# Should Firms Share Information About Expensive Customers? An Equilibrium Analysis 

Sameer Mathur ${ }^{1}$<br>Tel: (412)-622-0257<br>Email: smathur@cmu.edu

Kannan Srinivasan
Tel: (412)-268-7357 Fax: (412)-268-7357
Email: kannans@andrew.cmu.edu

Baohong Sun
Tel: (412)-268-6903 Fax: (412)-268-7357
Email: bsun@andrew.cmu.edu

Tepper School of Business, Carnegie Mellon University
5000 Forbes Avenue, Pittsburgh PA 15213

June 14, 2007

[^0]
# Should Firms Share Information About Expensive Customers? An Equilibrium Analysis 


#### Abstract

Advances in information technology increasingly allow firms to identify expensive, high-cost customers, who are not only individually less profitable for firms but also raise the average marginal cost incurred by firms and thus impose a negative externality on inexpensive customers. Should competing firms share information that identifies such customers? The answer to this question has important implications for firm profitability, consumer welfare, and privacy laws that currently constrain firms' ability to share information.

Considering consumers to be heterogeneous in terms of the cost they impose on firms, this paper presents an analytical model to examine the conditions in which two differentiated Bertrand competitors prefer to share information. The firms' incentives to share information differ according to the degree of product differentiation, the relative proportion of expensive customers in the market, the relative marginal cost of selling to expensive customers and the level of noise in the information.

When firms sell substitutable products a Prisoner's Dilemma results. The competing firms unilaterally benefit from sharing information, but the benefits from reneging on an information-sharing agreement are even higher; paradoxically, in equilibrium, both firms therefore keep their information private. A third-party agency such as an industry trade association might serve to supervise and coordinate information-sharing agreements between competing firms. In contrast, when the firms sell complementary products, they always have an incentive to share information.

Importantly, this paper establishes that information sharing decreases the welfare of expensive customers but increases that of inexpensive customers. Privacy laws thus protect expensive customers more than inexpensive customers. In certain conditions, the aggregate consumer welfare might increase, when firms share information identifying expensive customers. This research recommends relatively weaker, not stronger, privacy laws, which is counterintuitive to the recommendations of the popular press.


Keywords: Customer knowledge, Expensive customers, Information sharing, Trade association, Privacy laws, Game theory

## 1. Introduction

With the onset of the information revolution and advances in technology, companies possess increasingly detailed databases of individual-level customer information that enable them to better identify individual-level customer preferences and reveal who bought what, when, and for how much. Consistent with the recent trend of shifting from product management to customer portfolio management (Gupta and Lehmann 2005), companies often use customer information as a strategic asset to individually identify their best customers. However, not every customer is necessarily valuable, and firms can use individual-level databases to identify undesirable customers. As Selden and Colvin (2003) argue, retail markets contain a high-cost market segment, along with a typical low-cost, profitable market segment. This high-cost segment typically spends too little, complains too much, and/or ties up too many firm resources. Firms can cater their marketing-mix tools to avoid doing business with such customers, for example, by not sending them promotional catalogs.

In the case of Best Buy, an estimated 100 million of 500 million annual customer visits are undesirable; some customers buy products, apply for rebates, return the purchases, and then buy them back at returned-merchandise discounts. Alternatively, they may load up on loss leaders, then sell the goods at a profit on eBay. Other undesirable customers present rock-bottom price quotes from Web sites and demand that Best Buy make good on its lowest-price pledge. "They can wreck enormous economic havoc," noted Brad Anderson, CEO of Best Buy, in a Wall Street Journal interview (McWilliams 2004). Along similar lines, Federal Express has kicked off a marketing program to rate customers as the good, the bad, and the ugly, using data mining technology. "We want to keep the good, grow the bad, and have nothing to do with the ugly," says Sharanjit Singh, managing director for marketing analysis, in Business Week (Judge 1998). Federal Express has recognized that the cost of doing business with the ugly is greater than the revenue it provides. Such an adverse impact of a minority segment of customers is widespread across markets as diverse as clothing retailers and financial services providers, with less than $20 \%$ of customers typically responsible for more than $80 \%$ of losses (Weston 2004). Some customers are so costly that a firm may be better off by not serving them. We refer to such customers as expensive customers.

While many firms have experienced an increasing ability to employ their own customer databases, it is interesting to observe that some firms share individual-level customer information
with their competitors. For example, the financial services industry has a long history of using independent credit bureaus to report, monitor, and identify customers with a higher probability of financial default. In the direct marketing industry, more than 600 catalog marketers exchange information about purchases by individual customers with competitors (Deighton et al. 1996). The Abacus B2B Alliance comprises 352 businesses that share 75 million customer names and 1.2 billion transactions. After paying a monthly membership fee of $\$ 85$ and contributing at least 5000 business leads from its personal database, a participating company can label undesirable customers using the shared database (Miller 2003; Bult and Wansbeek 2004). Furthermore, the Retail Industry Leaders Association (RILA), which includes members such as Office Depot, Target, and Safeway, collectively maintains a shared data repository called RILA InfoShare, which serves as a site for reporting retail theft and serious incident information. InfoShare facilitates the communication of loss details among retail members, as well as with local, state, and federal law enforcement agencies (Jones 2006). These trends suggest that database co-ops are not uncommon, which in turn indicates that benefits may accrue from information sharing (Rendleman 2001).

However, various firms also keep their customer information private. The retail giant Wal-Mart does not sell its point-of-sales data to market research companies and thereby indirectly prevents its competitors from accessing its customer information (Heun 2001). Toys 'R Us has ended its practice of sharing data with the NPD Group, which produces industry analysis reports (Annicelli 2003).

Customer information management is becoming increasingly more important. Should a firm share its customer information with a competitor? On the one hand, information sharing can help a firm lower its average costs by identifying and excluding expensive customers. On the other hand, sharing customer information can concede the same benefits to competitors and thereby increase the price competition in the market. It is thus not clear whether firms would be better off sharing their knowledge about expensive customers. This question also raises timely issues about privacy laws and consumer welfare. In the Best Buy example, a minority of expensive customers significantly raises the marginal cost incurred by the firm, which gets reflected in a higher equilibrium price paid by all consumers, including the inexpensive customers. That is, the inexpensive consumers subsidize the expensive consumers. If information
sharing improves a firm's ability to exclude expensive customers, the overall consumer surplus may increase, even if the surplus of the expensive customers decreases.

Despite the recent media discussions about expensive customers (Gupta and Lehmann 2005, Selden and Colvin 2003), relatively little research has addressed the incentives for firms to share customer information. Therefore, the following research questions remain unanswered:

- When do competing firms have an incentive to share individual-level customer information?
- When competing firms share information, are expensive and inexpensive customers affected homogeneously?
- Do privacy laws, which limit firms from sharing customer information, protect all consumers uniformly?

To respond to these issues, we propose a model of Bertrand price competition between two firms. In our model, two firms sell competing but differentiated products to two segments of expensive and inexpensive customers. The firms, independently and ex ante, decide whether to share information about expensive customers, which leads to one of three possible pure equilibrium information-sharing scenarios: (1) both firms keep information private, (2) only one firm shares its information, or (3) both firms share information. In our model, pooling a competitor's information with its own information permits a firm to exclude expensive customers. The firms simultaneously set profit-maximizing prices, a la Bertrand, given their underlying information structure. We measure firm profits in the different equilibrium information-sharing scenarios and specifically characterize how (1) the product differentiation between the competing firms, (2) the relative proportion of the expensive and the inexpensive customers, (3) the higher marginal cost of selling to the expensive customers, (4) the noise in the firms' customer information moderates their incentives to share individual-level customer information. We also study the corresponding impact of information sharing on consumer surplus by characterizing how the surplus changes in the absence or presence of information sharing across the two market segments of expensive and inexpensive customers.

Our equilibrium analysis generates insights of interest to both academics and practitioners. First, we shed light on managing cost-related information assets in a competitive
environment. We show that although a superior knowledge about the cost imposed by individual customers can be a competitive advantage, a firm should not always protect its customer information from competitors. On the contrary, firms may mutually benefit from sharing individual-level customer information in certain competitive scenarios. We find that when the competing products are substitutes, the firms keep information private, despite having an incentive to share information. We also find that when the competing products are complements instead, the firms have an incentive to share information. Our research thus adds to the stream of literature in economics and marketing, such as Sakai (1985), Gal-Or (1986), Raith (1996), Raju and Roy (2000), and Chen et al (2001), that investigates incentives for information sharing in different market settings.

Second, we find a compelling reason for competing firms to join an industry trade group or trade association that supervises and regulates information-sharing agreements. We demonstrate that firms selling substitute products keep information private, since they face a Prisoner's Dilemma like situation with respect to information-sharing agreements. Even though both firms would make higher profits by sharing information, neither firm shares information in equilibrium, because reneging on an agreement leads to even higher profits. Given such a dilemma, a trade association can help regulate and coordinate information-sharing agreements among the competing firms. Our research thus also adds to the stream of literature that investigates the incentives and implications for firms to join trade associations (Kirby 1988; Vives 1990). We identify a potential supervisory role played by trade associations such as the National Retail Federation, in which they coordinate joint actions between competing firms.

Third, our research addresses a controversial public policy implication for privacy laws, which are designed to protect consumers from economic injury and unwanted intrusions. Firms' incentives to share customer information are moderated by privacy laws that legally constrain their ability to share information. Whereas advocates of stringent consumer privacy laws argue that consumers are affected adversely when firms share information, we propose a more balanced stance in which we note that such laws may not protect consumers equally. Our analysis establishes a heterogeneous change in the consumer surplus across the two customer segments, such that the consumer surplus of the expensive customers decreases, whereas that of the inexpensive customers increases, when competing firms share customer information. In this way, we demonstrate that the expensive customers impose a negative externality on the
inexpensive customers in the market. We therefore infer that privacy laws may indirectly cause an increase in the consumer surplus of expensive customers, at the expense of inexpensive customers. Hence, we conclude that privacy laws may protect expensive customers more than inexpensive customers. Contrary to the popular press, we argue in favor of not more but less stringent consumer privacy laws.

The rest of this article is organized as follows: In Section 2, we review related literature in marketing and economics. In Section 3, we develop our information-sharing model, and then in Section 4, we discuss the managerial implications and insights from our model. In Section 5, we conclude and discuss some limitations. ${ }^{2}$

## 2. Related Literature and Contributions

### 2.1. Exchange of Cost Information

Our research relates to prior literature on the exchange of cost information in a duopolistic industry (Fried 1984; Sakai 1985; Gal-Or 1986; Shapiro 1986; Raith 1996). Fried (1984) shows that each duopolist will choose to reveal its private information to its rival in a Cournot duopoly with a homogeneous good. Sakai (1985) considers various cost information structures in a Cournot duopoly and also finds that a firm is better-off when its cost is known to its rival. Gal-Or (1986) extends this idea to a Bertrand duopoly in which each firm observes its own cost with noise and may send noisy signals to its rival. She shows that in a Bertrand duopoly with substitute goods and cost uncertainty with IID marginal costs, not revealing information is a dominant strategy. This literature also evaluates the impact of information sharing on consumer surplus. For example, Sakai (1985) finds that consumer surplus decreases if firms share cost information in a Bertrand duopoly. ${ }^{3}$

[^1]The key difference between our paper and existing literature is that we consider heterogeneous, individual-level costs imposed by customers, unlike the aggregate-level cost information considered in previous studies. In addition, the key finding of extant research is that firms selling to homogeneous consumers have an incentive to keep their information private under Bertrand competition (Gal-Or 1986); we refine this research by incorporating heterogeneity in the costs imposed by consumers by assuming two market segments of expensive and inexpensive customers. Our model therefore leads to a richer solution space, in which Bertrand competitors may prefer sharing information in certain scenarios but keep information private in others. It also provides correspondingly richer findings with regard to consumer welfare. Unlike Sakai's (1985) model, in which the consumer surplus of homogeneous consumers falls when firms share information, we show that overall consumer surplus may either increase or decrease, depending on consumer heterogeneity and the relative proportion of the two market segments in the market.

### 2.2. Information Sharing and Trade Associations

Advances in today's information-intensive marketing environment have prompted recent papers in information management, including Chen et al.'s (2001) model of a state of imperfect targetability, in which firms' ability to predict the purchase behaviors of individual consumers to customize price or product offers is imperfect. These authors segment customers as either a firm's own loyal customers or common switchers; each firm imperfectly classifies its own loyal customers and switchers correctly; and the competing firms can improve their ability to identify price-sensitive switchers and price-insensitive loyal customers by sharing information. ${ }^{4}$ In contrast, we model a retail situation in which firms cannot price discriminate between market segments, nor can they charge expensive and inexpensive customers different prices. Also unlike

[^2]Chen et al. (2001), we consider the marginal costs imposed by different market segments on the firms to be heterogeneous.

Raju and Roy (2000) study the effect of market characteristics on the value of information by modeling firm profits under demand uncertainty and find that information is more valuable when demand uncertainty and product substitutability is higher, which suggests that information is of greater value in more competitive industries. In contrast, we address cost uncertainty, such that expensive customers are significantly more expensive to serve than are inexpensive customers and competing firms face uncertainty in identifying expensive customers.

Our work also relates to the stream of literature investigating why firms join industry trade associations or groups (Shapiro 1986; Kirby 1988; Vives 1990). For example, Shapiro (1986) analyses firms' decisions to form trade associations to share cost data under an oligopoly and shows that industry-wide information exchange is the unique point in the core of the trade association membership game, given conditions of linear demand and Cournot competition. Along similar lines, Kirby (1988) examines the incentives for firms that compete in an oligopolistic industry to share information about an unknown demand parameter when such sharing takes place on a quid pro quo basis. If the total cost functions are sufficiently convex, joining a trade association is Pareto preferred to a setting of private information. ${ }^{5}$ The key finding in both Shapiro (1986) and Kirby (1988) is that firms always benefit by joining industry trade associations. Our paper refines this research by incorporating heterogeneity into the cost imposed by expensive and inexpensive customers. In turn, our model is more realistic and richer, one where competing firms may or may not be better off by joining industry trade associations. In contrast to Shapiro (1986) and Kirby (1988), we find that if the proportion of expensive customers is too high and the competing products are strong substitutes, firms may benefit by choosing not to join industry trade associations.

### 2.3. Information Privacy

Recent technological developments in information collection and processing have heightened privacy concerns among consumers and led to widespread concerns about the collection of

[^3]personal information in various contexts. Consumers are concerned that though individual profiling by firms may benefit them if the firms can more precisely identify and cater to their needs, it also may be used by firms to price discriminate or exclude consumers with less attractive characteristics (Mussa and Rosen 1978; Moorthy 1984; Varian 1985; Tirole 1988). The use of personal information to profile individual consumers thus imposes an indirect or consequential externality, because some consumers suffer from either paying a relatively higher price or being excluded from enjoying a particular good or service. In this sense, the exploitation of personal information could lead to ex post inefficiencies. Moreover, some consumers may get priced out of the market when more information is available to the seller, even if it would be socially efficient for them to consume the item (Varian 1985; Fudenberg and Villas-Boas 2006). Furthermore, the option of sharing or selling consumer information for extra revenue may further reduce social efficiency. Taylor (2004) demonstrates that the seller may experience an excessive incentive to collect consumer information, at the expense of some of its own potential consumers, because of the loss in trade and increase in deadweight losses or the cost of compiling information. Hui and Png's (2006) review of the economics of privacy examines these issues in greater detail.

In assuming all consumers to be homogeneous in terms of their cost, the key finding from previous literature has been that consumer surplus typically decreases when competing firms share information (Sakai 1985; Shapiro 1986). We contribute to privacy literature by examining whom such privacy laws benefit, and we find a heterogeneous change in consumer surplus across our two heterogeneous cost market segments. While the consumer surplus of expensive consumers decreases, that of inexpensive consumers increases, and therefore, privacy laws that restrict firms' ability to share information benefit expensive customers at the cost of the dominant, inexpensive customers in a market.

## 3. Model

Consider a market consisting of two risk-neutral Bertrand competitors $i \in\{1,2\}$. We assume that their inverse demand functions are downward sloping, given as follows.

$$
p_{i}=\alpha-\beta q_{i}-\gamma q_{3-i}
$$

The firms sell $q_{i}>0$ quantities at prices $p_{i}>0$ respectively. We normalize $\beta=1$, without loss of generality, yielding $p_{i}=\alpha-q_{i}-\gamma q_{3-i}$. The competing products are substitutes or complements, depending on whether $\gamma>0$ or $\gamma<0$ respectively. We assume that the demands are linear in self and cross-price effects and that self-price effects are greater than cross-price effects. This implies that, $-1<\gamma<1$ (McGuire and Staelin 1983). We can equivalently rewrite the demand functions as

$$
q_{i}=a-b p_{i}+d p_{3-i}
$$

where $a=\frac{\alpha}{1+\gamma}, b=\frac{1}{1-\gamma^{2}}$ and $d=\frac{\gamma}{1-\gamma^{2}}$.

## Information Sharing under Bertrand Competition

Each firm independently adopts a policy of either sharing its customer information with its competitor or keeping its information private. A firm also could choose to give away information unilaterally, which means one of the following three information-sharing scenarios $Z \in\{p r, s h, g\}$ may occur: Both firms keep information private $(Z=p r)$; both firms share information $(Z=s h)$; or only one firm shares information, while the other firm keeps information private $(Z=g)$. We confine our analysis to pure strategy Nash equilibria.

A firm's decision to share information is not obvious. The firm can improve its ability to identify and exclude expensive customers by gaining access to its competitor's database of expensive customers. This lowers the firm's cost and leads to potentially higher profits. However, sharing its own information grants the competition the same benefit and potentially increases the price competition for inexpensive customers, which leads to potentially lower profits.

## Consumers

The consumers impose heterogeneous marginal costs on the firms. The market has two segments - a segment of relatively high marginal cost consumers, referred to as expensive customers and a segment of relatively low marginal cost consumers, referred to as inexpensive customers. Let $I$ denote an inexpensive customer and $E$ denote an expensive customer. The marginal costs of serving expensive and inexpensive customers are represented by $c_{E}$ and $c_{I}$ respectively, with
$c_{E} \gg c_{I}>0$. These marginal costs are constant and invariant across the two firms. Our model thus explicitly incorporates consumer cost heterogeneity.

Suppose the probability that a given customer $j$ is inexpensive is $\lambda$, while the corresponding probability that the customer is expensive is $1-\lambda$, or

$$
\begin{gathered}
\operatorname{Pr}(j=I)=\lambda \\
\operatorname{Pr}(j=E)=1-\lambda
\end{gathered}
$$

Invoking the law of large numbers, the proportion of inexpensive customers in the market is $\lambda$, while the corresponding proportion of expensive customers in the market is $1-\lambda$.

## Information Structure

The firms gets private signals from the customers in the market as being either expensive or inexpensive, based on the firm's customer database and market research. Specifically, each firm $i \in\{1,2\}$, has a privately known binary signal $s_{i} \in\{L, H\}$ about a given customer, concerning whether the customer is expensive $\left(s_{i}=L\right)$ or inexpensive $\left(s_{i}=H\right)$. Such a signal is based on the firm's customer database of purchase history and market research.

A firm never gets an $L$ signal from an inexpensive customer. However, the firm may get either a correct $L$ signal or an incorrect $H$ signal from an expensive customer. In order to interpret this information structure, consider the following hypothetical scenario: A customer purchases an expensive evening gown from a clothing retailer on Friday and then returns it on Monday morning. The same customer purchases a pair of designer jeans from another retailer and does not return them. In this case, the first retailer has an $L$ signal from this customer, while the second retailer has an $H$ signal from this customer.

Let $s_{1} s_{2}$ represent such a pair of signals about a given customer, where $s_{1}$ refers to firm 1's signal about this customer and $s_{2}$ refers to firm 2's signal about this customer. Since each signal is binary, with $s_{i} \in\{L, H\}$, this implies that there are four possible pairs of signals about the customer, leading to four possible states: $s_{1} s_{2} \in\{H H, H L, L H, L L\}$. For instance, $\left(s_{1} s_{2}=H H\right)$ refers to the state where both firms have a $H$ signal about this customer, while $\left(s_{1} s_{2}=H L\right)$ refers to the state where firm 1 has a $H$ signal about this customer, while firm 2 has a
$L$ signal about the same customer. The states $\left(s_{1} s_{2}=L H\right)$ and $\left(s_{1} s_{2}=L L\right)$ are analogously defined.

## Conditional Probability Beliefs about Expensive Customers

The firms have potentially incorrect signals about the expensive customers in the market. For instance, a firm may incorrectly have a $\left(s_{i}=H\right)$ signal about a given expensive customer $j$. We define the conditional probability of only firm $i$ having an incorrect $H$ signal about customer $j$ to be $\delta_{i}$, or

$$
\begin{aligned}
& \operatorname{Pr}\left(s_{1} s_{2}=H L \mid j=E\right)=\delta_{1} \\
& \operatorname{Pr}\left(s_{1} s_{2}=L H \mid j=E\right)=\delta_{2}
\end{aligned}
$$

Thus, $\delta_{1}$ is equivalent to firm 1 making a Type I error and represents the probability of firm 1 having an incorrect $H$ signal and firm 2 having a correct $L$ signal about a given expensive customer $j . \delta_{2}$ has an analogous meaning with respect to firm 2 . Note that since these are probabilities, we must have $\left(0 \leq \delta_{i} \leq 1\right)$.

Next, we assume that both the firms do not have incorrect $H$ signals from a given expensive customer $j$, or

$$
\operatorname{Pr}\left(s_{1} s_{2}=H H \mid j=E\right)=0
$$

Lastly, the conditional probability of both firms having a correct $L$ signal about customer $j$ simply follows by subtraction, as follows.

$$
\operatorname{Pr}\left(s_{1} s_{2}=L L \mid j=E\right)=1-\delta_{1}-\delta_{2}
$$

Note that $\left(0 \leq \delta_{1}+\delta_{2} \leq 1\right)$.
The information structure implies that the competing firms are uncertain about the segment of expensive customers, particularly those expensive customers from whom the firms receive incorrect $H$ signals. When a firm has access to only its own signals, it sells to only those customers from whom it receives $H$ signals. However, its signals are noisy and the firm ends up selling to both inexpensive customers and some expensive customers from whom it incorrectly receives $H$ signals. The parameter $\delta_{i}$ captures the probability of this misclassification and is a measure of the noise in firm $i$ 's information.

## Conditional Probability Beliefs about Inexpensive Customers

We assume that the firms do not have incorrect signals about the inexpensive customers in the market. This implies the following conditional probability beliefs about the inexpensive customers. The probability of firm $i$ receiving an incorrect $L$ signal about a given inexpensive customer $j$ is zero, or

$$
\begin{aligned}
& \operatorname{Pr}\left(s_{1} s_{2}=H L \mid j=I\right)=0 \\
& \operatorname{Pr}\left(s_{1} s_{2}=L H \mid j=I\right)=0 \\
& \operatorname{Pr}\left(s_{1} s_{2}=L L \mid j=I\right)=0
\end{aligned}
$$

This further means that the firms never make Type II errors and always correctly get a $H$ signal from a given inexpensive customer $j$, or

$$
\operatorname{Pr}\left(s_{1} s_{2}=H H \mid j=I\right)=1
$$

In other words, the firm does not face any uncertainty regarding the desirable segment of inexpensive customers in the market. We make this assumption for the following two reasons. First, this allows our model to concentrate on the firms' uncertainty regarding the expensive customers in the market and model the potential gains from sharing information about the expensive customers. Second, this keeps our model analytically tractable. Modeling Type II errors adds unnecessary complexity to our model, without offering additional significant insights.

## Bayesian Updating of Beliefs

When a firm does not know its competitor's signals about the customers, it can only use its own signals about the customers. Following the information structure discussed above, when a firm has a $L$ signal about a given customer, it correctly infers that this customer is expensive and does not sell to this customer. The firm sells to all the remaining customers in the absence of knowing its competitors signals. In other words, the firm sells to a customer, if it has a $H$ signal about this customer.

The information structure additionally implies that a firm's signals about expensive customers are noisy and the firm may potentially misclassify expensive customers to be inexpensive. When a firm does not have access to its competitor's information, the firm sells to only those customers from whom its gets $s_{i}=H$ signals. However, there is type I error and the
firm ends up selling to both expensive and inexpensive customers. In contrast, when a firm learns its competitor's customer information, the firm now only sells to inexpensive customers.

Suppose firm $i$ has a $H$ signal about customer $j$. Customer $j$ may be either expensive or inexpensive with some probability. We can use Baye's Rule to measure the posterior probability that customer $j$ is indeed an inexpensive customer.

Let us define $\lambda_{i H}$ as the probability that customer $j$ is inexpensive, given that firm $i$ has a $H$ signal about customer $j$, or $\lambda_{i H}=\operatorname{Pr}\left(j=I \mid s_{i}=H\right)$.

We can apply Baye's Rule to measure $\lambda_{i H}$, as $\operatorname{Pr}\left(j=I \mid s_{i}=H\right)=\frac{\operatorname{Pr}\left(j=I \Omega s_{i}=H\right)}{\operatorname{Pr}\left(s_{i}-H\right)}$, or

$$
\operatorname{Pr}\left(j=I \mid s_{i}=H\right)=\frac{\operatorname{Pr}\left(s_{i}=H \mid j=I\right) * \operatorname{Pr}(j=I)}{\operatorname{Pr}\left(s_{i}=H \mid j=I\right) * \operatorname{Pr}(j=I)+\operatorname{Pr}\left(s_{i}=H \mid j=E\right) * \operatorname{Pr}(j=E)}
$$

Following the information structure discussed above, the probability that firm $i$ has a $H$ signal, given that customer $j$ is inexpensive is 1 , or $\operatorname{Pr}\left(s_{i}=H \mid j=I\right)=1$. Further, the probability that firm $i$ has a $H$ signal, given that customer $j$ is inexpensive is $\delta_{i}$, or
$\operatorname{Pr}\left(s_{i}=H \mid j=E\right)=\delta_{i}$. Substituting in Baye's rule, we get

$$
\operatorname{Pr}\left(j=I \mid s_{i}=H\right)=\lambda_{i H}=\frac{\lambda}{\lambda+(1-\lambda) \delta_{i}}
$$

This gives us the probability that customer $j$ is an inexpensive customer, given that firm $i$ has a $H$ signal about this customer.

Let $\theta_{i}$ denote the proportion of customers from whom firm $i$ receives $H$ signals. This includes both correctly classified inexpensive customers and incorrectly classified expensive customers. We note from above that

$$
\begin{gathered}
\theta_{i}=\lambda+(1-\lambda) \delta_{i} \\
\lambda_{i H}=\frac{\lambda}{\theta_{i}}
\end{gathered}
$$

The corresponding probability that a customer $j$ is expensive, given that firm $i$ receives a $H$ signal from this customer, or $\operatorname{Pr}\left(j=E \mid s_{i}=H\right)$ is simply $\left(1-\lambda_{i H}\right)=\frac{1-\lambda}{\theta_{i}}$. Notice that $\lambda_{i H}>\lambda$, implying that the posterior probability of a customer being inexpensive, given that the firm receives a $H$ signal from this customer, is higher than the prior probability $\operatorname{Pr}(j=I)=\lambda$.

When $\delta_{i}=0$, then the firm has perfect information, $\theta_{i}=\lambda$ and $\lambda_{i H}=1$. When $\delta_{i}>0$, firm $i$ receives a noisy $H$ signal and cannot perfectly identify the customer. As the noise in the firm's signal increases, $\delta_{i}$ correspondingly increases.

We choose the above parsimonious information structure for analytical tractability, although the same intuition extends to a more complex information structure. We now consider the Bertrand pricing game between the two firms and compare the different information scenarios $Z \in\{p r, s h, g\}$.

## Game

The firms $i \in\{1,2\}$ play the following two-stage game.

1. In the first stage, before receiving any private information, the firms independently commit to either share or reveal their private information. Once the uncertainty is realized and private signals are received, a firm's information is revealed or not revealed to its competitor.
2. In the second stage, the firms non-cooperatively set prices that maximize individual expected profits, conditional on their available information.

## 4. The Impact of Information Sharing on Equilibrium Profits

We estimate the equilibrium profits made by Bertrand duopolists under the three informationsharing scenarios $Z \in\{p r, s h, g\}$, when firms keeps information private, when firms share information and when one firm gives away information while the other firm keeps it private, respectively.

We focus on the scenario where the two competing firms are symmetric. The insights from our analysis also extend to the case where the firms are asymmetric. When the firms are symmetric, we let $\delta=\delta_{i}$. The firms' conditional probability beliefs about expensive customers can be represented as follows: $\quad \operatorname{Pr}\left(s_{1} s_{2}=H L \mid j=E\right)=\delta ; \quad \operatorname{Pr}\left(s_{1} s_{2}=L H \mid j=E\right)=\delta$; $\operatorname{Pr}\left(s_{1} s_{2}=H H \mid j=E\right)=0$ and $\operatorname{Pr}\left(s_{1} s_{2}=L L \mid j=E\right)=1-2 \delta$. And since we must have $1-2 \delta \geq 0$, the noise parameter $\delta$ is constrained as follows:

$$
0 \leq \delta \leq 0.5
$$

When the firms are symmetric, we let $\lambda_{H}=\lambda_{i H}$, where $\lambda_{H}$ measures the conditional probability of inexpensive customers giving $H$ signals, or $\operatorname{Pr}(j=I \mid s=H)$, while $\left(1-\lambda_{H}\right)$ measures the conditional probability of expensive customers giving $H$ signals, or $\operatorname{Pr}(j=E \mid s=H)$. We also let $\theta=\theta_{i}$, where $\theta$ measures the fraction of consumers from whom a firm receives $H$ signals. These parameters are related as follows:

$$
\lambda_{H}=\frac{\lambda}{\theta}=\frac{\lambda}{\lambda+(1-\lambda) \delta}
$$

We now model how the firms' optimal decision to share or not share information gets moderated by the relative proportion of inexpensive customers in the market $\lambda$, the noise in the firms' information $\delta$, the marginal cost of selling to expensive customers $c_{E}$, and the product differentiation between the competing firms $\gamma$.

Both Firms Keep Information Private ( $Z=p r$ )
We first model the firms' Bertrand pricing problem when both firms keep information private. When information is kept private, a firm sells to consumers from whom it receives $H$ signals, which includes both inexpensive and expensive customers.

The demand function for inexpensive customers is $q_{i}=a-b p_{i}+d p_{3-i}$, where $a=\frac{\alpha}{1+\gamma}$, $b=\frac{1}{1-\gamma^{2}}$ and $d=\frac{\gamma}{1-\gamma^{2}}$ while the demand function for expensive customers is $q_{i}=\alpha-p_{i}$. Note that if we invert the inverse demand function $p_{i}=\alpha-q_{i}-\gamma q_{3-i}$, we get the demand function for inexpensive customers as $q_{i}=a-b p_{i}+d p_{3-i}$. If we consider the customers not served by firm $3-i$, setting $q_{3-i}=0$, the inverse demand function becomes $p_{i}=\alpha-q_{i}$, and we get the demand function for expensive customers as $q_{i}=\alpha-p_{i}$.

Suppose firm $i$ sets a price $p_{i}$. The firms' profit functions, when both firms keeps information private, are given as follows:

$$
\Pi_{i}=\theta\left[\lambda_{H}\left(p_{i}-c_{I}\right)\left(a-b p_{i}+d p_{3-i}\right)\right]+\theta\left[\left(1-\lambda_{H}\right)\left(p_{i}-c_{E}\right)\left(\alpha-p_{i}\right)\right]
$$

The profit-maximizing problem for firm $i \in\{1,2\}$ can thus be represented as

$$
\max _{p_{i}} \lambda\left(p_{i}-c_{I}\right) \frac{\alpha(1-\gamma)-p_{i}+\gamma p_{3-i}}{\left(1-\gamma^{2}\right)}+\delta(1-\lambda)\left(p_{i}-c_{E}\right)\left(\alpha-p_{i}\right)
$$

We proceed by simultaneously solving the first-order conditions $\frac{\partial \Pi_{i}}{\partial p_{i}}=0$ and verifying that the second-order conditions $\frac{\partial^{2} \Pi_{i}}{\partial p_{i}^{2}}<0$ are satisfied. We find that

$$
\begin{gathered}
\frac{\partial \Pi_{i}}{\partial p_{i}}=\frac{\lambda\left(\alpha(1-\gamma)-2 p_{i}+\gamma p_{3-i}+c_{I}\right)}{\left(1-\gamma^{2}\right)}+\delta(1-\lambda)\left(\alpha+c_{E}-2 p_{i}\right) \\
\frac{\partial^{2} \Pi_{i}}{\partial p_{i}^{2}}=-2\left(\frac{\lambda}{1-\gamma^{2}}+\delta(1-\lambda)\right)<0
\end{gathered}
$$

Since the firms are symmetric, they set the same equilibrium price, $p_{p r}$, given as follows.

$$
p_{p r}=\frac{\lambda\left(\alpha(1-\gamma)+c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}
$$

When firms keep information private, the equilibrium demand from inexpensive customers is $q_{p r, I}=\frac{\lambda\left(\alpha-p_{p r}\right)}{(1+\gamma)}$, while the equilibrium demand from expensive customers is $q_{p r, E}=\delta(1-\lambda)\left(\alpha-p_{p r}\right)$. The total equilibrium demand is given by their sum, as $q_{p r}=q_{p r, I}+q_{p r, E}$. Substituting $p_{p r}$, we find that $\left(\alpha-p_{p r}\right)=\left(\frac{\lambda\left(\alpha-c_{l}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)$. Thus, we get

$$
\begin{gathered}
q_{p r, I}=\frac{\lambda}{(1+\gamma)}\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right) \\
q_{p r, E}=\delta(1-\lambda)\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right) \\
q_{p r}=\left(\frac{\lambda}{(1+\gamma)}+\delta(1-\lambda)\right)\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)
\end{gathered}
$$

The equilibrium profit, when both firms keep information private, is given by
$\Pi_{p r}=\left(p_{p r}-c_{I}\right) q_{p r, I}+\left(p_{p r}-c_{E}\right) q_{p r, E}$, or

$$
\Pi_{p r}=\left(\alpha-p_{p r}\right)\left(\frac{\lambda\left(p_{p r}-c_{I}\right)}{1+\gamma}+\delta(1-\lambda)\left(p_{p r}-c_{E}\right)\right)
$$

Substituting $p_{p r}$ and simplifying yields the following profit function:

$$
\Pi_{p r}=\frac{\left[M\left(\alpha-c_{E}\right)+\lambda\left(\alpha-c_{I}\right)\right]\left[\left(\alpha-c_{I}\right)(M+\lambda(1-\gamma))-M(M+\lambda)\left(c_{E}-c_{I}\right)\right]}{\left(1-\gamma^{2}\right)[2 M+\lambda(2-\gamma)]^{2}}
$$

where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$

Lemma 1 and 2 described below, establish the boundary conditions on the parameter space, when the firms keep information private. Please see Appendix A for proofs.

LEMMA 1: A necessary condition for the firms to enter the market is $c_{E}<\left(1+\frac{\lambda}{M}\right) \alpha$, where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$.

The firms will enter the market only if they expect to earn positive profits. This requires that the firms make strictly positive sales $\left(q_{p r}>0\right)$ and earn a positive margin from selling to the inexpensive customers $\left(p_{p r}>c_{I}\right)$. The inequality in Lemma 1 is derived from these requirements. Note that Lemma 1 implies that the firms' marginal cost of selling to expensive customers cannot exceed a threshold. We also note that $c_{E}<\alpha$ is a sufficient condition for market entry.

LEMMA 2: When firms keep information private, they incur a loss from selling to the segment of expensive customers, when $\left(\frac{\lambda(1-\gamma)+M}{\lambda(2-\gamma)+M}\right) \alpha<c_{E}<\left(1+\frac{\lambda}{M}\right) \alpha$, where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$.

When a firm keeps information private, it sells to customers from whom the firm receives $H$ signals. But this includes both inexpensive customers and also some expensive customers who the firm misclassifies. The firm will enter the market only if it expects to earn a positive profit, as shown in Lemma 1. It always earns a profit from selling to the segment of inexpensive customers. However, depending on the marginal cost of selling to the expensive customers, the firm may incur a loss from selling to this segment. Lemma 2 establishes the boundary condition when this happens. Specifically, if $c_{I}<c_{E}<\left(\frac{\lambda(1-\gamma)+M}{\lambda(2-\gamma)+M}\right) \alpha$, the firm earns a positive margin by selling to the expensive customers, although this margin is lower than that earned from selling to the inexpensive customers. However, the firm incurs a negative
margin or in other words, a loss, by selling to the expensive customers if $\left(\frac{\lambda(1-\gamma)+M}{\lambda(2-\gamma)+M}\right) \alpha<c_{E}<\left(1+\frac{\lambda}{M}\right) \alpha$.

Both these scenarios - either lower profits or outright losses - arising from selling to expensive customers, suggest an incentive for the firm to avoid selling to this segment by sharing information with its competitor. Next, we model the firms' Bertrand pricing problem for the scenario when both firms share information.

## Both Firms Share Information ( $Z=s h$ )

The firms' profit functions, when both firms share information, are given as $\Pi_{i}=\lambda\left(p_{i}-c_{I}\right)\left(a-b p_{i}+d p_{3-i}\right)$, where $a=\frac{\alpha}{1+\gamma}, b=\frac{1}{1-\gamma^{2}}$ and $d=\frac{\gamma}{1-\gamma^{2}}$.

The profit-maximizing problem for firm $i \in\{1,2\}$ can thus be represented as

$$
\max _{p_{i}} \lambda\left(p_{i}-c_{I}\right) \frac{\alpha(1-\gamma)-p_{i}+\gamma p_{3-i}}{\left(1-\gamma^{2}\right)}
$$

We proceed by simultaneously solving the first-order conditions $\frac{\partial \Pi_{i}}{\partial p_{i}}=0$ and verifying that the second-order conditions $\frac{\partial^{2} \Pi_{i}}{\partial p_{i}^{2}}<0$ are satisfied. We find that

$$
\begin{gathered}
\frac{\partial \Pi_{i}}{\partial p_{i}}=\frac{\lambda\left(\alpha(1-\gamma)-2 p_{i}+\gamma p_{3-i}+c_{I}\right)}{\left(1-\gamma^{2}\right)} \\
\frac{\partial^{2} \Pi_{i}}{\partial p_{i}^{2}}=\frac{-2 \lambda}{1-\gamma^{2}}<0
\end{gathered}
$$

Since the firms are symmetric, they set the same equilibrium price, given as follows:

$$
p_{s h}=\frac{\alpha(1-\gamma)+c_{I}}{(2-\gamma)}
$$

Note that the firms earn a strictly positive margin by selling to inexpensive customers, as $p_{s h}>c_{I}$. Also, the equilibrium sales resulting when the firms share information are given by $q_{s h}=\frac{\left(\alpha-p_{s h}\right)}{(1+\gamma)}$. Since we must have $q_{s h}>0$, we see that $\alpha>p_{s h}$. Combining these two inequalities indicates that the equilibrium price is constrained as $\alpha>p_{s h}>c_{I}$. Also, substituting for $p_{s h}$ in the above expression for the equilibrium sales yields

$$
q_{s h}=\frac{\left(\alpha-c_{I}\right)}{(1+\gamma)(2-\gamma)}
$$

Notice that $q_{s h}>0$ also implies $c_{I}<\alpha$ and this is consistent with Lemma 1. Substitution into the profit function yields the profit when both firms keep information private, as follows:

$$
\Pi_{s h}=\frac{\lambda(1-\gamma)\left(\alpha-c_{I}\right)^{2}}{(1+\gamma)(2-\gamma)^{2}}
$$

We now establish the boundary conditions on the parameter space, when the firms share information. Please see Appendix A for proofs.

LEMMA 3: When firms share information, they always find it unprofitable to sell to expensive customers if $c_{E}>\frac{2 \alpha+c_{1}}{3}$.

When the firms share information, they have no incentive to sell to expensive customers, if the marginal cost of selling to them exceeds the price, or $c_{E}>p_{s h}$. Thus, a necessary condition for firms to only sell to inexpensive customers when they share information is that the marginal cost of selling to the inexpensive customers $c_{E}$ exceeds a lower bound given by $\frac{2 \alpha+c_{1}}{3}$.

We now compare the equilibrium prices resulting when the firms share information, with those when the firms keep information private.

LEMMA 4: When firms share information, the equilibrium price $p_{\text {sh }}$ is lower than the equilibrium price when firms keep information private $\left(p_{p r}>p_{s h}\right)$.

Lemma 4 establishes that inexpensive customers pay a lower price when the firms share information, as compared to the case where they kept information private. This lowering of the equilibrium price has interesting welfare implications, as discussed in the next section of our paper.

## Only One Firm Shares Information $(Z=g)$

We now turn our attention to the third remaining information sharing scenario, where one firm shares its information, while its competitor keeps information private. This analysis is necessary, in order to examine if either firm has any incentive to deviate from an information sharing agreement.

For example, imagine the scenario where firm 1 and 2 have an agreement to share information $(Z=s h)$. Now suppose Firm 2 breaks their agreement. This leads to a situation where firm 1 does not get access to firm 2's information, while firm 2 has access to firm 1 's information.

Under this scenario, the profit maximization problem of firm 1, is as follows,

$$
\max _{p_{g 1}} \lambda\left(p_{1}-c_{I}\right) \frac{\alpha(1-\gamma)-p_{1}+\gamma p_{2}}{\left(1-\gamma^{2}\right)}+(1-\lambda)\left(p_{1}-c_{E}\right)\left(\alpha-p_{1}\right)
$$

while the profit maximizing problem of firm 2 can be represented as follows:

$$
\max _{p_{g 2}} \lambda\left(p_{2}-c_{I}\right) \frac{\alpha(1-\gamma)-p_{2}+\gamma p_{1}}{\left(1-\gamma^{2}\right)}
$$

We proceed by simultaneously solving the first-order conditions $\frac{\partial \Pi_{i}}{\partial p_{i}}=0$ and verifying that the second-order conditions $\frac{\partial^{2} \Pi_{i}}{\partial p_{i}^{2}}<0$ are satisfied, similar to the earlier cases. Let $p_{g 1}$ and $p_{g 2}$ represent the equilibrium profit-maximizing prices selected by the firms. The equilibrium prices are given as follows.

$$
\begin{gathered}
p_{g 1}=\frac{2 \delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}\right)+\lambda(2+\gamma)\left(\alpha(1-\gamma)+c_{I}\right)}{4 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\left(4-\gamma^{2}\right)} \\
p_{g 2}=\frac{\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha(2-\gamma)+2 c_{I}+\gamma c_{E}\right)+\lambda(2+\gamma)\left(\alpha(1-\gamma)+c_{I}\right)}{4 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\left(4-\gamma^{2}\right)}
\end{gathered}
$$

Substitution into the profit functions yields the firm profits. Firm 1's profit, given that firm 1 shares information, but firm 2 keeps information private, is as follows.

$$
\Pi_{g 1}=\frac{\lambda\left(p_{g 1}-c_{I}\right)\left(\alpha(1-\gamma)-p_{g 1}+\gamma p_{g 2}\right)}{\left(1-\gamma^{2}\right)}+\delta(1-\lambda)\left(p_{g 1}-c_{E}\right)\left(\alpha-p_{g 1}\right),
$$

where $p_{g 1}$ and $p_{g 2}$ are given as above.
Similarly, firm 2 's profit, given that firm 1 shares information, but firm 2 keeps information private, is as follows.

$$
\Pi_{g 2}=\frac{\lambda\left(p_{g 2}-c_{l}\right)\left(\alpha(1-\gamma)-p_{g 2}+\gamma p_{g 1}\right)}{\left(1-\gamma^{2}\right)}
$$

Substituting for $p_{g 1}$ and $p_{g 2}$ and simplifying, yields

$$
\Pi_{g_{2} 2}=\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{\left[\alpha((2-\gamma) N+(2-\gamma) \lambda)+\gamma N c_{E}-(2 N+(2+\gamma) \lambda) c_{I}\right]^{2}}{\left[4(1-\gamma) N+\lambda\left(4-\gamma^{2}\right)\right]^{2}}
$$

where $N=\delta(1-\lambda)(1+\gamma)$.

## 5. Firm Incentives to Share Information

We establish the constraints under which firms find it Pareto optimal to share information by comparing the profits made by the firms when $I=s$ or $I=g$ relative to $I=p$. We consider the relative change in Bertrand profits when both firms share information, $\Delta \Pi_{\mathrm{sh}}=\left(\Pi_{\mathrm{sh}}-\Pi_{\mathrm{pr}}\right)$, relative to when both firms keep information private. We also investigate the incentive to deviate from an information sharing agreement, by examining the relative change in Bertrand profits when firm 1 shares information with firm 2 but firm 2 keeps its information private, $\Delta \Pi_{\mathrm{g} 1}=\left(\Pi_{\mathrm{g} 1}\right.$ $\left.-\Pi_{\mathrm{pr}}\right)$ and $\Delta \Pi_{\mathrm{g} 2}=\left(\Pi_{\mathrm{g} 2}-\Pi_{\mathrm{pr}}\right)$, relative to when both firms keep information private.

## PROPOSITION 1.

- The firms keep information private when the competing products are substitutes. However, there exists a Prisoner's Dilemma - they would be better off if both firms shared information.
- The firms share information when the competing products are complements.

Please see Appendix B for an analytical proof of Proposition 1.
We first establish the conditions necessary for a Prisoner's Dilemma to occur and also the conditions necessary for the firms to prefer sharing information to keeping information private. Next, we examine the case where the products are substitutes, separately from the case where the products are complements.

The three conditions necessary for the Prisoner's Dilemma with respect to information sharing to occur are as follows.

1. The profit when the firms share information is higher than the profit when the firms keep information private: $\left(\Pi_{\text {sh }}>\Pi_{\mathrm{pr}}\right)$ or $\left(\Delta \Pi_{\text {sh }}>0\right)$.
2. Each firm has an individual incentive to deviate from an information sharing agreement: $\left(\Pi_{\mathrm{g} 2}>\Pi_{\mathrm{sh}}\right)$ or $\left(\Delta \Pi_{\mathrm{g} 2}>\Delta \Pi_{\text {sh }}\right)$.
3. Each firm prefers keeping information private to giving information away: $\left(\Pi_{\mathrm{g} 1}<\Pi_{\mathrm{pr}}\right)$ or $\left(\Delta \Pi_{\mathrm{g} 1}<0\right)$.

We find that such a Prisoner's Dilemma occurs when the competing products are substitutes $(0<\gamma<1)$. The Nash equilibrium resulting from this Prisoner's Dilemma is that both firms keep information private. Interestingly, the firms would both make a higher profit if both shared information, since $\left(\Delta \Pi_{s}>0\right)$. However, mutually sharing information is not a Nash equilibrium, as each firm has an independent incentive to deviate from an information sharing agreement $\left(\Delta \Pi_{\mathrm{g} 2}>\Delta \Pi_{\mathrm{sh}}\right)$. Thus, there exists a Prisoner's Dilemma regarding information sharing when the above conditions are satisfied and the resulting Nash equilibrium is that both firms keep information private.

We now consider the conditions necessary for the firms to prefer sharing information to keeping information private.

1. The profit when the firms share information is higher than the profit when the firms keep information private: $\left(\Pi_{\text {sh }}>\Pi_{\mathrm{pr}}\right)$ or $\left(\Delta \Pi_{\text {sh }}>0\right)$.
2. Neither firm has an individual incentive to deviate from an information sharing agreement: $\left(\Pi_{\mathrm{g} 2}<\Pi_{\text {sh }}\right)$ or $\left(\Delta \Pi_{\mathrm{g} 2}<\Delta \Pi_{\text {sh }}\right)$.

We find that when the competing products are complements $(-1<\gamma<0)$, both these conditions are satisfied. The resulting Nash equilibrium is that both firms share information.

The intuition behind Proposition 1 lies in the trade-off faced by competing firms that sell differentiated products to a heterogeneous population of inexpensive and expensive customers. Sharing information causes an increase in price competition between the firms for the desirable segment of inexpensive customers, particularly when the competing products are substitutes. At the same time, sharing information improves the firms' ability to segment and sell to inexpensive customers in the market, which lowers their marginal costs. These cost savings are a function of the relative proportion of expensive customers in the market and the uncertainty associated with
identifying them. The gains from information sharing are higher when the relative proportion of expensive customers in the market is greater. The tug-of-war between lower marginal costs and higher price competition drives the equilibrium outcomes.

We demonstrate Proposition 1 using numerical simulations.

## Insert Figure 1

Figure 1 shows the incentive of the firms to share information, as a function of the product differentiation $(-1<\gamma<1)$. We consider a unit demand intercept $\alpha=1$ and normalize the marginal cost of selling to inexpensive customers as $c_{I}=0$ without loss of generality. Further, we fix the level of noise in the firms' information as $\delta=0.25$ and examine the case where the proportion of expensive customers in the market is $(1-\lambda)=10 \%$.

The left panel of Figure 1 plots the change in equilibrium price when firms share information, compared to when the firms keep information private $\left(p_{s h}-p_{p r}\right)$, with respect to the product differentiation $(-1<\gamma<1)$. We observe that the equilibrium prices when firms share information are always lower than those when the firms keep information private, or $p_{s h}-p_{p r}<0$. This numerically illustrates Lemma 4.

The right panel of Figure 1 plots the change in profits under information sharing, when firms share information, compared to when the firms keep information private, with respect to the product differentiation $(-1<\gamma<1)$. It illustrates that $\Delta \Pi_{\mathrm{g} 2}>\Delta \Pi_{\mathrm{sh}}>0>\Delta \Pi_{\mathrm{g} 1}$ for $(0<\gamma<1)$. This demonstrates the Prisoner's Dilemma, when the products are substitutes. The left half of Figure 1 also illustrates that $\Delta \Pi_{\text {sh }}>0>\Delta \Pi_{\mathrm{g} 2}$ for $(-1<\gamma<0)$. This demonstrates that the Nash equilibrium, when the products are complements, is that the firms share information.

## Comparative Statics

We additionally numerically illustrate the comparative statics for the firms' incentive to share information, with respect to the other model parameters. Specifically, we demonstrate how the relative proportion of expensive customers in the market (1- $\lambda$ ); the noise in the firms' information $(\delta)$ and the relative marginal cost of selling to expensive customers $c_{H}$, moderate the incentives to share customer information.

RESULT 1: The gain from sharing information increases as the proportion of expensive customers in the market (1- $\lambda$ ) increases, provided their relative proportion is small.

## Insert Figure 2

We numerically illustrate in Figure 2 that when the firms are substitutes ( $\gamma=0.5$ ), the gains from information sharing increase, as the relative proportion of expensive customers in the market (1- $\lambda$ ) increases up to $60 \%$. However, as the proportion of expensive customers increases beyond $60 \%$, the relative sales made by the firms start decreasing so much, that they begin to overwhelm the gains from sharing information. This accounts for the decrease in gain from sharing information. Analogously, the right panel of Figure 2 illustrates the case where the firms are complements ( $\gamma=-0.5$ ). We observe an increase in the gains from sharing information, while the relative proportion of expensive customers is less than $80 \%$ and a decrease in gains beyond that.

We also observe that the Prisoner's Dilemma exists for the entire range of proportion of inexpensive customers in the market $(0<\lambda<1)$, when the products are substitutes $(\gamma=0.5)$. Also, the firms prefer to share information when the products are complements $(\gamma=-0.5)$ for all values over the range $(0<\lambda<1)$. This offers additional evidence in support of Proposition 1 .

RESULT 2: The gain from sharing information increases as the noise in the firms' information ( $\delta$ ) increases.

## Insert Figure 3

In case firms selling substitute products $(\gamma=0.5)$ are able to coordinate and share information, we find that the benefits from information sharing increase with an increase in the noise in the firms' information, $(0<\delta<0.5)$. We numerically illustrate this in the left panel of Figure 3. When the firms sell complementary products $(\gamma=-0.5)$, the right panel of Figure 3 numerically illustrates a similar result, $\frac{\partial\left(\Delta \Pi_{s}\right)}{\partial \delta}>0$.

We also note that the Prisoner's Dilemma exists throughout the support of the noise parameter $(0<\delta<0.5)$, when the products are substitutes $(\gamma=0.5)$. Also, the firms prefer to share information when the products are complements $(\gamma=-0.5)$ throughout the support of the noise parameter $(0<\delta<0.5)$. This offers additional evidence in favor of Proposition 1.

RESULT 3: The gain from sharing information increases as the marginal cost of selling to expensive customers ( $c_{E}$ ) increases.

## Insert Figure 4

In case firms selling substitute products $(\gamma=0.5)$ are able to coordinate and share information, we find that the benefits from information sharing increase, as the marginal cost of serving the expensive customers increase. A similar result $\frac{\partial\left(\Delta \Pi_{s}\right)}{\partial c_{E}}>0$ is also numerically illustrated in Figure 4, when the firms share information, given that the products are complementary ( $\gamma=-0.5$ ).

We once again note that the Prisoner's Dilemma exists in the numerical simulation with respect to $c_{E}$, when the products are substitutes $(\gamma=0.5)$. Also, once again, the firms prefer to share information when the products are complements $(\gamma=-0.5)$. This offers even more evidence in favor of Proposition 1.

RESULT 4: The Nash equilibria of the information-sharing game are $Z=\{p r, s h\}$ and $Z=g$ is never a Nash equilibrium. There is no asymmetric pure strategy equilibrium.

An analytical proof of this result requires us to show that $(\mathrm{Z}=\mathrm{g})$ is a strictly dominated strategy and never Nash equilibrium. This proof is also analytically intractable. Our numerical simulations in Figures 1-4 illustrate that the Nash equilibrium of the information sharing game, when the firms are substitutes, is that they keep information private $(\mathrm{Z}=\mathrm{pr})$. Our simulations also illustrate that when the firms are complements, the Nash equilibrium of the information sharing game is that the firms share information $(\mathrm{Z}=\mathrm{sh})$. We never observe an asymmetric Nash Equilibrium $(Z=g)$.

## 6. The Impact of Information Sharing on Consumer Surplus

We measure the consumer surplus, corresponding to the profit maximizing prices set by two competing symmetric firms, as discussed above.

Consumer Surplus When Both Firms Keep Information Private ( $Z=p r$ )
The symmetric firms set the equilibrium price $p_{p r}$ and sell to a fraction of $\lambda$ inexpensive and $1-\lambda$ expensive customers. The demand function is given by $p_{i}=\alpha-q_{i}-\gamma q_{3-i}$. For the symmetric case, the demand function for inexpensive customers is given as $p=\alpha-(1+\gamma) q$, while the demand function for expensive customers is given as $p=\alpha-q$.

A given consumer's surplus is defined as the difference between her maximum willingness to pay and the price charged by the firm. Thus, we aggregately measure the surplus of all inexpensive customers, representing a fraction $\lambda$, of all sales, by integration. The consumer surplus of inexpensive customers, when two symmetric firms keep information private, is given as follows.

$$
C S_{p r, I}=\int_{p_{p r}}^{\alpha} \frac{\lambda(\alpha-p)}{(1+\gamma)} d p=\frac{\lambda\left(\alpha-p_{p r}\right)^{2}}{2(1+\gamma)}
$$

Analogously, the consumer surplus of expensive customers, when two symmetric firms keep information private, is given as follows.

$$
C S_{p r, E}=\int_{p_{p r}}^{\alpha} \delta(1-\lambda)(\alpha-p) d p=\frac{\delta(1-\lambda)\left(\alpha-p_{p r}\right)^{2}}{2}
$$

The aggregate consumer surplus across all consumers, when both firms keep information private, is the sum, $C S_{p r}=C S_{p r, I}+C S_{p r, E}$, given as follows.

$$
C S_{p r}=\left(\frac{\lambda}{(1+\gamma)}+\delta(1-\lambda)\right) \frac{\left(\alpha-p_{p r}\right)^{2}}{2}
$$

Substituting for $p_{p r}$ yields

$$
C S_{p r}=\frac{1}{2}\left(\frac{\lambda}{(1+\gamma)}+\delta(1-\lambda)\right)\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)^{2}
$$

## Consumer Surplus When Both Firms Share Information ( $Z=s h$ )

The symmetric, competing firms set the equilibrium price $p_{s h}$ and sell only to inexpensive customers and do not sell to any expensive customers. Under this scenario, the consumer surplus of expensive customers is zero, while the consumer surplus of inexpensive customers is measured by integration as follows.

$$
C S_{s h}=\int_{p_{s h}}^{\alpha} \frac{\lambda(\alpha-p)}{(1+\gamma)} d p=\frac{\lambda\left(\alpha-p_{s h}\right)^{2}}{2(1+\gamma)}
$$

Substituting for $p_{s h}$ yields

$$
C S_{s h}=\frac{\lambda\left(\alpha-c_{I}\right)^{2}}{2(1+\gamma)(2-\gamma)^{2}}
$$

## Consumer Surplus When Only One Firm Shares Information ( $Z=g$ )

Result 4 demonstrates that $(Z=g)$ is never a Nash Equilibrium. Since this information-sharing scenario never arises in equilibrium, we do not measure the corresponding consumer surplus.

## Comparing the Change in Consumer Surplus of Expensive and Inexpensive Customers

We now examine the impact of information sharing on consumer surplus across the two segments of expensive and inexpensive customers. Specifically, we compare the consumer surplus of each segment when $(Z=s h)$ with that when $(Z=p r)$.

Recall from Proposition 1 that the firms

PROPOSITION 2: When firms share information, the consumer surplus of inexpensive customers increases, while that of the expensive customers decreases.

Please see Appendix C for an analytical proof of Proposition 2.
The intuition behind Proposition 2 is that the segment of expensive customers in the market raises the marginal cost incurred by the firms. If information sharing improves the firms' ability to exclude expensive customers and sell to inexpensive customers, the consumer surplus of expensive customers decreases.

The expensive customers impose a negative externality on the inexpensive customers, in the sense that the increase in the firms' marginal costs translates into a higher equilibrium price for all consumers in the market. Sharing information results in lower equilibrium prices, as shown in Lemma 4. This correspondingly causes an increase in the consumer surplus of the inexpensive customers.

## Insert Figures 5-8

We numerically illustrate Proposition 2 in Figures 5-8. Figure 5 demonstrates that the consumer surplus of inexpensive customers increases, while that of the expensive customers decreases, as a function of the product differentiation $(-1<\gamma<1)$ in the market, holding other model parameters constant. Figure 6 demonstrates a similar increase in the surplus of inexpensive customers and decrease in the surplus of expensive customers, as a function of the proportion of expensive customers $(0<1-\lambda<1)$ in the market, both when the competing products are substitutes $(\gamma=0.5)$ and complements $(\gamma=-0.5)$. Figure 7 illustrates Proposition 1, but with respect to variation in the noise $(0<\delta<1)$ in the firms' information, both when the competing products are substitutes $(\gamma=0.5)$ and complements $(\gamma=-0.5)$. Finally, Figure 8 illustrates Proposition 1 with respect to variations in the marginal cost of selling to expensive customers. Figures 5-8 collectively reinforce that when firms share information, the inexpensive customers are better off, even though the expensive customers become worse-off.

## The Impact of Information Sharing on Aggregate Consumer Surplus

We examine the corresponding impact of information sharing on aggregate consumer welfare, by estimating the consumer surplus when both firms keep information private ( $Z=p r$ ) and share information $(Z=s h)$. By aggregate consumer surplus, we refer to the sum of the consumer surplus of expensive and inexpensive customers in the market. Note that once again we ignore the case when only one firm shares information while the other firm keeps information private $(Z=g)$, because as per Result 4, this is never an equilibrium outcome. We examine whether $\Delta C S_{s h}=\left(C S_{s h}-C S_{p r}\right)>0$. This investigation highlights the aggregate impact of information sharing between firms on consumers.

PROPOSITION 3: When firms share information, the aggregate consumer surplus typically increases.

We find that a formal proof of this proposition is analytically intractable. We illustrate this proposition through numerical simulations in Figures 9-12.

Insert Figures 9-12

Figure 9 illustrates the change in aggregate consumer surplus, as a function of the product differentiation $(-1<\gamma<1)$ in the market. We observe that the aggregate consumer surplus typically increases. An exception to this is that the aggregate consumer surplus decreases for $(0.6<\gamma<1)$.

Figure 10 demonstrates that the aggregate consumer surplus increases, as a function of the proportion of expensive customers $(0<1-\lambda<1)$ in the market, both when the competing products are substitutes $(\gamma=0.5)$ and complements $(\gamma=-0.5)$. Figure 11 illustrates Proposition 3, but with respect to variation in the noise $(0<\delta<1)$ in the firms' information, both when the competing products are substitutes $(\gamma=0.5)$ and complements $(\gamma=-0.5)$. Finally, Figure 12 illustrates Proposition 3 with respect to variations in the marginal cost of selling to expensive customers. The important point to emphasize here is that consumers can be aggregately better off when firms share information.

## 7. Implications and Insights

### 7.1. Management of Information Assets

Customer information has become an increasingly important corporate asset in the modern information-intensive business environment. Our analysis examines how firms' knowledge about individual customers affects the nature of their strategic interactions and thus offers some noteworthy implications regarding the management of information assets in a competitive scenario. Should a firm share its individual-level customer information with that of its competitors?

Our work establishes that the firm's decision to share information is driven by a tug-ofwar between a decrease in marginal costs arising from sharing information about expensive
customers on the one hand and a decrease in profits from increased price competition on the other hand. Whether competing firms should share their knowledge of individual customers in a market is driven critically by the product differentiation between the firms. Our research demonstrates that even though a superior knowledge of individual customers may be a competitive advantage, it does not mean that the firm always protects its customer information from its competitors. Rather, firms can benefit mutually from sharing individual customer information.

Specifically, we find support for information sharing when the competing products are complements, but find that firms keep information private when they sell substitute products. The counterintuitive outcome of our analysis is that firms selling substitute products may be able to increase their mutual profits by entering into an information-sharing agreement. However, the limitation of such an agreement lies in an individual firm's incentive to deviate from its commitment to share information, which gives rise to the need for an external mechanism to regulate information sharing.

### 7.2. To Join or Not to Join an Industry Trade Group?

An industry trade group or trade association is one institution through which information gets collected, organized, and transmitted among firms. For example, the National Retail Federation (NRF) is the world's largest retail trade association, with membership that comprises all retail formats and channels of distribution including department, specialty, discount, catalog, Internet, grocery, and independent stores, as well as chain restaurants and the industry's key trading partners of retail goods and services. The NRF's statistical program typically involves three steps: (1) Collect individual company data, usually about production, demand, or cost; (2) compile industry wide totals; and (3) distribute aggregate reports to member firms and others (Vives 1990). The decision to join an industry trade group remains a strategic consideration for many firms that involves various micro-level considerations. For example, information sharing under the umbrella of an industry trade group may affect the degree of competitiveness in the market or increase the covariance between firms' actions. At a macro level, this issue has important implications for antitrust regulations, legislation, and public policy in terms of whether it improves efficiency and social surplus.

Should a firm join a trade association? Our research offers some noteworthy insights into this question in a competitive scenario. Our equilibrium analysis demonstrates a Prisoner's Dilemma if the competing products are substitutes. We find that the firms can increase their profits by sharing information when the decrease in marginal costs brought about by not selling to expensive customers, dominates the increase in price competition. However, the profits that the firm can accrue, if it deviates from an information-sharing agreement, are even higher than those it earns from sharing information. This situation gives rise to a Prisoner's Dilemma. Ironically, the firms are therefore unable to sustain an information-sharing agreement without external supervision, and in equilibrium, they end up keeping information private.

We suggest that industry trade groups might oversee the sharing of market information among firms that sell competing products. In particular, they may help overcome the information-sharing Prisoner's Dilemma. (If we expand the strategy space to include the choice of deciding to join a trade association prior to the information-sharing decision, both firms will choose to join the association to obviate the Prisoner's Dilemma due to defection.) Joining an industry trade group therefore could lead to a win-win situation for the competing firms as well as the dominant majority of inexpensive customers in the market. Although firms may have other exogenous reasons that dictate their decision to join or not join industry trade associations, in the context of information sharing, our research identifies a compelling reason in support of firms joining trade associations.

### 7.3. Whom Do Privacy Laws Protect?

Electronic data-mining systems gather and process information about consumers 24 hours a day, tracking prior purchases, personal files and public records, and Internet surfing. They thus create overall databases of individual behavior that contain details about everything, from activities and interests to credit and product purchasing habits. Garfinkel (2001) cautions consumers to be concerned about how and where their data are collected, tracked, and stored; the laws that protect privacy; and who owns, manipulates, ensures the safety of, and manages the vast amount of individually identifiable personal data. Other advocates of stringent privacy laws argue that consumers are affected adversely when firms share their information. Privacy and security experts advise consumers about why and how to protect themselves from information predators
(Sontag 2001; Luna 2004), and opinion polls consistently show that the public would prefer more privacy and is concerned about its erosion.

Our research attempts to temper the panic and mitigate the dismal picture painted by privacy experts by advocating a more neutral stance. We demonstrate that privacy laws meant to protect consumers from economic injury have instead had a heterogeneous impact on consumer welfare. For example, we find a heterogeneous change in consumer surpluses across our two customer segments when competing firms share customer information: The consumer surplus of the expensive customers decreases, whereas that of the inexpensive customers increases. We infer from this finding that the expensive customers impose an indirect negative externality on the inexpensive customers. The adverse impact of expensive customers thus has two facets: (1) These customers impose a direct cost on the firms by raising their marginal costs and lowering their profitability and (2) they impose an indirect cost on the inexpensive customers.

Firm incentives to share customer information are moderated by privacy laws, which constrain their ability to share information legally. Our analysis justifies a more moderate stance toward privacy laws. Brin (1999) supports our argument, arguing for less, not more, privacy protection. He claims that attempts to protect privacy usually benefit only a minority of the powerful or corrupt. The counterintuitive insight emerging from our research is that society as a whole may be better off with relatively weaker privacy laws. Although we acknowledge that our suggestion must be tempered, in the sense that we refer to one specific notion of private information, we hope to provoke the idea that strict privacy laws may not increase consumer welfare.

## 8. Conclusion and Limitations

The advancement of technology, particularly database management, data-mining techniques, and network communications, has dramatically increased firms' ability to collect, process, and share individual customer information. It is therefore more important than ever for companies to assess the individual value of their customers objectively and acknowledge the fallacy that the customer is always right. Some customers are so costly that a company may be better off by not serving them. Firms with access to progressively more comprehensive, detailed, and precise databases of micro-level customer information can use increasingly sophisticated data mining and segmentation techniques to identify expensive customers. On a related note, advancements in
information collection in the modern digital age are causing increasing and widespread privacy concerns among contemporary consumers about individual profiling by firms and data sharing among firms. Policymakers and consumer protection groups actively question the current state of privacy laws and their impact on consumer welfare.

Our article addresses these issues and investigates information sharing by firms that offer differentiated products. Expensive customers who seek a high level of service impose a high cost on the firm, and when the firm offers a single price, inexpensive customers, who seek a lower level of service, end up subsidizing the expensive customers. We present an analytical model to examine the conditions in which two profit-maximizing Bertrand competitors become motivated to share information about the cost of serving various consumers and find that its implications differ depending on the degree of product differentiation between the competing firms in the market. In particular, when the competing products are substitutes, our model predicts a Prisoner's Dilemma. The competing firms are unilaterally better off by sharing information as opposed to keeping it private, but the benefits from reneging on an information-sharing agreement are even higher, so paradoxically, both firms keep their information private in equilibrium.

We thus infer that syndicates, such as industry trade associations, can play a supervisory role in coordinating joint information-sharing agreements between competing firms. On a related note, the sharing of customer information among firms remains constrained by privacy laws. Whereas advocates of stringent privacy laws claim that consumers are always adversely affected when firms share information, we question the validity of this hypothesis. Our analysis shows that information sharing increases the welfare of the firms' inexpensive customers but decreases the consumer welfare of expensive customers. Therefore, privacy laws that limit firms from legally sharing customer information heterogeneously affect consumers, such that they protect and benefit the expensive customers more than they do inexpensive customers. This inference is counterintuitive to the comments published in the popular press. Furthermore, our research recommends relatively weaker, not stronger, privacy laws.

Our analysis contains some limitations and constraints. First, in practice, information sharing enhances firms' ability to identify and exclude expensive customers. However, we consider a limiting case of this improvement, in which information sharing allows firms to identify and exclude expensive customers perfectly. Therefore, our results should be interpreted
as a useful benchmark and solution to this limiting case. Second, we model a duopoly market that sells to two market segments, which are heterogeneous in the cost they impose on the firms. In practice, markets have many firms and a continuum of consumers. Nevertheless, we expect our key insights to extend to more general settings. Third, we model Bertrand price competition, in which firms simultaneously decide what price to charge. We do not consider any asymmetry that may result from order of entry effects or the sequence of decision-making. Therefore, our model insights might to be affected if we were to consider a Stakelberg market setting. For example, a new entrant may benefit more than an incumbent from information sharing and joining a trade association. We leave this issue for further research.

Figure 1: Incentive to share information, as a function of product differentiation $(-1<\gamma<1)$
$\%$ Expensive Customers $(1-\lambda)=10 \%$
Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


Figure 2: Incentive to share information, as a function of \% expensive customers in the market ( $0<1-\lambda<1$ )

Product Differentiation $\gamma=0.5$ (Substitutes) $O R-0.5$ (Complements)
Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


Legend: $-\Delta \Pi_{s} ; \quad \cdots \cdots \cdots \cdots \cdot \Delta \Pi_{\mathrm{g} 1} ; \quad-----\Delta \Pi_{\mathrm{g} 2}$

Figure 3: Incentive to share information, as a function of noise in the information ( $0<\delta<0.5$ )
Product Differentiation $\gamma=0.5$ (Substitutes) $O R-0.5$ (Complements)
Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ;(1-\lambda)=10 \%$


Figure 4: Incentive to share information, as a function of the relative marginal cost of serving expensive customers ( $c_{E}$ )

Product Differentiation $\gamma=0.5$ (Substitutes) $O R-0.5$ (Complements)
Normalize $\alpha=1 ; c_{I}=0 ;(1-\lambda)=10 \%$
$(1-f) \alpha+f c_{I}<c_{E}<\alpha$ Implies $0.6<c_{E}<1$


Legend: $\Delta \Pi_{s} ; \quad \cdots \cdots \cdots \cdots \cdot \Delta \Pi_{\mathrm{g} 1} ; \quad-----\Delta \Pi_{\mathrm{g} 2}$

Figure 5: Change in consumer surplus of inexpensive and expensive customers, when firms share information, as a function of product differentiation, $(-1<\gamma<1)$.
$\%$ Expensive Customers $(1-\lambda)=10 \%$
Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


Figure 6: Change in consumer surplus of inexpensive and expensive customers, when firms share information, as a function of the $\%$ expensive customers in the market $(0<1-\lambda<1)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma=-0.5)$.

Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


Legend: - Substitutes $(\gamma=0.5) \quad----$ Complements $(\gamma=-0.5)$

Figure 7: Change in consumer surplus of inexpensive and expensive customers, when firms share information, as a function of noise in the information $(0<\delta<0.5)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma=-0.5)$.

Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ;(1-\lambda)=10 \%$


Figure 8: Change in consumer surplus of inexpensive and expensive customers, as a function of the relative marginal cost of serving expensive customers $\left(c_{E}\right)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma=-0.5)$.

Normalize $\alpha=1 ; c_{I}=0 ;(1-\lambda)=10 \%(1-f) \alpha+f c_{I}<c_{E}<\alpha$ Implies $0.6<c_{E}<1$


[^4]Figure 9: Change in aggregate consumer surplus when firms share information, as a function of product differentiation, $(-1<\gamma<1)$.
\%Expensive Customers $(1-\lambda)=10 \%$
Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


Figure 10: Change in aggregate consumer surplus when firms share information, as a function of the $\%$ expensive customers in the market $(0<1-\lambda<1)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma=-0.5)$.

Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ; \delta=0.25$


$$
\text { Legend: }- \text { Substitutes }(\gamma=0.5) \quad---- \text { Complements }(\gamma=-0.5)
$$

Figure 11: Change in aggregate consumer surplus when firms share information, as a function of noise in the information $(0<\delta<0.5)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma=-0.5)$.

Normalize $\alpha=1 ; c_{E}=0.75 ; c_{I}=0 ;(1-\lambda)=10 \%$


Figure 12: Change in aggregate consumer surplus, as a function of the relative marginal cost of serving expensive customers $\left(c_{E}\right)$, when the products are substitutes $(\gamma=0.5)$ or complements $(\gamma$ $=-0.5$ ).

Normalize $\alpha=1 ; c_{I}=0 ;(1-\lambda)=10 \%(1-f) \alpha+f c_{I}<c_{E}<\alpha$ Implies $0.6<c_{E}<1$


Legend: - Substitutes $(\gamma=0.5) \quad----$ Complements $(\gamma=-0.5)$

## References

Annicelli, C. 2003. Toys 'R' Us ends data sharing. Playthings (19 August).
Bakos, Y.J. 1997. Reducing buyer search costs: Implications for electronic marketplaces. Management Science 43(12) 1676-1692.

Brin, D. 1999. The Transparent Society: Will Technology Force Us to Choose Between Privacy and Freedom? Perseus Books Group.

Bult, J.R., T. Wansbeek. 2004. Lists and prospecting: Who's who among the B2B co-op databases. Catalog Age (1 May)

Chen, Yuxin, Chakravarthi Narasimhan, Z. John Zhang. 2001. Individual marketing with imperfect targetability. Marketing Science 20(1) 23-41.

Clarke, Richard. 1983. Collusion and the incentives for information sharing. Bell Journal of Economics 14 383-394

Deighton, J., D Petters, M Rogers. 1996. Next generation marketing: Data sharing. Direct Marketing (March)

Desai, Preyas S. 2001. Quality segmentation in spatial markets: When does cannibalization affect product line design? Marketing Science 20(3) 265-283.

Fried, D. 1984. Incentives for information production and disclosure in a duopolistic environment. Quarterly Journal of Economics 99 367-381.

Fudenberg, D. J. M. Villas-Boas. 2006. Behavior-Based Price Discrimination and Customer Recognition in Handbook of Economics and Information Systems. Elsevier Publishing.

Gal-Or, Esther. 1986. Information transmission-Cournot and Bertrand equilibria. The Review of Economic Studies 53(1) 85-92.

Garfinkel Simpson. 2001. Database Nation: The Death of Privacy in the 21st centuary. O'Reilly Media, Inc.

Gupta, Sunil, Donald Lehmann. 2005. Managing Customers as Investments: The Strategic Value of Customers in the Long Run. Philadelphia, PA: Wharton School Publishing.

Heun, C. T. 2001. Wal-Mart and other companies reassess their data sharing strategies Information Week (21 May)

Hotelling, Harold. 1929. Stability in competition. The Economic Journal 39(153) 41-57.
Hui, Kai-lung, LP.L. Png 2006. The Economics of Privacy in Handbook of Economics and Information Systems. Elsevier Publishing.

Jones, P. 2006. Tackling organized retail crime. Retail Merchandiser (January).
Judge, P. C. 1998. Do you know who your most profitable customers are? Business Week September 14,.
Kirby, Alison, J. 1988. Trade associations as information exchange mechanisms. RAND Journal of Economics 99 138-146

Libey, Donald R., Christopher Pickering. 2005. Libey and Pickerfing on RPM and Beyond. MeritDirect Press.

Luna, J. J. 2004. How to be Invisible: The Essential Guide to Protecting Your Personal Privacy, Your Assets, and Your Life. Thomas Dunne Books.

McGuire, Timothy W., Richard Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. Marketing Science 2(2) 161-191.

McWilliams, G. 2004. Minding the store: Analyzing customers, Best Buy decides not all are welcome. The Wall Street Journal. (November 8)

Miller, Paul. 2003. Re-evaluating co-op database options. Catalog Age.
Moorthy, K.S. 1984. Market segmentation, self-selection and product-line design. Marketing Science 3(2) 288-307.

Novshek, W., Hugo Sonnenschein. 1982. Fulfilled expectations Cournot duopoly with information acquisition and release. Bell Journal of Economics 13 214-218

Mussa, M, S Rosen. 1978. Monopoly and product quality. Journal of Economic Theory 18 301-317.
Raith, Michael. 1996. A general model of information sharing in oligopoly. Journal of Economic Theory 71 260-289.

Raju, Jagmohan S., Abhik Roy. 2000. Market information and firm performance. Management Science 46 (8) 1075-84

Rendelman, John. 2001. Customer data means money. Information Week. (20 August)
Sakai, Y. 1985. The value of information in a simple duopoly model. Journal of Economic Theory 36 3654.

Selden, Larry, Geoffery Colvin. 2003. Angel Customers and Demon Customers: Discover Which Is Which and Turbo-Charge Your Stock. Portfolio Hardcover.

Shaffer, Greg, Z. John Zhang. 2002. Competitive one-to-one promotions. Management Science 48 11431160

Shapiro, Carl. 1986. Exchange of cost information in oligopoly. The Review of Economic Studies 53(3) 433-446.

Sontag, Larry 2001. It's None of Your Business: A Complete Guide to Protecting-Your Privacy, Identity and Assets. PMI Enterprises.

Taylor, C.R. 2004. Consumer privacy and the market for customer information. RAND Journal of Economics 35 631-650.

Tirole, Jean. 1988. The Theory of Industrial Organization. Cambridge, MA: The MIT Press.
Tyagi, Rajeev K. 2004. Technological advances, transaction costs and consumer welfare. Marketing Science 23(3) 355-364.

Varian, H.R. 1985. Price discrimination and social welfare American Economic Review 75 870-885.
Vives, Xavier. 1984. Duopoly information equilibrium: Cournot and Bertrand. Journal of Economic Theory 34 71-94.

Vives, Xavier. 1990. Trade association disclosure rules, incentives to share information and welfare. RAND Journal of Economics 21(3) 409-430.

Weston, L.P. 2004. Are you a bad customer? MSN Money.

## Appendix A

## Proof of Lemma 1

The firms should earn a positive margin by selling to inexpensive customers, or $p_{p r}-c_{I}>0$.
Substituting for $p_{p r}$ in the inequality $p_{p r}>c_{I} \Leftrightarrow \lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}-2 c_{I}\right)>0$
$\Leftrightarrow \lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\left(\alpha-c_{I}\right)+\left(c_{E}-c_{I}\right)\right)>0$.
Also, we must have strictly positive equilibrium sales or $q_{p r}>0$.
Note that $q_{p r}=\left(\frac{\lambda}{(1+\gamma)}+\delta(1-\lambda)\right)\left(\alpha-p_{p r}\right)$. Since we always have $\frac{\lambda}{(1+\gamma)}+\delta(1-\lambda)>0$, this implies that $\alpha>p_{p r}$. Substituting for $p_{p r}$ and simplifying yields a second inequality $\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)>0$.
Reconsider the two inequalities:

1. $\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}-2 c_{I}\right)>0$
2. $\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)>0$

Since $\left(\alpha+c_{E}-2 c_{I}\right)=\alpha-c_{E}+2\left(c_{E}-c_{I}\right)$, we see that the first inequality is always true whenever the second inequality is true. Thus, we only need inequality 2 .
Rearranging the terms of Inequality 2 and simplifying yields
$c_{E}<\left(1+\frac{\lambda}{\delta(1-\lambda)\left(1-\gamma^{2}\right)}\right) \alpha$
We can write this as $c_{E}<\left(1+\frac{\lambda}{M}\right) \alpha$, where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$.
This is the necessary condition for market entry. This proves Lemma 1.
As an aside, we also note that $c_{E}<\alpha$ is a sufficient condition for market entry. QED

## Proof of Lemma 2

The firms incur a loss from selling to the segment of expensive customers if their margin from selling to them is negative, or $p_{p r}-c_{E}<0$. Substituting for $p_{p r}$ in the inequality $p_{p r}<c_{E}$ yields $\lambda \alpha(1-\gamma)+\lambda c_{I}+\delta(1-\lambda)\left(1-\gamma^{2}\right) \alpha<\lambda(2-\gamma) c_{E}+\delta(1-\lambda)\left(1-\gamma^{2}\right) c_{E}$
$\Leftrightarrow(1-\gamma)\left(\alpha-c_{E}\right)(\lambda+\delta(1-\lambda)(1+\gamma))<\lambda\left(c_{E}-c_{I}\right)$
Simplifying this inequality, we get
$c_{E}>\left(\frac{\lambda(1-\gamma)+M}{\lambda(2-\gamma)+M}\right) \alpha$, where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$
Applying Lemma 1, we see that the firms incur a loss from selling to the segment of expensive customers, if $\left(\frac{\lambda(1-\gamma)+M}{\lambda(2-\gamma)+M}\right) \alpha<c_{E}<\left(1+\frac{\lambda}{M}\right) \alpha$. QED

## Proof of Lemma 3

When firms share information, they find it unprofitable to sell to expensive customers if $c_{E}>p_{s h}$. Note that the price $p_{s h}=\frac{\alpha(1-\gamma)+c_{1}}{(2-\gamma)}$ is decreasing in $\gamma$, as $\frac{\partial p_{s h}}{\partial \gamma}=-\frac{\left(\alpha-c_{1}\right)}{\left(2-\gamma^{2}\right)}<0$. Since
$(-1<\gamma<1)$, this implies that $\operatorname{Max}\left(p_{s h}\right)=\left.p_{s h}\right|_{(\gamma=-1)}=\frac{2 \alpha+c_{1}}{3}$. The firms always find it unprofitable to sell to expensive customers if $c_{E}>\operatorname{Max}\left(p_{s h}\right)$ or $c_{E}>\frac{2 \alpha+c_{I}}{3}$. QED

## Proof of Lemma 4

We need to prove that the equilibrium price when the firms share information is lower than the equilibrium price when they keep information private, or $p_{p r}>p_{s h}$.
Recall $p_{p r}=\frac{\lambda\left(\alpha(1-\gamma)+c_{t}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}, p_{s h}=\frac{\alpha(1-\gamma)+c_{1}}{(2-\gamma)}$
Note that for any general positive variables, $a, b, x, y>0$, we have $\frac{a+x}{b+y}>\frac{a}{b} \Leftrightarrow \frac{a}{b}<\frac{x}{y}$
Let $a=\alpha(1-\gamma)+c_{I} ; b=(2-\gamma) ; x=\frac{\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{F}\right)}{\lambda} ; y=\frac{2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}{\lambda}$
Thus, $p_{p r}=\frac{a+x}{b+y} ; p_{s h}=\frac{a}{b}$
$p_{p r}>p_{s h} \Leftrightarrow \frac{\alpha(1-\gamma)+c_{I}}{(2-\gamma)}<\frac{\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}\right)}{2 \delta(1-\lambda)\left(1-\gamma^{2}\right)} \Leftrightarrow \frac{\alpha(1-\gamma)+c_{I}}{(2-\gamma)}<\frac{\left(\alpha+c_{E}\right)}{2} \Leftrightarrow \gamma \alpha+(2-\gamma) c_{E}>2 c_{I}$
Thus, $p_{p r}>p_{s h} \Leftrightarrow \gamma \alpha+(2-\gamma) c_{E}>2 c_{I}$
From Lemma 4, $\alpha>c_{I}$.
Thus, $\gamma \alpha+(2-\gamma) c_{E}>\gamma c_{I}+(2-\gamma) c_{E}=(2-\gamma)\left(c_{E}-c_{I}\right)+2 c_{I}>c_{I}$,
or $\gamma \alpha+(2-\gamma) c_{E}>2 c_{I} \Leftrightarrow p_{p r}>p_{s h}$ QED

## Appendix B

## Proof of Proposition 1

Prisoner's Dilemma when the firms are substitutes $(0<\gamma<1)$.
The Prisoner's Dilemma occurs if three conditions are satisfied:

1. A firm earn more profits by sharing information, or $\Pi_{s h}>\Pi_{p r}$.
2. A firm has an incentive to deviate from the scenario where both firms share information. This occurs if $\Pi_{g 2}>\Pi_{s h}$.
3. A firm also prefers to deviate and keep information private, whenever a competitor deviates from sharing information, or $\Pi_{p r}>\Pi_{g 1}$.
Firms share information when they are complements $(-1<\gamma<0)$. This occurs if two conditions are satisfied:
4. A firm earn more profits by sharing information, or $\Pi_{s h}>\Pi_{p r}$. (same as above)
5. Neither firm has an incentive to deviate from sharing information, or $\Pi_{s h}>\Pi_{g 2}$. (converse of condition 2 above)
When is $\Pi_{s h}>\Pi_{p r}$ ?
Recall that $\Pi_{s h}=\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{\left[\alpha-c_{1}\right]^{2}}{[2-\gamma]^{2}}$ and
$\Pi_{p r}=\frac{\left[M\left(\alpha-c_{E}\right)+\lambda\left(\alpha-c_{t}\right)\right]\left[\left(\alpha-c_{t}\right)(M+\lambda(1-\gamma))-M(M+\lambda)\left(c_{E}-c_{t}\right)\right]}{\left(1-\gamma^{2}\right)[2 M+\lambda(2-\gamma)]^{2}}$
where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$

We first prove that $\frac{\partial\left(\Delta \Pi_{s h}\right)}{\partial \delta}>0$, where $\Delta \Pi_{s h}=\Pi_{s h}-\Pi_{p r}$ and $(0<\delta<0.5)$.
Substituting $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$, we get
$\Delta \Pi_{s h}=\Pi_{s h}-\Pi_{p r}=$
$\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{\left[\alpha-c_{1}\right]^{2}}{[2-\gamma]^{2}}-$
$\frac{\left[\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)+\lambda\left(\alpha-c_{c}\right)\right]\left[\left(\alpha-c_{c}\right)\left(\delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda(1-\gamma)\right)-\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\right)\left(c_{E}-c_{t}\right)\right]}{\left(1-\gamma^{2}\right)\left[2 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda(2-\gamma)\right]^{2}}$
Differentiating this partially with respect to $\delta$ and simplifying yields:

$$
\begin{aligned}
& \frac{\partial\left(\Delta \Pi_{s h}\right)}{\partial \delta}=48 \alpha^{3} c_{E}^{3} M^{7}(2-\gamma)^{2}(1-\gamma)(1-\lambda)(S-\lambda) \lambda^{6} * \\
& {[6-(6-\gamma) \gamma] *[2-(2-\gamma) \gamma] *[2 M+(2-\gamma) \lambda] *} \\
& {\left[2 M^{2}+3 M(2-\gamma) \lambda+2 *(2-\gamma) \lambda^{2}\right] *\left[2 M^{2}+M(4-\gamma) \lambda+(2-(1-\gamma) \gamma) \lambda^{2}\right]}
\end{aligned}
$$

where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)>0$ and $S=\delta(1-\lambda)(1+\gamma)>0$
Note that every product term on the right hand side is always positive.
Thus, we conclude that $\frac{\partial\left(\Delta \Pi_{s h}\right)}{\partial \delta}>0$.
Notice that $\frac{\partial\left(\Pi_{s b}\right)}{\partial \delta}=0$. Since $\Delta \Pi_{s h}=\Pi_{s h}-\Pi_{p r}$, this implies that $\frac{\partial\left(\Pi_{p r}\right)}{\partial \delta}<0$.
Since we have $(0<\delta<0.5)$, this further implies that $\operatorname{Max}\left[\Pi_{p r}\right.$ ] occurs when $\delta=0$.
Let us now consider $\lim _{\delta \rightarrow 0} \Pi_{p r}$.
We have
$\Pi_{p r}=\frac{\left[\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)+\lambda\left(\alpha-c_{l}\right)\right]\left[\left(\alpha-c_{l}\right)\left(\delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda(1-\gamma)\right)-\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\right)\left(c_{E}-c_{l}\right)\right]}{\left(1-\gamma^{2}\right)\left[2 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda(2-\gamma)\right]^{2}}$
Taking limits $\delta \rightarrow 0$, we notice that $\lim _{\delta \rightarrow 0} \Pi_{p r}=\Pi_{s h}$
where $\Pi_{s h}=\frac{\lambda(1-\gamma)}{(1+\gamma)}\left[\frac{\left[\alpha-c_{c}\right]^{2}}{[2-\gamma]^{2}}\right.$
In other words, as $\delta$ becomes smaller and smaller over its range $(0<\delta<0.5)$ and $\delta \rightarrow 0$, $\Pi_{p r} \rightarrow \Pi_{s h}$.
Combining this result with $\frac{\partial\left(\Pi_{p r}\right)}{\partial \delta}<0$, it follows that $\Pi_{s h}>\Pi_{p r}$.
Note that this result is independent of the value of $\gamma$, where $(-1<\gamma<1)$ or any other model parameter besides $\delta$. From this we conclude that $\Pi_{s h}>\Pi_{p r}$, both when the firms are substitutes and when the firms are complements.
When is $\Pi_{g 2}>\Pi_{s}$ ?
Recall that $\Pi_{s h}=\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{\left[\alpha-c_{l}\right]^{2}}{[2-\gamma]^{2}}$
Also, we can write $\Pi_{g 2}=\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{[R]^{2}}{[K]^{2}}$
where
$R=\alpha((2-\gamma) N+(2-\gamma) \lambda)+\gamma N c_{E}-(2 N+(2+\gamma) \lambda) c_{I}$
$K=4(1-\gamma) N+\lambda\left(4-\gamma^{2}\right)$
$N=\delta(1-\lambda)(1+\gamma)>0$

Note that $N>0$ always. So, we also have $K>0$. However, because of a negative sign, it is not clear if $R>0$ or $R<0$
We can write $\Pi_{g 2}-\Pi_{s}=\frac{\lambda(1-\gamma)}{(1+\gamma)} \frac{(2-\gamma)^{2} R^{2}-\left(\alpha-c_{t}\right)^{2} K^{2}}{K^{2}[2-\gamma]^{2}}$
Thus, $\Pi_{g 2}-\Pi_{s}>0 \Leftrightarrow(2-\gamma)^{2} R^{2}-\left(\alpha-c_{I}\right)^{2} K^{2}>0$
$\Leftrightarrow\left((2-\gamma) R-\left(\alpha-c_{I}\right) K\right)\left((2-\gamma) R+\left(\alpha-c_{I}\right) K\right)>0$
Define $A=(2-\gamma) R-\left(\alpha-c_{I}\right) K$ and $B=(2-\gamma) R+\left(\alpha-c_{I}\right) K$
Thus $\Pi_{g 2}-\Pi_{s}>0 \Leftrightarrow A B>0$
Now substituting for $R$ and $K$ in $A=(2-\gamma) R-\left(\alpha-c_{I}\right) K$ and simplifying, yields $A=\gamma N\left[\gamma\left(\alpha-c_{E}\right)+2\left(\alpha-c_{I}\right)\right]$
From Lemma 1, $\alpha>c_{I}$. Thus $A>\gamma N(2-\gamma)\left(c_{E}-c_{I}\right)$
When the firms are substitutes, with $(0<\gamma<1)$, we get $\gamma N(2-\gamma)\left(c_{E}-c_{I}\right)>0$ which implies that $A>0$.
However, if the firms are complements, with $(-1<\gamma<0)$, we get $\gamma N(2-\gamma)\left(c_{E}-c_{I}\right)<0$ and this is not the case.
Now substituting for $R$ and $K$ in $B=(2-\gamma) R+\left(\alpha-c_{I}\right) K$ and simplifying, yields
$B=\delta(1-\lambda)\left[\gamma(1+\gamma)(2-\gamma) c_{E}-2(1+\gamma)(4-3 \gamma) c_{I}+\alpha\left(8-7 \gamma^{2}+\gamma^{3}\right)\right]+2 \lambda\left(4-\gamma^{2}\right)\left(\alpha-c_{I}\right)$ Note that $\left(8-7 \gamma^{2}+\gamma^{3}\right)=8-\gamma^{2}(7-\gamma)>0$ for all $(-1<\gamma<1)$.
Also, from Lemma 1, $\alpha>c_{I}$
Thus $\alpha\left(8-7 \gamma^{2}+\gamma^{3}\right)>\left(8-7 \gamma^{2}+\gamma^{3}\right) c_{I}$

$$
\Leftrightarrow \gamma(1+\gamma)(2-\gamma) c_{E}-2(1+\gamma)(4-3 \gamma) c_{I}+\alpha\left(8-7 \gamma^{2}+\gamma^{3}\right)>\gamma(1+\gamma)(2-\gamma)\left(c_{E}-c_{I}\right)
$$

This implies that
$B>\delta(1-\lambda)\left[\gamma(1+\gamma)(2-\gamma)\left(c_{E}-c_{I}\right)\right]+2 \lambda\left(4-\gamma^{2}\right)\left(\alpha-c_{I}\right)$
When the firms are substitutes, with $(0<\gamma<1)$, we get $B>0$.
However, if the firms are complements, with $(-1<\gamma<0)$, this is not the case.
Thus, we see that when the firms are substitutes, with $(0<\gamma<1)$, we have both $A>0$ and $B>0$.
Since $\Pi_{g_{2}}-\Pi_{s}>0 \Leftrightarrow A B>0$, we see that $\Pi_{g_{2}}-\Pi_{s}>0$ when the firms are substitutes, but not when the firms are complements. QED

When is $\Pi_{p r}>\Pi_{g 1}$ ?
Recall that
$\Pi_{p r}=\frac{\left.\left[M\left(\alpha-c_{E}\right)+\lambda\left(\alpha-c_{1}\right)\right]\left(\alpha-c_{1}\right)(M+\lambda(1-\gamma))-M(M+\lambda)\left(c_{E}-c_{f}\right)\right]}{\left(1-\gamma^{2}\right)[2 M+\lambda(2-\gamma)]^{2}}$
where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)$
Also $\Pi_{g 1}=\frac{\lambda\left(p_{g 1}-c_{l}\right)\left(\alpha(1-\gamma)-p_{g 1}+\gamma p_{g 2}\right)}{\left(1-\gamma^{2}\right)}+\delta(1-\lambda)\left(p_{g 1}-c_{E}\right)\left(\alpha-p_{g 1}\right)$
where
$p_{g 1}=\frac{2 \delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha+c_{E}\right)+\lambda(2+\gamma)\left(\alpha(1-\gamma)+c_{1}\right)}{4 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\left(4-\gamma^{2}\right)}$

$$
p_{g 2}=\frac{\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha(2-\gamma)+2 c_{I}+\gamma c_{E}\right)+\lambda(2+\gamma)\left(\alpha(1-\gamma)+c_{I}\right)}{4 \delta(1-\lambda)\left(1-\gamma^{2}\right)+\lambda\left(4-\gamma^{2}\right)}
$$

We need to prove that $\Pi_{p r}-\Pi_{g 1}>0$
Substituting and simplifying the algebra yields

$$
\begin{aligned}
& (2 M+\lambda(2+\gamma))^{2}\left(4 M+\lambda\left(4-\gamma^{2}\right)\right)^{2} *\left(\Pi_{p r}-\Pi_{g 1}\right) \\
& =\delta(1-\delta)(M+\lambda) \gamma(1-\gamma)\left[\gamma\left(\alpha-c_{I}\right)+2\left(c_{E}-c_{I}\right)\right] * \\
& {\left[8(1+\gamma)\left(1-\gamma^{2}\right) \delta^{2}(1-\lambda)^{2}\left(\alpha+c_{E}-2 c_{I}\right)\right]}
\end{aligned}
$$

This implies that

$$
\begin{aligned}
& (2 M+\lambda(2+\gamma))^{2}\left(4 M+\lambda\left(4-\gamma^{2}\right)\right)^{2}\left(\Pi_{p r}-\Pi_{g 1}\right) \\
& =8 \delta^{3}(1-\delta)(1-\lambda)^{2}(M+\lambda) \gamma\left(1-\gamma^{2}\right)^{2}\left(\alpha+c_{E}-2 c_{I}\right)\left[\gamma\left(\alpha-c_{I}\right)+2\left(c_{E}-c_{I}\right)\right]
\end{aligned}
$$

Thus, we have

$$
\left(\Pi_{p r}-\Pi_{g 1}\right)=\frac{8 \delta^{3}(1-\delta)(1-\lambda)^{2}(M+\lambda) \gamma\left(1-\gamma^{2}\right)^{2}\left(\alpha+c_{E}-2 c_{I}\right)\left[\gamma\left(\alpha-c_{I}\right)+2\left(c_{E}-c_{I}\right)\right]}{(2 M+\lambda(2+\gamma))^{2}\left(4 M+\lambda\left(4-\gamma^{2}\right)\right)^{2}}
$$

where $M=\delta(1-\lambda)\left(1-\gamma^{2}\right)>0$
Notice that every term on the right hand side is positive.
Thus, we get $\left(\Pi_{p r}-\Pi_{g 1}\right)>0$ or $\Pi_{p r}>\Pi_{g 1}$ QED.
We see that when the firms are substitutes, we have $\Pi_{s h}>\Pi_{p r} ; \Pi_{g 2}>\Pi_{s h}$ and $\Pi_{p r}>\Pi_{g 1}$. This proves the existence of the Prisoner's Dilemma when the firms are substitutes. We also see that when the firms are complements, $\Pi_{s h}>\Pi_{p r}$ and $\Pi_{s h}>\Pi_{g 2}$. Thus, the firms prefer to share information when they are complements. Collectively, this proves Proposition 1.

## Appendix C

## Proof of Proposition 2

Does the consumer surplus of inexpensive customers increase, when firms share information? The consumer surplus of inexpensive customers, when firms keep information private is
$C S_{p r, I}=\frac{1}{2}\left(\frac{\lambda}{(1+\gamma)}\right)\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)^{2}$
The consumer surplus of inexpensive customers, when firms share information is $C S_{s h}=\frac{\lambda\left(\alpha-c_{I}\right)^{2}}{2(1+\gamma)(2-\gamma)^{2}}$
The consumer surplus of inexpensive customers increases when firms share information, if $C S_{s h}-C S_{p r, I}>0$

$$
C S_{s h}-C S_{p r, I}=\frac{1}{2}\left(\frac{\lambda}{(1+\gamma)}\right)\left(\frac{\left(\alpha-c_{I}\right)^{2}}{(2-\gamma)^{2}}-\left(\frac{\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)^{2}\right)
$$

Thus $C S_{s h}-C S_{p r, l}>0 \Leftrightarrow$
$\left(\alpha-c_{I}\right)^{2}\left(\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)\right)^{2}-$
$(2-\gamma)^{2}\left(\lambda\left(\alpha-c_{I}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)\right)^{2}>0$
This implies that $C S_{s h}-C S_{p r, I}>0 \Leftrightarrow\left(A^{2}-B^{2}\right)=(A-B)(A+B)>0$, where
$A=\left(\alpha-c_{I}\right)(\lambda(2-\gamma)+2 R)$
$B=(2-\gamma)\left(\lambda\left(\alpha-c_{I}\right)+R\left(\alpha-c_{E}\right)\right)$
where $R=\delta(1-\lambda)\left(1-\gamma^{2}\right)>0$
Define $X=A+B$ and $Y=A-B$
$X=2(\lambda(2-\gamma)+R)\left(\alpha-c_{I}\right)+(2-\gamma) R\left(\alpha-c_{E}\right)$
$Y=R\left[2\left(\alpha-c_{I}\right)-(2-\gamma)\left(\alpha-c_{E}\right)\right]$
Thus $C S_{s h}-C S_{p r, I}>0 \Leftrightarrow X Y>0$
We have $Y=R\left[\gamma \alpha-2 c_{I}+(2-\gamma) c_{E}\right]$
Since $\alpha>c_{I}$, we get $Y>(2-\gamma)\left(c_{E}-c_{I}\right)>0$
Thus, $Y>0$ and $C S_{s h}-C S_{p r, I}>0 \Leftrightarrow X>0$
Notice that we always have $X>0$ whenever $\left(\alpha-c_{E}\right)>0$ or $c_{E}<\alpha$
But what happens when $c_{E}>\alpha$ ?
We have $X=[(4-\gamma) R+2 \lambda(2-\gamma)] \alpha-$
$(2-\gamma) R c_{E}-2(\lambda(2-\gamma)+R) c_{L}$
We see that $X>0$ provided $c_{E}<\frac{(4-\gamma) R+2 \lambda(2-\gamma)}{(2-\gamma) R} \alpha \Leftrightarrow c_{E}<\left(\frac{(4-\gamma)}{(2-\gamma)}+\frac{2 \lambda}{R}\right) \alpha \Leftrightarrow$ $c_{E}<\left(\frac{(4-\gamma)}{(2-\gamma)}+\frac{2 \lambda}{\delta(1-\lambda)\left(1-\gamma^{2}\right)}\right) \alpha$
Note that the right hand side approaches its minimum value as the fraction of expensive customers in the market approaches $100 \%$ or $\lambda \rightarrow 0$. As $\lambda \rightarrow 0$ we get $c_{E}<\left(\frac{(4-\gamma)}{(2-\gamma)}\right) \alpha$
Also note that as $-1<\gamma<1, \frac{5}{3}<\frac{(4-\gamma)}{(2-\gamma)}<3$
This implies that $X>0$ when $c_{E}<\frac{5}{3} \alpha$
Thus, $C S_{s h}-C S_{p r, I}>0$ provided $c_{E}<\frac{5}{3} \alpha$ QED
Does the consumer surplus of expensive customers decrease, when firms share information?
The consumer surplus of expensive customers, when firms keep information private is

$$
C S_{p r, E}=\frac{1}{2} \delta(1-\lambda)\left(\frac{\lambda\left(\alpha-c_{1}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)^{2}
$$

Notice that $C S_{p r, I}>0$
The consumer surplus of expensive customers, when firms share information is zero, or $C S_{s h, E}=0$.
Thus, the change in consumer surplus of expensive customers is

$$
C S_{s h, E}-C S_{p r, E}=\frac{-1}{2} \delta(1-\lambda)\left(\frac{\lambda\left(\alpha-c_{1}\right)+\delta(1-\lambda)\left(1-\gamma^{2}\right)\left(\alpha-c_{E}\right)}{\lambda(2-\gamma)+2 \delta(1-\lambda)\left(1-\gamma^{2}\right)}\right)^{2}<0 \text { QED }
$$


[^0]:    ${ }^{1}$ Sameer Mathur is a doctoral student in Marketing; Kannan Srinivasan is H.J. Heinz II Professor of Management, Marketing and Information Systems; and Baohong Sun is Associate Professor of Marketing at Carnegie Mellon University.

[^1]:    ${ }^{2}$ Our model differs from the consumer heterogeneity models in prior literature, in that we model consumers as heterogeneous with respect to the cost they impose on the firm, an approach that enables us to consider the strategic implications of expensive customers in the market. Researchers in economics and marketing have a long history of modeling consumers as either (1) horizontally differentiated by taste, a method that dates back to Hotelling's (1929) location model, or (2) vertically differentiated by valuation for quality, beginning with Mussa and Rosen's (1978) model of product quality under a monopoly. Subsequent research has modeled consumers as simultaneously horizontally and vertically differentiated, such as in Desai (2001), Shaffer and Zhang (2002), and Tyagi (2004). Our model complements this long history of analytical modeling, because we model consumers as heterogeneous in terms of their cost rather than restricting our analysis to heterogeneity in taste and/or valuation.
    ${ }^{3}$ Also note that we consider cost uncertainty rather than demand uncertainty. There exists an additional literature pertaining to firm incentives to share information about demand shocks, such as Novshek and Sonnenschein (1982) and Clarke (1983), who show that information-sharing incentives do not exist in a Cournot duopoly with

[^2]:    homogeneous goods, given demand uncertainty. Vives (1984) further establishes that firms have an incentive to share information in Cournot competition with complementary goods and in Bertrand competition with substitute goods. Subsequently, Raith (1996) reconciled these previous results in a general model of information sharing in an oligopoly.
    ${ }^{4}$ They find that competing firms can mutually benefit from exchanging individual customer information at the nascent stage of individual marketing, when firms' targetability is low. However, as individual marketing matures and firms' targetability improves, further improvements in targetability intensify price competition and lead to a Prisoner's Dilemma.

[^3]:    ${ }^{5}$ In a related paper, Vives (1990) studies the effects of the different disclosure rules used by trade associations on the incentives for firms to share information and the welfare of consumers, firms, and society. He finds that a policy of exclusionary disclosure preserves incentives to share information and that information sharing decreases the total surplus with Bertrand competition and demand uncertainty.

[^4]:    Legend:

