generated decreases from perfect information when information acquisition is inflexible to less than perfect information when the manufacturer can sequentially decide how much information to acquire. This result stands in contrast to the case of ex-ante mandatory information sharing, where the equilibrium amount of information generated increases from zero when information acquisition is inflexible to a positive amount when the manufacturer can sequentially decide how much information to acquire. Overall, this suggests that the equilibrium amount of information acquired depends upon the interaction between the flexibility in information acquisition (sequential versus inflexible) and the flexibility in information sharing (ex-post voluntary versus ex-ante mandatory).

5.2.2. Equilibrium Ex-ante Profits

Let us then examine the firms' equilibrium ex-ante payoffs. To this end, note that the retailer's updated posterior belief is characterized in Proposition 5 when $I = y$ is realized, and given by $\hat{\beta}_r(\otimes)$ when $I = \bar{y}$. We will focus on the comparison with the mandatory sharing case in Section 4.2.

Proposition 6: *Under ex-ante voluntary information sharing and when information acquisition is sequential, in equilibrium:*

(i) If both β and d are intermediate, the manufacturer's equilibrium ex-ante payoff is higher than that under mandatory information sharing.

(ii) If β is sufficiently small, the manufacturer's (retailer's) equilibrium ex-ante payoff can be lower (higher) than that under mandatory information sharing.

This proposition indicates that, when information acquisition is sequential, ex-post voluntary information sharing can benefit the manufacturer in comparison to mandatory information sharing only if the prior belief on product quality is immediate, but can hurt the manufacturer if the prior quality belief is sufficiently low. To understand this result, note that under voluntary information sharing, the manufacturer can selectively disclose favorable information but withhold unfavorable one. This flexibility induces the manufacturer to collect more (unfavorable) information, extending the lower bound $\underline{\beta}$ of information acquisition to zero. This increase in information generation has two offsetting effects on the manufacturer's payoff. On the one hand, it increases the likelihood that the upper bound $\overline{\beta}$ is reached in which case the retailer would order $x = 1$, i.e., $\Phi(\beta|\underline{\beta}, \overline{\beta}) = \frac{\beta - \underline{\beta}}{\overline{\beta} - \underline{\beta}}$ decreases with the lower bound $\underline{\beta}$. This increases the manufacturer's expected payoff Π' when the information state is $I = y$. On the other hand, collecting more unfavorable information may drive down the retailer's updated posterior belief when no information is disclosed by the manufacturer. That is, $\hat{\beta}_r(\otimes)$ is lower when the lower bound is extended to $\underline{\beta} = 0$. This in turn has a negative effect on the retailer's ordering quantity and thus on Π'' when the information state is $I = \bar{y}$.

Therefore, whether or not voluntary sharing leads to a higher equilibrium ex-ante payoff for the manufacturer hinges on whether or not the retailer is induced to order less when in equilibrium its updated posterior belief is $\hat{\beta}_r(\otimes)$. Recall that in the mixed-strategy equilibrium when the retailer may order an amount of zero with probability $1 - \rho$, $\hat{\beta}_r(\otimes)$ needs to be sufficiently close to β_l from above. Note also that $\hat{\beta}_r(\otimes)$ is positively related to the prior belief β. As a result, when the prior belief β is not sufficiently small, the mixed-strategy equilibrium will not arise and the increase in the acquired information under voluntary sharing does not lead to lower retail ordering. Nevertheless, when the prior belief β become sufficiently large, the maximum ordering of one unit would be induced in equilibrium even under mandatory sharing. Thus, it is only when β is intermediate that the manufacturer is better off under voluntary sharing.

In contrast, when β is sufficiently small, the mixed-strategy equilibrium would occur if the wholesale price (and hence β_l) is sufficiently high. Voluntary information sharing may then induce a lower amount of retail ordering, hurting the manufacturer's equilibrium ex-ante profit.Moreover, when this effect becomes sufficiently important, the manufacturer is induced to charge a lower

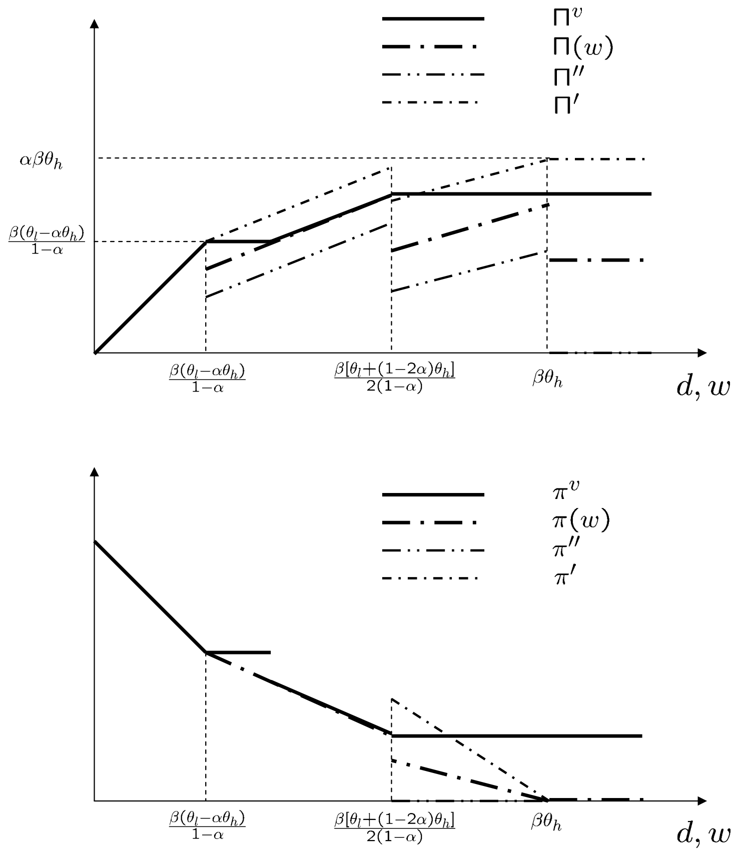

Figure 4: The Firms' Expected Payoffs under Voluntary Sharing ($\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$)

wholesale price than its competitive margin d to prevent this demand recession problem. However, note that in the analogous scenario under mandatory information sharing, the manufacturer would in equilibrium charge $\omega = d$. This explains why there may exist equilibrium conditions under which the manufacturer is worse off, while the retailer is better off, when the former can selectively disclose its acquired information, i.e., a "lose-win" situation. The firms' expected payoffs with voluntary sharing are shown in Figure 4 for the case when $\beta < \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$.

In summary, the economic forces underlying the effect of the flexibility in information sharing (i.e., ex-post voluntary versus ex-ante mandatory) on the manufacturer's equilibrium ex-ante profits are similar to those when information acquisition is inflexible (i.e., Section 5.1): Sharing information voluntarily with the retailer leads to an increasing incentive for the manufacturer to acquire information, whereas generating more information is a "double-edged sword" that can in equilibrium improve the manufacturer's ex-ante payoff when and only when on average more retailer ordering is induced. These results allow us to determine the conditions under which the manufacture should pursue the flexibility in disclosing its acquired information to the retailer. Counter-intuitively, the manufacturer may not necessarily prefer to maintain its flexibility in information sharing. In particular, it is proposed that the manufacturer's preference for the ex-ante commitment to mandatorily share information is influenced by the prior belief on product quality. When the prior belief is sufficiently low and thus the product is more likely to be a bad fit, the manufacturer may want to commit to mandatorily share the acquired information with the retailer and thereby choose to give up the flexibility to voluntarily disclose information. Mandatorily committing to disclose the collected information to the retailer can then serve as a self-disciplining device to acquire less information, which may paradoxically induce the retailer to order higher amounts when the likelihood of product failure is high. For example, in many fashion markets only a small fraction of designs may ultimately succeed in any given season. In such environments, ex-ante mandatory commitments by manufacturers to share the to-be-acquired information can be valuable. Not surprisingly, a number of successful mandatory information sharing arrangements have been documented in markets such as fashion apparels and consumer electronics (Hammond et. al 1991).

6. Concluding Remarks

Today's markets are increasingly characterized by sequentially gathering and voluntarily sharing consumer information between parties in vertical relationships. Yet the majority of the existing literature focus on either non-sequential information acquisition or mandatory sharing, each of which is typically analyzed in isolation and normally in horizontal oligopolies. Our paper is distinct in that it jointly examines both sequential information acquisition and either mandatory or voluntary information transmission by a manufacturer in a vertical system. This is theoretically significant and allows us to understand the strategic interaction between these two decisions, and how it is influenced by both the flexibility in information acquisition and the flexibility in the sharing format. The economic forces that drive the equilibria of the interactive acquisition and sharing decisions in our paper involve the strategic effects on retailer actions and on double marginalization.

We derive some interesting results that can shed light on manufacturers' optimal information acquisition and sharing strategies. First, with the flexibility to sequentially control information generation, a manufacturer should exercise self-restriction in information acquisition and refrain from acquiring perfect information, even if it is costless to do so, in order to manipulate downstream retailer behavior. However, when sequential information acquisition becomes infeasible, the manufacturer should acquire either more or less information, depending upon whether the acquired information is to be voluntarily or mandatorily shared with downstream retailers, respectively. In contrast, irrespective of the level of flexibility in information acquisition, the flexibility to selectively decide whether or not to share the acquired information should generally lead the manufacturer to collect more information. The follows from the manufacturer's ability under voluntary sharing to credibly disclose favorable information while withholding unfavorable one. Moreover, our analysis demonstrates that mandatory sharing arrangements may actually be preferred by manufacturers when the prior belief on quality is sufficiently low.

It is instructive to discuss the effects of the costs of information acquisition. The effect of fixed costs is straightforward, which simply reduces the ex-ante payoffs of the manufacturer. If the fixed costs of information acquisition become sufficiently large, then no information will be collected irrespective of the information sharing scenarios. Positive marginal costs will also reduce the amount of information collected as they determine the optimal stopping thresholds for information collection. However, the impact on manufacturer payoffs may differ across the ex-ante mandatory and ex-post voluntary sharing arrangements. Higher marginal costs can substitute for manufacturer self-restriction in information collection. Given that voluntary sharing involves greater amount of equilibrium information collection, this suggests that mandatory sharing is more likely to be preferred in environments with higher marginal costs of acquisition. Similarly, one may expect to see reductions in information acquisition when the costs of information sharing are positive.

One problem which seems to be good candidate for future investigation is the role of explicit and costly mechanisms that will induce truth-telling in ex-post voluntary sharing arrangements. Such an analysis would be similar to Ziv (1993), and will shed light on the trade-off between information sharing and signaling incentives for firms in vertical relationships. Another direction for future research is to take the downstream retailer's perspective, and study the acquisition and transmission of information (e.g., demand) that is more accessible to retailers. One can also investigate bilateral information transmission in a more comprehensive framework in which firms in a vertical system have access to different types of consumer/market information (e.g., demand versus quality), which can be acquired and exchanged to influence each other's decision making. Of course, all these issues, including but not confined to sequential information acquisition and voluntary sharing that are currently studied in this paper, are not necessarily unique to vertical relationships and can be examined analogously in contexts with horizontal oligopolies.

APPENDIX

Proof of Lemma 1: Let us define $\Psi^S(N) \equiv \Phi^S(\hat{\beta}(N)|\hat{\beta}_L, \hat{\beta}_H)$ as the conditional probability (on $S \in \{G, B\}$) that the net number of "good" signals reaches N_H before N_L, as a function of the starting point $N \in [N_L, N_H]$. To obtain the solution to the function $\Psi^S(N)$, note that by definition we have:

$$\Psi^G(N) = \gamma\Psi^G(N+1) + (1-\gamma)\Psi^G(N-1), \text{ where } N \in \{N_L+1, \ldots, N_H-1\}.$$

This is a second-order difference equation, where the boundary conditions can be obtained from the definition of $\Psi^S(N)$: $\Psi^G(N_L) = 0$ and $\Psi^G(N_H) = 1$. We can then obtain the solution:

$$\Psi^G(N) = \frac{1-(\frac{1-\gamma}{\gamma})^{(N-N_L)}}{1-(\frac{1-\gamma}{\gamma})^{(N_H-N_L)}}, \text{ where } N \in \{N_L, \ldots, N_H\}. \tag{i}$$

Similarly, we have the second-order difference equation conditional on $S = B$: $\Psi^B(N) = (1-\gamma)\Psi^B(N+1) + \gamma\Psi^B(N-1)$, where $N \in \{N_L+1, \ldots, N_H-1\}$, $\Psi^B(N_L) = 0$, and $\Psi^B(N_H) = 1$. The solution is thus given by:

$$\Psi^B(N) = \frac{1-(\frac{\gamma}{1-\gamma})^{(N-N_L)}}{1-(\frac{\gamma}{1-\gamma})^{(N_H-N_L)}}, \text{ where } N \in \{N_L, \ldots, N_H\}. \tag{ii}$$

Note that by definition we have $\hat{\beta}_L \equiv \hat{\beta}(N_L) = \frac{\dot{\beta}}{\dot{\beta}+(1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{N_L}}$, and $\hat{\beta}_H \equiv \hat{\beta}(N_H) = \frac{\dot{\beta}}{\dot{\beta}+(1-\dot{\beta})(\frac{1-\gamma}{\gamma})^{N_H}}$, which can be inverted to obtain N_L as a function of $\hat{\beta}_L$ and N_H as a function of $\hat{\beta}_H$, respectively. Plugging these inverted solutions into (i) and (ii), respectively, we therefore have $\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^G(0) = \frac{\hat{\beta}_H(\dot{\beta}-\hat{\beta}_L)}{\dot{\beta}(\hat{\beta}_H-\hat{\beta}_L)}$, and $\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \Psi^B(0) = \frac{(1-\hat{\beta}_H)(\dot{\beta}-\hat{\beta}_L)}{(1-\dot{\beta})(\hat{\beta}_H-\hat{\beta}_L)}$. Finally, we have $\Phi(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) \equiv \dot{\beta}\Phi^G(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) + (1-\dot{\beta})\Phi^B(\dot{\beta}|\hat{\beta}_L, \hat{\beta}_H) = \frac{\dot{\beta}-\hat{\beta}_L}{\hat{\beta}_H-\hat{\beta}_L}$.

Q.E.D.

Proof of Proposition 1: Suppose that the manufacturer acquire information when $I = y$. The manufacturer's expected payoff is hence given by $\Pi = \beta\omega$ if $\omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi = \alpha\beta\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \theta_h$, and $\Pi = 0$ if otherwise. This expected payoff is higher than that when no information is acquired at all, if and only if $\beta > \alpha$ and $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, or $\beta\theta_h < \omega \leq \theta_h$.

Let us then derive the equilibrium wholesale price. Consider first the case when $d \leq \beta\theta_h$. If furthermore $\beta < \alpha$, the manufacturer's expected payoff with information acquisition is always lower than that when no information is acquired at all. If instead $\beta > \alpha$, information acquisition is desirable if and only if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. However, charging a wholesale price in the range $\omega \in \left(\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\right]$, irrespective of the subsequent information acquisition decision, is ex ante dominated by charging $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$. Similarly, in the case $d > \beta\theta_h$, it is always dominant for the manufacturer to charge $\omega = \beta\theta_h$. As a result, the equilibrium wholesale price is the same as that in the benchmark without information sharing. The proposition follows.

Q.E.D.

Proof of Proposition 2: Let us first characterize whether or not the manufacturer would collect an extra piece of information when the updated belief is $\hat{\beta} \in (0,1)$. When $\hat{\beta} \geq \beta_h$, the retailer would order $x = 1$ and it is obvious that no further information would be acquired. When $\hat{\beta} < \beta_l$, the retailer would order $x = 0$, if the manufacturer immediately stops collecting further information, which is obviously sub-optimal. When $\beta_l < \hat{\beta} < \beta_h \leq 1$, the retailer would order $x = \alpha$ if no further information is to be collected. The manufacturer can do better by continuing the acquisition process until either β_h or β_l is reached, by which time the retailer would order $x = 1$ or $x = \alpha$, respectively. When $\beta_l \leq \hat{\beta} < 1 < \beta_h$, the retailer would order $x = \alpha$ that is the maximum possible level, which implies that the manufacturer would stop the acquisition process. Finally, when $\hat{\beta} = \beta_l$ and $\beta_h \leq 1$, the manufacturer would not collect more information if and only if $\alpha\omega > \Phi(\beta_l|0,\beta_h)\omega$, which is indeed true given that $\Phi(\beta_l|0,\beta_h) = \frac{\beta_l}{\beta_h} = \frac{\theta_l - \alpha\theta_h}{\theta_h - \alpha\theta_h}$ and $\theta_l < \alpha(2-\alpha)\theta_h$.

Therefore, if $\omega \leq \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$ (i.e., $\beta \geq \beta_h$), then in equilibrium the manufacturer would not start the information acquisition process and the retailer would order $x = 1$. It follows that $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$.

If $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l - \alpha\theta_h}{1-\alpha}\}$ (i.e., $\beta_l \leq \beta < \beta_h \leq 1$), then in equilibrium the manufacturer would continue the information acquisition process until the updated posterior belief $\hat{\beta}$ reaches either $\underline{\beta} = \beta_l$ or $\overline{\beta} = \beta_h$. The manufacturer's expected payoff is given by $\Pi' = \Phi(\beta|\beta_l,\beta_h)\omega + [1 - \Phi(\beta|\beta_l,\beta_h)]\alpha\omega = \frac{(1-\alpha)\beta\theta_h(\theta_l - \alpha\theta_h)}{\theta_h - \theta_l} + \frac{[\alpha(2-\alpha)\theta_h - \theta_l]\omega}{\theta_h - \theta_l}$, and the retailer's given by $\pi' = \beta[\Phi^G(\beta|\beta_l,\beta_h)\theta_l + [1 - \Phi^G(\beta|\beta_l,\beta_h)]\alpha\theta_h] - [\Phi(\beta|\beta_l,\beta_h)\omega + [1 - \Phi(\beta|\beta_l,\beta_h)]\alpha\omega] = \alpha\beta\theta_h - \alpha\omega$.

If $\frac{\theta_l - \alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$ (i.e., $\beta_l \leq \beta < 1 < \beta_h$), then in equilibrium the manufacturer would not collect any information and the retailer's order is given by $x = \alpha$, which leads to $\underline{\beta} = \overline{\beta} = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$.

Finally, if $\omega > \beta\theta_h$ (i.e., $\beta < \beta_l$), then in equilibrium the manufacturer would continue the information acquisition process until either it is learned almost with certainty that the true quality state is bad or the updated posterior belief $\hat{\beta}$ reaches β_l, i.e., $\underline{\beta} = 0$ and $\overline{\beta} = \beta_l$. The manufacturer's expected payoff is given by $\Pi' = \Phi(\beta|0,\beta_l)\alpha\omega = \alpha\beta\theta_h$, and the retailer's given by $\pi' = \beta\Phi^G(\beta|0,\beta_l)\alpha\theta_h - \Phi(\beta|0,\beta_l)\alpha\omega = 0$.

Q.E.D.

Proof of Proposition 3: Note that there are two possible realizations of the information state. When $I = \bar{y}$, no useful information is available and the manufacturer's expected payoff, conditional on $\omega \leq d$, is given by $\Pi'' = \omega$ if $\omega \leq \frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha}$, $\Pi'' = \alpha\omega$ if $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \leq \beta\theta_h$, and $\Pi'' = 0$ if otherwise. When $I = y$, the manufacturer would acquire information if and only if $\frac{\beta(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l - \alpha\theta_h}{1-\alpha}\}$ or $\omega > \beta\theta_h$, where its expected payoff is given by $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ or $\Pi' = \alpha\beta\theta_h$, respectively; if otherwise, we have $\Pi' = \Pi''$. To derive the equilibrium wholesale price and the firms' ex-ante payoffs, let us consider two alternative parameter ranges.

Range (i): $\beta \leq \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, we have $\beta\theta_h \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. Therefore, when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < d \leq \beta\theta_h$, the manufacturer would charge $\omega = d$ if $[\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]d}{\theta_h-\theta_l}]/2 + \alpha d/2 \geq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if otherwise. The firms' equilibrium ex-ante payoffs are then given by, respectively:

$$\Pi_i^m = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}; \\ \min\left\{\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]d}{2(\theta_h-\theta_l)}, \alpha\beta\theta_h\right\}, & \text{if otherwise.} \end{cases}$$

$$\pi_i^m = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \leq \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}; \\ \alpha\beta\theta_h - \alpha\min\{d, \beta\theta_h\}, & \text{if otherwise.} \end{cases}$$

Note that $\Pi_i^m > \Pi^{ns}$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \beta\theta_h$, and $\pi_i^m < \pi^{ns}$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$.

Range (ii): $\beta > \frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, we have $\beta\theta_h > \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \leq d$, is then $\Pi(\omega) = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi(\omega) = \alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, and $\Pi(\omega) = \alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ is lower than ω when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and higher than $\alpha\omega$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. Moreover, there exists a $\tilde{\beta} \in \left(\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h}, 1\right)$, such that $\frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]\omega}{2(\theta_h-\theta_l)}$ is lower than $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ if and only if $\beta > \tilde{\beta}$.

If $\beta \geq \tilde{\beta}$, the manufacturer would charge $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} \leq d \leq \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$. As a result, the manufacturer in equilibrium would not acquire any information even when $I = y$, and its equilibrium ex-ante payoff Π_{ii}^m is the same as that without information sharing (i.e., Π^{ns}). If $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$, the manufacturer would charge $\omega = \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < d \leq \frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}$, $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < d \leq \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, $\omega = \beta\theta_h$ if $d > \beta\theta_h$, and $\omega = d$ if otherwise. Therefore, when $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$, the manufacturer in equilibrium would acquire information when $I = y$ if and only if $\frac{\beta(\theta_l-\alpha\theta_h)[(1+\alpha(2-\alpha))\theta_h-2\theta_l]}{(1-\alpha)[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]} < d < \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, where its equilibrium ex-ante payoff Π_{ii}^m is higher than that without information sharing (i.e., Π^{ns}).

When $\frac{\theta_l-\alpha\theta_h}{(1-\alpha)\theta_h} < \beta < \tilde{\beta}$ and $\max\left\{\frac{\theta_l-\alpha\theta_h}{1-\alpha}, \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}\right\} < d < \frac{(\theta_l-\alpha\theta_h)[(\alpha(3-2\beta)-\alpha^2(1-\beta)+\beta)\theta_h-(1+\alpha)\theta_l]}{2\alpha(1-\alpha)(\theta_h-\theta_l)}$, note that the optimal wholesale price $\omega = \frac{\theta_l-\alpha\theta_h}{1-\alpha}$ is lower than that when information sharing is infeasible (i.e., $w = d$). This implies that the retailer's equilibrium ex-ante payoff under mandatory information sharing and sequential information acquisition can be higher than that without information sharing (i.e., π^{ns}).

Q.E.D.

Proof of Proposition 4: Let us first prove that acquiring information is the unique equilibrium strategy in the sub-game when $I = y$. Suppose that the manufacturer acquire information, which leads to $\hat{\beta}_m = 1$ or $\hat{\beta}_m = 0$, with probability β or $1-\beta$, respectively. Because the manufacturer's expected payoff, conditional on the wholesale price ω, increases with the retailer's updated posterior belief $\hat{\beta}_r$, the manufacturer's optimal information disclosure strategy is given by $m(1) = 1$ and $m(0) = \otimes$. Knowing this, the retailer's updated posterior belief is given by $\hat{\beta}_r(1) = 1$ or $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$, when the received message is $m = 1$ or $m = \otimes$, respectively. This is because, the message $m = \otimes$ would be delivered either when the manufacturer acquires no information at all or when the manufacturer's updated posterior belief is zero (i.e, $\hat{\beta}_m = 0$). As a result, it is indeed an equilibrium strategy for the manufacturer to acquire information when $I = y$, because $\hat{\beta}_r(1) > \hat{\beta}_r(\otimes)$. To prove the uniqueness, let us suppose instead that the manufacturer acquire no information when $I = y$. The retailer's updated posterior belief is then given by $\hat{\beta}_r(\otimes) = \beta$. But then the manufacturer can deviate and choose to acquire information. The manufacturer can send a "$\otimes$" message and do no worse when the acquired information indicates $S = B$, and can do better when the acquired information indicates $S = G$, than acquiring no information at all. Therefore, acquiring information is a dominant strategy for the manufacturer when $I = y$.

Given the manufacturer's optimal information acquisition and disclosure strategies, there are two possible scenarios on the retailer's updated posterior belief: $\hat{\beta}_r \in \{1, \frac{\beta}{2-\beta}\}$. In particular, if the realized information state is $I = y$ and the manufacturer subsequently learns that $\hat{\beta}_m = 1$, it will disclose the information to the retailer who then has a updated posterior belief $\hat{\beta}_r(1) = 1$, the ex-ante probability of which is $\beta/2$. If the realized information state is $I = y$ and the manufacturer subsequently learns that $\hat{\beta}_m = 0$, or the realized information state is $I = \bar{y}$, the retailer would receive the message "$\otimes$" and update its belief toward $\hat{\beta}_r(\otimes) = \frac{\beta}{2-\beta}$, the ex-ante probability of which is $1 - \beta/2$. The manufacturer's optimal wholesale price setting takes into account these two scenarios.

Range (i): $\beta < \frac{2(\theta_l - \alpha\theta_h)}{\theta_l + (1-2\alpha)\theta_h}$. In this case, we have $\frac{\hat{\beta}_r(\otimes)(\theta_l - \alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(\otimes)\theta_h < \frac{\hat{\beta}_r(1)(\theta_l - \alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(1)\theta_h$. The manufacturer's expected payoff is $\Pi(\omega) = \omega$ when $\omega \le \frac{\hat{\beta}_r(\otimes)(\theta_l - \alpha\theta_h)}{1-\alpha}$, where the retailer always orders a size of one for either of its updated posterior beliefs. Similarly, the manufacturer's expected payoff is $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega$ when $\frac{\hat{\beta}_r(\otimes)(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \le \hat{\beta}_r(\otimes)\theta_h$, $\Pi(\omega) = \beta\omega/2$ when $\hat{\beta}_r(\otimes)\theta_h < \omega \le \frac{\hat{\beta}_r(1)(\theta_l - \alpha\theta_h)}{1-\alpha}$, and $\Pi(\omega) = \alpha\beta\omega/2$ when $\frac{\hat{\beta}_r(1)(\theta_l - \alpha\theta_h)}{1-\alpha} < \omega \le \hat{\beta}_r(1)\theta_h$. Note that the manufacturer's expected payoff is increasing in ω within each of the above four ranges, and discontinuously drops at the boundary points. Note also that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega \to \frac{\hat{\beta}_r(\otimes)(\theta_l - \alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = \beta\omega/2 + \alpha(1-\beta/2)\omega \to \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}$ when $\omega \to \hat{\beta}_r(\otimes)\theta_h$, $\Pi(\omega) = \beta\omega/2 \to \frac{\beta(\theta_l - \alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\hat{\beta}_r(1)(\theta_l - \alpha\theta_h)}{1-\alpha}$, and $\Pi(\omega) = \alpha\beta\omega/2 \to \alpha\beta\theta_h/2$ when $\omega \to \hat{\beta}_r(1)\theta_h$. Moreover, we have $\frac{\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}$, $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \frac{\beta(\theta_l - \alpha\theta_h)}{2(1-\alpha)}$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \alpha\beta\theta_h/2$. Noticing that $\beta\omega/2 + \alpha(1-\beta/2)\omega = \frac{\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega = \frac{2\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]}$, one can then obtain the manufacturer's equilibrium ex-ante payoff, when $\beta < \frac{2(\theta_l - \alpha\theta_h)}{\theta_l + (1-2\alpha)\theta_h}$:

$$\Pi_i^{iv} = \begin{cases} \min\left\{d, \frac{\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)}\right\}, & \text{if } d \le \frac{2\beta(\theta_l - \alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]}; \\ \min\left\{[\alpha(2-\beta)+\beta]d/2, \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)}\right\}, & \text{if otherwise.} \end{cases}$$

Comparing Π_i^{iv} with Π^{ns}, we have $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)[\alpha(2-\beta)+\beta]} < \frac{\beta(\theta_l-\alpha\theta_h)}{\alpha(1-\alpha)}$, $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \alpha d$, $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} < \alpha\beta\theta_h$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $\beta < \frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h}$. Moreover, $[\alpha(2-\beta)+\beta]d/2 > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $d > \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]}$, and $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2(2-\beta)} > \alpha d$ if and only if $d < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}$. Therefore, when $\beta < \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, one obtains that $\Pi_i^{iv} > \Pi^{ns}$ if and only if $\beta > \frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h}$ and $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}$.

Range (ii): $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$. In this case, we have $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \hat{\beta}_r(\otimes)\theta_h < \hat{\beta}_r(1)\theta_h$. As in the parameter range (i), the manufacturer's expected payoff is $\Pi(\omega)=\omega$ when $\omega \le \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$. Similarly, the manufacturer's expected payoff is $\Pi(\omega)=\beta\omega/2+\alpha(1-\beta/2)\omega$ when $\frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega)=\alpha\omega$ when $\frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \hat{\beta}_r(\otimes)\theta_h$, and $\Pi(\omega)=\alpha\beta\omega/2$ when $\hat{\beta}_r(\otimes)\theta_h < \omega \le \hat{\beta}_r(1)\theta_h$. Note that $\Pi(\omega)=\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}$ when $\omega \to \frac{\hat{\beta}_r(\otimes)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega)=\beta\omega/2+\alpha(1-\beta/2)\omega \to \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\hat{\beta}_r(1)(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega)=\alpha\omega \to \frac{\alpha\beta\theta_h}{2-\beta}$ when $\omega \to \hat{\beta}_r(\otimes)\theta_h$, and $\Pi(\omega)=\alpha\beta\omega/2 \to \alpha\beta\theta_h/2$ when $\omega \to \hat{\beta}_r(1)\theta_h$. Moreover, we have $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\alpha\beta\theta_h}{2-\beta}$, and $\frac{\alpha\beta\theta_h}{2-\beta} > \alpha\beta\theta_h/2$. Therefore, one can obtain the manufacturer's equilibrium ex-ante payoff:

$$\Pi_{ii}^{iv} = \max\left\{\min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)}\right\}, \min\left\{[\alpha(2-\beta)+\beta]d/2, \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}\right\}, \min\left\{\alpha d, \frac{\alpha\beta\theta_h}{2-\beta}\right\}\right\}$$

.

Comparing Π_{ii}^{iv} with Π^{ns}, we have $\frac{\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)(2-\beta)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \alpha d$, $\frac{\alpha\beta\theta_h}{2-\beta} < \alpha\beta\theta_h$, and, given $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$. Therefore, when $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, $\Pi_{ii}^{iv} > \Pi^{ns}$ if and only if $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $[\alpha(2-\beta)+\beta]d/2 > \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, and $\frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} > \alpha d$, which give rise to $\beta < \frac{2\alpha}{1+\alpha}$, $d > \frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]}$, and $d < \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$, respectively.

Noticing that $\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)} > \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$ if and only if $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, one can then combine the ranges (i) and (ii) to obtain that $\Pi^{iv} > \Pi^{ns}$ if and only if $\frac{4\theta_l-2\alpha(3-\alpha)\theta_h}{2\theta_l+[1-\alpha(4-\alpha)]\theta_h} < \beta < \frac{2\alpha}{1+\alpha}$ and $\frac{2\beta(\theta_l-\alpha\theta_h)}{(1-\alpha)[\alpha(2-\beta)+\beta]} < d < \min\left\{\frac{\beta[\alpha(2-\beta)+\beta]\theta_h}{2\alpha(2-\beta)}, \frac{[\alpha(2-\beta)+\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}\right\}$. This completes the proof.

Q.E.D.

Proof of Proposition 5: Following the discussion in the text, there exists a pure-strategy equilibrium where $\hat{\beta}_r(\otimes) \ge \beta_h$ if and only if $\beta \ge \beta_h$, $\beta_l \le \hat{\beta}_r(\otimes) < 1 < \beta_h$ if and only if $\beta_l \le \beta < 1 < \beta_h$, $\beta_l \le \hat{\beta}_r(\otimes) < \beta_h \le 1$ if and only if $\frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h} \le \beta < \beta_h \le 1$, or $\hat{\beta}_r(\otimes) < \beta_l$ if and only if $\beta < \beta_l$, and a mixed-strategy equilibrium where $\hat{\beta}_r(\otimes) = \beta_l$ if and only if $\beta_l \le \beta < \frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h}$ and $\beta_h \le 1$.

Therefore, if $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ (i.e., $\beta \ge \beta_h$), then in equilibrium the manufacturer would not start the information acquisition process and the retailer would order $x=1$. It follows that $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = m(\overline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \omega$, and $\pi' = \beta\theta_l - \omega$.

If $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ (i.e., $\frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h} \le \beta < \beta_h \le 1$), then in equilibrium the manufacturer will continue to collect information until its updated posterior belief reaches either $\hat{\beta}_m = \beta_h$ in which case the acquired information would be disclosed and the retailer would order $x=1$, or $\hat{\beta}_m = 0$ in which case the acquired information would be withheld and the retailer would order $x = \alpha$. It follows that $\underline{\beta} = 0$, $\overline{\beta} = \beta_h$, $m(\underline{\beta}) = \otimes$, $m(\overline{\beta}) = \overline{\beta}$, and $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+(1-\beta/\beta_h)/2} = \frac{(1-\alpha)\beta\omega}{2(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)}$. The manufacturer's expected payoff is then given by $\Pi' = \Phi(\beta|0,\beta_h)\omega+[1-\Phi(\beta|0,\beta_h)]\alpha\omega = \beta(\theta_l-\alpha\theta_h)+\alpha\omega$, and the retailer's given by $\pi' = \beta[\Phi^G(\beta|0,\beta_h)\theta_l + [1-\Phi^G(\beta|0,\beta_h)]\alpha\theta_h] - [\Phi(\beta|0,\beta_h)\omega + [1-\Phi(\beta|0,\beta_h)]\alpha\omega] = \alpha\beta\theta_h - \alpha\omega$.

If $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ (i.e., $\beta_l \leq \beta < \frac{2(1-\alpha)\omega}{\theta_l+(1-2\alpha)\theta_h}$ and $\beta_h \leq 1$), then in equilibrium there exists no pure-strategy equilibrium. In the (partially) mixed-strategy equilibrium, the upper bound where the manufacturer stops the information acquisition process is given by $\overline{\beta} = \beta_h$, and the manufacturer discloses the upper bound to the retailer (i.e., $m(\overline{\beta}) = \overline{\beta}$). However, the manufacturer randomizes the lower bound of information acquisition between $\underline{\beta} = 0$ and $\underline{\beta} = \beta_l$. The probability λ that the manufacturer continues to collect information until $\hat{\beta}_m = 0$ is such that the retailer is indifferent between ordering $x = \alpha$ and $x = 0$ when the message "$\otimes$" is received, i.e., $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+\lambda(1-\beta/\beta_h)/2} = \beta_l$. This leads to $\lambda = \frac{\beta_h(\beta-\beta_l)}{\beta_l(\beta_h-\beta)} = \frac{(1-\alpha)(\beta\theta_h-\omega)}{(1-\alpha)\omega-\beta(\theta_l-\alpha\theta_h)}$. In addition, the manufacturer would withhold (disclose) the acquired information when the lower bound $\underline{\beta} = 0$ ($\underline{\beta} = \beta_l$) is reached, i.e., $m(0) = \otimes$ and $m(\beta_l) = \beta_l$. Moreover, when the retailer receives the "$\otimes$" message, it would randomize between ordering $x = \alpha$ and $x = 0$, with a probability of ρ and $1-\rho$, respectively. To make the manufacturer indifferent between $\underline{\beta} = 0$ and $\underline{\beta} = \beta_l$, it is required that $\alpha\omega = \Phi(\beta_l|0,\beta_h)\omega + [1-\Phi(\beta_l|0,\beta_h)]\rho\alpha\omega$, which leads to $\rho = \frac{\alpha\beta_h-\beta_l}{\alpha(\beta_h-\beta_l)} = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)}$. It follows that the manufacturer's expected profit is given by $\Pi' = \Phi(\beta|\beta_l,\beta_h)\omega + [1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$, and the retailer's given by $\pi' = \beta\{\lambda[\Phi^G(\beta|0,\beta_h)\theta_l+[1-\Phi^G(\beta|0,\beta_h)]\alpha\theta_h]+(1-\lambda)[\Phi^G(\beta|\beta_l,\beta_h)\theta_l+[1-\Phi^G(\beta|\beta_l,\beta_h)]\alpha\theta_h]\}-[\Phi(\beta|\beta_l,\beta_h)\omega+[1-\Phi(\beta|\beta_l,\beta_h)]\alpha\omega] = \frac{[\alpha^2\theta_h+(1-2\alpha)\theta_l](\beta\theta_h-\omega)}{\theta_h-\theta_l}$.

If $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$ (i.e., $\beta_l \leq \beta < 1 < \beta_h$), then in equilibrium the manufacturer would not collect any information and the retailer's order is given by $x = \alpha$, which leads to $\underline{\beta} = \overline{\beta} = \beta$, $m(\underline{\beta}) = m(\overline{\beta}) = \hat{\beta}_r(\otimes) = \beta$, $\Pi' = \alpha\omega$, and $\pi' = \alpha\beta\theta_h - \alpha\omega$.

Finally, if $\omega > \beta\theta_h$ (i.e., $\beta < \beta_l$), then in equilibrium the manufacturer would continue the information acquisition process until either it is learned almost with certainty that the true quality state is $S = B$ or the updated posterior belief reaches β_l, i.e., $\underline{\beta} = 0$ and $\overline{\beta} = \beta_l$. In addition, the manufacturer would withhold (disclose) the acquired information when the lower (upper) bound is reached, i.e., $m(\underline{\beta}) = \otimes$ and $m(\overline{\beta}) = \overline{\beta}$. This implies that $\hat{\beta}_r(\otimes) = \frac{\beta/2}{1/2+(1-\beta/\beta_l)/2} = \frac{\beta\omega}{2\omega-\beta\theta_h} < \beta_l$. The manufacturer's expected payoff is then given by $\Pi' = \Phi(\beta|0,\beta_l)\alpha\omega = \alpha\beta\theta_h$, and the retailer's given by $\pi' = \beta\Phi^G(\beta|0,\beta_l)\alpha\theta_h - \Phi(\beta|0,\beta_l)\alpha\omega = 0$.

Q.E.D.

Proof of Proposition 6: There are two realizations of the information state I. When $I = \bar{y}$, the manufacturer's expected payoff, conditional on $\omega \leq d$, is given by $\Pi'' = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi'' = \alpha\omega$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$ or $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, $\Pi'' = \rho\alpha\omega$ if $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, where $\rho = \frac{\alpha(2-\alpha)\theta_h-\theta_l}{\alpha(\theta_h-\theta_l)}$, and $\Pi'' = 0$ if otherwise. When $I = y$, the manufacturer's expected payoff is given in Proposition 3. That is, $\Pi' = \omega$ if $\omega \leq \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi' = \beta(\theta_l-\alpha\theta_h)+\alpha\omega$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \leq \min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, $\Pi' = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{\theta_h-\theta_l} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ if $\min\{\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\} < \omega \leq \min\{\beta\theta_h, \frac{\theta_l-\alpha\theta_h}{1-\alpha}\}$, $\Pi' = \alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \leq \beta\theta_h$, and $\Pi' = \alpha\beta\theta_h$ if $\omega > \beta\theta_h$.

Range (i): $\beta < \frac{\theta_l - \alpha\theta_h}{(1-\alpha)\theta_h}$. In this case, we have $\beta\theta_h < \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \le d$, is then $\Pi(\omega) = \omega$ if $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l - \alpha\theta_h) + 2\alpha\omega]/2$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$, $\Pi(\omega) = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l}$ if $\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)} < \omega \le \beta\theta_h$, and $\Pi(\omega) = \alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l - \alpha\theta_h) + 2\alpha\omega]/2 \to \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$, and $\Pi(\omega) = \frac{(1-\alpha)\beta\theta_h(\theta_l-\alpha\theta_h)}{2(\theta_h-\theta_l)} + \frac{[\alpha(2-\alpha)\theta_h-\theta_l]\omega}{\theta_h-\theta_l} \to \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)}$ when $\omega \to \beta\theta_h$. Moreover, we have $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$, $\alpha\beta\theta_h/2 < \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)} < \alpha\beta\theta_h$, and $\frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)} > \frac{\beta\theta_h[\alpha(3-\alpha)\theta_h-(1+\alpha)\theta_l]}{2(\theta_h-\theta_l)}$ if $[1-\sqrt{(1-\alpha)^3}]\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$. Therefore, when $[1-\sqrt{(1-\alpha)^3}]\theta_h < \theta_l < \alpha(2-\alpha)\theta_h$, the firms' equilibrium ex-ante profits are given by, respectively:[17]

$$\Pi_i^v = \begin{cases} \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \le \frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}; \\ \min\left\{[\beta(\theta_l-\alpha\theta_h)+2\alpha d]/2, \frac{\beta(\theta_l-\alpha^2\theta_h)}{2(1-\alpha)}\right\}, & \text{if otherwise.} \end{cases}$$

$$\pi_i^v = \begin{cases} \beta\theta_l - \min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, & \text{if } d \le \frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}; \\ \alpha\beta\theta_h - \alpha\min\left\{d, \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}\right\}, & \text{if otherwise.} \end{cases}$$

One can then readily obtain that $\Pi_i^v < \Pi_i^m$ if d is sufficiently large. Similarly, we have $\pi_i^v > \pi_i^m$ if $d > \frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)}$.

Range (ii): $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$. In this case, we have $\frac{\beta[\theta_l+(1-2\alpha)\theta_h]}{2(1-\alpha)} > \frac{\theta_l-\alpha\theta_h}{1-\alpha}$. The manufacturer's expected payoff, conditional on $\omega \le d$, is then $\Pi(\omega) = \omega$ if $\omega \le \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l-\alpha\theta_h)+2\alpha\omega]/2$ if $\frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha} < \omega \le \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, $\Pi(\omega) = \alpha\omega$ if $\frac{\theta_l-\alpha\theta_h}{1-\alpha} < \omega \le \beta\theta_h$, and $\Pi(\omega) = \alpha\beta\theta_h/2$ if $\omega > \beta\theta_h$. Note that $\Pi(\omega) = \omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ when $\omega \to \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$, $\Pi(\omega) = [\beta(\theta_l-\alpha\theta_h)+2\alpha\omega]/2 \to \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}$ when $\omega \to \frac{\theta_l-\alpha\theta_h}{1-\alpha}$, and $\Pi(\omega) = \alpha\omega \to \alpha\beta\theta_h$ when $\omega \to \beta\theta_h$. Moreover, we have $\frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \alpha\beta\theta_h$ given $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, and $\frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)} < \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}$ if and only if $\beta > \frac{2\alpha}{1+\alpha}$. The manufacturer' equilibrium ex-ante payoff is given by $\Pi_{ii}^v = \max\left\{\min\left\{d, \frac{\beta(\theta_l-\alpha\theta_h)}{1-\alpha}\right\}, \min\left\{[\beta(\theta_l-\alpha\theta_h)+2\alpha d]/2, \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2(1-\alpha)}\right\}, \min\{\alpha d, \alpha\beta\theta_h\}\right\}$. It is then obvious that given $\beta > \frac{2(\theta_l-\alpha\theta_h)}{\theta_l+(1-2\alpha)\theta_h}$, we have $\Pi_{ii}^v > \Pi_{ii}^m$ if $\beta < \frac{2\alpha}{1+\alpha}$ and $\frac{(1+\alpha)\beta(\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)} < d < \frac{[2\alpha+(1-\alpha)\beta](\theta_l-\alpha\theta_h)}{2\alpha(1-\alpha)}$, and $\Pi_{ii}^v = \Pi_{ii}^m$ if otherwise.

Q.E.D.

[17] The case when $\alpha\theta_h < \theta_l < [1-\sqrt{(1-\alpha)^3}]\theta_h$ is similar, but the computation is more involved.

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